Confinement in Magnetic Fusion

1. Stars "burn" with self-sustaining fusion reaction

2. Star is a plasma (ionized gas) that is confined due to gravity.

3. The hydrogen bomb works, to produce fusion energy, because adequate confinement is achieved by compressing D-T to high densities.
However energy uncontrolled and leads to lethal explosion

4. Inertial fusion has the potential to achieve controlled fusion by miniaturizing H-bomb reaction to produce energy densities that nearby solid walls can handle

5. Magnetic confinement is attempting to achieve controlled fusion by confining plasma with a magnetic field, rather than by gravity

Lawson criterion for confinement needed to produce fusion

(a) nucleons have the potential to fuse to produce heavier nucleons and a release of energy

But, "spark" needed to "ignite" system

Among the reactions of most interest are:

$$d + t \rightarrow {}^4He + n + 17.6 \text{ MeV}$$

$$d + He^3 \rightarrow {}^4He + p + 18.3 \text{ MeV}$$

$$d + d \rightarrow \begin{cases} 3He + n + 3.3 \text{ MeV} \\ t + p + 4.0 \text{ MeV} \end{cases}$$

(a) The d-d reaction is the one with the potential to deliver energy for $\sim 10^9 y$, but is more difficult to implement than d-t

(b) The d-He^3 reaction has nice feature of neutron free reaction, but also more difficult d-t

(c) d-t fuel of choice (on Earth) even with t decaying radio-actively (12 yr half-life) why is this?

Let us consider a nuclear reaction occuring in same volume, V. The d-t reaction produces a 14. MeV neutron that leaves the system (very small E&M reaction to prevent loss) but the 3.5 MeV alpha particle is absorbed by the system

Let $n_t = \frac{n_e}{2}$

$n_d = \frac{n_e}{2}$

$$PP = \frac{\text{Power Produced}}{\text{Volume}} = n_t \, n_d \, \overline{\sigma_{fus} v} \, W_\alpha$$

$$= \frac{n^2}{4} \overline{\sigma_{fus} v} \, W_\alpha$$

$$W_\alpha = 3.5 \text{ MeV} \times \left(1.6 \times 10^{-12} \frac{\text{erg}}{\text{eV}} \times 10^{6} \frac{\text{eV}}{\text{MeV}} \right)$$

$$\approx 5 \times 10^{-6} \text{ erg}$$

To be self-sustaining, rate of power density production is matched by power density loss (PL)

$$PL = \frac{\frac{3}{2}(n_e T_e + n_i T_i)}{\tau_L} = \frac{3nT}{\tau_L} \; : \; \frac{1}{\tau_L} \equiv \text{energy loss rate}$$

For self-sustained reaction Power Loss = Power Produced

$$PL = PP$$

$$\frac{3nT}{\tau_L} = \frac{n^2}{4} \overline{\sigma v} \, W_\alpha$$

Therefore:

$$n\tau_L = \frac{12 \, T}{\overline{\sigma v} \, W_\alpha}$$ Lawson Criteria

Note the plummeting of cross-section at lower energies ($\lesssim 5$ keV). Fusion is due to tunnelling of nucleons through Coulomb barrier, but reaction rate exponentially slow

2. disparity of reaction rates, and temperature peaking values for different fusion reactions,

3. Also recall energy output for each reaction varies leading to (together with point 2) different $n\tau$ curves.

Source of fuel

Deuterium .016 % of sea-water

a. Tritium with half-life of 12 years

Must be bred.

Indeed in H-bombs Lithium produces in-situ tritium due to

$$n + {}^{6}Li \rightarrow t + {}^{4}He + 4.8 \text{ MeV}$$

$$n + {}^{7}Li \rightarrow t + {}^{4}He + n - 2.5 \text{ MeV}$$

natural abundance

${}^{6}Li$ - 7.5 %

${}^{7}Li$ - 92.5 %

Lithium blankets forseen in d-t burn reactors

b. D - He^3

He^3 natural element, but extremely rare ("evaporates" from Earth's gravitational field. Found in rocks)

Embedded in moon, Confined in Jupiter's atmosphere (can it be mined?)

c. d-d - $\sim 10^9$ y worth of nuclear energy stored in sea-water

Is confinement goals achievable?

