DOE/ET-53088-153 IFSR #153

ION ACOUSTIC TURBULENCE AND ANOMALOUS TRANSPORT

Wendell Horton
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

August 1984

ION ACOUSTIC TURBULENCE AND ANOMOLOUS TRANSPORT

W. Horton
University of Texas
Austin, Texas 78712

Abstract

Theoretical interpretation of ion acoustic turbulence is shown to require the use of renormalized turbulence thoery for calculating the turbulent spectra and transport coefficients. In the one dimensional problem, the physics of solitons, double layers and ion phase space holes is shown to influence the dynamics.

I. Ion Acoustic Problem

Ion acoustic turbulence is driven in a collisionless plasma by a wide range of mechanisms including the injection of laser beams or particle beams and through the passage of high currents. In this review we consider principally the case where the ion acoustic waves given by $\omega(\underset{\sim}{k}) = kc_s/(1+k^2\lambda_{De}^2)^{1/2}$ with the ion sound speed $c_s = (T_e/m_i)^{1/2}$ and the Debye length $\lambda_{De} = (T_e/4\pi ne^2)^{1/2}$ are driven unstable by a current $\underset{\sim}{j} = -en\underset{\sim}{v}_d$. The drift of the thermal electrons through the ions with velocity $\underset{\sim}{v}_d$ produces linear instability with $\gamma^e(\underset{\sim}{k}) = (\frac{\pi}{8})^{1/2}(\frac{m_e}{m_i})^{1/2}(\underset{\sim}{k}\cdot\underset{\sim}{v}_d-\omega_k)$ from the positive slope on the electron velocity distribution for $v_d > \omega_k/k = c_s$. For $T_e \gg T_i$ and $v_d \gg c_s$ the ion Landau damping $\gamma^i(\underset{\sim}{k}) = -(\frac{\pi}{8})^{1/2}(\frac{T_e}{T_i})^{3/2}\omega_k \exp(-T_e/2T_i)$ is negligible and all $\underset{\sim}{k}$ modes within the polar angle $\vartheta \leqslant \vartheta_o$ with respect to $\underset{\sim}{v}_d$ where $\cos\vartheta_o = c_s/v_d$ are linearly unstable.

Examples of current driven ion acoustic turbulence occur in shocks[1], magnetic reconnection[2-5], plasma return currents[6] arising from the current neutralization of injected particle beams and turbulent heating experiments.[7,8]

The principal effect of ion acoustic turbulence[6-11] is to produce an anomalous resistivity $\eta = m_e\nu_{eff}/n\,e^2$ and the associated turbulent heating[7,8] through the scattering of the electrons from the ion acoustic fluctuations. Numerous experimental studies of turbulent heating show the presence of ion-acoustic turbulence from the measurements of the fluctuation spectra. The principal characteristics of the system are the presence of a turbulent heating pulse with $\omega_{pi}\Delta t \sim 100$ during which a strong resistivity and d.c. electric field $\langle E\rangle$ arise with $e\langle E\rangle=\nu_{eff}m_e v_d$. The effective collision frequnecy

$\nu_{eff} \lesssim \omega_{pi}$ typically exceeds the collisional resistivity by several orders of magnitude. The turbulent heating pulse produces a conversion of drift electron kinetic energy $\frac{1}{2} n_e m_e v_d^2$ into thermal energy $n_e T_e$ through $\langle j \cdot E \rangle \Delta t = n_e m_e v_d^2 \nu_{eff} \Delta t$. The momentum of the drifting electrons $n_e m_e v_d$ is taken up by the ion-acoustic waves propagating in the direction of $\underset{\sim}{v}_d$ and then transferred to a small fraction of fast ions[12-14] $n_f/n_e \sim (m_e/m_i)^{1/4}$ with $T_{if} \sim \frac{1}{5} T_e$. The production of the fast ion population arises from the turbulent trapping of the ions by the finite amplitude ion acoustic waves.[15]

The detailed interpretation of a turbulent heating experiment requires that account be taken of the losses of the electron thermal energy, of the fast ion component and of the waves from the region of turbulent heating. The turbulent heating and loss processes taken together then determine the long-time macroscopic evolution of the system.[16] Current penetration and net thermal deposition or efficiency are determined from the macroscopic balance analysis. Here we are only concerned with the microscopic laws for the anomalous resistivity arising from ion acoustic turbulence. The microscopic laws are the basis for the macroscopic confinement and heating studies.[7,16]

II. Collective Interactions Between Electrons and Ions

The ion acoustic waves and the associated solitons and double layers, are the mechanism for the interactions between electrons and ions in a collisionless plasma. During the slow ion-acoustic motion the electrons remain near equilibrium with $-en_e\underset{\sim}{E}(\underset{\sim}{x},t)=\nabla(n_eT_e)\simeq T_e\nabla n_e$ and the ions are accelerated by the electric field according to

$\partial_t \mathbf{v}+\mathbf{v}\cdot\nabla\mathbf{v}=(e_i/m_i)\mathbf{E}(\mathbf{x},t)$. The ion density changes by $\partial_t n_i(\mathbf{x},t)=-\nabla\cdot(n_i\mathbf{v})$. The self-consistent long range Coulomb electric field is determined by

$$\nabla\cdot\mathbf{E} = -\nabla^2\varphi(\mathbf{x},t) = 4\pi e(n_i-n_e). \tag{1}$$

For small amplitude oscillations these closed fluid equations yield $\omega_{\mathbf{k}}=kc_s/(1+k^2\lambda_{De}^2)^{1/2}$ and for wavelengths long compared with the Debye length $\omega_k=|\mathbf{k}|c_s$. For large amplitudes the steeping nonlinearity $v\partial_x v$ of the fluid equations balances with the dispersion from $\omega_k=kc_s(1-\frac{1}{2}k^2\lambda_{De}^2)$ to produce ion-acoustic solitons propagating with speeds $c>c_s$. We discuss the soliton components in Sec. IV. To describe the interactions of the particles with waves requires the Vlasov equations.

For finite amplitude potential fluctuations $\varphi_{\mathbf{k}\omega}\exp(i\mathbf{k}\cdot\mathbf{x}-i\omega t)$ the fluctuation $\delta f_{\mathbf{k}\omega}(\mathbf{v})\exp(i\mathbf{k}\cdot\mathbf{x}-i\omega t)$ of the particle distribution function satisfies

$$(\omega-\mathbf{k}\cdot\mathbf{v})\delta f_{\mathbf{k}\omega}(\mathbf{v})= -\frac{e}{m}\varphi_k\mathbf{k}\cdot\frac{\partial\langle f\rangle}{\partial\mathbf{v}} - \frac{e}{m}\sum_{k_1\neq k}\varphi_{k_1}\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{v}}\delta f_{k-k_1} \tag{2}$$

where $k = \mathbf{k}\omega$. The mean distribution $\langle f\rangle$ evolves by

$$\frac{\partial\langle f\rangle}{\partial t} = -\frac{e}{m}\frac{\partial}{\partial\mathbf{v}}\cdot\langle\sum_{\mathbf{k}\omega} i\mathbf{k}\varphi^*_{\mathbf{k}\omega}\delta f_{\mathbf{k}\omega}(\mathbf{v})\rangle. \tag{3}$$

