DOE/ET/53088-136 IFSR #136

MAGNETIC FLUCTUATIONS, FIELD REVERSAL MAINTENANCE, AND ANOMALOUS THERMAL TRANSPORT IN THE REVERSED FIELD PINCH

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July 1984

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Abstract

A theory of magnetic fluctuations, field reversal maintenance (dynamo activity), and anomalous thermal transport in the Reversed Field Pinch is proposed. Nonlinear generation of and coupling to $m \geqq 2$ modes is advanced as an m=1 tearing mode saturation mechanism. The mechanism by which nonlinear m=1 tearing modes sustain the toroidal magnetic field is elucidated. The predicted fluctuation levels and scalings are consistent with those required for maintaining the B_z configuration. Heat transport is estimated using stochastic magnetic field diffusion arguments.

Recent experimental results have indicated a correlation of macroscopic magnetic fluctuations with field reversal maintenance[1] and anomalous thermal transport[2] in the Reversed Field Pinch (RFP). Typically, the dominant observed magnetic fluctuations have poloidal mode number m=1, toroidal mode numbers $10 < n < 20$, with frequency width $\Delta\omega \sim S^{-1/3}(S=\tau_R/\tau_A)$, and are therefore associated with m=1 resistive internal kink (tearing) modes.[3] In this Letter, a theory of nonlinear, multiple helicity m=1 tearing mode interaction and saturation is proposed. The theory is used to develop models of the dynamics of magnetic field configuration maintenance (dynamo) and electron energy confinement in the RFP which are in qualitative agreement with several experimental observations.

In the RFP, a spectrum of tearing modes with m=1, $10 < n < 20$ are destabilized by the resistive diffusion of the magnetic field configuration away from a minimum energy Taylor equilibrium state.[4] The neighboring magnetic islands resonant at $q = 1/n'$, $q = 1/n''$ overlap for $\delta B/B_0 \geq 1/n^2Rq'$, thus <u>resonantly</u> generating a current sheet and magnetic island with m=2, $n=n'+n''$. The m=2, $n=n'+n''$ mode is linearly stable, with $\Delta'<0$, and is further stabilized by flattening of the equilibrium current gradient by global m=1 modes. The (stable) driven m=2 modes nonlinearly <u>absorb</u> energy from the primary m=1 modes, which saturate when the rate of coupling of energy to m=2 balances the rate of equilibrium magnetic energy release by m=1's. This progressive current filamentation process (cascade) proceeds with the generation of $m \geq 3$, and eventually terminates when the (m=1) driving energy is depleted by resistive dissipation at small scales and quasilinear profile modification associated with the generation of large m,n stable modes. These phenomena are analyzed theoretically using renormalized spectral equations.

Obtaining quantitative results is facilitated by the observation that in the region of (resonant) tearing mode interaction, located well within the reversal surface, $B_{0z} >> B_{0\vartheta}$. Thus, reduced resistive MHD[5] is an adequate plasma model. Hence, the magnetic and kinetic energy evolution equations are:

$$\frac{\partial}{\partial t} E^M = \int dx\ (J\nabla\phi\times\underline{n}\cdot\nabla\psi) + \int dx\ (J\nabla_{\parallel}^{(0)}\phi) - S^{-1}\int dx\ (J)^2 \tag{1}$$

$$\frac{\partial}{\partial t} E^K = -\int dx\ (\phi\nabla\psi\times\underline{n}\cdot\nabla J) + \int dx\ (\phi\nabla_{\parallel}^{(0)}J) - \int dx\ \phi(\nabla_y\psi)\langle J\rangle' \tag{2}$$