$n\tau_L \gtrsim 10^{14}\ \mathrm{cm}^{-3}\,\mathrm{s}$ (d-t)

Radiation losses

Binary collisions of electrons among themselves, and ions, produce bremmstrahlung radiation, that escapes from the plasma

$$n\tau_{brem} = 2.8\times10^{15}\ T_{10k}^{1/2} / Z_{eff}$$

$$\gtrsim 20 \times n\tau_{DT} \text{ (at 10-20 keV)}$$

$$\gtrsim 10 \times n\tau_{DHe} \text{ (at 70 - 150 keV)}$$

$$\approx 2 \times n\tau_{DD} \text{ (at} \sim \text{100 - 200 keV)}$$

(an important loss mechanism in d-d reaction)

(can be useful in spreading out power deposition)

Synchrotron Radiation

electrons radiate due to their circular motion in a magnetic field

$$n\tau_{sync} = 1.9 \times 10^{15} \beta / T_{10k} \quad cm^{-3} s$$

This appears important even for D-T in $\beta = 10\%$ operation

$$\beta = \frac{\text{particle pressure}}{\text{magnetic Pressure}} \times 100 \text{ (percent)}$$

However, radiation has relatively short mean free path and will be reabsorbed by plasma

With reabsorption, no constraint for d-t reactors.

Requires d-d, and d-He^3 reactors to operate at moderately high beta values

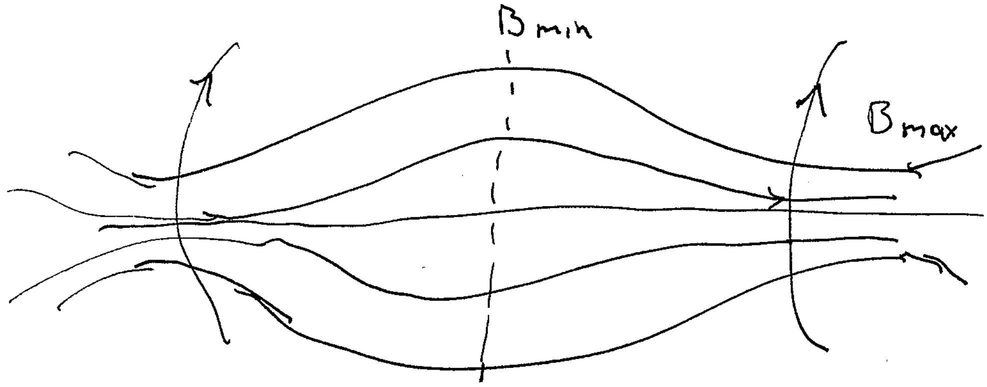
($\beta > 15\%$)

In magnetic devices two types of magnetic traps.

1. Open (mirror machines)

2. closed (toroidal machines)

The viability of open magnetic mirror machines very sensitive to collisions:

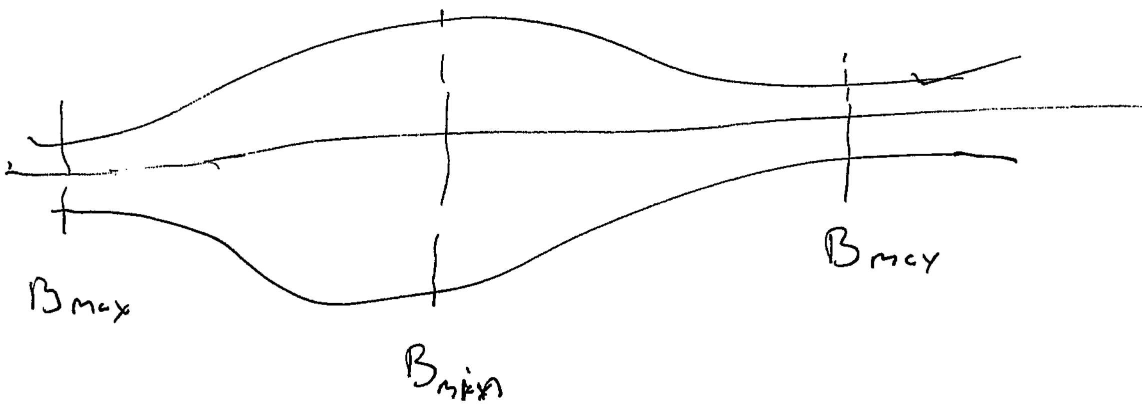


Mirror Confinement Principle based on magnetic moment conservation

$\mu = \frac{m v_\perp^2}{2B}$ constant as particle moves through mirror field

(Basic adiabatic invariant)

