Confinement in Magnetic Fusion

1. Stars "burn" with self-sustaining fusion reaction

2. Star is a plasma (ionized gas) that is confined due to gravity.

3. The hydrogen bomb works to produce fusion energy because adequate confinement is achieved by compressing D-T to high densities. However, energy uncontrolled and leads to lethal explosion.

4. Inertial fusion has the potential to achieve controlled fusion by miniaturizing H-bomb reaction to produce energy densities that nearby solid walls can handle.

5. Magnetic confinement is attempting to achieve controlled fusion by confining plasma with a magnetic field, rather than by gravity.
Lawson criterion for confinement needed to produce fusion

Nucleons have the potential to fuse to produce heavier nucleons and a release of energy. But, "spark" needed to "ignite" system.

Among the reactions of most interest are:

\[ d + t \rightarrow ^{4}\text{He} + n + 17.6 \text{ MeV} \]

\[ d + ^{3}\text{He} \rightarrow ^{4}\text{He} + p + 18.3 \text{ MeV} \]

\[ d + d \rightarrow ^{3}\text{He} + n + 3.3 \text{ MeV} \]

\[ e + p \rightarrow 4.0 \text{ MeV} \]

(a) The d-d reaction is the one with the potential to deliver energy for n/100, but is more difficult to implement than d-t.

(b) The d-3He reaction has nice feature of neutron free reaction, but also more difficult d-t.

(c) d-t fuel of choice (on Earth) even with t decaying radioactively (12 yr half-life). Why is this?
Let us consider a nuclear reaction occurring in some volume, $V$. The d-t reaction produces a 14 MeV neutron that leaves the system (very small fission reaction to prevent loss) but the 3.5 MeV alpha particle is absorbed by the system.

Let $n_t = \frac{n_e}{2}$

$$n_d = \frac{n_e}{2}$$

$$\text{PP} = \frac{\text{Power Produced}}{\text{Volume}} = n_t n_d \frac{\sigma v W_d}{V}$$

$$= \frac{n^2}{4} \frac{\sigma v W_d}{V}$$

$$W_d = 3.5 \text{ MeV} \times (1.6 \times 10^{-12} \text{ erg} \times 10^6 \text{ ev})$$

$$= 5 \times 10^{-6} \text{ erg}$$

To be self-sustaining, rate of density power production is matched by power density loss (PL)

$$PL = \frac{\frac{3}{2} (n_e T_e + n_i T_i)}{\tau_L} = \frac{3n T}{\tau_L}$$

\(\tau_L\) = energy loss rate
For self-sustained reaction

\[ \text{Power Loss} = \text{Power Produced} \]

\[ P_L = P_{\text{prod}} \]

\[ \frac{3nT}{\tau_3} = \frac{n^2}{4} \sigma V W_0 \]

Therefore:

\[ n \tau_3 = \frac{12}{\sigma V W_0} \]  

Lawson Criteria

Note the plummeting of cross-section at lower energies (\( \lesssim 5 \text{keV} \)).

Fusion is due to tunnelling of nuclei through Coulomb barriers but reaction rate exponentially slow.

2. Disparity of reaction rates, and temperature peaking values for different fusion reactions.

3. Also recall energy output for each reaction varies leading to (together with point 2) different \( \nu T \) curves.
Source of fuel

- Deuterium: 0.016% of seawater. Tritium with half-life of 12 years must be bred.
- Indeed in H-bombs lithium produces in-situ tritium due to
  \[ n + ^6\text{Li} \rightarrow t + ^4\text{He} + 4.8 \text{ MeV} \]
  \[ n + ^7\text{Li} \rightarrow t + ^4\text{He} + n - 2.5 \text{ MeV} \]
  - natural abundance
    \[ ^6\text{Li} = 7.5\% \]
    \[ ^7\text{Li} = 92.5\% \]
  - Lithium blankets for seen in D-T burn reactors
- D-T, \( ^3\text{He} \)
  - \( ^3\text{He} \) natural element, but extremely rare ("evaporates" from Earth's gravitational field. Found in rocks)
  - Embedded in moon, confined in Jupiter's atmosphere (can it be mined?)
- D-D: \( 10^9 \) y worth of nuclear energy stored in seawater
Is confinement goals achievable?

