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ENHANCEMENT OF ION TRANSPORT DUE TO (A) ELECTRON COLLISIONS AND (B) NON-MAXWELLIAN ION DISTRIBUTIONS

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This paper considers the enhancement of ion transport, namely ion heat conduction (q_{ir}) and ion viscosity ($P_{\| r}$), due firstly, to electron collisions and secondly, to non-Maxwellian ion distributions (f_{io}).

(a) Electron Collisions

All previous calculations of ion transport in tokamaks have neglected electron collisions. This is a good approximation for $T_e > T_i$ but this is not the case when T_e is appreciably less than T_i. Also the frictional force experienced by ions in the tail of the ion distribution is proportional to v^{-2} for i-i collisions but proportional to v for i-e collisions. As a result electron collisions will be relatively more important for ion transport processes which involve significant energy-weighting, such as q_{ir} and $P_{\| r}$.

Pfirsch-Schluter Ion Heat Conduction

From Herdan and Liley[1] the ion heat conduction parallel to a magnetic field with electron collisions included is given by

$$q_{i\|} = \frac{3.1\ p_i \nabla_{\|} T_i}{m_i(\nu_{ii} + \frac{15}{4}(\frac{m_e}{m_i})\ \nu_e)} \tag{1}$$

or written as the equality of two energy weighted forces

$$\frac{5}{2} p_i \nabla_\parallel T_i = 0.81[\nu_{ii} + \frac{15}{4}(\frac{m_e}{m_i})\nu_e] m_i q_{i\parallel} \qquad (2)$$

The right-hand side of Eq. (2) is the parallel component of the energy weighted friction force $G_\parallel$ defined by

$$G_\parallel \equiv \int m v_\parallel (\frac{1}{2} m_i v^2 - \frac{5}{2} T_i) C(f_i) d^3v.$$

The radial Pfirsch-Schluter heat conduction is given by

$$(q_{ir})_{ps} = -\frac{1}{eB_{\vartheta o}} \overline{(2\tilde{G}_\parallel (r/R)\cos\vartheta)} \qquad (3)$$

where the overbar means the ϑ-average. Since

$$\tilde{q}_{i\parallel} = -\frac{5p_i T_i'}{eB_{\vartheta o}} \frac{r}{R} \cos\vartheta , \qquad (4)$$

substituting from Eqs. (2) and (4) into Eq. (3) gives

$$(q_{ir})_{ps} = -2n_i (\frac{r}{R})^2 \nu_{ii} \rho_{i\vartheta}^2 T_i' [1 + \frac{15}{4}(\frac{m_e}{m_i})\frac{\nu_e}{\nu_{ii}}] \qquad (5)$$

Note that

$$\frac{15}{4}\frac{\nu_e}{\nu_{ii}}\left(\frac{m_e}{m_i}\right) = \frac{15\sqrt{2}}{4}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_e}{T_e}\right)^{3/2} \geqslant 1$$

for $T_i/T_e \geqslant 4$.

Banana Regime Ion Heat Conduction

Since the dominant term in the collision operator C_{ie}, the dynamic friction term, involves energy scattering, the contribution to $(q_{ir})_{Ban}$ due to C_{ie} will be smaller by the extra factor $(r/R)^{1/2}$ compared with the pitch-angle scattering contribution due to C_{ii}. Making the assumption that $z^i_{eff}{>>}1$, where $z^i_{eff} \equiv (n_i+\sqrt{2}\ n_z Z^2)/n_i$, the Spitzer type problem which has to be solved to obtain the circulation in velocity space due to C_{ie} is simplified and one obtains for $(q_{ir})_{Ban}$.

$$(q_{ir})_{Ban} = -0.7 n_i\left(\frac{r}{R}\right)^{1/2}\rho^2_{i\vartheta}\nu_{ii}T_i'\left[\left(z^i_{eff}+18\,\frac{\nu_e}{\nu_{ii}}\left(\frac{m_e}{m_i}\right)\left(\frac{r}{R}\right)^{1/2}\right)\left(1-0.48\,\frac{T_e}{T_i}\right)\right] \qquad (6)$$

where

$$z^i_{eff} \equiv \frac{n_i + \sqrt{2}\,\sum_Z n_z Z^2}{n_i} \qquad (7)$$

The result given in Eq. (6) was obtained by just adding the extra collision operator C_{ie} to the normal neoclassical theory without considering what other physical processes may be present. Since $T_e<T_i$, the ions are being cooled by the electrons and either there is a balancing heating mechanism or $\partial f_i/\partial t \neq 0$ due to $\partial T_i/\partial t$ and possibly other time

derivatives. Whatever the balancing process it will in general affect the transport as well. As an example, if one assumes in lowest order the cooling by electrons is balanced by beam particle heating such that

$$(C_{ie} + C_{i,NB})(f_{io}) = 0, \tag{8}$$

where f_{io} is the lowest order Maxwellian part of f_i, the two collision operators do not balance when operating on f_{i1}. Here f_{i1} is the perturbation of f_i, first order in $\rho_{i\vartheta}/L$, caused by $\partial f_{io}/\partial r$. The factor in square brackets in Eq. (6) for this particular case becomes

$$[z^{i}_{eff} + 9.6 \frac{\nu_e}{\nu_{ii}} (\frac{m_e}{m_i})(\frac{r}{R})^{1/2}] \tag{9}$$