where ϕ is the stream function, ψ the flux function, $J = \nabla_{\perp}^2\psi$ the current, $U = \nabla_{\perp}^2\phi$ the vorticity, $E^M = \int dx\ (\nabla\psi)^2$, $E^k = \int dx\ (\nabla\phi)^2$, $\nabla_{\parallel}^{(0)} = |B_0|^{-1}B_0\cdot\nabla$, and a slab approximation is used to treat the inhomogeneous radial dependence. Renormalized spectrum equations are derived by using <u>iteratively</u> calculated fields for (driven) mode $\underline{k}''$ to close triple moments representing the triad interaction of a test mode $\underline{k}$, background mode $\underline{k}'$, and beat mode $\underline{k}''$. Here $\underline{k} = (m,n)$. The driven fields are[6,7]: $\psi_{\underline{k}''}^{(2)} = S_1(1+\Delta'_{\underline{k}''}G_N(x''))/\Delta\omega_{\underline{k}''}$, $J_{\underline{k}''}^{(2)} = S_1\Delta'_{\underline{k}''}\delta(x'')/\Delta\omega_{\underline{k}''}$, $\phi_{\underline{k}''}^{(2)} = i\Delta'_{\underline{k}''}G_N(x'')S_1/k''_{\parallel}$, $U_{\underline{k}''}^{(2)} = S_2/\Delta\omega_{\underline{k}''}(1+x''^2/x''^2_A)$, where the notation is that used in Ref. 7, and $x_A = L_s\Delta\omega_{\underline{k}}/k_y$. $(\Delta\omega_{\underline{k}})^{-1}$ is the mode interaction time, to be calculated. The structure of the driven fields reflects the radial inhomogeneity and nonlocality of the problem, and the fact that the nonlinear interaction of kink-tearing modes, with resonant and exterior regions, will in turn generate driven modes (turbulence) with resonant and exterior regions. Using the nonlinear beat fields (assuming $\Delta\omega < 1/\tau_A$, $x_A < a$), it follows that the renormalized spectral equations are:

$$\frac{\partial}{\partial t} E^K_{\underline{k}} = i\int dx\ \phi_{-\underline{k}}\, k_{\parallel} J_{\underline{k}} - \int dx\ \langle \phi b_r \rangle_{\underline{k}} \langle J \rangle'$$

$$+ \int dx \sum_{\underline{k}'} \left(1 - \frac{n'^2}{n''^2}\right) \Delta'_{\underline{k}''} \frac{\delta(x'')}{\Delta\omega_{\underline{k}''}} \left[\langle b_r^2 \rangle_{\underline{k}'} \mathscr{E}^K_{\underline{k}} + \langle v_r^2 \rangle_{\underline{k}} \mathscr{E}^M_{\underline{k}'}\right] \tag{3}$$

$$\frac{\partial}{\partial t} E^M_{\underline{k}} = i\int dx\ J_{-\underline{k}} k_{\parallel} \phi - \int dx\ \frac{\langle J^2 \rangle_{\underline{k}}}{S}$$

$$+ \int dx \sum_{\underline{k}'} \frac{n^2}{n''^2} \Delta'_{\underline{k}''} \frac{\delta(x'')}{\Delta\omega_{\underline{k}''}} \left[\langle b_r^2 \rangle_{\underline{k}} \mathscr{E}^K_{\underline{k}'} + \langle v_r^2 \rangle_{\underline{k}'} \mathscr{E}^M_{\underline{k}}\right]$$

$$- \int dx\ \Delta'_{\underline{k}} \frac{\delta(x)}{\Delta\omega_{\underline{k}}} \sum_{\substack{\underline{p},\underline{q} \\ (\underline{p}+\underline{q}=\underline{k})}} \left[\langle v_r^2 \rangle_{\underline{p}} \mathscr{E}^M_{\underline{q}} + \langle b_r^2 \rangle_{\underline{q}} \mathscr{E}^K_{\underline{p}}\right] \tag{4}$$

where $b_r = \nabla_y \psi$, $v_r = \nabla_y \phi$, $\mathscr{E}^K_{\underline{k}} = \langle (\nabla_x \phi)^2 \rangle_{\underline{k}}$, and $\mathscr{E}^M_{\underline{k}} \doteq \langle (\nabla_x \psi)^2 \rangle_{\underline{k}}$. By adding Eqs. (3), (4) and summing over $\underline{k}$, it is easy to verify that the renormalized spectral equations conserve energy.

The m=1 saturation amplitude is determined by the balance of the rate of magnetic energy release from the equilibrium configuration with the rate of energy scattering to m=2 via nonlinear interaction with neighboring m=1's. This balance is extracted from Eqs. (3) and (4) by adding the equations, summing over n (with m=1), noting $\Delta' < 0$ for $m \geqq 2$, and using standard properties of tearing modes, thus yielding:

$$-\sum_n \int dx \, \langle \phi b_r \rangle_{1,n} \langle J \rangle' = \sum_{n,n'} \int dx \, |\Delta'_{2,n+n'}| \frac{\delta(x'')}{\Delta\omega_{2,n+n'}} \langle b_r^2 \rangle_{1,n'} \mathscr{E}^K_{1,n} \tag{5}$$