\[ n_e L \geq 10^{14} \text{ cm}^{-3} \text{ s (c-d)} \]

Radiation losses

Binary collisions of electrons among themselves, and ions, produce bremsstrahlung radiation, that escapes from the plasma.

\[ N_{\text{brem}} = 2.8 \times 10^{15} \frac{\gamma^2}{T_{10k} / Z_{\text{eff}}} \]

\[ \geq 20 \times N_{\text{DT}} \text{ (at 10-20 keV)} \]

\[ \geq 10^6 \times N_{\text{DHe}} \text{ (at 70-150 keV)} \]

\[ \geq 2 \times N_{\text{DD}} \text{ (at 100-200 keV)} \]

(an important loss

mechanism in d-d reaction)

(can be useful in spreading)

(out power deposition)
Synchrotron Radiation

electrons radiate due to their circular motion in a magnetic field

\[ \eta_{\text{sync}} = 1.9 \times 10^{25} \beta / \text{Tol} \text{ cm}^{-3} \text{s} \]

This appears important even for D-T in \( \beta = 10\% \) operation

\[ \beta = \frac{\text{particle pressure}}{\text{magnetic Pressure}} \times 100 \text{ (percent)} \]

However, radiation has relatively short mean free path and will be reabsorbed by plasma

With reabsorption, no constraint for D-T reactors.

Requires d-d, d-He, and d-He reactors to operate at moderately high beta values

\( (\beta > 15\%) \)
In magnetic devices two types of magnetic traps:

1. Open (mirror machines)
2. Closed (toroidal machines)

The viability of open magnetic mirror machines very sensitive to collisions:

Mirror Confinement Principle based on magnetic momentum conservation

\[ m = \frac{M V^2}{2B} \] constant as particle moves through mirror field

(Basic adiabatic invariant)
Confinement Condition

\[ E = \frac{m V_{10}^2}{2} + \frac{m V_{40}^2}{2} \quad \leq \quad \frac{m V_{1mx}^2}{2} \quad B_{mx} = B_{min} \]

\[ \frac{V_{10}^2}{B_{min}} = \frac{V_{1mx}^2}{B_{max}} \]

\[ V_{1mx}^2 = \frac{B_{max} V_{10}^2}{B_{min}} \quad \text{and} \quad V_{1mx}^2 = B_{max} V_{10} \quad (R = \frac{B_{max}}{B_{min}}) \]

Confinement Condition

\[ m \frac{V_{10}^2}{2} + m \frac{V_{40}^2}{2} < m \frac{R V_{40}^2}{2} \]

\[ V_{10}^2 < (R-1) V_{40}^2 \]

or \[ V_{0}^2 = V_{10}^2 + V_{40}^2 < RV_{40}^2 \]

How long, \( z_1 \), can this condition be fulfilled?
Time particle can stay
confined in open system
is roughly a collision time
for 90° deflection (if \( R \approx 2 \))

\[ v_{90} \approx v_0 \approx \frac{e^4}{m} \]

\[ \alpha = \frac{e^4}{m} \left( \frac{1}{M} \right)^{1/2} \]

\[ v_{90} = 10^7 \left( \frac{e^4}{M} \right)^{1/2} \]

\[ n \approx \frac{10^7}{e^4} \left( \frac{1}{M} \right)^{1/2} \]

\[ N \approx \frac{T^2}{e^4} \left( \frac{M}{T} \right)^{1/2} \]

\[ \frac{N \chi}{T^2} = \frac{10^7}{e^4} \left( \frac{1}{M} \right)^{1/2} \]

