

Parameter Optimization Studies for a Tandem Mirror Neutron Source

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Abstract A basic plasma physics tandem mirror experiment is proposed to develop the potential uses of magnetic mirror confined plasmas for a neutron source. We consider parameter variations from the currently operating symmetric mirror plasma trap GDT in an attempt to optimize the neutron source intensity while minimizing the expense and complications of the system. The combined radial and axial plasma loss rates are analyzed and shown to yield an optimal operational point that minimizes the required auxiliary heating power.

Keywords Mirror confinement · Tandem mirror · Neutron source · Nuclear fusion

Introduction

The fundamental understanding of mirror confined plasma has progressed rapidly in the last 5 years with breakthroughs in performance from both the Gas Dynamic Trap and the GAMMA-10 systems. There are intrinsic advantages to the linear, symmetric mirror confinement geometry over the toroidal geometry for particle confinement and plasma heating.

Briefly, some of the advantages of the linear, symmetric confinement system are:

1. Infinite confinement times for certain classes of charge particle orbits which may be very important for the

alpha particles born in the core of the plasma with a relatively large gyroradius. The recent example of the intrinsically long single particle confinement time is given by the 300 s electron confinement time in the RT1 dipole experiment at the University of Tokyo [1].

2. No toroidal curvature that is responsible for the uniform unfavorable magnetic curvature over large volumes of the plasma. A torus of major radius R has an effective gravity from the $g_{\text{eff}} = c_s^2/R$ over the whole outer-half of the torus. This produces a magnetic type of Rayleigh–Taylor instability that limits the plasma pressure to low values in MHD and produces long fingers of vortices in the drift wave turbulence that rapidly transport particles and energy radially across the machine's minor radius $r = a$. The favorable and unfavorable magnetic field curvature in the mirror system are next to each other over the distance L_t and, furthermore, vanishes by symmetry in the core of the plasma. The result is MHD stable plasma up to a dimensionless plasma pressure of $\beta = 8 \pi p/B^2$ of order unity. This allows a much higher plasma pressure to be confined for the standard magnetic field and keeps the volume of the neutron source less than 100 m^3 .
3. There are no large plasma currents as in the tokamak system. For steady state tokamak operation the large toroidal plasma I_p current must be driven, which presents an unsolved technology problem. Even when the toroidal current is driven there are spontaneous magnetic reconnection events that can disrupt the core plasma confinement. Examples, for a burning plasma in a tokamak are given in Horton et al. [2] and Hu et al. [3].
4. Mirrors have a natural open magnetic field line divertor that removes the plasma transported to the

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- edge of the cylindrical confinement region. We show a simulation in “[Burning Plasma Pulse](#)” where the core plasma is burning and the cooler plasma is transported from the hot core to outer shell where the plasma escapes out the machine ends into the expansion tanks. The natural scrape-off layer effect is from the higher axial loss rates from pitch-angle scattering in to the velocity space loss cones in the outer radial zone.
5. The collimated loss of plasma out the ends of the cylinder then gives the potential of direct conversion of the kinetic energy in the plasma to electric power. On the GAMMA-10 device a direct electric convertor has been installed by the Osaka University group. The converter powers a small neon light bulb from the separation of the escaping ions and electrons in cusp magnetic field geometry.

Simonen points out [4] that the results from the Gas Dynamic Trap in Russia and the GAMMA-10 tandem mirror in Japan have renewed interest in magnetic mirror concepts. The Russian Gas Dynamic Trap experiment achieved plasma beta $\beta = 0.6$ in an axisymmetric magnetic mirror. The Japanese Gamma-10 experiment demonstrated the suppression of radial transport due to drift-wave turbulence. Simonen notes that the simple axisymmetric configuration has applications ranging from a source for material testing with plasma and neutrons to a driver for fusion-fission hybrid to the possibility of a fusion power plant.

Ivanov et al. [5] review the status of the experiments on the axially symmetric magnetic mirror device gas dynamic trap (GDT). The plasma has been heated by skewed injection of 20-keV, 3.5-MW, 5-ms deuterium/hydrogen neutral beams at the center of the device, which produces anisotropic fast ions. Neither enhanced transverse losses of the plasma nor anomalies in the fast ion scattering and slowing down were observed. Extension of neutral beam injection pulse duration from 1 to 5 ms resulted in an increase in the on-axis transverse beta (ratio of the transverse plasma pressure to magnetic field pressure) from 0.4 at the fast ion turning points near the end mirrors to about 0.6. The measured beta value is rather close to or even higher than that expected in different versions of the GDT-based 14-MeV neutron source for fusion materials testing. The density of fast ions with the mean energy of 10–12 keV reached $5 \times 10^{19} \text{ m}^{-3}$ near the turning points. The electron temperature at the same time reached 200 eV. The radial plasma losses were suppressed by sheared plasma rotation in the periphery driven by biasing of end wall segments and the radial limiter in the central solenoid.