where $\Delta'_{2,n+n'} = -|\Delta'_{2,n+n'}|$ and $\Delta\omega_{\underline{k}} \cong \sum_{\underline{k}'} |\Delta'_{\underline{k}''}| \delta(x'') \langle b_r^2 \rangle_{\underline{k}'} / \Delta\omega_{\underline{k}''}$. Equation (5) states that saturation is determined by the balance of growth due to $\langle J \rangle'$ relaxation with scattering to m=2. Since m=1 tearing modes do not exhibit a Rutherford phase[8] and because the islands overlap for relatively modest fluctuation levels ($\delta B/B \sim 10^{-3}$), linear tearing mode eigenfunctions are used to calculate the driving term. Hence, $\int dx \, \langle \phi b_r \rangle_{1,n} \langle J \rangle' \cong \int dx \gamma_{1,n} \mathscr{E}^K_{1,n}$, where $\gamma_{1,n}$ is the linear growth rate. Consistent with the notion that the nonlinear time scale $\Delta\omega_{\underline{k}}$ is determined by the dominant fluctuations (m=1 modes in this case), it follows that $\Delta\omega_{m=1} \sim (\delta B/B_0)_{rms}/|\Delta'_2 a|^{1/2}$. Thus, the root mean square m=1 magnetic fluctuation level at saturation is $(\delta B/B_0)_{rms} \approx (\gamma_{m=1}\tau_A)/|\Delta'_2 a|^{1/2}$, where $\gamma_{m=1}\tau_A \sim S^{-1/3}$. However, quasilinear flattening of the equilibrium current gradient may cause some reduction in the driving term (as compared to $\gamma_{m=1}$). Finally, it should be noted that $(\delta B/B)_{rms}$ refers to the value of $(\delta B/B)_{rms}$ at the resonant surface, located in the RFP core.

A physical interpretation of the proposed m=1 saturation mechanism is that the (primary) m=1 modes are stabilized by a nonlinear $\underline{J} \times \underline{B}$ force, resulting from interaction with stable modes, which opposes the growth of m=1 vorticity. This can be seen by examining the renormalized m=1 vorticity equation, which is

$$\frac{\partial}{\partial t} \frac{\partial^2 \phi}{\partial x^2}{}_{m=1,n} = i k_\parallel J_{m=1,n} + a_{\substack{m=1\\n}} \frac{\partial^2 \phi}{\partial x^2}{}_{m=1,n}$$

$$a_{\substack{m=1\\n}} = \sum_{\underline{k}'} \left(1 - \frac{n'^2}{n''^2}\right) \Delta'_{\substack{1+m'\\n+n'}} \frac{\delta(x'')}{\Delta\omega}_{\substack{1+m'\\n+n'}} \langle b_r^2 \rangle_{\underline{k}'} \tag{6}$$

For $m'=1$, $\Delta'_{m=2} < 0$, it is apparent that the nonlinear interaction opposes the growth of m=1 vorticity. Furthermore, the $(b_r)_{rms}$ saturation level can be obtained from the balance of the stabilizing $\underline{J}\times\underline{B}$ force with the driving forces, as approximated by $\gamma_{1,n}\phi''_{1,n}$. Note that this is the opposite case to that described in Ref. 6, where $\Delta'_{\underline{k}''} > 0$ and the resulting nonlinear destabilization triggers the onset of major disruption.

In order to qualitatively test the theory proposed in this Letter, numerical solution of the nonlinear, incompressible MHD equations is used to compare the single and multiple (nonlinearly coupled) helicity temporal evolution of m=1, n=14 mode kinetic and magnetic energy. The results are shown in Fig. 1. It is apparent that multiple helicity coupling effects play a very significant role in the nonlinear evolution and saturation of m=1 tearing modes in RFP. The computational results are discussed further in Ref. 9.

It is apparent from Eqs. (3) and (4) that energy extracted from m=1 modes is scattered, by incoherent emission, to m=2 ($\Delta'_2<0$). Subsequent interaction with m=1 then scatters energy to $m\geqq3$. Hence, a cascade to small scales (large $\underline{k}$) results. For $-\Delta'_m$ increasing with m, (for $m\geqq2$), it follows that energy extracted from m=1 is ultimately expended driving large m, stable modes and dissipated by resistive diffusion at small scales. Note that this problem, the direction and rate of cascade are determined by $\Delta'_{\underline{k}}$, a measure of stability at wavenumber $\underline{k}$. Hence, the cascade is actually a progressive current filamentation process, where nonlinear interaction generates small scale current sheets and islands.