In the limit $\varphi_k\to 0$ we write from Eq. (2)

$$\delta f_{k\omega}(\mathbf{v}) = g^o_{\mathbf{k}\omega}(v)\left(\frac{-e}{m}\varphi_k\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{v}}\right)\langle f\rangle \equiv \underset{k}{\longrightarrow}\overset{k}{\downarrow}\langle f\rangle$$

and obtain the self-consistent linear modes $\varepsilon^{\ell}_{\underset{\sim}{k}}(\omega)\varphi_k=0$ from Eq. (1) with

$$\varepsilon_k(\omega) = 1 + \frac{\omega_p^2}{k^2}\int d\underset{\sim}{v}\; g^o_{k\omega}\underset{\sim}{k}\cdot\frac{\partial\langle f\rangle}{\partial\underset{\sim}{v}} \cong 1 + \frac{\omega_{pe}^2}{k^2\lambda_{De}^2} - \frac{\omega_{pi}^2}{\omega^2} + i\varepsilon''_k \tag{4}$$

with

$$\varepsilon''_k = -\frac{\omega_p^2}{k^2}\int d\underset{\sim}{v}\;\pi\delta(\omega-\underset{\sim}{k}\cdot\underset{\sim}{v})\underset{\sim}{k}\cdot\frac{\partial\langle f\rangle}{\partial\underset{\sim}{v}}$$

where $g^0_{k\omega}(\underset{\sim}{v}) = (\omega-\underset{\sim}{k}\cdot\underset{\sim}{v}+io^+)^{-1}$ is the linear particle propagator and ε''_k gives the response of the resonant particles which we now consider in more detail.

A nearly resonant electron $\omega\cong\underset{\sim}{k}\cdot\underset{\sim}{v}$ is accelerated in one wave period $1/\omega_k$ by

$$\Delta\vec{V} = \frac{e\underset{\sim}{k}\varphi_k}{m_e(\omega-\underset{\sim}{k}\cdot\underset{\sim}{v}_e)} = \frac{e\underset{\sim}{k}\varphi_k}{m_e\omega_k}\;.$$

The electron is accelerated out of resonance when $\Delta V>\omega/k$ which requires

$$\Delta V > \frac{\omega}{k} \;\Rightarrow\; \frac{e\varphi_k}{T_e} > \left(\frac{\omega_k}{kv_e}\right)^2 \simeq \frac{m_e}{m_i}\;. \tag{5}$$

The acceleration out of resonance is not described by the linear propagator $g^o_{\underset{\sim}{k}\omega}(\underset{\sim}{v})$. The expansion of $\Delta V(\underset{\sim}{k},\underset{\sim}{v})$ in powers of $g^o_{\underset{\sim}{k}\omega}(\underset{\sim}{v})$ fails for amplitudes greater than the critical amplitude given in Eq. (5). In fact, we know for a single resonant particle the nonlinear motion can be expressed in terms of the pendulum equation $k\ddot{x}=-\omega_b^2\sin kx$ with can be expressed in terms of the pendulum equation $k\ddot{x}=-\omega_b^2 \sin kx$ with $\omega_b^2=ek^2\varphi_k/m_e$. For two waves the motion becomes stochastic at the very low amplitude $v_t(k)=(e\varphi_k/m_e)^{1/2} \gtrsim (\omega/k) = c_s$. Study of the test particle with the Hamiltonian $H = \frac{1}{2} m_e v^2 - e\varphi(\underset{\sim}{x},t)$ gives the conditions for stochastic motion for given wave spectra.

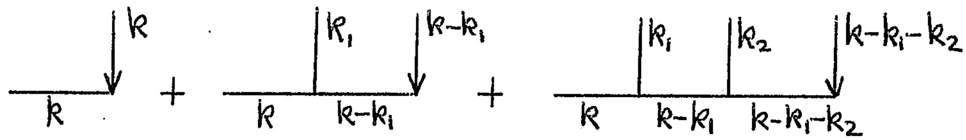
The condition for stochastic motion and the form of the velocity space diffusion $\underset{\approx}{D}(\underset{\sim}{v})$ depends on the dimensionality d of the system. The resonant domains satisfy $\omega=\underset{\sim}{k}\cdot\underset{\sim}{v}$ which is a d–1 dimensional velocity subspace. For d=1 there is a unique resonant velocity, for d=2 the resonant velocities occur on a line and for d=3 the resonant velocities lie on a surface. For this reason the d=1 system evolves qualitatively differently, forming a quasilinear plateau, whereas the d=2 (simulations) and d=3 (laboratory experiments) do not form a plateau but evolve through a turbulent heating pulse as shown in Fig. 1.

III. Limitations of Weak Turbulence Theory

It is the assumption of weak turbulence theory[11] that the dispersion in $\Delta\underset{\sim}{k}$ and $\Delta\underset{\sim}{v}$ in the linear resonance $\omega=\underset{\sim}{k}\cdot\underset{\sim}{v}$ is sufficient to justify the use of the small amplitude expansion in powers of $g_k^o(\underset{\sim}{v}) = (\omega-\underset{\sim}{k}\cdot\underset{\sim}{v}+io^+)^{-1}$. For ion acoustic turbulence the dispersion is known to increase the range of validity of the expansion in $g_k^o(\underset{\sim}{v})$ well above the single particle limit (3), but not to the amplitude levels of primary interest for ion acoustic turbulence.

To concisely discuss the weak turbulence theory and its limits of validity we introduce the diagrammatic notation of Refs. 17–20. The third order perturbation expansion used in weak turbulence theory is

$$\delta f_{k}^{WT}(\underset{\sim}{v}) = \qquad (6)$$



where $g_k^o(\underset{\sim}{v}) = \overline{}_{k}$ and $\sum_{k_1} (-e/m)\varphi_{k_1}\underset{\sim}{k}_1\cdot\partial_{\underset{\sim}{v}} = |^{k_1}$ with the summation $\sum_{k_1}$ deleted for $\downarrow^{k_1}$. Terms of order $|(e/m)g_k^o\varphi_k\underset{\sim}{k}\cdot\partial_{\underset{\sim}{v}}|^4$ are dropped. The charge density $\rho_k^{WT} = \sum e\int\delta f_k^{WT}d\underset{\sim}{v}$ is third order and yields the nonlinear Poisson field equation

$$\varepsilon_k^{\ell}\varphi_k + \sum_{\substack{k_1+k_2\\=k}} \varepsilon^{(2)}_{k_1,k_2}\varphi_{k_1}\varphi_{k_2} + \sum_{\substack{k_1+k_2+k_3\\=k}} \varepsilon^{(3)}_{k_1,k_2,k_3}\varphi_{k_1}\varphi_{k_2}\varphi_{k_3}=0 \qquad (7)$$

determining φ_k. Here ε_k^{ℓ} is given in Eq. (4) and $\varepsilon^{(2)}_{1,2}$ and $\varepsilon^{(3)}_{1,2,3}$ are the nonlinear dielectrics given in Eqs. (2.21) and (2.22) of Ref. 19.

The approximate radius of convergence of expansion (7) is determined by requiring that the third order contribution in (7) be less than the first order.

