Beat-Wave Current Drive in a Magnetized Plasma

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Abstract

It is shown that large currents can be efficiently driven in magnetically confined plasmas by the ponderomotive force created by the beating of two electromagnetic waves. The beating waves can be cyclotron waves propagating parallel to the magnetic field or light waves propagating obliquely to the magnetic field.

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The ultimate feasibility of tokamak reactors may critically depend on the possibility of maintaining the toroidal current in the plasma in steady-state or quasi steady-state operation. Present devices rely entirely on inductive current generation which is not appropriate for steady-state operation. Various schemes have been proposed for non-inductive current drive using injection of neutral particles[1], rotating magnetic fields[2], electromagnetic waves[3], etc. Of particular relevance is the current drive by radio-frequency waves because the power sources are readily available, the basic theoretical aspects are reasonably understood, and the scheme is attractive from an engineering point of view.[4]

Two linear mechanisms have been discussed for current generation by radio-frequency waves.[3,5] In the first mechanism[3], electromagnetic waves are linearly converted to an electrostatic mode with phase velocity parallel to the magnetic field much larger than the electron thermal velocity. Resonant electrons are accelerated and a steady state in velocity space is reached by a balance between the quasilinear diffusion caused by the electrostatic mode and collisional drag.[3,6] This is the basic mechanism proposed for current generation by lower hybrid waves.[3] The second mechanism[5] is based upon perpendicular electron heating by cyclotron waves propagating parallel to the magnetic field. Because of the Doppler effect, the wave frequency can be chosen such that only electrons travelling in one toroidal direction are in resonance with the wave and absorb energy. This destroys the spatial symmetry of the momentum exchange between electrons and ions and a parallel current is generated.

Both lower hybrid and electron cyclotron driven currents have been experimentally observed.[7,8] However, the experiments indicate that the efficiency of current generation decreases sharply with the plasma density and so far no substantial wave-driven currents have been obtained for densities $n \geqslant 10^{14}\mathrm{cm}^{-3}$, characteristic of fusion reactors. The reason for this decrease in efficiency has been hotly debated. One simple possibility is that at high densities the balance between quasilinear wave diffusion and collisional drag cannot be achieved.[9] This can be an obstacle for the practical implementation of wave current drive, in particular for high field and high density compact devices.

In this letter we propose a nonlinear mechanism for current drive by electromagnetic waves. This work has been motivated by an apparent discrepancy between the large values of the driven current observed in lower hybrid experiments and the much lower values predicted by the theory. Cannobio has suggested that this effect could be due to the ponderomotive force.[9] Here we show that the ponderomotive force can be directly used to drive currents. Two strong electromagnetic waves are made to beat at the characteristic frequency of an electrostatic plasma mode. Particles trapped in the potential well of the ponderomotive force are accelerated and cause an elongation of the distribution function generating current.[10] The mechanism is similar to the one of the beat-wave accelerator[11]; but there are differences. In the laser beat-wave accelerator, the electrostatic (plasmon) mode is produced by forward scattering and its velocity is meant to be the speed of light. For successful current drive, the phase velocity of the driven electrostatic mode has to be a few times the thermal velocity of the electrons. This can be achieved by backward scattering or even by forward scattering of fast electromagnetic waves under proper conditions.[10] In particular, we consider two possibilities for nonlinear current drive: i) two cyclotron waves propagating parallel to the magnetic field $\vec{B}$ and beating to produce an electrostatic magnetized plasma mode and ii) two laser beams propagating obliquely to $\vec{B}$ and backscattering at the upper hybrid resonance layer. These two processes can be efficient in high magnetic field devices such that $\Omega > \omega_p$, where Ω and ω_p are respectively the gyrofrequency and the plasma frequency of the electrons. In the following we consider explicitly the current generation by the beating of cyclotron waves and also indicate the results for current generation by the laser excitation of the upper hybrid mode. Detailed calculations will be presented in a future publication.