Zhmoginov and Fisch [6] show that it may be possible in a mirror machine to transfer the kinetic energy in the alpha particles directly to the ions by the proper design of RF antennas to create a finite amplitude ion cyclotron wave

with the required frequency and wavenumbers for the resonant wave-particle interactions.

Molvik et al. [7] propose to design a dynamic trap neutron source (DTNS), which burns approximately 100 g of tritium per full-power year. Such a DTNS would provide a neutron spectrum similar to that of ITER and satisfying the missions of the materials community to test fusion materials.

In the next section, we use a high resolution space-time transport code called ITB developed earlier by Horton and Zhu [8] to describe the competition between the radial and end losses of an axisymmetric mirror system. In “[Confine-ment Dynamics](#)” we give the drift wave turbulence transport formulas, and we consider optimization of the system parameter vector $P_\alpha = [a, L_{cc}, L_t, B_{cc}, B_{\max}, P_{ICRF}, P_{ECRF}, P_{cc}, N_{BI}, \theta_{NBI}, P_{plug,NBI}]$. The parameters start at those for the present GDT and also we discuss how to use genetic algorithms (GA) adopted from earlier research [9] to find optimal combinations of system parameters within this multidimensional parameter vector space. The equivalent neutron flux of order $10^{18}/\text{s}$ over the targets of 1 m^2 corresponding to 2 MW/m^2 is expected in for these values. In the last section, we give the conclusions.

The Combined Axial and Radial Transport Code for the Mirror System

With the transport code we varying the ECH heating power in response to the core electron temperature so as to maintain a targeted core electron temperature. The simulation varies the core heating power every Δt in response to the measured core electron temperature. We propose that this real-time coupled diagnostic and auxiliary heating system be developed and tested on the University of Texas Helimak plasma that is created in Argon with microwave heating and then exported for experiments on G-10 and Gas Dynamics Trap.

The transport equations for the four fields n_e , n_α and T_e , T_i are written as follows:

$$\frac{\partial}{\partial t} n_e + \frac{1}{r} \frac{\partial}{\partial r} (r \langle n_e v_r \rangle) = S_{n_e} - J_{||e} \quad (1)$$

$$\frac{\partial}{\partial t} n_\alpha + \frac{1}{r} \frac{\partial}{\partial r} (r \langle n_\alpha v_{r\alpha} \rangle) = n_{DNT} \langle \sigma v \rangle_{DT} - J_{||\alpha} \quad (2)$$

where $J_{||}(r, t)$ are the axial fluxes due to the pitch angle scattering. The pressures p_a from T_e and T_i evolve according to

$$\frac{\partial}{\partial t} \left(\frac{3}{2} p_a \right) + \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(q_a + \frac{5}{2} T_a \Gamma_a \right) \right] = P_{aux} + P_a - P_{a||} \quad (3)$$

The sources S_a , P_a and sinks Q_a , $J_{\parallel a}$, $P_{\parallel a}$ are specified from both the auxiliary fueling and heating and the self-nuclear fusion heating and power transfers between the ions and electrons. The axial losses J_{\parallel} and P_{\parallel} are from pitch-angle scattering for which the confinement time is $\tau_{\parallel} = \exp(c_i)(A_p c_i \tau_{ii} + R_m L/v_{th,i})$ and $A_p = \frac{\sqrt{\pi}}{4} \frac{R_m}{R_m+1} \ln(2R_m+2) = 1.5$ where $R_m = B_{\text{plug}}/B$ is the mirror ratio and A_p is the Pastukhov parameter.

Reference parameters are defined in Table 1 such that $c_i = \phi_i/T_i = 5.75$, and $c_e = \phi_e/T_e = 8.0$ as given by Hua and Fowler. The central cell electron density is denoted by n , temperatures by T_e and T_i . The practical formula for the ion-ion collision time is

$$\tau_{ii} = 1/v_i = \frac{66.82 \epsilon_0^2 m_i^{1/2} T_i^{3/2}}{n_i e^4 \ln A_i} \sim 5.2 \text{ ms} \frac{T_i(\text{keV})^{3/2}}{n_{19} \ln A_i / 20}, \quad (4)$$

using Eq. (11.24) of Goldston and Rutherford [10]. For $n = 10^{20} \text{ m}^{-3}$ and $T_i = 10 \text{ keV}$, we have $\tau_{ii} = 4.5 \text{ ms} \gg R_m L/v_{th,i} = 0.43 \text{ ms}$.