To determine the contributions of $\varepsilon_k^{(2),(3)}$ to the dynamics the assumption is made that due to the $\Delta\underset{\sim}{k}$ dispersion the coupling between the modes is weak. The long time scale $\omega_k T>>1$ evolution of the spectrum $I_{\underset{\sim}{k}}(T)=\langle|\varphi_{\underset{\sim}{k}}|^2\rangle$ is determined by calculating $\langle\varphi_1\varphi_2\varphi_3\rangle$ perturbatively in terms of $\langle\varphi_1\varphi_2\varphi_3\varphi_4\rangle$ and closing with the gaussian or quasi-normal approximation

$$\langle\varphi_1\varphi_2\varphi_3\varphi_4\rangle = \langle\varphi_1\varphi_2\rangle\langle\varphi_3\varphi_4\rangle+\langle\varphi_1\varphi_3\rangle\langle\varphi_2\varphi_3\rangle+\langle\varphi_1\varphi_4\rangle\langle\varphi_2\varphi_3\rangle \quad . \qquad (8)$$

The evolution of $I_{\underset{\sim}{k}}(T)$ is given by

$$\frac{\partial\varepsilon^{\ell}_{\underset{\sim}{k}}}{\partial\omega_{\underset{\sim}{k}}}\frac{dI_{\underset{\sim}{k}}}{dT} = -\varepsilon''_{\underset{\sim}{k}}I_{\underset{\sim}{k}}+\text{Im}\sum_{k_1}\left[\frac{4\varepsilon^{(2)}_{\underset{\sim}{k}_1,\underset{\sim}{k}-\underset{\sim}{k}_1}\varepsilon^{(2)}_{-\underset{\sim}{k}_1,\underset{\sim}{k}}}{\varepsilon^{\ell}_{\underset{\sim}{k}-\underset{\sim}{k}_1}(\omega_{\underset{\sim}{k}}-\omega_{\underset{\sim}{k}_1})} - 2\varepsilon^{(3)}_{\underset{\sim}{k}_1,-\underset{\sim}{k}_1,\underset{\sim}{k}}\right]I_{\underset{\sim}{k}_1}I_{\underset{\sim}{k}}$$

$$+ 2 \text{ Im} \sum_{\underset{\sim}{k}_1} \frac{|\varepsilon^{(2)}_{\underset{\sim}{k}_1, \underset{\sim}{k}-\underset{\sim}{k}_1}|^2}{\varepsilon^{\ell *}_{\underset{\sim}{k}}} I_{\underset{\sim}{k}_1} I_{\underset{\sim}{k}-\underset{\sim}{k}_1} \tag{9}$$

where $\varepsilon^{\ell}_{\underset{\sim}{k}}=\varepsilon'_{\underset{\sim}{k}}+i\varepsilon''_{\underset{\sim}{k}}$ and $\varepsilon^{(2),(3)}$ are evaluated at $\omega=\omega_{\underset{\sim}{k}}$.

The integrals in Im $\varepsilon^{(3)}$ are too complicated to discuss here. studied extensively in Refs. 21, 22. The results are that the third order contribution is comparable with the linear contribution,

$$\text{Im} \sum_{\underset{\sim}{k}_1} \varepsilon^{(3)}_{\underset{\sim}{k}_1, -\underset{\sim}{k}_1, \underset{\sim}{k}} |\varphi_{\underset{\sim}{k}_1}|^2 \simeq (W/nT_e)(kv_e/\omega_k)^2 \simeq \text{Im } \varepsilon^{\ell}_{\underset{\sim}{k}},$$

for

$$\frac{W^{crit}}{nT_e} \sim \left(\frac{\omega_k}{kv_e}\right)^2 \sim \frac{m_e}{m_i} . \tag{10}$$

For $W > W^{crit}$ the third order contribution from resonant electrons dominates in Eq. (7) and Eq. (9). We conclude that weak turbulence theory is not valid for ion acoustic turbulence with $W>W^{crit}$ because of the third order truncation in Eq. (6).

Problems in addition to the electron divergence discussed above occur when applying weak turbulence theory to ion acoustic turbulence. The prinicipal problems in using weak turbulence theory are summarized briefly as follows:

1. The electron–wave interaction series is divergent for $W/nT_e>m_e/m_i$.
2. The weak turbulence wave spectrum $I(\underset{\sim}{k})$ determined by induced wave scattering requires a long wavelength cut-off[23] for finite $W/nT_e=\int d\underset{\sim}{k}(1+k^2\lambda^2_{De})I(\underset{\sim}{k})$. A long wavelength cut-off is

observed in collisionless experiments[6,9,10] and simulations.[13,14]

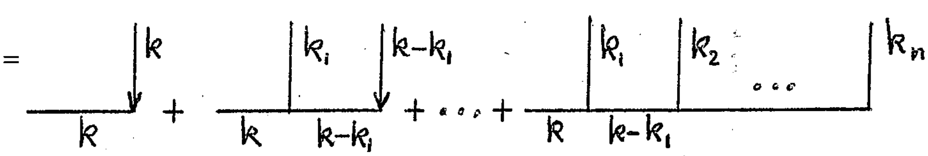
3. Weak turbulence theory predicts no significant production of fast ions during a turbulent heating pulse. (Once the fast ion component is established quasilinear theory adequately describes its evolution).

4. Weak turbulence theory fails to describe the finite lifetime of the waves. A related problem is the absence of a three wave resonance contribution to the theory since $\delta(\omega_{\underset{\sim}{k}}-\omega_{\underset{\sim}{k}_1}-\omega_{\underset{\sim}{k}-\underset{\sim}{k}_1})=0$ for the linear $\omega(\underset{\sim}{k})$.

III. Renormalized Turbulence Theory

To eliminate the divergence of the small amplitude expansion for $\delta f_k(\underset{\sim}{v})$ it is necesary to select the dominant secular terms in the $\omega=\underset{\sim}{k}\cdot\underset{\sim}{v}$ resonance at nth order in the expansion and sum their contribution to all orders. The nth order contribution is

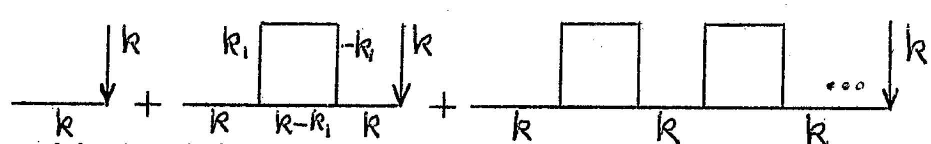
$f_k^{(n)}(\underset{\sim}{v})$ =



(11)

and the highest order multiple resonance in g_k^o is given by

$\delta f_k^{(n)}(v)$ =



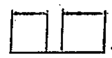
(12)

+ subdominant terms

$$= -\left(\frac{e}{m}\right)\left[\omega-\underset{\sim}{k}\cdot\underset{\sim}{v} + \left(\frac{e}{m}\right)^2 \frac{\partial}{\partial\underset{\sim}{v}} \cdot \sum_{k_1} I_{k_1}\underset{\sim}{k}_1\underset{\sim}{k}_1\, g^o_{k-k_1} \frac{\partial}{\partial v}\right] \cdot \frac{\partial\langle f\rangle}{\partial\underset{\sim}{v}}$$

+ subdominant terms.