Confinement Condition



$$E = \frac{m v_{\parallel 0}^2}{2} + \frac{m v_{\perp 0}^2}{2}\Bigg|_{B = B_{min}} < \frac{m v_{\perp mx}^2}{2}\Bigg|_{B_{max}}$$

$$\frac{v_{\perp 0}^2}{B_{min}} = \frac{v_{\perp mx}^2}{B_{mx}}$$

$$v_{\perp mx}^2 = \frac{B_{mx}}{B_{min}} v_{\perp 0}^2 \equiv R\, v_{\perp 0}^2 \quad \left(R \equiv \frac{B_{mx}}{B_{min}}\right)$$

$\therefore$ Confinement condition

$$\frac{m v_{\parallel 0}^2}{2} + \frac{m v_{\perp 0}^2}{2} < m \, R \frac{v_{\perp 0}^2}{2}$$

$$v_{\parallel 0}^2 < (R-1) v_{\perp 0}^2$$

or $$v_0^2 = v_{\parallel 0}^2 + v_{\perp 0}^2 < R\, v_{\perp 0}^2$$

How long, τ_L, can this condition be fulfilled?

Time particle can stay confined in open system is roughly a collision time for 90° deflection (if $R \simeq 2$)

$$\nu_{90°} \simeq \overline{v\sigma} n$$

$$\sigma \simeq r_{90°}^2 \ln\Lambda \text{ with } \frac{e^2}{r_{90}} \simeq T$$

$$\therefore r_{90}^2 = \frac{e^4}{T^2} \text{ ; recall } \ln\Lambda \simeq 10-20$$

$$\nu_{90°} = 10n \frac{e^4}{T^2}\left(\frac{T}{M}\right)^{1/2} \simeq \frac{1}{\tau_L} \left\{ v \simeq \left(\frac{T}{M}\right)^{1/2} \right\}$$

$$\therefore n\tau_L = \frac{T^2}{e^4}\left(\frac{M}{T}\right)^{1/2} \frac{1}{10} = \frac{M^{1/2} T^{3/2}}{10 e^4}$$

$$T = 1.6 \times 10^{-12} \frac{\text{erg}}{\text{ev}} \times 10^4 \frac{\text{ev}}{10\text{kev}} \,\mathcal{T}(10\text{keV})$$

$$M_{DT} \simeq 4 \times 10^{-24} \text{ gm}$$

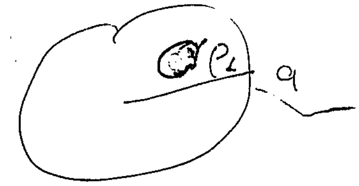
$$n\tau_L = \frac{(1.6 \times 10^{-8})^{3/2} (4 \times 10^{-24})^{1/2}\, \mathcal{T}^{3/2}(10\text{ kev})}{10(4.8 \times 10^{-10} \text{ esu})^4}$$

$$\simeq \frac{2 \times 10^{-12} \times 2 \times 10^{-12}}{10 \cdot \cancel{4}00 \times 10^{-40}} = 10^{13}\, \mathcal{T}^{3/2}(10\text{kev})$$

e.g. at 50 keV $\quad n\tau \simeq 10^{14} \text{ cm}^{-3}\text{ s}$

Close: Possible use as power amplifier

If we can plug up ends (making confinement device toroidal) particles will then only diffuse across field lines in a random manner



$$\tau_L \approx \frac{1}{2\nu_{90^\circ}} \frac{a^2}{\rho_L^2}$$

a = radius of device

Larmor radius squared $\} = \rho_L^2 = \dfrac{MTc^2}{e^2 B^2}$ $\qquad a = 100\,\text{cm}$, $\quad B = 10^4\, B$ gauss