Experimental evidence has linked m=1 tearing modes with dynamo events (increases in $|\langle B_z\rangle|$) near the RFP wall. Here, we consider the dynamo effect produced by (dominant) m=1 modes. Using Ohm's law, it follows that ΔB_z, the change in $\langle B_z\rangle$ due to turbulence and resistive diffusion, is

$$\Delta B_z = \frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{1}{S\Delta\omega}\frac{\partial\langle B_z\rangle}{\partial r} + \sum_{\underline{k}} |(\tilde{\xi}^*_{\underline{k}}\times\tilde{B}_{\underline{k}})_\vartheta|\right) \tag{7}$$

where $\tilde{\xi}$ is the displacement and $|(\tilde{\underline{V}}\times\tilde{\underline{B}})_\vartheta|$ is the turbulence-induced poloidal electric field. Away from the resonant surface, the exterior (ideal MHD) equations apply; thus $\tilde{\underline{B}} = \underline{\nabla}\times(\tilde{\xi}\times\langle\underline{B}\rangle)$. Assuming a kink displacement $\tilde{\xi}_{\underline{k}} = \tilde{\xi}_{\underline{k}}(r)(\underline{r}+i\underline{r}\times\underline{b}_0)$, it follows that near the reversal point $r_0(\langle B_z\rangle = 0)$

$$\Delta B_z = \frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{1}{S\Delta\omega}\frac{\partial\langle B_z\rangle}{\partial r} + \sum_n \frac{\langle B_\vartheta\rangle}{r}\left(1-nq(r)\right)|\tilde{\xi}_{1,n}|^2\right) \tag{8}$$

and near the wall, $(\tilde{\xi}_{1,n}(r) \to 0$ as $r\to r_{wall})$,

$$\Delta B_z \cong \frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{1}{S\Delta\omega}\frac{\partial\langle B_z\rangle}{\partial r} + \sum_{\underline{n}} \langle B_z\rangle \frac{\partial}{\partial r}\left(|\tilde{\xi}_{1,n}(r)|^2\right)\right) . \tag{9}$$

Hence, for $\partial\langle B_z\rangle/\partial r < 0$, $q(r) < 1/n$ near the reversal surface, and $\partial|\tilde{\xi}_{1,n}(r)|^2/\partial r < 0$ and $\langle B_z\rangle$ reversed near the wall, respectively; $\Delta B_z \to 0$ for $(\tilde{\xi})_{rms} \sim (S\Delta\omega)^{-1/2} \sim S^{-1/3}$. Noting that $(\tilde{\xi})_{rms} \sim (\delta B/B)_{rms} \sim S^{-1/3}$, it follows that dynamo activity induced by

$m=1$ tearing modes can offset resistive diffusion of $\langle B_z \rangle$ at the reversal surface and near the wall, thus maintaining the magnetic configuration. Furthermore, the fluctuation scalings required for $\Delta B_z \rightarrow 0$ are consistent with those actually predicted for saturated $m=1$ modes. Indeed, it is worthwhile to observe that a model of $m=1$ saturation based on the balance of dynamo activity with resistive diffusion away from the Taylor minimum-energy equilibrium state predicts (using Eq. (7)) saturated state magnetic fluctuation levels also scaling as $\delta B/B \sim S^{-1/3}$. Finally, it is interesting to note that in contrast to solar dynamo models, where the symmetry breaking necessary for dynamo activity is a consequence of finite helicity fluid turbulence[10], the necessary symmetry breaking here is a consequence of the radial structure of $k_{\parallel}a$ ($k_{\parallel}a=1-nq(r) > 0$, near reversal point) and $\tilde{\xi}_{\underline{k}}$ ($\partial|\tilde{\xi}_k|^2/\partial r < 0$, near wall) imposed by the equilibrium magnetic configuration structure, boundary conditions, and ideal MHD energetics. Also, in the case of the solar dynamo, thermal energy is <u>converted</u> to magnetic enegy. In the RFP, global $m=1$ tearing modes convert magnetic energy density in the core poloidal field (near r_s) to magnetic energy density in the toroidal field. Thus, the RFP dynamo process is actually a <u>redistribution</u> of magnetic energy.