We use a simple fluid model to discuss the beating of two cyclotron waves. Consider two strong waves, (ω_1, k_1) and (ω_2, k_2) propagating in the direction parallel to $\vec{B}$. We assume that the amplitude of the waves are approximately constant and that the matching conditions $\omega_1 - \omega_2 = \omega$ and $k_1 \pm k_2 = k$ are fulfilled over a certain distance somewhere in the plasma. Here ω and k are respectively the frequency and wavenumber of an electrostatic plasma mode. The low-frequency oscillations are described by the equations

$$\frac{\partial n}{\partial t} + \nabla \cdot (n_0 \vec{v}) = -\nabla \cdot [n^{(1)} \vec{v}^{(2)} + n^{(2)} \vec{v}^{(1)}] , \qquad (1)$$

$$\frac{\partial \vec{v}}{\partial t} - \frac{e}{m} \nabla\varphi + \Omega\left(\vec{v} \times \frac{\vec{B}}{B}\right) + \gamma \frac{v_t^2}{n_0} \nabla n$$

$$= -\vec{v}^{(1)} \cdot \nabla\vec{v}^{(2)} - \vec{v}^{(2)} \cdot \nabla\vec{v}^{(1)} - \frac{e}{m}\left(\vec{v}^{(1)} \times \vec{B}^{(2)} + \vec{v}^{(2)} \times \vec{B}^{(1)}\right) \quad (2)$$

and

$$\nabla^2\varphi = \frac{n_e}{\varepsilon_0} , \quad (3)$$

where $n, \vec{v}$, and φ are respectively the perturbed density, velocity, and electrostatic potential of the low-frequency mode, n_0 is the equilibrium density, v_t is the thermal speed of the electrons, and $n^{(i)}$, $\vec{v}^{(i)}$, and $\vec{B}^{(i)}$, i=1,2, are respectively the perturbed density, velocity, and magnetic field of the high-frequency cyclotron modes. Using the appropriate expressions[12] for $n^{(i)}$, $\vec{v}^{(i)}$, and $\vec{B}^{(i)}$, i=1,2, it is easy to show that $n^{(i)}\vec{v}^{(j)} = \vec{v}^{(i)} \cdot \nabla\vec{v}^{(j)} = 0$, $i \neq j$. Thus, only the $\vec{v} \times \vec{B}$ term in Eq. (2) provides a nonlinear force. Calculating this term, substituting in Eq. (2), and carrying out the phase average over the high-frequency oscillations, Eqs. (1)-(3) can be reduced to a single equation for the low-frequency density perturbation

$$\frac{\partial^2 n}{\partial t^2} - \gamma v_t^2 \frac{\partial^2 n}{\partial z^2} + \omega_p^2 n = -n_0\lambda\sin(kz-\omega t) , \quad (4)$$

where

$$\lambda = v_{osc}^{(1)} v_{osc}^{(2)} \left(\frac{k_1}{1-\frac{\Omega}{\omega_2}} \pm \frac{k_2}{1-\frac{\Omega}{\omega_1}}\right) k \ , \tag{5}$$

$v_{osc}^{(i)} = \frac{eE^{(i)}}{m\omega_i}$, i=1,2, and $E^{(i)}$ is the amplitude of the corresponding high-frequency wave. Equation (4) is the equation for a harmonic oscillator driven at its resonance frequency. The solution is given by

$$n(t) = n_0 \Lambda t \ \sin(kz-\omega t-\frac{\pi}{2}) \ ; \tag{6}$$

where $\Lambda = \lambda/2\omega$ and $\omega^2 = \omega_p^2 + \gamma k^2 v_t^2$. Equation (6) is a generalization of the result obtained by Rosenbluth and Liu for the beating of light waves.[13] As the electrostatic energy increases, particles in the bulk of the distribution function begin to be trapped and the growth is curbed by strong nonlinear Landau damping and nonlinear frequency shift not included in our simple fluid model.[10,13] The saturation time can be estimated by requiring that the oscillation velocity in the electrostatic potential, $|2e\varphi/m|^{1/2}$, be of the order of the thermal velocity. Using Eqs. (3), (5), and (6), we find $\tau_s \approx k^2 v_t^2 \omega/\lambda\omega_p^2$. For $v_{osc}^{(1)} = v_{osc}^{(2)} = v_{osc}$, this reduces to $\tau_s \approx (v_t/v_{osc})^2 \omega_p^{-1}$.