$$n\tau_L = 10^{13} \frac{T^{3/2}}{T} \frac{a^2 10^4 (4.8 \times 10^{-10})^2 10^8 B^2}{4\times10^{-24} \quad 9\times10^{20} \; (1.6\times10^{-12+4})}$$

$$= 10^{13} a^2 B^2 T^{-1/2} \quad \frac{20}{50} \quad \frac{10^{-8}}{10^{-12}}$$

$$= 4\times10^{16} a^2 B^2 T^{1/2}$$

Really looks good:

Diffusion Losses negligible

But is simplified classical picture correct?

~~Are~~ Are there other mechanisms controlling transport?

Particle Orbits in a $\underset{\sim}{E}$ & $\underset{\sim}{B}$ - field

$E << B$ (cgs units)

$$\frac{d\underset{\sim}{v}}{dt} = \frac{e}{m}\left(\underset{\sim}{E}(\underset{\sim}{r},t) + \frac{\underset{\sim}{v}}{c}\times\underset{\sim}{B}(\underset{\sim}{r},t)\right)$$

$$\frac{d\underset{\sim}{r}}{dt} = \underset{\sim}{v}$$

Suppose $\underset{\sim}{E}$ and $\underset{\sim}{B}$ are uniform in space, $\underset{\sim}{E}\cdot\underset{\sim}{b} = 0$ and time independent

$$\frac{d\underset{\sim}{v}}{dt} = \frac{e}{m}\underset{\sim}{E} + \frac{\underset{\sim}{v}}{c}\times\underset{\sim}{B}$$

We can go to frame where $\underset{\sim}{E}_\perp = 0$

$$\underset{\sim}{v} = \underset{\sim}{u} + c\underset{\sim}{E}\times\underset{\sim}{b}/B = \underset{\sim}{u} + \underset{\sim}{v}_E; \quad \underset{\sim}{v}_E = \frac{c\underset{\sim}{E}\times b}{B}$$

$$\frac{d\underset{\sim}{v}}{dt} = \frac{d\underset{\sim}{u}}{dt} = \frac{e}{m}\left(\underset{\sim}{E} + c\frac{(\underset{\sim}{E}\times\underset{\sim}{b})}{B}\times\underset{\sim}{B}/c + \underset{\sim}{u}\times\underset{\sim}{B}/c\right)$$

$$(E\times b)\times B = b\,\underbrace{E\cdot B}_{0} - E_\perp |B|$$

$\therefore$

$$\frac{du}{dt} = \frac{e}{m}\left[E - \frac{E}{B}E_\perp + \frac{u}{c}\times B\right]$$

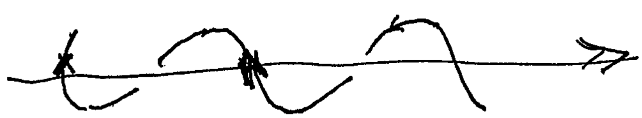
$$= \frac{e}{m}\,\frac{u\times B}{c}$$

Thus in frame

$v = c\,\dfrac{E\times b}{B} \equiv V_E$ electric field does not appear

$$\frac{du}{dt} = \frac{e}{mc}\,u\times b$$

We know that motion is 'helical' around magnetic field



B, $B \parallel \hat{z}$

$u\cdot b = u_\parallel = \text{const.}$

$\rho_L = \dfrac{u_\perp}{\omega_c}$

$\omega_c = \dfrac{eB}{mc}$

clockwise gyration for positive charge particle, as B-field points to us.

$$u_\perp = u_{\perp 0}\left[\cos(\phi-\omega_c t)\,\hat{x} + \sin(\phi-\omega_c t)\,\hat{y}\right]$$

$$\rho_L(t) = \rho_{L0}\left[-\sin(\phi-\omega_c t)\,\hat{x} + \cos(\phi-\omega_c t)\,\hat{y}\right]$$

$u_\parallel \equiv u\cdot b \equiv \text{constant}$; $\quad z = z_0 + u_\parallel t$