Three-dimensional filamentary structures of a relativistic electron beam in fast ignition plasmas

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The filamentary structures and associated electromagnetic fields of a relativistic electron beam have been studied by three-dimensional particle-in-cell simulations in the context of fast ignition fusion. The simulations explicitly include collisions in return plasma current and distinctly examine the effects of beam temperature and collisions on the growth of filamentary structures generated.
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The transport of laser generated electron beams in overdense plasmas is an important topic to study specially in the context of fast ignition (FI).[1] Particle-in-cell (PIC) simulations and calculations have shown that the typical energy of electrons in these beams is of the order of a few MeV, which implies giga-ampere currents needed to transport petawatt energy fluxes.[2,3] However, the transportation of beam currents greater than the Alfvén current limit is not possible as the self-generated magnetic field associated with these currents bends electron trajectories strongly thereby preventing the forward propagation of the beam. In vacuum, the Alfvén current limit is $I_{\mathrm{alf}}=17\gamma$ kA, where $\gamma=1/\sqrt{1-\beta^2}$ is the relativistic Lorentz factor associated with the beam. A current greater than this limit can only propagate as a charge neutralized current. This sort of situation occurs in the FI scenario, where the charge of the relativistic electron beam is compensated by the return plasma current and the propagation of the electron beam is possible. But this configuration, where two opposite streams of plasma currents are counterpropagating, is susceptible to the electrostatic two-stream instability and the transverse filamentation instability, often referred to as the Weibel instability. Due to these instabilities, the electron beam splits into many filaments.[4–6] This filamentation can significantly affect the beam energy deposition at the center of the fuel target and thus is one of the central issues for the FI concept. Recently, Honrubia *et al.*[7] have carried out three-dimensional (3D) hybrid simulations of the fast electron transport and resistive beam filamentation in inertial fusion plasma. Califano *et al.*[8] have reported three-dimensional magnetic structures generated due to the Weibel instability of a relativistic electron beam in collisionless plasmas. Three-dimensional PIC simulations of the Weibel instability in astrophysical context have also been reported.[9] Moreover, the evidence of Weibel-like dynamics and the resultant filamentation of electron beams have been observed experimentally.[10]

The role of collisions on the Weibel instability is rather nontrivial and has been studied analytically in linear and quasilinear regime of the instability.[11,12] The eigenmodes of a collisional plasma have been computed recently, where it was shown that the frictional nature of collisions can change the dynamics of the eigenmodes of the system significantly.[12] A more focused discussion about the collisional effects on the Weibel instability has also been addressed recently, where it was shown that even small collisions in the background plasma can trigger the Weibel instability of a warm electron beam that would be otherwise stable in collisionless plasmas.[13] However, until now the full scale PIC simulations on the filamentation of the electron beams have mostly concentrated on the collisionless Weibel instability and have not explicitly included collisions in the return plasma current.

Motivated by these findings, we carry out three-dimensional PIC simulations on the transport of a relativistic electron beam in the FI plasmas by explicitly including the effect of collisions in the return plasma current. The rationale behind choosing 3D geometry is twofold: the first reason is to study the realistic geometry and the second is to show that in this geometry, the Weibel instability cannot be suppressed due to the transverse beam temperature as proposed initially.[14] It has been shown before analytically that in 3D geometry the Weibel instability survives even with high transverse beam temperature due to the presence of an "oblique mode," which is the manifestation of coupling between the Weibel and the two-stream instabilities.[15] We conjecture the onset of the anomalous collisionality of the background plasma due to the existence of the oblique mode. It arises due to the turbulence of electrostatic fields generated by the two-stream instability. We choose four cases for the simulations: (A) cold electron beam in a collisionless background plasma, (B) cold electron beam in a collisional background plasma, (C) warm electron beam in a cold collisionless background plasma, and (D) warm electron beam in a collisional background plasma, thus highlighting the influence of these physical processes separately. Hence, for the first time we have studied the filamentation of the relativistic beam investigating the role of each physical process clearly in full 3D PIC simulations. In our simulations, the electron beam propagates along the negative $\hat{z}$ axis with relativistic velocity