Experimental evidence has linked heat transport along stochastic magnetic fields produced by $m=1$ tearing modes with confinement in the RFP.[11] Quasilinear magnetic field line diffusion[12] predicts a (collisionless) thermal conductivity $\chi_E = V_{T_e} \sum_{\underline{k}} \langle b_r^2 \rangle_{\underline{k}}\, \delta(k_{\parallel})$. The use of a quasilinear estimate is consistent with the large radial extent of the modes – turbulent broadening of the $\underline{k} \cdot \underline{B}_0$ resonance is not a significant effect here. Using the saturated state magnetic fluctuation levels obtained above, it follows that in the core region, $\chi_E \sim \alpha v_{T_e} a\ S^{-2/3}$, where $\alpha \sim .01$ and V_{T_e} is the electron thermal velocity. Balancing heat loss with ohmic heating yields $T_e \sim I_p^{.7}$, where I_p is the plasma current and the proportionality of density to I_p has been assumed (consistent with experimental results). For $S = 10^5$, $\beta \sim 10\%$ is predicted. The scaling of temperature with current and the estimated β are in reasonable agreement with experimental results. However, it should be noted that other loss processes, such as resistive pressure-driven turbulence[13,14,15], and particularly between the reversal point and wall, rippling modes, may contribute to heat transport. Preliminary results[14,15] indicate that stochastic magnetic fields produced by resistive interchange modes result in an anomalous thermal conductivity $\chi_E \simeq (1/4\pi)^{1/2}(\varepsilon/q)^3 v_{T_e} a\beta^{3/2}/S$. Balance of thermal loss with ohmic heating indicates that $T_e \sim I_p$, for density proportional to current, and $\beta \sim 10\%$. Note that the thermal loss prediction for resistive pressure-driven turbulence is qualitatively similar to that for m=1 tearing modes, but smaller in magnitude. Both indicate that $\beta \sim 10\%$.

The authors would like to acknowledge stimulating conversations with Drs. H. R. Strauss, I. H. Hutchinson, A. Hasegawa, A. Aydemir, R. Nebel, and D. Schnack, and to thank Drs. R. Gerwin and E. Caramana for emphasizing the need to analyze dynamo activity at several different radii. P. D. and Z. A. are also grateful to Dr. C. S. Liu for his encouragement during the course of this work. This research was supported by the Office of Fusion Energy under contract DOE/ET/53088 with the Institute for Fusion Studies and Contract No. DEAC05-840R21400 with the Martin Marietta Corporation.

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Figure Caption

Fig. 1 -

Comparison of temporal evolution of m=1, n=14 magnetic and kinetic energy for single helicity and multiple helicity (nonlinearly coupled) cases.

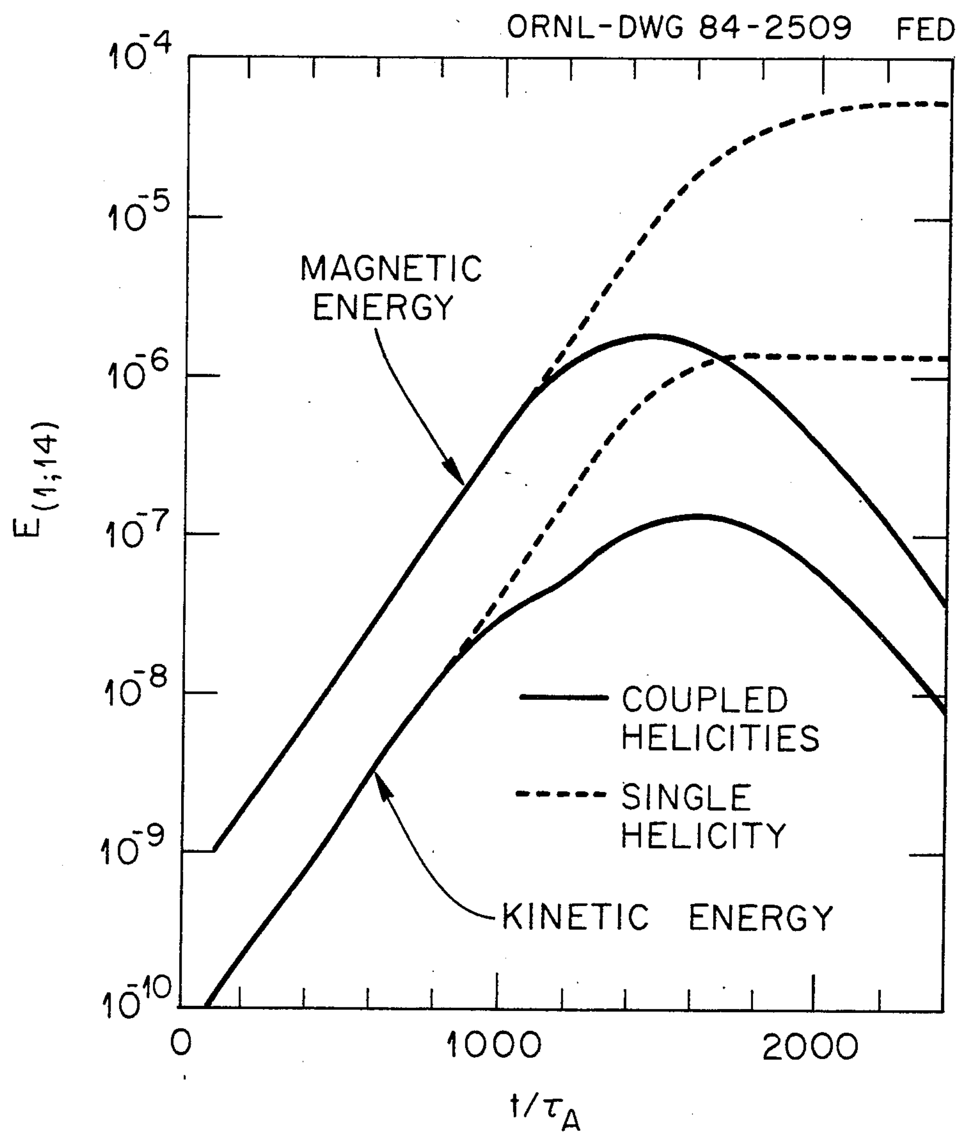
ORNL-DWG 84-2509 FED
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
$E_{(1;14)}$
MAGNETIC
ENERGY
COUPLED
HELICITIES
SINGLE
HELICITY
KINETIC ENERGY
0
1000
2000
t/τ_A