Ion Viscosity ($P_{\parallel r}$)

To illustrate qualitatively how C_{ie} contributes to ion viscosity consider the neoclassical formula for $P_{\parallel r}$, namely,

$$P_{\parallel r} = \int \frac{d\vartheta}{2\pi} \int m_i v_\parallel v_{dr} f_i d^3v,$$

where

$$v_{dr} = - \frac{(\mu B + v_\parallel^2)\sin\vartheta}{\Omega_{ci} R} .$$

Putting $v_\parallel/v = \zeta$, this can be written in the form

$$P_{\parallel r} = -\frac{1}{2\Omega R}\int \frac{d\vartheta}{\pi}\sin\vartheta \int v_{\parallel}(1/2)mv^2(1+\zeta^2)f_i d^3v$$

If one omits the ζ^2 term, the d^3v integral is simply the total ion heat flow parallel to $\underset{\sim}{B}$ namely $\hat{q}_{i\parallel} = q_{i\parallel}+(5/2)\ T_i\ V_{i\parallel}$, and the ϑ-integral picks the $\sin\vartheta$ component of $\hat{q}_{i\parallel}$ to be denoted by $\hat{q}_{i\parallel s}$. Hence

$$P_{\parallel r} \gtrsim -\hat{q}_{i\parallel s}/2\Omega_{ci}R.$$

From the ion heat balance equation to first order in $\rho_{i\vartheta}/L$

$$\frac{B_{\vartheta}}{B}\frac{\partial}{r\partial\vartheta}(\hat{q}_{i\parallel s}\sin\vartheta) - \frac{3}{2}\frac{E_r}{B}\frac{\partial(\tilde{p}_{is}\sin\vartheta)}{r\partial\vartheta} = -\tilde{Q}_{iec}\cos\vartheta$$

where $\tilde{Q}_{iec}$ is the $\cos\vartheta$ component of Q_{ie}, the energy transfer from ions to electrons.

The $\tilde{p}_{is}$ term leads to a contribution to $P_{\parallel r}$ given by $m_i\Gamma_i E_r/B_{\vartheta}$, where Γ_i is neoclassical ion diffusion, a contribution which has been known for many years.[2] The new contribution considered here comes from $\tilde{Q}_{iec}$ giving

$$P_{\parallel r} \gtrsim \frac{rm\ \tilde{Q}_{iec}}{2eB_{\vartheta}R}$$

More rigorous neoclassical theory[3] leads to the formula

$$P_{\parallel r} = -\frac{m_i^2}{eB_{\vartheta o}}\int \overline{\frac{h^3v_{\parallel}^2}{2}C(f_i)d^3v} \qquad (10)$$

where $h \equiv 1+(r/R)\cos\vartheta$. The part of f_i which is even in $v_\parallel$ and varies as $\cos\vartheta$ is given by

$$\tilde{f}_i^+ = f_{io}\ \{\frac{(m_i V_\parallel^2 \frac{r}{R}\cos\vartheta - e\tilde{\Phi})}{T_i}$$

$$- \frac{m_i(\mu B + v_\parallel^2)}{eB_{\vartheta o}} \frac{r}{R} \cos\vartheta\ \frac{m_i}{T_i}[V_\parallel' + \frac{V_\parallel T_i'}{T_i}\ (\frac{v^2}{v_{T_i}^2} - c)]\} \tag{11}$$

where $\tilde{\Phi}$ is the part of the electrostatic potential which varies with ϑ and the constant c is 2.33 for the banana regime. The dominant terms are those containing the factor $(\mu B+v_\parallel^2)$ because of the extra energy dependence and substituting from Eq. (11) into Eq. (10) the contribution due to i–e and i–NB collisions (the balanced case given by Eq. (8)) is

$$(P_{\parallel r})_{ie} = -\frac{6}{5}\, n_i\ (\frac{r}{R})\rho_{i\vartheta}^2(\frac{m_e}{m_i}\ \nu_e)m_i(V_\parallel' + \frac{2V_\parallel T_\parallel'}{T_i}) \tag{12}$$

From references 3 the corresponding term due to i–i collisions is

$$(P_{\parallel r})_{ii} = -0.1\ n_i(\frac{r}{R})^2 \rho_{i\vartheta}^2 \nu_{ii} m_i V_\parallel' \tag{13}$$

For the case $T_i'/T_i \simeq V_\parallel'/V_\parallel$, the ratio of the magnitude of the two contributions in Eqs. (12) and (13) is

$$\frac{(P_{\parallel r})_{ie}}{(P_{\parallel r})_{ii}} = 36\sqrt{2}\ (\frac{T_i}{T_e})^{3/2}(\frac{m_e}{m_i})^{1/2}$$

which is greater than unity even for $T_i = T_e$.

