RELATION OF FLUID DRIFT TO OSCILLATION CENTER DRIFT

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March 1984

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Abstract

The fluid theory and oscillation center theory of the ponderomotive drift give apparently different results. It is shown that this difference is due to a ponderomotive diamagnetic effect.

In oscillation-center theory[1] particle motion in a high-frequency electromagnetic field is analyzed by splitting the position into slowly and rapidly varying parts, $\underset{\sim}{x} = \underset{\sim}{x}_s + \underset{\sim}{x}_f$. Application of expansion and averaging techniques yields the average drift of the particle. In their most general form these drifts can be obtained from a Hamiltonian.[2-4] Alternatively one can apply averaging techniques to the cold fluid equations, thereby obtaining a fluid drift[5] which differs from the oscillation drift. This note contains a discussion of how these two analyses fit together. It is shown that there is a ponderomotive diamagnetic drift which in combination with the oscillation center drift yields the fluid drift.

To illustrate the diamagnetic effect we discuss dynamics in the presence of a linearly polarized electrostatic field,

$$\underset{\sim}{E} = \hat{x}\,\mathscr{E}(x)\exp(-i\omega t) + c.c. \tag{1}$$

A straightforward analysis of the cold-fluid equation, $\partial\underset{\sim}{v}/\partial t + \underset{\sim}{v}\cdot\nabla\underset{\sim}{v} = (e/m)(\underset{\sim}{E}+\underset{\sim}{v}\times\hat{z}B_0/c)$ shows that there is an average fluid drift[5]

$$\langle\underset{\sim}{v}_2(x)\rangle = \hat{y}\,\frac{e^2\omega^2}{m^2\Omega(\omega^2-\Omega^2)^2}\,\frac{\partial}{\partial x}\,|\mathscr{E}|^2 \ . \tag{2}$$

where $\Omega \equiv eB_0/mc$ is the gyrofrequency.

In the particle picture we consider a cold particle with position

$$\underset{\sim}{x} = (x_s+x_f)\hat{x} + (y_s+y_f)\hat{y} \tag{3}$$

for small oscillations, $\underset{\sim}{x}_f \cdot \nabla \ell n |\mathscr{E}| \ll 1$. The first-order equations give the oscillation in the position,

$$x_f = \frac{e\mathscr{E}(x_s)}{m(\Omega^2-\omega^2)} \, e^{-i\omega t} + c.c. \qquad (4a)$$

and

$$y_f = \frac{ie\Omega\mathscr{E}(x_s)}{m\omega(\omega^2-\Omega^2)} \, e^{-i\omega t} + c.c. \qquad (4b)$$

The second-order equation for the drift obtained by expansion and averaging is

$$\dot{y}_s = \frac{e^2}{m^2\Omega(\omega^2-\Omega^2)} \, \frac{\partial}{\partial x_s} \, |\mathscr{E}(x_s)|^2 \ . \qquad (5)$$

This result agrees with previous[2-3] formulas for the ponderomotive drift, but apparently disagrees with the fluid result (2).

The resolution of this effect is due to a ponderomotive diamagnetic effect. This effect arises because the particle oscillation velocity,

$$\dot{y}_f = \frac{e\Omega\mathscr{E}(x_s)}{m(\omega^2-\Omega^2)} \, e^{-i\omega t} + c.c. \qquad (6)$$

varies with position. Thus, a contribution to the mean fluid velocity comes from averaging the velocity of particles at the same physical position but different oscillation center positions.

$$\langle \underset{\sim}{v}_{2D}(x) \rangle = \langle \dot{y}_f(x_s = x - x_f) \rangle \hat{y} \cong - \langle \frac{\partial \dot{y}_f}{\partial x_s}(x) x_f(x) \rangle \hat{y}$$

$$= \hat{y} \frac{e^2 \Omega}{m^2 (\omega^2 - \Omega^2)^2} \frac{\partial}{\partial x} |\mathscr{E}(x)|^2 . \tag{7}$$

The fluid drift of Eq. (2) is obtained by adding the oscillation center drift of Eq. (5) to the ponderomotive diamagnetic drift of Eq. (7). The situation is analogous to the usual diamagnetic drift due to a temperature gradient, which is present even when the guiding centers do not move.

This effect has not been left out of oscillation-center theory. The terms of Refs. 2-3 which gives rise to the effect are the terms $\eta_0 \frac{1}{2} \{w_1, \{w_1, F\}\}_0 - \langle \eta_1 \{w_1, F\} \rangle$ of Eq. (89) of Ref. 3 which effect the transformation from the oscillation center distribution to the physical distribution.

Acknowledgments

References

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