Fig. 1

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Abstract

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$$+ \int dx \sum_{\underline{k}'} \left(1 - \frac{n'^2}{n''^2}\right) \Delta'_{\underline{k}''} \frac{\delta(x'')}{\Delta\omega_{\underline{k}''}} \left[\langle b_r^2 \rangle_{\underline{k}'}\, \mathcal{E}^K_{\underline{k}} + \langle v_r^2 \rangle_{\underline{k}}\, \mathcal{E}^M_{\underline{k}'}\right] \tag{3}$$

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where $b_r = \nabla_y \psi$, $v_r = \nabla_y \phi$, $\mathcal{E}^K_{\underline{k}} = \langle (\nabla_x \phi)^2 \rangle_{\underline{k}}$, and $\mathcal{E}^M_{\underline{k}} = \langle (\nabla_x \psi)^2 \rangle_{\underline{k}}$. By adding Eqs. (3), (4) and summing over $\underline{k}$, it is easy to verify that the renormalized spectral equations conserve energy.

The m=1 saturation amplitude is determined by the balance of the (equilibrium) magnetic energy release rate with the rate of energy scattering to m=2 via nonlinear interaction with neighboring m=1's. This balance is extracted from Eqs. (3), (4) by adding the equations, summing over n (with m=1), noting $\Delta' < 0$ for $m \geq 2$, and using standard properties of tearing modes, thus yielding:

$$-\sum_{n} \int dx \, \langle \phi b_r \rangle_{1,n} \langle J \rangle' = \sum_{n,n'} \int dx \, |\Delta'_{2,n+n'}| \, \frac{\delta(x'')}{\Delta\omega_{2,n+n'}} \langle b_r^2 \rangle_{1,n'} \, \mathcal{E}^K_{1,n} \quad (5)$$

where $\Delta'_{2,n+n'} = -|\Delta'_{2,n+n'}|$ and $\Delta\omega_{\underline{k}} \cong \sum_{\underline{k}'} |\Delta'_{\underline{k}''}| \delta(x'') \langle b_r^2 \rangle_{\underline{k}'} / \Delta\omega_{\underline{k}''}$. Equation (5) states that saturation is determined by the balance of growth due to $\langle J \rangle'$ relaxation with scattering to m=2. Since m=1 tearing modes do not exhibit a Rutherford phase[8] and because the islands overlap for relatively modest fluctuation levels, linear tearing mode eigenfunctions are used to calculate the driving term. Hence, $\int dx \, \langle \phi b_r \rangle_{1,n} \langle J \rangle' \cong \int dx \gamma_{1,n} \, \mathcal{E}^K_{1,n}$, where $\gamma_{1,n}$ is the linear growth rate. It follows directly that the root mean square m=1 magnetic fluctuation saturation level is $(\delta B/B_0)_{rms} \approx (\gamma_{m=1}\tau_A)/|\Delta'_2 a|^{1/2}$, where $\gamma_{m=1}\tau_A \sim S^{-1/3}$. It also follows that $\Delta\omega \sim (\delta B/B_0)_{rms} \sim S^{-1/3}$. It should be noted that quasilinear flattening of the equilibrium current gradient may cause some reduction in the driving term (as compared to $\gamma_{m=1}$). The temporal behavior of single and multiple helicity growth rates are compared in Ref. 9.

