

Comment on “Derivation of paleoclassical key hypothesis” [Phys. Plasmas 14, 040701 (2007)]

A. Thyagaraja^{a)} and C. M. Roach

EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon OX14 3DB, United Kingdom

R. D. Hazeltine

Institute for Fusion Studies, University of Texas at Austin, Austin, Texas 78712, USA

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The paleoclassical hypothesis, derived in Callen [Phys. Plasmas **14**, 040701 (2007)], proposes that electron guiding centers experience additional diffusion which is absent from neoclassical theory. This is claimed to be associated with the diffusion of poloidal magnetic flux, and to be most significant in cold resistive plasmas. In this comment we explain why the paleoclassical hypothesis contradicts electrodynamics. [DOI: [10.1063/1.2828096](https://doi.org/10.1063/1.2828096)]

In recent works Callen^{1–3} has proposed a novel hypothesis relating the transport of charged particles to the resistive diffusion of magnetic fields in tokamaks. This “paleoclassical hypothesis” (PCH) posits that charged particles, irrespective of their usual collisional radial transport due to finite orbit widths and Coulomb collisions, are transported with the magnetic flux at the rate associated with resistive diffusion of the magnetic field. It follows that the particle diffusivity in a tokamak cannot be lower than $D_\eta \approx \eta / \mu_0 \approx (c / \omega_{pe})^2 \nu_e$. Here η is the collisional plasma resistivity, c is the speed of light, ω_{pe} is the electron plasma frequency, μ_0 is the permeability of vacuum, and ν_e is the Braginskii electron collision frequency. In this comment we show that PCH contradicts the well-established principles of electrodynamics.

The PCH prediction of additional transport is most surprising when $\mathbf{B}(\mathbf{x})$ is independent of t . In a tokamak $\mathbf{B} = \nabla \Psi(R, Z, t) \times \nabla \phi + R \mathbf{B}_\phi(R, Z, t) \nabla \phi$, where (R, ϕ, Z) are conventional cylindrical coordinates. Stationary \mathbf{B} corresponds to static surfaces of constant poloidal flux Ψ . A steady magnetic field can be maintained by driving the toroidal current j_ϕ with a current source $S_\phi(R, Z)$ (e.g., lower hybrid current drive) to balance the Ohmic dissipation. Ohm’s law is given by

$$\frac{1}{R} \frac{\partial \Psi}{\partial t} = -\eta(j_\phi - S_\phi) \\ = \eta \left\{ \frac{1}{\mu_0} \left[\frac{1}{R} \frac{\partial^2 \Psi}{\partial Z^2} + \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \Psi}{\partial R} \right) \right] + S_\phi \right\}, \quad (1)$$

where the plasma resistivity η is an isotropic flux function, and Ψ has a steady state solution when $j_\phi = S_\phi$.

The Lorentz force equation,

$$m \frac{d\mathbf{v}}{dt} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (2)$$

governs particle orbits in plasmas, and describes the motions of electron guiding centers. In a steady state tokamak (and ignoring turbulent perturbations and short range collisional

fields) the equilibrium vector fields \mathbf{E} and \mathbf{B} are fixed, and at constant \mathbf{B} , only the equilibrium \mathbf{E} which drives the plasma current, depends on resistivity. \mathbf{E} has a small impact on particle orbits, and is responsible for the Ware pinch of trapped particles. Particle orbits and the drift kinetic equation follow directly from the Lorentz force equation, and these together with the physics of collisions are the pillars of neoclassical theory. PCH assumes that Eq. (1) describes the Brownian motion of poloidal flux, even at steady state. Even if this flux Brownian motion were physical, it can have no influence on the particle orbits; these orbits depend only on the local magnetic field, which is constant by assumption.

A key point here is that in classical physics, Ψ (a component of the steady vector potential) has no relevance to individual charged particle motions; it is merely a convenient way of representing \mathbf{B} . Only the latter appears in the equations governing the particle’s motion. A particle undergoing infrequent collisions would not be influenced by Brownian motion of poloidal flux; it must remain on its Lorentz orbit. PCH can only generate additional diffusion of electron guiding centers by adding new terms to the Lorentz force equation. Such terms have not been unveiled in Ref. 1. Additional forces that might legitimately appear in the Lorentz equation are those due to the short range electric fields due to inter-particle collisions and those due to perturbations of the equilibrium magnetic field. Particle “collisions”—manifested by the rapidly fluctuating Coulomb fields between particles—are a prominent example of such additional forces; we discuss them explicitly below. Perturbed electromagnetic fields are not included in the PCH.