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$\upsilon_{bz} \gg \upsilon_{bx}, \upsilon_{by}$. The bulk cold background plasma is represented by ambient plasma electrons, while the plasma ions are considered as a fixed charge-neutralizing background with density $n_0 = n_b + n_p$, where n_b and n_p are the densities of the electron beam and the background plasma, respectively. The 3D simulation box has spatial dimensions $X \times Y \times Z = 20\lambda_s \times 20\lambda_s \times 20\lambda_s$, where $\lambda_s = c/\omega_{pe}$; c and ω_{pe} are the velocity of light in vacuum and the total electron plasma frequency respectively. The 3D simulation domain is sampled with the mesh of $160 \times 80 \times 20$ cells. All simulations are performed with 64 particles per cell. The density ratio between the plasma and beam electrons is $n_p/n_b = 9$, whereas the beam and the background plasma electrons have velocities $\upsilon_{bz} = 0.9c$ and $\upsilon_{pz} = -0.1c$, respectively. If we take typical plasma density ($n_0 \sim 10^{22}$ cm^{-3}), then the current density of the electron beam is $J_{bz} \sim 10^{12}$ A cm^{-2} and the total current carried is ~50 kA, which is larger than the Alfvén current limit. The evolution of the field energies for every component F_i of the fields **E** and **B** is recorded at every diagnostic step of the simulation summed over all the grid cells as

$$\int_V F_i^2 dxdydz = \sum (eF_i/m_e c\omega_{pe})^2 h_x h_y h_z, \quad (1)$$

where h_x, h_y, and h_z are the spatial grid sizes along x, y, and z, respectively, and $eF_i/m_e c\omega_{pe}$ represents the relativistic field normalizations. The relativistic electron beam has a transverse temperature of $T_b \approx 70$ keV and the ambient plasma collision frequency is assumed to be $\nu_{ei}/\omega_{pe} = 0.15$ for these simulations. The background plasma is always cold while the beam electrons do not face any collisions. Initially the beam current is fully compensated by the return plasma current. The quasineutrality is maintained over all simulation time as the field evolutions due to the Weibel instability occur on a time scale slower than the electron plasma frequency. We choose periodic boundary conditions in the simulations. The collisional processes are simulated with a newly implemented collision module in the relativistic PIC code Virtual Laser Plasma Laboratory (VLPL).[16]

Figure 1 shows the structure of the beam filaments during the nonlinear regime at a time $T = 16(2\pi/\omega_{pe})$ for the case (A). The filaments transverse spatial sizes are comparable to the plasma skin depth λ_s, while their longitudinal spatial extents are larger than the plasma skin depth. This could be understood as follows: in the linear regime of the beam filamentation, the transverse extent of the filaments is governed by the Weibel instability and its growth rate maximizes around the plasma skin depth in k-space. However, the longitudinal extent is determined by the two-stream instability which maximizes at wavelengths larger than the plasma skin depth. As a result, the filaments have small spatial extent in the transverse direction and are elongated in the propagation direction. Later on, during the nonlinear stage of the instability, the filaments merge together and their radii grow. The growth rate of the filamentation instability depends on the beam-to-plasma densities ratio, and is expressed as $\Gamma = (\sqrt{3}/2^{4/3})(n_b/n_p\gamma_b)^{1/3}$, where γ_b is the relativistic Lorentz factor of the electron beam.[17] The Weibel instability generates very strong quasistatic magnetic field,