The selection and summation of these terms is given by Choi and Horton[18,19] and called the simply renormalized propagator $\tilde{g}_k(v)$. The approximation is also studied by Misguich and Balescu[24] and called the Weak Coupling Approximation.

Other higher order but less secular terms can also be included in the summation (11) for the renormalized propagator. In the next approximation, called the doubly renormalized propagator $\tilde{\tilde{g}}_k(\mathbf{v})$, the additional fourth order term  is added to . Retaining the largest class of terms that can be formally summed in the propagator series leads to the nonlinear operator equation[18,25]

$$\left[\omega-\mathbf{k}\cdot\mathbf{v}+\left(\frac{e}{m}\right)^2 \frac{\partial}{\partial\mathbf{v}}\cdot\sum_{k_1} I_{k_1}\mathbf{k}_1\mathbf{k}_1\, g_{k-k_1}(\mathbf{v})\cdot\frac{\partial}{\partial\mathbf{v}}\right]g_k(\mathbf{v}) = I \tag{13}$$

for the fully renormalized propagator $g_k(v)=(\omega-\mathbf{k}\cdot\mathbf{v}+i\nu_k)^{-1}$. The simply renormalized propagator (12) can be calculated exactly in terms of the spectrum $I_{\mathbf{k}\omega}$, whereas the nonlinear operator equation (13) for the fully renormalized propagator is not solvable.

In addition to the propagator series selected in Eq. (12) which has the interaction $g_k^o\mathbf{E}_k\cdot\partial_{\mathbf{v}}$ as a terminal line, there are terms in the series where the contribution $g_k^o\mathbf{E}_k\cdot\partial_{\mathbf{v}}$ is not terminal. The series formed by these terms leads to the renormalized vertex operator

$$\xi_k(\mathbf{v}) = \frac{\partial}{\partial\mathbf{v}} + \left(\frac{e}{m}\right)^2 \sum_{k_1} I_{k_1}\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{v}}\, g_{k-k_1}\frac{\partial}{\partial\mathbf{v}}\, g_{-k_1}\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{v}} \tag{14}$$

as shown by Balescu and Misguich.[20] The vertex contribution is well known for drift waves.[26]

With these selective summations of the perturbation series the renormalized fluctuating distribution function is

$$\delta f_k(\underset{\sim}{v}) = -\frac{e}{m}\varphi_k g_k(\underset{\sim}{v})\underset{\sim}{k}\cdot\underset{\sim}{\xi}_k\langle f\rangle \ . \tag{15}$$

In addition to the coherent part $\delta f_k(\underset{\sim}{v})$ of the fluctuation given by (15) there is an incoherent contribution given by Dupree[27] arising from non-wave fluctuations called clumps and holes. The role of holes in d=1 ion acoustic turbulence is considered in Sec. IV.

Now we reconsider the convergence of the perturbation expansion of Eq. (11) in powers of $(e/m)g_k\varphi_k\underset{\sim}{k}\cdot\partial_{\underset{\sim}{v}}$ using $g_k=(\omega-\underset{\sim}{k}\cdot\underset{\sim}{v}+i\nu_k)^{-1}$ given by Eq. (13). The new expansion operator is bounded by

$$\varepsilon = \max \left\| \frac{(e/m)\varphi_k}{\omega-\underset{\sim}{k}\cdot\underset{\sim}{v}+i\nu_k}\,\underset{\sim}{k}\cdot\frac{\partial}{\partial\underset{\sim}{v}} \right\| = \left|\frac{ek\varphi_k}{m\nu_k(\nu_k/k)}\right| \leqslant 1 \tag{16}$$

where to obtain the inequality the smallest value of ν_k allowed by Eq. (13) is $\nu_k \geqslant (e/m)^2k^2\varphi_k^2/\nu_k\Delta v_k^2$ and $\Delta v_k=\nu_k/k$ to give $\nu_k^2 \geqslant (e/m)k^2|\varphi_k|$.

The new or renormalized perturbation series appears at least asymptotically convergent for the levels of turbulence required to describe strong ion-acoustic turbulence. The renormalized expansion eliminates the divergence problem (1) of Sec. III.

For electrons, the turbulent collision operator $i\nu_k$ in the renormalization series (12) describes angular scattering of the electrons by the ion acoustic fluctuations. Noting that $\underset{\sim}{k}\cdot\underset{\sim}{v}/\omega_k \simeq (m_e/m_i)^{1/2}$ for thermal electrons the impulse $\Delta\underset{\sim}{v}=e\underset{\sim}{k}\varphi_k/m_e\nu_k$

imparted to a resonant electron is essentially perpendicular to $\underset{\sim}{v}$. The electron propagator reduces to

$$(g_k^e)^{-1} = \omega - \underset{\sim}{k} \cdot \underset{\sim}{v} + \nu_{eff} \left(\frac{v_e}{v}\right)^3 \nabla^2_{\Omega_v} \tag{17}$$

where $\Omega_{\underset{\sim}{v}}$ is the solid angle in velocity space and

$$\nu_{eff} = \frac{\pi v_e}{4} \int d\underset{\sim}{k}_1 \; |\underset{\sim}{k}_1| \; \frac{e^2 I_{\underset{\sim}{k}_1}}{T_e^2} \simeq c_1 \omega_{pe} \left(\frac{W}{nT_e}\right) \tag{18}$$

with the constant c_1 depending on the shape of the spectrum. For turbulence with $W/nT_e \sim (m_e/m_i)^{1/2}$ the effective Lorentz collision frequency from the turbulence is $\nu_{eff} \sim \omega_{pi}$ in agreement with strong turbulence experiments.[6–9]

Analysis of the electron contribution to the renormalized dielectric function[28] $\tilde{\varepsilon}_{\underset{\sim}{k}}(\omega, W)$ shows that at long wavelengths $\lambda = 2\pi/k > v_e/\nu_{eff}$ the Landau resonance is disrupted by the turbulent scattering. In the region $k\lambda_{De} \leqslant \nu_{eff}/\omega_{pe} \sim W/nT_e$ the ion Landau damping dominates and the wave energy is absorbed. The absorption leads to a low k cut-off of the spectrum I(k) and eliminates the long wavelength divergence problem (2) of Sec. III. The details of the $\tilde{\varepsilon}_k(\omega, W)$ and I(k) calculations are given in Refs. 18,19,28. A typical wavenumber spectrum is shown in Fig. 2

The ion interaction with the turbulence is dominated by the high energy ions since $\omega_{\underset{\sim}{k}}/k \gg v_i = (T_i/m_i)^{1/2}$. The renormalized ion propagator is calculated from velocity diffusion taking $D_{\parallel}(k)\hat{v}_d\hat{v}_d + D_{\perp}(k)(I - \hat{v}_d\hat{v}_d)$.