The diffusion equation for poloidal flux in a tokamak [Eq. (1)] describes the dissipation of plasma current and the poloidal component of the magnetic field strength due to resistivity. In steady state, when $\mathbf{B}(\mathbf{x})$ and constant Ψ surfaces are stationary, this dissipation must be balanced by sources. To define a particular magnetic field line, we must prescribe $\mathbf{B}(\mathbf{x})$ and a single point on the field line. If $\mathbf{B}(\mathbf{x})$ is independent of t , then the magnetic field line passing through the location of a particle at a particular time t is stationary and does not undergo Brownian motion. When the magnetic field lines passing through fixed points in space are

^{a)}Electronic mail: a.thyagaraja@ukaea.org.uk.

stationary—that is when $\mathbf{B}(\mathbf{x})$ is stationary—there is no physical mechanism to explain the PCH diffusion of poloidal flux.

We note that Lorentz force equations are the exact characteristics of the hyperbolic Vlasov equation,

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{e}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f = 0. \quad (3)$$

It follows that any “drift kinetic equation” (DKE) derived from averaging over the Larmor gyrations, using the adiabatic invariance of the magnetic moment and the exact invariance of energy and p_{ϕ} , cannot contradict the predictions of the Lorentz–Newton and the equivalent Vlasov equations.

The above argument can now be extended to cover the case when the “test charges” collide with some “background.” For example, this needs to be nothing more than elastic scattering between the test charges and the background ions with an effective collision frequency, ν_{eff} . This circumstance can be described by the Einstein–Langevin Brownian motion theory wherein the Lorentz–Newton equations are changed to the Einstein–Langevin–Lorentz equations (cf. Ref. 4),

$$m \frac{d\mathbf{v}}{dt} + m\nu_{\text{eff}}\mathbf{v} = e\mathbf{F}_{\text{Langevin}}(t) + e(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (4)$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}. \quad (5)$$

Here we have the “Einstein” drag force on the left and the white noise random “Langevin” field on the right, in addition to the steady magnetic field. In principle, this Langevin field $\mathbf{F}_{\text{Langevin}}(t)$ represents short-range random collisions. As before, we assume that the plasma is maintained in a steady state by suitable sources. It is well-known that this system is equivalent to a Fokker–Planck equation, rather than the Vlasov equation. The full kinetic equation can be reduced⁵ using the exact procedures of neoclassical theory to a DKE with a corresponding linear collision operator.

It is straightforward to demonstrate that the test particle distribution diffuses in velocity space, and that, when the collision frequency is smaller than the gyrofrequency or bounce frequency, the velocity-space diffusion yields a real space diffusivity which is proportional to the squared orbit width (Larmor radius or banana width). This result differs strongly from the “field diffusivity” η/μ_0 which appears in Eq. (1). According to both classical and neoclassical kinetics (in their respective domains of applicability) the particle diffusivity must scale like $1/B^2$ (“gyro-Bohm”) while the field diffusivity is independent of B . Clearly the B -independent transport rates predicted by PCH violate this fundamental ordering, and would predict finite particle transport even

with very large fields! Of course turbulent perturbations to electric or magnetic fields can generate anomalous transport that exceeds neoclassical predictions without contradicting basic physical principles; however, no turbulent fields appear in PCH.

Our principal conclusion is that the paleoclassical hypothesis contradicts both the Lorentz force equation and direct asymptotic solutions of the generally accepted kinetic equations for the charged particle distribution functions in the classical and neoclassical regimes.⁶ As we have shown, this contradiction arises from two distinct invalid notions: (1) that the diffusion equation [Eq. (1)] for the poloidal magnetic flux Ψ implies Brownian motion of the magnetic field even when $\mathbf{B}(\mathbf{x})$ is independent of t , and (2) that charged particles must also experience these Brownian motions. If $\mathbf{B}(\mathbf{x})$ is independent of t , the field line through any given point in space is stationary and cannot undergo Brownian motion. Furthermore, the forces required to generate paleoclassical motions of particles are missing from the Lorentz force equation. Charged particles can indeed diffuse due to collisions in accordance with neoclassical predictions at rates fixed by their drift orbits and collision frequencies.

While electron heat transport has been found in some experiments to lie close to the paleoclassical level,³ this in no way validates the model’s physical basis. Indeed any micro-turbulence model implying a step-length of $\delta_e = c/\omega_{pe}$ (there are many reasons to think this is plausible for electrons) and a turbulent decorrelation rate of the order ν_e , would lead to similar numerical values for the “effective turbulent diffusivity,” $\nu_e \delta_e^2 \approx \eta/\mu_0$. As a final aside we point out that a challenging observation for PCH is posed by runaway electron populations which clearly do not experience paleoclassical transport and have coexisted with cold resistive plasmas in the JET for up to $\sim O(100)$ resistive diffusion times.⁷

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