Formulas for thermonuclear heating power P_{α} , radial power loss P_{radial} , and end loss power P_{\parallel} are:

$$P_{\alpha} = n_D n_T \langle \sigma v \rangle_{DT} E_{\alpha} = \frac{1}{4} n^2 \langle \sigma v \rangle_{DT} E_{\alpha} = P_{\alpha/e} + P_{\alpha/i} \quad (5)$$

$$P_{\text{radial}} = \frac{3n(T_e + T_i)\chi}{2a^2} \quad (6)$$

$$P_{i\parallel} = \frac{n_i \phi_i}{\tau_{\parallel}} = \frac{c_i n_i T_i}{\tau_{\parallel}}. \quad (7)$$

$$P_{e\parallel} = \frac{n_e \phi_e}{\tau_{\parallel e}} = \frac{c_e n_e T_e}{\tau_{\parallel e}}. \quad (8)$$

Table 1 Machine parameters

Parameter	PoP value	Reactor value	Scale-up for PoP
a	1 m	1.5 m	1.5
L	7 m	30 m	4.28
n	10^{20} m^{-3}	10^{20} m^{-3}	1
B_{cc}	.28 T	3 T	10.7
B_{plug}	2.5 T	18 T	7.2
T_e	600 eV	10 keV	16
T_i	32.5 eV	15 keV	461
Gas type	Deuterium	Deuterium and tritium	1.5
Volume	22 m^3	212 m^3	9.6
Surface area	44 m^2	283 m^2	6.4
$c_i = \phi_i/T_i$	5.75	7.8	1.35
$c_e = \phi_e/T_e$	8	8.5	1.06
P_{ECH}	.8 MW/ m^3	.8 MW/ m^3	1
R_m	9	9	1
β_e	.066	.27	4

with ϕ_i and ϕ_e measured in electron volts converted to Joules for P_{\parallel} as power density in MW/ m^3 .

Two reference models are given in Table 1 for the Proof of Principle (PoP) machine, the DT neutron source(DTNS) machine and the associated scale-up factor for going from the PoP to the DTNS in the fourth column of the table.

The two standard drift wave turbulent diffusivities χ of energy are

$$\chi^B = c^B \frac{T_e}{B} = c^B \frac{130 \text{ V}}{.28 \text{ T}} = c^B 464 \text{ m}^2/\text{s} \quad (9)$$

$$\chi^{gB} = c^{gB} \frac{\rho_s}{L_{Te} eB} \frac{T_e}{eB} = c^{gB} \frac{5.9 \text{ mm}}{1 \text{ m}} \frac{130 \text{ V}}{.28 \text{ T}} = c^{gB} 2.7 \text{ m}^2/\text{s} \quad (10)$$

We note that the parametric dependences of T_e/B and $T_e^{3/2}/(a B^2)$ for the radial transport explain the energy confinement time scalings in the ITER database.

In Fig. 2 we show the radial profiles of a plasma with time-constant auxiliary heaters $P_e(r)$ and $P_i(r)$ for specified radial profiles.

The transport code evolves the profiles of heating and cooling. In Fig. 2, we show the results using a real-time control of the auxiliary heaters $P_e(t)$ and $P_i(t)$ that change with a feedback rate αn and αT_e every $\Delta t \sim 1 \text{ s}$ such that the temperature is returned to the target temperature T_e^* and T_i^* and density n^* . For the radial fluxes we use drift wave formulas. The simplest procedure is to use either the Bohm and gyro-Bohm formulas while recognizing that the toroidal confinement database indicates confinement that falls between these two laws in its dependence at the parameter.