The explicit formulas for $\tilde{g}^{i}_{\underset{\sim}{k}\omega}(\underset{\sim}{v},\underset{\sim}{v}')$ are given in Sec.6 of Ref. 19. The turbulent propagator $g_{\underset{\sim}{k}\omega}(\underset{\sim}{v},\underset{\sim}{v}')$ and the Fourier transform $g_{\underset{\sim}{k}}(\underset{\sim}{v},\underset{\sim}{v}',\tau)$ form a Markov semi-group with

$$\int d\underset{\sim}{v}' g_{\underset{\sim}{k}}(\underset{\sim}{v},\underset{\sim}{v}',\tau_1) g_{\underset{\sim}{k}}(v',\underset{\sim}{v}'',\tau_2) = g_k(\underset{\sim}{v},\underset{\sim}{v}'',\tau_1+\tau_2)$$

for $\tau_1+\tau_2>0$ and zero for τ_1 or $\tau_2<0$. From the analysis of the two point phase space correlation function $f(\underset{\sim}{x}_1\underset{\sim}{v}_1;\underset{\sim}{x}_2\underset{\sim}{v}_2,\tau)$ Dupree[27] shows that neighboring trajectories in phase space diverge exponentially with the separation increasing as $k|\delta x|+|\delta v/v_t|\sim\exp[(k^2D_k)^{1/3}t]$. The result is an exponential sensitivity to the initial data which gives rise to the statistical Markovian behavior for times greater than the Lyapunov time $t>\tau_c=(k^2D_k)^{-1/3}$ describing the maximum lifetime of wave-like correlations.

The renormalized propagator $g_{\underset{\sim}{k}}(\underset{\sim}{v},\underset{\sim}{v}',\tau)$ describes a dissipative dynamics which opens new regions of $\underset{\sim}{k}$, $\underset{\sim}{v}$, ω space to wave-particle interactions. Although the net dissipation is constrained by frequency sum rules[29], the interactions now extend well outside the region $\omega=\underset{\sim}{k}\cdot\underset{\sim}{v}$. The evolution of the background ion distribution is given by

$$\frac{\partial\langle f\rangle}{\partial t} = \frac{\partial}{\partial\underset{\sim}{v}}\cdot\int d\underset{\sim}{v}'\ \underset{\approx}{D}(\underset{\sim}{v},\underset{\sim}{v}',\tau)\cdot\frac{\partial}{\partial\underset{\sim}{v}'}\langle f(\underset{\sim}{v}',t)\rangle \qquad (19)$$

with the nonlocality of $\underset{\approx}{D}(\underset{\sim}{v},\underset{\sim}{v}')$ being strong for $W/nT\gtrsim 10^{-2}$. Calculations[19] of the ion acceleration from Eq. (19) show that the renormalization eliminates problem (3) of Sec. III.

The charge density $\rho_k^{(n)}=\sum e\int\delta f_k^{(n)}d\underset{\sim}{v}$ computed from the renormalized perturbation expansion yields the mode coupling equation (7) where the formulas for $\varepsilon^{(n)}_{k_1 \ldots k_n}$ contain $g_k(\underset{\sim}{v},W)$. Iterating the equation and neglecting the fourth cumulant (8) also leads to Eq. (9). For long wavelengths, however, the procedure fails due to the divergence from

$$\varphi^{(2)}_{\underset{\sim}{k}} = - \frac{\varepsilon^{(2)}_{\underset{\sim}{k}_1, \underset{\sim}{k}-\underset{\sim}{k}_1} \varphi_{\underset{\sim}{k}_1} \varphi_{\underset{\sim}{k}-\underset{\sim}{k}_1}}{(\omega_{\underset{\sim}{k}} - \omega_{\underset{\sim}{k}_1} - \omega_{\underset{\sim}{k}-\underset{\sim}{k}_1})(\partial \varepsilon / \partial \omega_{\underset{\sim}{k}})} \rightarrow \infty \qquad \text{as} \quad k, k_1 \rightarrow 0 \tag{20}$$

and the higher order iterations of this fluctuation propagator.

The correlations from the near three wave resonance (20) at long wavelengths leads to the formation of intrinsic high order correlation as contained in solitons. The soliton components of the field are discussed in Sec. IV.

Assuming the fields remain sufficiently random that a statistical description remains valid we may sum the divergence from $\varepsilon^{(2)}_{k_1 k_2} \varphi_{k_1} \varphi_{k_2} / \varepsilon^{\ell}_{k}$ to all orders to obtain a renormalized fluctuation propagator $\varepsilon^{n\ell}_{k}$. The renormalized dispersion function satisfies the equation

$$\varepsilon^{n\ell}_{k} = \varepsilon^{\ell}_{k} - \sum_{\substack{k_1 \\ = \underset{\sim}{k}_1 \omega_1}} \left[\frac{4 \varepsilon^{(2)}_{k_1, k-k_1} \varepsilon^{(2)}_{-k_1, k}}{\varepsilon^{n\ell}_{k-k_1}} - 2 \varepsilon^{(3)}_{k_1, -k_1, k} \right] I_{k_1} \tag{21}$$

and the spectral equation becomes

$$|\varepsilon^{n\ell}_{k}|^2 I_k = 4 \sum_{k_1} |\varepsilon^{(2)}_{k_1, k-k_1}|^2 I_{k_1} I_{k-k_1}. \tag{22}$$

Equations (21) and (22) are the equivalent of the Direct Interaction Approximation (DIA) of fluid turbulence with respect to the mode coupling $\varepsilon^{(2)}_{k_1, k_2}$. The equations in this form were first analyzed for ion acoustic turbulence by Tsytovich.[30] Our mode simulation studies indicate that the effects of the three wave coupling terms are less important, because of the small $\underset{\sim}{k}$ space volume for divergence (20), than the induced wave scattering process in determining the shape of the wavenumber spectrum $I(\underset{\sim}{k})=\int_{-\infty}^{+\infty} d\omega/2\pi \; I(\underset{\sim}{k},\omega)$ which is a cone in the direction of $\underset{\sim}{v}_d$.

The near three wave resonance is important in determining the frequency line width through Eqs. (21) and (22). The line width $\Delta\omega_{\underset{\sim}{k}}$ is given approximately by $\Delta\omega_{\underset{\sim}{k}} = \mathrm{Im}\; \varepsilon_{\underset{\sim}{k}}^{n\ell}/\partial\varepsilon^{n\ell}/\partial\omega_{\underset{\sim}{k}}$ and the spectral distribution is approximately

$$I_{\underset{\sim}{k}}(\omega) = \frac{2\Delta\omega_{\underset{\sim}{k}} I_{\underset{\sim}{k}}}{(\omega-\omega_{\underset{\sim}{k}})^2+\Delta\omega_{\underset{\sim}{k}}^2} \; . \tag{23}$$

The complete solution of Eqs. (21) and (22) for the spectrum $I_{\underset{\sim}{k}\omega}$ and nonlinear dielectric $\varepsilon_{\underset{\sim}{k}}^{n\ell}(\omega)$ remains a difficult problem. The approximate solution (23) with $\Delta\omega_k \sim \omega_k(W/nT_e)$ eliminates the finite lifetime problem (4) of Sec. III.

The renormalized turbulence theory given here is the direct renormalization of weak turbulence theory. An advantage of the theory is its ability to predict the quantities of interest to experiments and simulations. A number of comparisons have been made such as with Stenzel[6], Kawai et al.[9], and Slusher et al.[10], and the simulations of Biskamp et al.[13,14] The scaling of the anomalous resistivity predicted by the theory is shown in Fig. 3.