To check the results of our fluid model, we have carried out computer simulations employing a $1\,\frac{2}{2}$ – D (one space and three velocity and field components) particle code. Two right-hand circularly polarized waves with wave numbers k_1 and k_2 are imposed on an initially uniform thermal plasma. The waves propagate in the x-direction. The plasma parameters are chosen such that $\Omega/\omega_p \simeq 1.6$ and $v_t/c \simeq 0.1$. The frequencies of the two electromagnetic waves are given by $\omega_1/\omega_p = 3.5$

and $\omega_2/\omega_p = 2.4$, such that they are in the upper fast branch of the dispersion curve for cyclotron waves. The system has typically a length $L_x = 512\Delta$, the wave numbers of the electromagnetic waves are given by $k_1 = 2\pi \times 29/512\Delta$ and $k_2 = 2\pi \times 15/512\Delta$. The number of particles in each species is 5120, and the particle size is 1Δ with a Gaussian shape, where Δ is the unit spatial grid distance and is equal to the Debye length. The intensity of the electromagnetic waves is chosen such that $v_{osc}/c = 0.01$.

The electron distribution function at $t = 0$ and after a time $t = 22\omega_p^{-1}$ are shown in Figs. 1(a) and 1(b), respectively. In this case, the matching conditions are such that the phase velocity of the electrostatic wave is about $3v_t$. Accordingly, the electron distribution function develops a positive tail leading to current generation. The electrostatic energy density and the driven current density are shown in Figs. 1(c) and 1(d), respectively. The current density is normalized to nev_t. Both the electrostatic energy density and current density tend to nonlinearly saturate after the initial growth. At a time $\tau_s \approx 24\omega_p^{-1}$, the current density reaches a peak value $j/nev_t \approx 0.2$ and oscillates afterwards. A similar value for j is obtained for $v_{osc}/c = 0.1$ at a much shorter time. The value of the saturation time in the numerical simulations is somewhat smaller than the one predicted by our rough model, i.e., $\tau_s \approx (v_t/v_{osc})^2\omega_p^{-1} = 100\omega_p^{-1}$. The nonlinear frequency shift may also play a role in the saturation mechanism.[10]

After the saturation value of the current density is reached, the source can be turned off and the current be left to collisionally relax until the next pulse. For a plasma with $T_e \approx 5keV(v_t/c{\approx}0.1)$ and

$n = 10^{14}cm^{-3}$ and for $v_{osc}/c = 0.01$, the collision and saturation times are about 50μsec and $5 \times 10^{-5}\mu$sec, respectively. Thus, steady-state operation can be obtained by a sequence of very short pulses. The width of each pulse cannot always be made of the order of τ_s because the minimum width of the pulse in gyratrons is limited by the transit time of the electrons in the resonant cavity. For $v_{osc}/c = 0.01$, this time is longer than the saturation time τ_s. For smaller values of v_{osc}/c, i.e., for smaller intensities of the beating waves, the two times can be made of the same order.

The ratio between the current density and the power density dissipated to drive the current, j/P, is considered to be an important parameter to measure the local efficiency of different schemes for current drive. However, in the beat-wave scheme, the crucial parameter is the intensity of the electromagnetic waves required to induce the nonlinear process.[10,11] The maximum intensities that are achievable with lasers range from approximately $10^{14}W/cm^2$ for CO_2 to 10^{16} for Nd-Yag lasers. For microwaves, the maximum intensities are limited to approximately $10^8W/cm^2$ by breakdown problems in the waveguides. In our simulation, we have obtained a saturated current of $j \approx 0.2nev_t$ for $v_{osc}/c = 0.01$ and $v_t/c = 0.1$. Taking $n = 10^{14}cm^{-3}$, we obtain $j \approx 10kA/cm^2$. This is much larger than the average value of the current density in actual devices, i.e., $j < 1kA/cm^2$. The required intensity to obtain such a current is $I \approx 1.4 \times 10^8(v_{osc}/c)^2/\lambda_0^2$, where I is given in W/cm^2 and λ_0 is the wavelength of the electromagnetic waves $(\lambda_0^{(1)} \approx \lambda_0^{(2)} \approx \lambda_0)$ in microns. Considering a typical gyratron, f = 60GHz; $\lambda_0 = 0.5cm$, the value of I is approximately $5 \times 10^6W/cm^2$. This value is lower than the maximum allowed for microwave circuits and it is achievable with focusing structures at the mouth of the waveguides. In actual applications, the current pulses can be decreased almost an order of magnitude. This can be done by decreasing the intensity of the electromagnetic waves which also leads to an increase in the saturation time τ_s.