A physical interpretation of the proposed m=1 saturation mechanism is that the (primary) m=1 modes are stabilized by a nonlinear $\underline{J} \times \underline{B}$ force, resulting from interaction with stable modes, which opposes the growth of m=1 vorticity. This can be seen by examining the renormalized m=1 vorticity equation, which is

$$\frac{\partial}{\partial t} \frac{\partial^2 \phi}{\partial x^2}{}_{m=1,n} = ik_{\parallel} J_{m=1,n} + a_{\substack{m=1 \\ n}} \frac{\partial^2 \phi}{\partial x^2}{}_{m=1,n}$$

$$a_{\substack{m=1 \\ n}} = \sum_{\underline{k}'} \left(1 - \frac{n'^2}{n''^2}\right) \Delta'_{\substack{1+m' \\ n+n'}} \frac{\delta(x'')}{\Delta\omega_{\substack{1+m' \\ n+n'}}} \langle b_r^2 \rangle_{\underline{k}'} \quad (6)$$

For $m'=1$, $\Delta'_{m=2} < 0$, it is apparent that the nonlinear interaction opposes the growth of m=1 vorticity. Furthermore, the $(b_r)_{rms}$ saturation level can be obtained from the balance of the stabilizing $\underline{J}\times\underline{B}$ force with the driving forces, as approximated by $\gamma_{1,n}\phi''_{1,n}$. Note that this is the opposite case to that described in Ref. 6, where $\Delta'_{\underline{k}''} > 0$ and the resulting nonlinear destabilization triggers the onset of major disruption.

It is apparent from Eqs. (3),(4) that energy extracted from m=1 modes is scattered, by incoherent emission, to m=2 ($\Delta'_2<0$). Subsequent interaction with m=1 then scatters energy to $m\geq3$. Hence, a cascade to small scales (large $\underline{k}$) results. For $-\Delta'_m$ increasing with m, (for $m\geq2$), it follows that energy extracted from m=1 is ultimately expended driving large m, stable modes and dissipated by resistive diffusion at small scales. Note that in this problem, the direction and rate of cascade are determined by $\Delta'_{\underline{k}}$, a measure of stability at wavenumber $\underline{k}$. Hence, the cascade is actually a progressive current filamentation process, where nonlinear interaction generates small scale current sheets and islands.

Experimental evidence has linked m=1 tearing modes with dynamo events (increases in $|\langle B_z\rangle|$) near the RFP wall. ΔB_z, the change in $\langle B_z\rangle$ due to turbulence and resistive diffusion, is

$$\Delta B_z = \frac{1}{r}\frac{\partial}{\partial r} r \left(\frac{1}{S\Delta\omega}\frac{\partial\langle B_z\rangle}{\partial r} + \sum_{\underline{k}} \langle\tilde{\xi}\times\tilde{\underline{B}}\rangle_{\vartheta\underline{k}}\right) \tag{7}$$

where $\tilde{\xi}$ is the displacement and $\langle\tilde{\underline{v}}\times\tilde{\underline{B}}\rangle_{\vartheta}$ is the turbulence-induced electric field in the poloidal direction. Away from the resonant surface, the exterior (ideal MHD) equations apply; thus

$\tilde{\underline{B}} = \underline{\nabla} \times (\tilde{\underline{\xi}} \times \langle\underline{B}\rangle)$. Assuming a kink displacement $\tilde{\underline{\xi}}_{\underline{k}} = \tilde{\underline{\xi}}_{\underline{k}}(r)(\underline{r}+i\underline{r}\times\underline{b}_0)$, it follows that

$$\Delta B_z = \frac{1}{r}\frac{\partial}{\partial r} r \left\{\frac{1}{S\Delta\omega}\frac{\partial\langle B_z\rangle}{\partial r}\right.$$

$$+ \sum_n \left[\frac{\langle B_z\rangle^2}{rB_0} |\tilde{\xi}_{1,n}(r)|^2 - \frac{n}{R}\frac{\langle B_z\rangle\langle B_\vartheta\rangle}{B_0} |\tilde{\xi}_{1,n}(r)|^2\right.$$

$$\left.\left. + \tilde{\xi}_{1,-n}(r)\frac{1}{r}\frac{\partial}{\partial r}\left(r\tilde{\xi}_{1,n}(r)\langle B_z\rangle\right)\right]\right\} . \tag{8}$$