There are other statistical theories of Vlasov turbulence principally those developed by applying the direct interaction approximation to the Vlasov-Poisson equations. The first attempt with the DIA was given by Orzsag and Kraichnan.[31] Subsequently, a systematic Vlasov turbulence theory based on the DIA is given by DuBois and Espedal[32] and developed further by DuBois.[33] The theory contains additional nonlocal velocity space correlations arising from the shielding clouds of particles contained in the response function $R_{12}=\delta f_{k_1}/\delta\varphi_{k_2}$ and the fluctuation propagator $1/\varepsilon_{k}^{n\ell}$. The new contributions can be interpreted in terms of quasiparticles as in many-body field theories. An example of these quasiparticle contributions and their relation to the test particle propagator $g_k(\underset{\sim}{v})$ analyzed here is shown diagrammatically in Figs. 1 and 2 of DuBois.[33] It remains a difficult problem to evaluate the effects of these nonlocal shielding or polarization contributions contained the DIA theory of ion acoustic turbulence.

IV. Non-Wave Constituents

Renormalized turbulence theory retains the basic description of weak turbulence theory of a gas of interacting waves and particles. By renormalization of the particle propagator $g_{\underset{\sim}{k}\omega}(\underset{\sim}{v},W)$ and the fluctuation propagator $\varepsilon_{\underset{\sim}{k}}^{n\ell}(\omega,W)$ the secularities from the bare resonant interactions are eliminated. The renormalization sums to all order the divergent contributions to the perturbation series. The renormalized theory is capable of calculating the quantities of interest in plasma turbulence when high order correlations are not required. The closure of the heirarchy of correlation functions in renormalized turbulence theory loses the high order correlations contained in solitons, double layers and phase space ion holes.

A one-dimensional, long wavelength ion acoustic wave steepens due to $v\partial_x v$ until wave dispersion at the Debye scale balances the steepening. The balance of dispersion and the steepening leads to the spectrum

$$\varphi_s(k,\omega) = \pi k\delta(\omega-kc)\operatorname{csch}\left[\pi k\left(\frac{kc_s}{\omega-kc_s}\right)^{1/2}\right]\exp(-ikx_o) \tag{24}$$

of correlated fluctuation components describing a $\operatorname{sech}^2(K(x-ct))$ soliton. A similar situation applies to the double layer where the localized potential now contains a net jump $\Delta\varphi$ across the structure.

The ion acoustic solitons preserve their identity for many soliton-soliton and soliton-wave collisions. This coherence property has been used to construct an ideal gas model[34] for plasma fluctuations composed of randomly distributed solitons. The soliton component of the fluctuation spectrum for such a gas is given by $I(k,\omega) = k^2 f_s(\omega/k)\operatorname{csch}^2[\pi k(kc_s/(\omega-kc_s))^{1/2}]$ where $f_s(v)dv$ is the number of solitons with the speed $v \geqslant c_s$ in the range dv. The Gibbs ensemble with $E = H(\varphi,\partial_x\varphi)$ may be used to estimate $f_s(v)$.

Recently studies[35] have considered the effect of adding linear dissipative terms modeling the growth and damping $\gamma(k)$ taken from linear particle resonances into the soliton equation. Numerical simulations show that a mixture of solitons and wave components are produced from the unstable growth of noise in this dissipative soliton system.

A more realistic model for coherent structures in ion acoustic turbulence is given by the Kadomtsev-Petviashivili[36] equation for two-dimensional solitons. For the cone of waves propagating with the mean angle $\vartheta \ll 1$ with respect to the drift velocity $\underset{\sim}{v}_d$ the dispersion in wave frequencies is

$$\omega(\underset{\sim}{k}) = c_s\left(k_z - k_z^3 + \frac{k_\perp^2}{2k_z}\right) .$$

Taking into account the wave steepening leads to

$$\frac{\partial^2\varphi}{\partial z\partial t} + \frac{\partial^2\varphi}{\partial z^2} + \frac{1}{2}\nabla_\perp^2\varphi + \frac{\partial^4\varphi}{\partial z^4} + \frac{\partial}{\partial z}\left(\varphi\frac{\partial\varphi}{\partial z}\right) = \begin{cases} \hat{\gamma}\dfrac{\partial\varphi}{\partial z} \\ F(z,t) \end{cases} \tag{25}$$

with the growth rate $\hat{\gamma}\varphi$ for the unstable dissipative system and $F(z,t)$ for the forced conservative system.

There are exact two-dimensional soliton solutions of Eq. (25). The equation is used to prove the transverse stability[36] of the one-dimensional solitons propagating parallel to $v_d\hat{z}$. The time scale t_s for the formulation of the soliton is given by

$$\frac{1}{t_s} = c_s\langle k_z\rangle\left(\frac{W}{nT_e}\right)^{1/2}$$

and balances the transverse dispersion of the waves for

$$\frac{W}{nT_e} \simeq \left(\frac{\langle k_\perp^2\rangle}{\langle k_z^2\rangle}\right)^2 \simeq \vartheta^4 \ . \tag{26}$$

The interaction of three obliquely propagating solitons is strong when the $\omega,\vec{k}$ defined by $\mathrm{sech}^2(\vec{k}\cdot\vec{x}-\omega t)$ satisfy the three-wave resonance condition.[38] When the resonance condition is satisfied two solitons collide to produce a third soliton.

Experimental studies[39,40] showing the oblique collisions of ion acoustic solitons have been performed confirming the resonance condition and other properties. With a large number of obliquely colliding solitons the system may evolve to a chaotic state containing strong correlations.

Another important type of correlation that lives for times long compared with the life time of the wave constituients is the phase space hole.[41,42] The role of ion holes has been demonstrated in d=1 particle simulations of current-driven ion acoustic turbulence.[43] The simulations show that the production of holes starts for v_d below the threshold for unstable waves from a nonlinear instability.

Making a hole $\delta f_i(x,v) < 0$ in the ion phase space distribution of size $\Delta v \simeq (e\delta\Phi/m_i)^{1/2}$ and $\Delta x \sim 10\lambda_D$ leads to a long lived fluctuation $\delta\Phi(x-vt)$ for $v \leqslant v_i$ with

$$\left(\nabla^2 - \frac{1}{\lambda_D^2}\right)\,\delta\Phi \simeq -\frac{1}{\lambda_D^2}\,\delta\Phi = -4\pi e_i \delta f_i \Delta v \ . \tag{27}$$

The negative potential $\delta\Phi$ of the hole reflects electrons $v_e \rightarrow -v_e$ and in the process gains momentum $2m_e\Delta v_e$. Calculating the imbalance of the right and left going momentum transfer due to the drift v_d of the electron distribution leads to the hole growth rate[41]

$$\gamma^{hole} = -8\left(\frac{\Delta v}{\Delta x}\right)\Delta x^4 \omega_{pi}^2 \omega_{pe}^2 \frac{\partial f_e}{\partial v} \frac{\partial f_i}{\partial v} \simeq \omega_{bi}\left(\frac{\Delta x}{\lambda_D}\right)^2 \left(\frac{v v_d}{v_i v_e}\right) \tag{28}$$

for holes with $0 < v < v_d$.