The MHD stability of the axisymmetric mirror reactor is maintained by the large expander cells at each end of the device. The schematic of the expander cells from Pratt and Horton [11] is shown here in Fig. 1. This design follows

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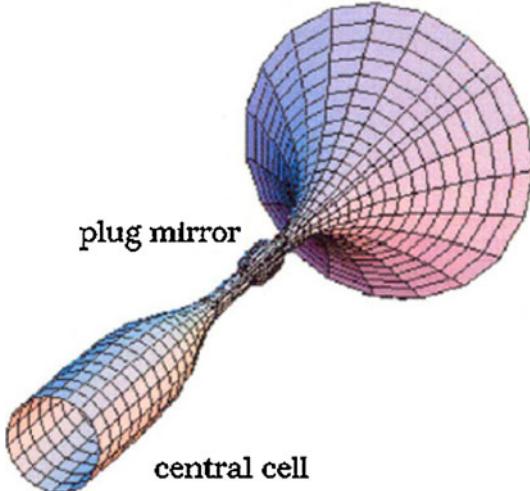


Fig. 1 A flux surface of the KSTM from Ref. [11]

that proposed by Post [12] and has now been investigated in detail by Pratt and Berk for interchange stability [13]. The system is stable to the flute interchange mode when sufficiently good connection exists between the end cell plasma and the central cell plasma. This requires sufficient density in the end cell.

Confinement Dynamics

Now we consider the combination of radial transport and the axial transport, we find that there is an optimal auxiliary heating power P_* and the associated plasma electron temperature T_e^* to obtain the highest confinement at the lowest auxiliary injected power for the mirror system. Here we derive the values P_* , T_e^* from analytic analysis keeping the other system parameters fixed. To allow multiple cost functions in addition to injected power, over all system variables we propose to use the Genetic Algorithm (GA) for optimizing object functions as carried out for space physics model in Spencer et al. [9]. This GA method requires large computational effect, however.

Optimization of the System

We define the “object function” as the injected power that equals the loss power in the steady state. To find the lowest injected power we vary the electron temperature which is a key parameter for the neutron source [14]. To find the optimal heating power P_* for fixed other parameters for the tandem mirror system we calculate the minimum of P_{loss} given by

$$P_{\text{loss}} = (\text{radial flux})(2\pi aL) + (\text{end loss})(\pi a^2 B/B_{\max}) \quad (11)$$

with the radial flux given by

$$q_r^e = (c_B \chi_B + c_{gB} \chi_{gB}) \frac{nT_e}{a} \quad (12)$$

where $\chi_B = T_e/eB$ and $\chi_{gB} = (\rho_s/a)\chi_B$. The simplest result (for $c_{gB} = 0$) is

$$T_e^* = (T_e)_{\min P_{\text{loss}}} = \left(\frac{c_{\parallel}}{4c_B} \right)^{2/5} (nZ\Lambda a^2 B)^{2/5} \quad (13)$$

for which confinement time is

$$\left(\frac{1}{\tau_E} \right)_{\min P_{\text{loss}}} = \frac{(4c_B)^{3/5} c_{\parallel}^{2/5}}{(4a^2 B)^{3/5}} + \frac{c_{\parallel}^{4/5} (4c_B)^{1/5} (nZ\Lambda)^{3/5}}{(4a^2 B)^{1/5}} \quad (14)$$

and the required injected power at this electron temperature is

$$P_{\min}/n_e = 1.7 \frac{c_B^{1/5} c_{\parallel}^{4/5} (nZ\Lambda)^{4/5}}{(4a^2 B)^{1/5}} (20\% + 80\%) \quad (15)$$

with 20% from the radial transport and 80% from the axial transport.

A Set of Analytically Derived Parameters

From these consideration we arrive at the neutron source parameters

$$\tau_E^* = 0.1 \text{ s} \quad (16)$$

$$W = \frac{3}{2} n T_e^* \pi a^2 L \leq 10^8 \text{ J} \quad (17)$$

$$P_{\text{inj}} \leq 10^9 \text{ W} \quad (18)$$

$$\pi a^2 B \leq 12 \text{ m}^2 \quad T = 12 \text{ Wb} \quad (19)$$

$$L \leq 30 \text{ m} \quad (20)$$

$$V_p = 100 \text{ m}^3 [\text{cp}700 \text{ m}^3] \quad (21)$$

$$p = 3 \times 10^5 \text{ Pa} \quad (22)$$

$$p_{\text{mag}} = 8 \times 10^7 \text{ Pa} \quad (23)$$

$$W_p \leq 30 \text{ MJ} \quad (24)$$

$$n_e \tau_E = 10^{12} \text{ s/cm}^3 \quad (25)$$

$$Q_{\text{fus}} \sim 0.1 \text{ to } 0.5 \quad (26)$$

$$P_{\text{fusion}}/P_{\text{inj}} \sim 10 \text{ MW}/100 \text{ MW} \quad (27)$$

Burning Plasma Pulse

When the auxiliary heating power exceeds the critical power P_* there is a burning pulse or wave that propagates from the core of the plasma. Here we show an example of the burning pulse and the associate axial loss rate.