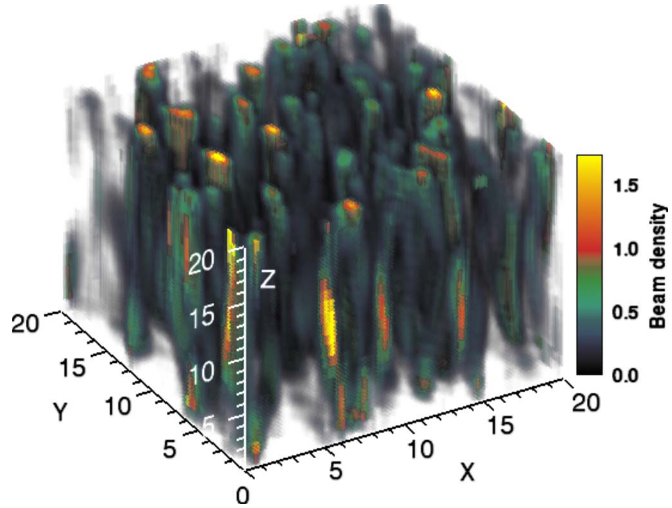

FIG. 1. (Color online) 3D structure of the beam filaments generated in the PIC simulation at a time $T = 16(2\pi/\omega_{pe})$ in the nonlinear regime for the case (A), where the electron beam is cold and the plasma is collisionless. See text for explanations.

which can be seen from the surface plot of the transverse (B_x, B_y) and longitudinal (B_z) magnetic field B_x shown in Fig. 2. Evidently, the amplitude of the transverse components is larger at least by one order of magnitude compared to the longitudinal component. The contour lines on the bottom surface in this figure show that each of these tiny filaments is surrounded by strong axial magnetic fields. The simulation case (B) is shown in Fig. 3. We observe that the transverse spatial width of the filaments is larger while the longitudinal extent remains unchanged. Due to collisions, the growth rate of the Weibel instability saturates at large plasma skin depth in k-space. At later times, the filaments merge together and their transverse spatial extents grow even larger, which is evident from the simulation results. The filament structures of the simulation case (C), where the electron beam has a

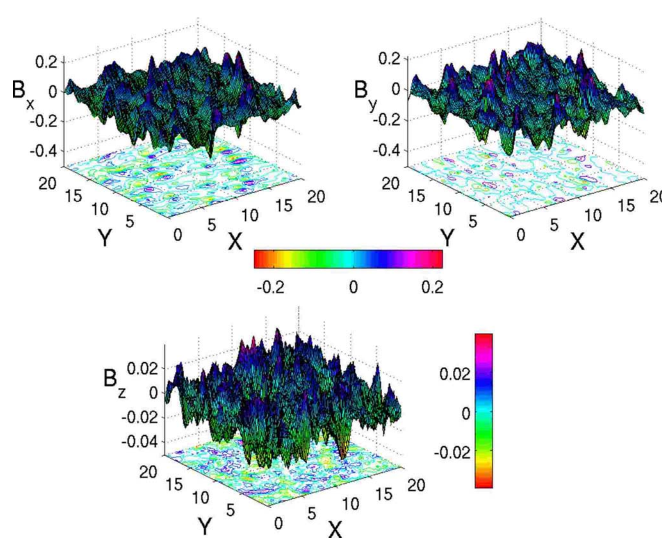

FIG. 2. (Color online) Illustration of the transverse (B_x, B_y) and longitudinal (B_z) magnetic fields at the same time as Fig. 1. The contour lines on the bottom surface give a two-dimensional projection.

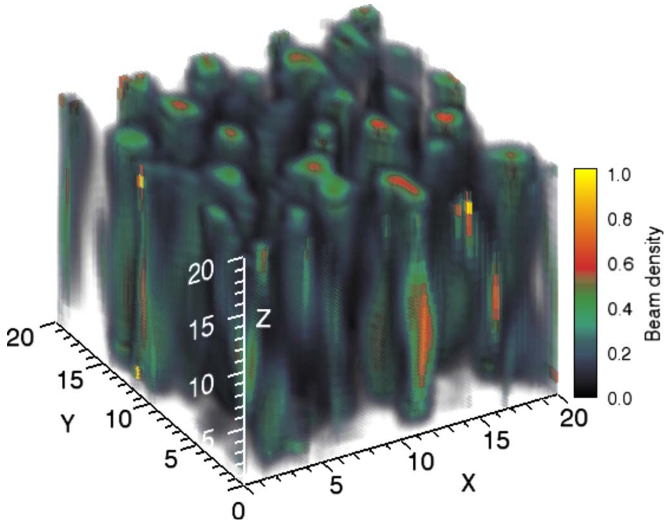

FIG. 3. (Color online) 3D structure of the beam filaments from the PIC simulation at a time $T=8(2\pi/\omega_{pe})$ for case (B), cold electron beam in a collisional plasma. The filament sizes are larger than collisionless case due to collisional dissipation.