The hole turbulence appears as fluctuation components with $\omega \sim kv_i$, a region of heavy damping for linear waves. Probably, the most important aspect of the hole turbulence phenomena is the possibility that it relaxes the onset conditions for the occurrence of ion acoustic turbulence and the associated transport processes. There are, in fact, numerous experiments with indications of ion acoustic turbulence where the conditions on T_i/T_e and v_d/c_s for unstable waves are not satisfied.

We conclude that although the phenomena of wave-particle interactions in the context of renormalized turbulence theory has given formulas for calculating $\underset{\sim}{k}, \underset{\sim}{v}, \omega$ spectra and transport coefficients, as given in Figs. 1-3 for example, the role of solitons and phase space holes, especially as they interact with the wave fluctuation spectrum, remains to be evaluated.

References

1. R. Z. Sagdeev, in Review of Plasma Physics, Vol. 4, M. A. Leontovich, Ed. (Consultants Bureau, New York, 1966) pp. 23-90.
2. F. V. Coroniti and A. Eviator, Ap. J. Supp. 33, 189 (1977).
3. J. D. Huba, N. T. Gladd, and K. Papadopoulos, J. Geophys. Res. 83, 5217 (1978).
4. K. Papadopoulos, in Dynamics of the Magnetosphere, S. I. Akasofu, Ed. (D. Reidel 1979) 289.,
5. R. L. Stenzel, W. Gekelman and N. Wild, J. Geophys. Res. 87, 111 (1982).
6. R. L. Stenzel, Phys. Fluids 21, 91 (1978).
7. R. D. Bengtson, K. W. Gentle, J. Jancarik, S. S. Medley, P. Nielsen and P. Phillips, Phys. Fluids 18, 710 (1975).
8. S. M. Hamberger and J. Jancarik, Phys. Fluids 15, 825 (1972).
9. Y. Kawai, Ch. Hollenstein and M. Guyot, Phys. Fluids 21, 970 (1978).
10. R. E. Slusher, C. M. Surko, D. R. Moler and M. Porkolab, Phys. Rev. Lett. 36, 674 (1976).
11. A. A. Galeev and R. Z. Sagdeev, in Reviews of Plasma Physics, Vol. 7, M. A. Leontovich, Ed. (Consultants Bureau, New York, 1979) pp. 141-180.
12. G. E. Vekstein, D. D. Ryutov and R. Z. Sagdeev, Zh. Eksp. Teor. Fiz. 60 (1971) 2142 [Sov. Phys.-JETP 33 (1971) 1152].
13. C. T. Dum, R. Chodura and D. Biskamp, Phys. Rev. Lett. 32 (1974) 1231.
14. D. Biskamp, R. Chodura and C. T. Dum, Phys. Rev. Lett. 34 (1975) 131.
15. D. Choi and W. Horton, Phys. Fluids 18 (1975) 858.
16. P. L. Mascheroni, Phys. Fluids 20, 634 (1977).
17. L. M. Altshul and V. I. Karpman, Zh. Edksp. Teor. Fiz. 49 (1965) 515 [Sov. Phys.-JETP 22 (1966) 361].
18. D. Choi and W. Horton, Phys. Fluids 17 (1974) 2048.
19. W. Horton and D. I. Choi, Phys. Rep. 49, 273-410 (1979).
20. R. Balescu and J. H. Misquich in Statistical Physics and Chaos in Fusion Plasmas, C. W. Horton and L. E. Reichl, Ed. (J. Wiley, 1984) pp. 295-310.

21. V. L. Sizonenko and K. N. Stepanov, Zh. Eksp. Theor. Fiz. Pis. Red. 9, 282 (1969) [J. Exp. Theor. Phys. Lett 9, 165 (1969)]; Nuclear Fusion 19, 155 (1970).

22. D. Choi and W. Horton, Phys. Fluids 20 (1977) 628.

23. B. B. Kadomtsev, Plasma Turbulence (Academic Press, London, 1965) p. 71.

24. J. H. Misguich and R. Balescu, J. Plasma Physics 19, 147 (1978); Plasma Physics 20, 781 (1978).

25. L.I. Rudakov and V. N. Tsytovich, Plasma Physics 13 (1971) 213.

26. T. H. Dupree and D. J. Tetreault, Phys. Fluids 21, 425 (1978).

27. T. H. Dupree, Phys. Fluids 15, (1972) 334.

28. R. A. Koch and W. Horton, Phys. Fluids 18, (1975) 861.

29. C. W. Horton, in Long-Time Prediction in Dynamics, C. W. Horton, L. E. Reichl and V. G. Szebehely, Eds. (J. Wiley, New York, 1983) p. 310-312.

30. V. N. Tsytovich, Plasma Physics 13 (1971) 74.

31. S. A. Orszag and R. H. Kraichnan, Phys. Fluids 10, 1720 (1967).

32. D. F. DuBois and M. Espedel, Plasma Physics 20, 1209 (1978).

33. D. F. DuBois, Phys. Rev. A, 865 (1981).

34. J. D. Meiss and W. Horton, Jr., Phys. Rev. Lett 48, 1362 (1982).

35. J. D. Meiss, T. Kamimura and W. Horton, "Turbulence in the Forced RLW Equation" (unpublished).

36. B. B. Kadomtsev and V. I. Petviashvili, Doklady Akademii Nauk SSSR, 192, 753 (1970) [Sov. Phys. Doklady 15, 539 (1970)].

37. B. B. Kadomtsev and V. I. Petviashvili, Sov. Phys. Doklady 18, 115 (1973).

38. J. W. Miles, J. Fluid Mech. 79, 171 (1979).

39. Y. Nishida and T. Nagasawa, Phys. Rev. Lett. 45, 1626 (1980).

40. I. Tsukabayashi and Y. N. Nakamura, Phys. Lett. 85A, 152 (1981).

41. T. H. Dupree, Phys. Fluids 26, 2460 (1983).

42. D. J. Tetreault, Phys. Fluids 26, 3247 (1983).

43. R. H. Berman, D. J. Tetreault, T. H. Dupree and T. Boutrois-Ghali, Phys. Rev. Lett. 48, 1249 (1982).

Figure Captions

1. Turbulent heating pulse computed from mode simulation with renormalized turbulence theory using $v_d=v_e(0)$, $m_i/m_e=1600$ and $T_e(0)/T_i(0)=50$.
2. Wavenumber spectrum from analytic solution of renormalized mode coupling equation taking square box angular distribution with cut-off given by $\cos\vartheta_o=c_s/v_d$.
3. Anomolous resistivity at the maximum of turbulent heating pulse as a function of $u=v_d$ and T_e/T_i.