To examine dynamo activity near the wall, it is useful to note that $\tilde{\xi}_{1,n}(r) \to 0$ as $r \to r_{wall}$. Therefore

$$\Delta B_z \cong \frac{1}{r}\frac{\partial}{\partial r} r \left\{\frac{1}{S\Delta\omega}\frac{\partial\langle B_z\rangle}{\partial r} + \sum_n \langle B_z\rangle \frac{\partial}{\partial r}\left(|\tilde{\xi}_{1,n}(r)|^2\right)\right\} . \tag{9}$$

Hence, for $\partial|\tilde{\xi}_{1,n}(r)|^2/\partial r < 0$, $\partial\langle B_z\rangle/\partial r < 0$, and $\langle B_z\rangle$ reversed, $\Delta B_z \to 0$ for $(\tilde{\xi})_{rms} \sim (L_\xi/L_{B_z})^{1/2}(S\Delta\omega)^{-1/2} \sim S^{-1/3}$, where L_ξ and L_{B_z} are the displacement and B_z scale lengths, respectively. Noting that $(\tilde{\xi})_{rms}) \sim (\delta B/B)_{rms} \sim S^{-1/3}$, it follows that dynamo activity induced by m=1 tearing modes can offset resistive diffusion of $\langle B_z\rangle$ near the wall, thus maintaining field reversal. Furthermore, the fluctuation scalings <u>required</u> for $\Delta B_z \to 0$ are consistent with those actually <u>predicted</u> for saturated m=1 modes. Indeed, it is worthwhile to observe that a model of m=1 saturation based on the balance of dynamo activity with resistive diffusion away from the Taylor minimum-energy equilibrium state predicts (using Eq. (7)) saturated state magnetic fluctuation levels also scaling as $\delta B/B \sim S^{-1/3}$. Finally, it is interesting to note that in contrast to solar dynamo models, where the symmetry breaking necessary for dynamo activity is a consequence of finite helicity fluid turbulence[10], the necessary symmetry breaking here is a consequence of the radial displacement structure ($\partial|\hat{\xi}_k|^2/\partial r < 0$ near wall) imposed by boundary conditions and ideal MHD energetics. Also, in the case of the solar dynamo, thermal energy is converted to magnetic enegy. In the RFP, global m=1 tearing modes convert magnetic energy density in the core poloidal field (near r_s) to magnetic energy density in the toroidal field (near the wall). Thus, the RFP dynamo process is actually a redistribution of magnetic energy.

Experimental evidence has linked heat transport along stochastic magnetic fields produced by m=1 tearing modes with confinement in the RFP.[11] Quasilinear magnetic field line diffusion[12] predicts a (collisionless) thermal conductivity $\chi_E = V_{T_e} \sum_{\underline{k}} \langle b_r^2 \rangle_{\underline{k}}\ \delta(k_\parallel)$. The use of a quasilinear estimate is consistent with the large radial extent of the modes – turbulent broadening of the $\underline{k}\cdot\underline{B}_0$ resonance is not a significant effect here. Using the saturated state magnetic fluctuation levels obtained above, it follows that $\chi_E \sim \alpha v_{T_e} a\, S^{-2/3}$, where $\alpha \sim .01$ and V_{T_e} is the electron thermal velocity. Balancing heat loss with ohmic heating yields $T_e \sim I_p^{.7}$, where I_p is the plasma current and the proprotionality of density to I_p has been assumed (consistent with experimental results). For $S = 10^5$, β~10% is predicted. The scaling of temperature with current and the estimated β are in reasonable agreement with experimental results. However, it should be noted that other loss processes, such as resistive pressure-driven turbulence[13,14], may contribute to heat transport, particularly between the reversal point and wall. Preliminary results[14] indicate that stochastic magnetic fields produced by resistive interchange modes result in an anomalous thermal conductivity $\chi_E \simeq (1/4\pi)^{1/2}(\varepsilon/q)^3 v_{T_e} a\beta^{3/2}/S$. Balance of thermal loss with ohmic heating indicates that $T_e \sim I_p$, for density proportional to current, and β~10%. The similarity of the thermal loss prediction for resistive pressure-driven turbulence to that for m=1 tearing modes indicates that turbulence within (tearing) and beyond (pressure driven) the field reversal point gives rise to comparable thermal transport. Hence, it appears plausible that RFP transport is insensitive to small variations in ϑ and F.

The authors would like to acknowledge stimulating conversations with Drs. H. R. Strauss, A. Hasegawa, I. H. Hutchinson, and A. Aydemir. P. D. and Z. A. are also grateful to Dr. C. S. Liu for his encouragement during the course of this work. This research was supported by DE-FG05-80ET-53088.

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