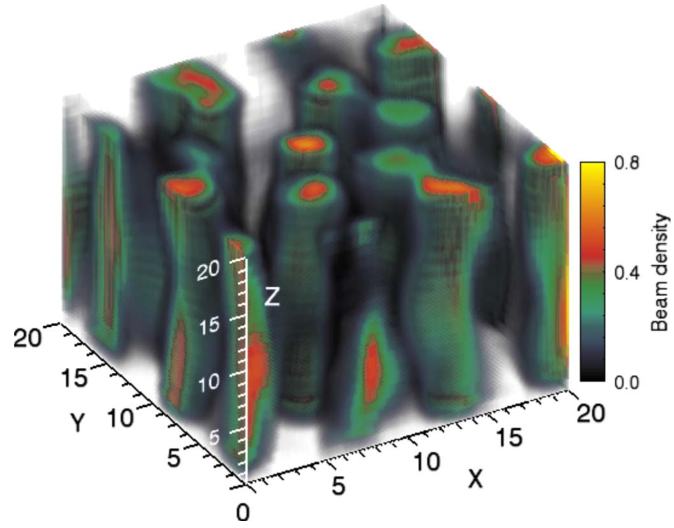

FIG. 5. (Color online) Structure of the electron beam filaments derived from the PIC simulations for the case (D), where the warm electron beam is propagating in a collisional background plasma.

transverse beam temperature $T_b \approx 70$ keV, is shown in Fig. 4 at a time $T=10(2\pi/\omega_{pe})$. The Weibel instability could be suppressed by the high transverse beam temperature. However, in 3D geometry the coupling of the Weibel and the two-stream instabilities can manifests itself into the so-called oblique mode, which persists even when the beam temperature is high. We may also consider the occurrence of the beam filamentation in this case due to "anomalous plasma collisionality" as shown in Ref. 13. The two-stream mode actually generates electrostatic turbulence in the plasma. Stochastic fields associated with this turbulence scatter the beam and plasma electrons and lead to an effective collisionality in the return current. This anomalous effect revives the Weibel instability. The phenomena of anomalous resistivity of a plasma generated due to the turbulence of electric and magnetic fields have been noted earlier also.[3] The allowance for finite background plasma temperature decreases the growth rate of the instability. The interplay of collisions in different regimes of beam and background plasma temperatures on the Weibel instability can be found in literature.[11] One may also note here that this filamentation is lacking complete evacuation of the background plasma electrons contrary to the previous cases, presumably due to the strong thermal effects. Figure 5 shows the beam filaments of the last simulation case (D), where the electron beam is hot and the background plasma is collisional. Here again we see strong filamentation and the spatial widths of the beam filaments are also larger due to collective collisional plasma effects. Moreover, we have observed almost complete evacuation of the background plasma in this case.