To estimate the average j/P parameter, let us represent the current density by $j = \Delta n e(\omega/k)$, where Δn is the average density increase related to the trapped particles. The power lost by collisional dissipation is given by $P = \Delta n[m(\omega/k)^2/2]\nu$, where ν is the electron–ion collision frequency. Thus,

$$\frac{j/(n_0 e v_t)}{P/(n_0 m v_t^2 \nu)} \approx 2\,\frac{v_t}{\omega/k} \; . \tag{7}$$

This is a number of order unity, comparable to the value of the j/P parameter for the scheme of current drive by lower hybrid waves.[3,4]

Finally, let us briefly discuss the possibility of driving currents by the nonlinear excitation of the upper hybrid electrostatic mode. The upper hybrid waves near the center of a tokamak plasma cannot be easily excited by linear conversion of electromagnetic waves propagating inwards because of the evanescent domain between the cut–off and upper hybrid resonance layers. This problem can be alleviated by using the beat of two high–frequency light waves. Consider two laser beams (ω_1, k_1) and (ω_2, k_2) propagating at an angle α with the magnetic field, such that $\cos\alpha \ll 1$. Because the phase velocity of the light waves is approximately c, the electrostatic mode generated by the beating of the laser beams has also a phase velocity close to c for forward scattering. However, using backscattering, it is possible to obtain a Doppler-shifted phase velocity $(\omega-\Omega)/k\cos\alpha$ close to v_t, in the direction of $\vec{B}_0$. By properly choosing the matching conditions $\omega_1-\omega_2 = \omega_{uh}$ and $k_1+k_2 = k$ and the angle α, it is possible to excite the upper hybrid electrostatic mode with a parallel Doppler-shifted phase velocity of a few times the thermal velocity and drive current. Using a fluid model similar to the one described before, we find that the electrostatic fluctuations increase linearly with time according to Eq. (6), with Λ replaced by

$$\Lambda_{uh} = \frac{1}{4} v_{osc}^{(1)} v_{osc}^{(2)} \frac{(\omega^2-\Omega^2\cos^2\alpha)}{\omega^2} \left[\frac{k_1\omega_2^2(\omega_1^2-\Omega^2) + k_2\omega_1^2(\omega_2^2-\Omega^2)}{(\omega_1^2-\Omega^2)(\omega_2^2-\Omega^2)}\right] \frac{k}{\omega} , \tag{8}$$

where ω and k are related by the dispersion relation for (nearly) electrostatic upper hybrid waves.[13]

In conclusion, we have shown that a large amount of current can be nonlinearly driven in magnetized plasmas by the beating of two strong electromagnetic waves. The required intensities are readily available, the efficiency is close to the one predicted by other methods[3,4,5], and the scheme is appropriate for localized current control. Since the two cyclotron modes have frequencies above the upper cutoff frequency, there are no accessibility problems and the process should also be efficient at high plasma densities, provided that $\Omega > \omega_p$. The practical implementation of this scheme would require the use of pulsed gyratrons at a high repetition rate, as opposed to the long pulse gyratrons being currently developed for steady-state applications. The ponderomotive force exerted by the beatwaves is also effective for plasma stabilization against the ballooning instabilities and the drift wave instabilities, etc. The details will be discussed in the future.

After this work was finished, we learned about a similar scheme independently and simultaneously proposed by Cohen.[15] The present paper was read at the International Conference for Plasma Physics in 1984.

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Figure Caption

Fig. 1 -

Electron distribution function at $t = 0$ (a) and at $t = 22\omega_p^{-1}$ (b), electrostatic field energy (c), and electronic current density (d). The scale of the horizontal axis in (a) and (b) is in units of the thermal speed of v_t of the electron. The scale of the horizontal axis in (c) and (d) is in units of ω_p^{-1}. The scale of the vertical axis in (d) is in units of $n_0 e v_t$. The broken line in (c) corresponds to the increase in electrostatic energy predicted by the fluids theory.