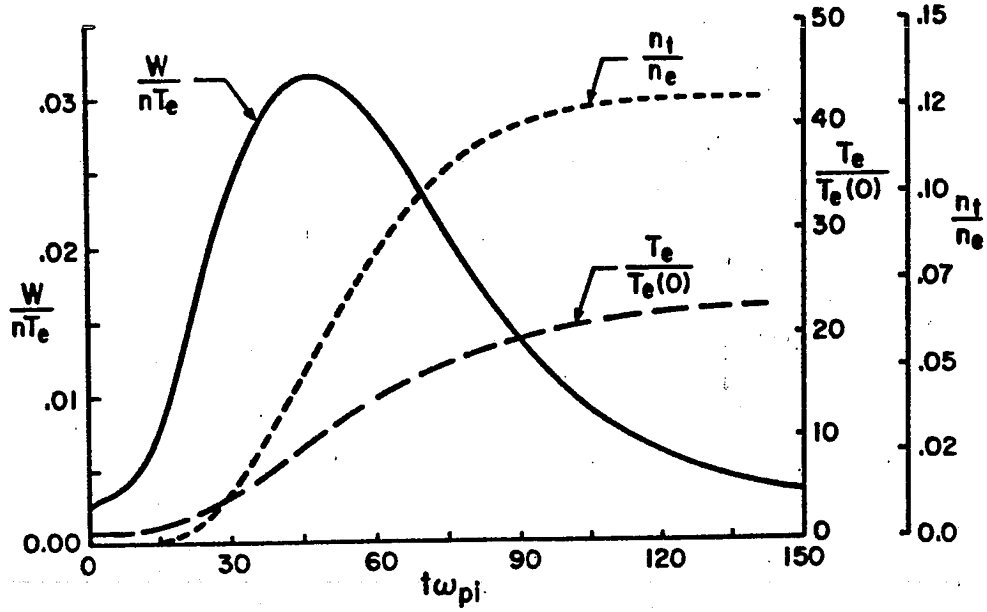


FIGURE 1

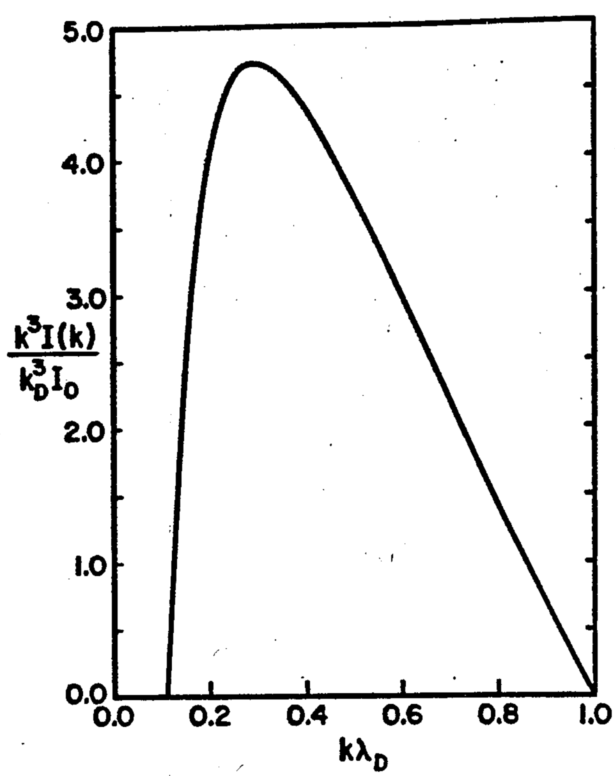
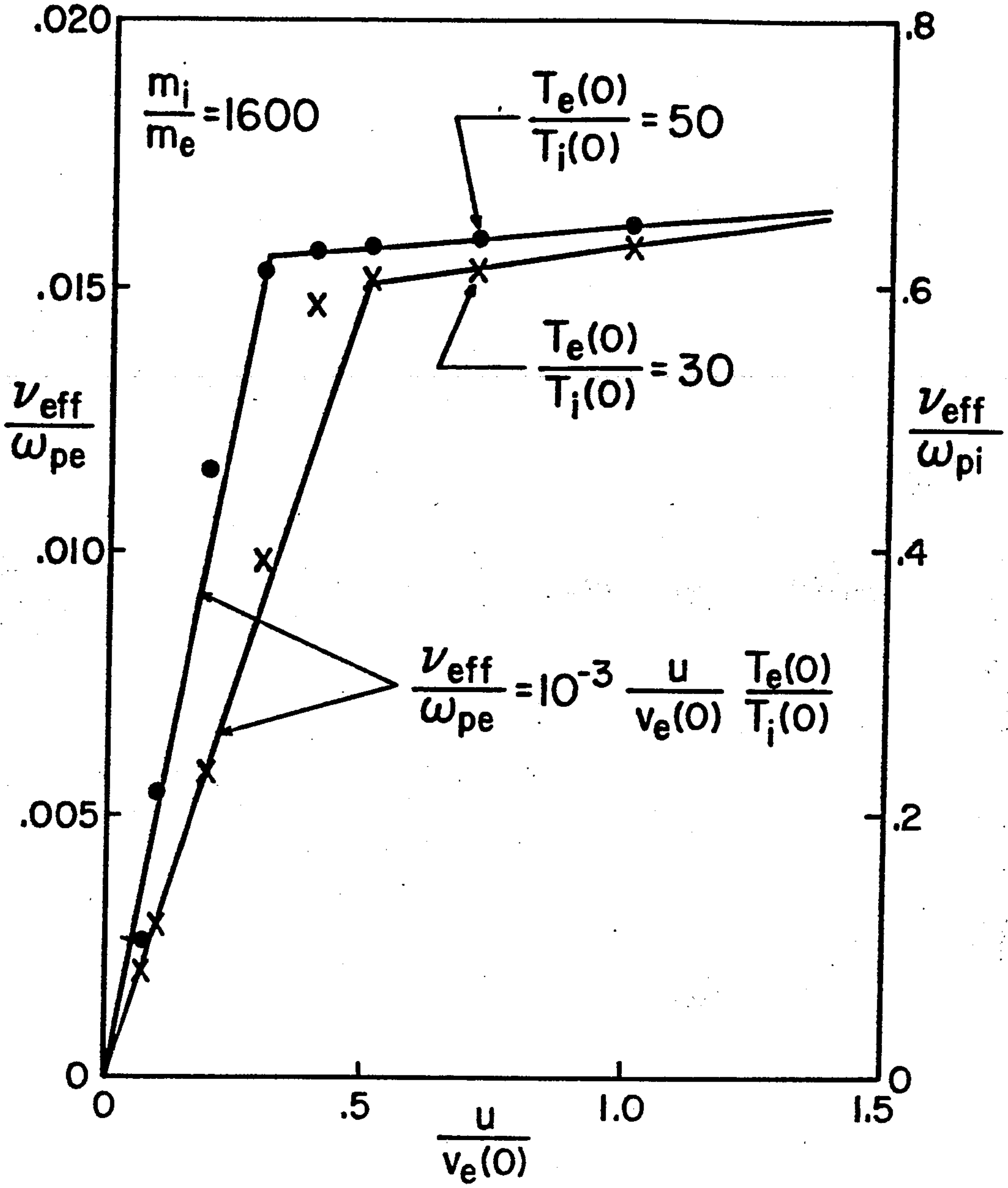


FIGURE 2



.020
.015
.010
.005
0
.8
.6
.4
.2
0
$\frac{m_i}{m_e} = 1600$
$\frac{T_e(0)}{T_i(0)} = 50$
$\frac{T_e(0)}{T_i(0)} = 30$
$\frac{\nu_{eff}}{\omega_{pe}}$
$\frac{\nu_{eff}}{\omega_{pi}}$
$\frac{\nu_{eff}}{\omega_{pe}} = 10^{-3} \frac{u}{v_e(0)} \frac{T_e(0)}{T_i(0)}$
0
.5
1.0
1.5
$\frac{u}{v_e(0)}$

FIGURE 3