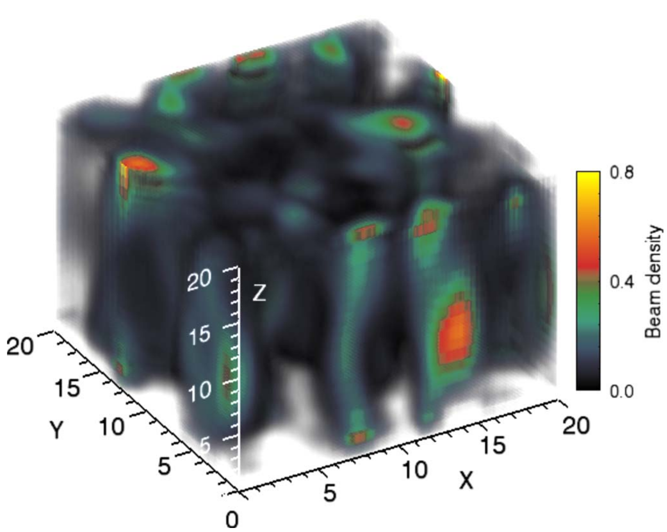

FIG. 4. (Color online) Beam filamentation persists in the case (C), where the electron beam is warm with $T_b \approx 70$ keV propagating in a collisionless plasma, at a time $T=10(2\pi/\omega_{pe})$ in the nonlinear regime.

We have studied the evolution of the associated electric and magnetic field energies of the beam plasma system, which is shown in Fig. 6, for the four different simulation cases. The vertical axis in Fig. 6 represents the normalized field energies in logarithmic scales, whereas the horizontal axis is for time scaled by $2\pi/\omega_{pe}$. In each of these cases, we see a stage of linear instability, where the field energies build up exponentially in time and it is followed by the nonlinear saturation of the instability. The nonlinear saturation occurs dominantly due to the magnetic trapping of the particles.[18] During this stage the filaments merge rapidly into each other due to the magnetic attraction between filaments. One may note that the simulation cases (B), (C), and (D) display similar trend of field energy build up. This similarity is very interesting for the case (C), where the electron beam is warm and has the sufficiently high temperature to kill the Weibel instability. Yet, we still see the filamentation. Clearly it happens due to the presence of the two-stream or oblique mode. Based on the similarity between cases (B) and (C) and as shown in Ref. 13, we attribute the filamentation of the beam to the "effective plasma collisionality" of the beam-plasma system. In our simulations, where the ions are fixed, this

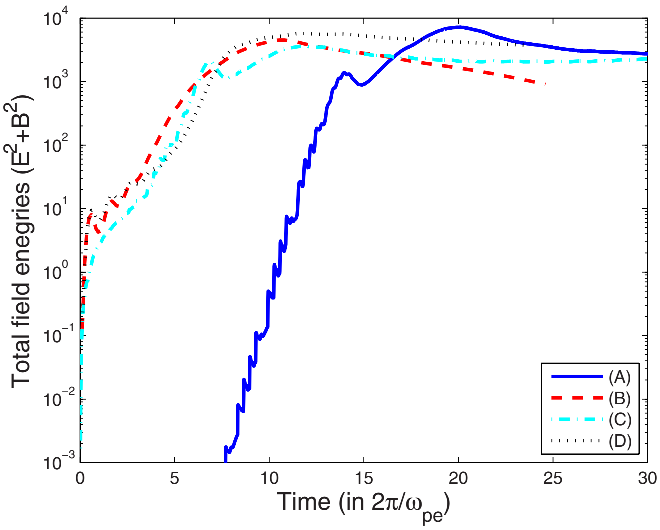

FIG. 6. (Color online) Time evolution of field energies (E^2 and B^2) for the different simulation cases: (A), (B), (C), and (D). See text for explanations.

anomalous plasma collisionality could be of the same order as the collisionality chosen in the system.

In summary, we have studied the filamentation instability of a relativistic electron beam in fast ignition plasma by three-dimensional particle-in-cell simulations, distinctly examining the effects of high beam temperature and plasma collisions. The important result of this study is that the beam temperature does not suppress the filamentation instability even in the absence of collisions in beam plasma system. An explanation on the persistence of the Weibel instability in 3D geometry is offered. It is attributed to the anomalous collisionality of the beam-plasma system due to the two-stream mode. The nonlinear regime of the filaments merging and the growth of field energies have been studied. We wish to point out here that in these simulations, we have taken an infinite width beam. For finite width beams, the results presented here will remain qualitatively same, however the presence of oblique mode would cause beam spraying during the linear stage of the instability, resulting into less energy being deposited in the forward direction, which could be detrimental for the FI.

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