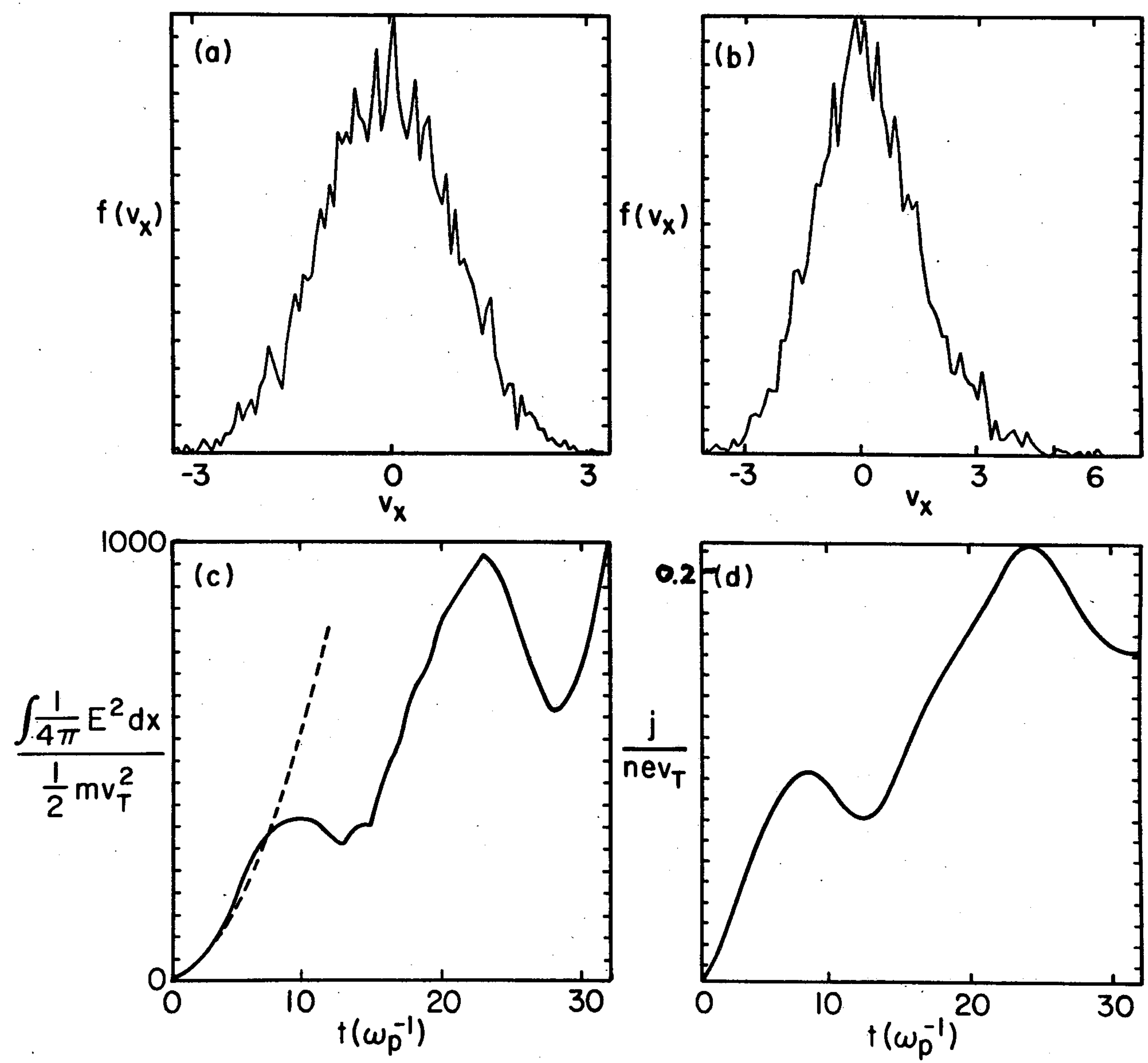

FIG. 1