(b) Non-Maxwellian Ion Distributions

There is experimental evidence that, at least for discharges where $\rho_{i\vartheta}$ is large, that f_{io} is non-Maxwellian. (See Ware[4] for a summary of the experimental evidence.) The measurements on PDX with active charge exchange show f_{io} has two components. There is a low energy component (f_c) with effective temperature $[T_c \equiv -(\partial \ell n f_{io}/\partial \varepsilon)^{-1}]$ decreasing as expected with radius and a higher energy component (f_H) whose effective temperature (T_H) remains approximately constant with radius. It has been shown[4] that given the low energy component f_c and assuming self collisions are dominant, the ions in f_H will diffuse outwards in radius and downwards in energy with f_H maintaining the same exponential dependence on energy, i.e. same effective temperature. The approximate solution obtained was

$$f_H = f_{Ho}\left(\frac{T_H - T_{co}}{T_H - T_c}\right)^{1/4} e^{-\int_{r_o}^{r} \frac{dr}{\lambda}} e^{-\frac{mv^2}{2T_H}}, \qquad (14)$$

where

$$1/\lambda = \frac{eB_\vartheta}{mv_{T_H}} \left(\frac{R}{r}\right)^{1/4} \left[\frac{4\left(1 - \frac{T_c}{T_H}\right)}{Z^i_{eff}}\right]^{1/2} \qquad (15)$$

The impurity ions will be strongly coupled with the low energy part f_c and the temperature measured from Doppler broadening will be close to T_c.

Using the measurements of f_{io} for r=26cms in PDX, even allowing for only i-i collisions, gives a heat conduction which is four times that which would be obtained using the standard neoclassical formula and the temperature T_c and its gradient. The dominant contribution comes from f_H and is given by

$$(q_{ir})_{ii} = 1.04\left(\frac{r}{R}\right)^{1/2} n_H(\nu_{ii})_H \rho^2_{\vartheta H}\left(\frac{T_H}{\lambda}\right)\left(1+\frac{\hat{W}}{T_H}\right) e^{-\frac{\hat{W}}{T_H}} \qquad (16)$$

where

$$n_H \simeq n_i\left(\frac{T_H}{T_c}\right)^{3/2} \exp\left[\hat{W}\left(\frac{1}{T_H}-\frac{1}{T_c}\right)\right],$$

$$(\nu_{ii})_H = \frac{4\sqrt{\pi}\, n_i e^4 \ell n\Lambda}{3m_i^{1/2} T_H^{3/2}}, \qquad \rho^2_{\vartheta H} = \frac{2m_i T_H}{e^2 B^2_\vartheta}.$$

λ is given by Eq. (15) and $\hat{W}$ is the ion energy where the discontinuity in the slope of $\ell n f_{io}$ occurs.

This is not a very useful formula for an INTOR code since it contains the unknowns T_c and $\hat{W}$. These cannot be predicted unless one solves a two–dimensional drift–kinetic equation as a sub–routine, the dimensions being particle energy and minor radius. However, the error in neglected the effect of the two–component ion distributions is very large since the contribution due to electron collisions can be even larger than that given by Eq. (16). For the PDX parameters at r=26cms $(q_{ir})_{ie}$ is 1.5 times $(q_{ir})_{ii}$ and the total $(q_{ir})_{ii} + (q_{ir})_{ie}$ is ten times the standard neoclassical formula with T_c.

Assuming

$$(C_{ie}+C_{i,NB})(f_H) = 0, \tag{17}$$

which is suggested by the PDX experimental results since i-e and i-NB collisions should be dominant for the f_H ions, one obtains

$$(q_{ir})_{ie} = 1.8\left(\frac{r}{R}\right)n_H\left(\frac{m_e}{m_i}\,\nu_e\right)\rho^2_{\vartheta H}\left(\frac{T_H}{\lambda}\right) , \tag{18}$$

This also requires the unknowns $\hat{W}$ and T_c.

Conclusions

1. If f_{io} is expected to be Maxwellian (i.e. $\rho_{i\vartheta}/a \ll 1$), the Chang-Hinton formula for q_{ir} can be used. If $T_e < T_i$ an enhancement factor such as given by Eq. (9) is required.

2. If there is appreciable ion heating (NB or ICRH) or appreciable gas-puffing, f_{io} will be non-Maxwellian. The standard neoclassical formulae for ion transport will be poor approximations.

3. Because the presence of an enhanced tail to f_{io} has been found to substantially increase ion transport, one can infer that slowing-down beam ions will also make an important contribution to both q_{ir} and $P_{\parallel r}$.

References

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