Figure 2 shows the evolution of the particle end loss rate as the plasma heats up from below 1–20 kev for the parameters of the experiment. The vertical solid line at

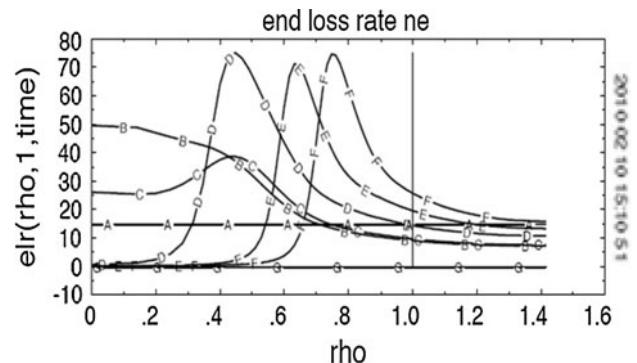


Fig. 2 The evolution of the particle end loss rate as the plasma heats up from below 1–20 kev. The snapshots are marked by letters with $A t = 0$, $B t = 1 \text{ s}$, $C t = 2 \text{ s}$, $D t = 3 \text{ s}$, $E t = 4 \text{ s}$, and $F t = 5 \text{ s}$. The units are based on the parameters of (16)–(27)

$\rho = r/a = 1$ defines the cool edge plasma where atomic recombination would begin.

The radial transport provides a stable operating point in temperature space once the temperature exceeds a minimum critical value.

The requirements for optimizing the neutron source rather than energy multiplication as in the ITER project are different as explained by Ryutov [14]. The mirror system has definite advantages over the torus as a compact efficient neutron source. The neutron source with a flux of $f_n = 1.4 \times 10^{14} \text{ n/cm}^2\text{s}$ or 3MW/m^2 can be produced by the injection of 200 kev triton beam into a thermal plasma with $T_e < 1$ kev. The system can be less than $L = 10$ m and radius less than $a = 30$ cm according to Ryutov's design [14]. The high efficiency of the system compared with a torus comes from the theory and experiments showing that a mirror confined plasma is stable up to a relative plasma to magnetic pressure of order unity. In the torus the large effective gravity, as explained earlier, limits the plasma pressure to less than 10% of the magnetic pressure.

The theory of MHD stability from the favorable magnetic curvature in the expander tanks is verified in the experiments reported by Anikeev et al. [15] and earlier by Bagryanskii et al. [16].

The capital cost of a mirror neutron source is a small fraction of the cost of the ITER project while the answers to the best choices of materials for surviving the unprecedented neutron fluxes in the next step demo reactor (Table 2).

When the electron temperature deviates significantly from the optimal value T_e^* , the required power increases. For $T_e = T_e^*/2$, the required power is $1.2P_*$ and radial loss is only 5% and the axial loss is 95% of the injected power. When the electron temperature increases to $T_e = 1.5T_e^*$, we find that power required increases to $1.67P_*$ and the radial loss is 41% and the axial loss is 59%. When $P > P_*$ the higher T_e operating point is stable and the lower T_e

Table 2 Radial transport parameters

Parameter	PoP value	Reactor value	Ratio
ρ_s	5.9 mm	13 mm	2.2
c_s	79 km/s	1.5×10^3 km/s	19
a/ρ_s	170	113	.66
$\rho_s c_s$	464 m ² /s	2×10^4 m ² /s	43
c^B	.1	.003	.03
c^{gB}	18.4	2.8	.15
c^{ETG}	4.0	.06	.015
χ^B	46 m ² /s	60 m ² /s	1.3
χ^{gB}	24 m ² /s	24.3 m ² /s	1
χ^{ETG}	.1 m ² /s	.24 m ² /s	2.4

operating point would require feed back control to guard against thermal collapse.

Conclusions

The combination of axial and radial transport for the symmetric mirror system leads to an attractive DT burning plasma. The global confinement time τ_E is shown to be a hybrid of the core radial transport coefficient and the axial pitch angle scattering time $\tau_{||}$. An optimization is presented in Eq. 15 that minimizes the power loss P_{\min} . The dependence on the system parameters is derived for the simple Bohm transport formula which shows that 20% of the power lost is from radial transport and 80% from axial losses.

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