\[ T = 1.6 \times 10^{12} \times 10^4 \frac{eV}{keV} \]

\[ M = 4 \times 10^{-24} \text{ gm} \]

\[ N \chi = \frac{1.6 \times 10^{-8}}{10 \left( 4.8 \times 10^{-10} \text{ eV} \right)^4} \]

\[ = 2 \times 10^{-12} \times 10^{-12} \times 10^4 \]

\[ = 10^{13} \left( \frac{1}{1000} \right)^{3/2} \]

\[ e.g., \text{ at } 50 \text{ kev} \quad N \chi \approx 10^{14} \text{ cm}^{-3} \text{ s} \]

Close: Possible use as power amplifier.
If we can plug up ends (making confinement device toroidal), particles will then only diffuse across field lines in a random manner.

\[ \tau_2 \approx \frac{1}{2 \pi^{1/2}} \frac{a^2}{\beta^2} \]

\( a \) = radius of device

Hamer radius? \( \ell = \frac{M^2 C^2}{\epsilon^2 B^2} \quad A = 100 \text{ cm} \quad \beta = 10^{-4} \text{ B gauss} \)

\[ \frac{\beta^2}{10^4 (4.8 \times 10^{-19})} \frac{10^8 B^2}{4 \times 10^{-24}} \frac{a^2}{9 \times 10^{-20}} \left( \frac{1.6 \times 10^{-12} + \ell}{\ell} \right) \]

\[ \Upsilon = 10^{13} \frac{T^{3/2}}{T} \]

\[ \Upsilon = 10^{13} a^2 B^2 T^{3/2} \frac{20}{50} \frac{10^{-8}}{10^{-6}} \]

\[ \Upsilon = 4 \times 10^{16} a^2 B^2 T^{3/2} \]

Really looks good:
Diffusion Losses negligible
But is simplified classical picture correct?
Are there other mechanisms controlling transport?
Particle Orbits in a $E \times B$ Field

$E \ll B$ (cgs units)

$$\frac{dv}{dt} = \frac{e}{m} \left( E \frac{v}{c} + \frac{v \times B}{c} \right)$$

$$\frac{dr}{dt} = v$$

Suppose $E$ and $B$ are uniform in space, $E \cdot B = 0$ and time independent.

$$\frac{dv}{dt} = \frac{e}{m} E + \frac{v \times B}{c}$$

We can go to frame where $E_\perp = 0$

$$v = \frac{v}{c} + \frac{c E \times B}{B} = \frac{v}{c} + \frac{v_E}{c}, \quad v_E = \frac{c E \times B}{B}$$

$$\frac{dv}{dt} = \frac{dv}{dt} \left( E \frac{v}{c} + \frac{c E \times B}{B} \right)$$

(1)
\[(F \times b) \times B = \frac{e}{c} \mathbf{E} \cdot \mathbf{B} - \mathbf{E} \times [\mathbf{B}]\]

\[
\frac{du}{dt} = \frac{e}{m} \left[ \frac{\mathbf{E}}{B} \left( \frac{\mathbf{E}}{B} \right) + \frac{\mathbf{u} \times \mathbf{B}}{c} \right]
\]

\[
= \frac{e}{m} \frac{\mathbf{u} \times \mathbf{B}}{c}
\]

Thus in frame
\[
\chi = c \frac{\mathbf{E} \times \mathbf{b}}{B} \equiv \frac{\mathbf{E}}{B} \]

electric field does not appear.

\[
\frac{du}{dt} = \frac{e}{mc} \mathbf{u} \times \mathbf{b}
\]

We know that motion is 'helical' around magnetic field.

![Helical motion diagram]

\[
\mathbf{u} \cdot \mathbf{b} = \mathbf{u} \cdot \mathbf{b} = \text{const.}
\]

\[
\mathbf{p} = \frac{\mathbf{u} \times \mathbf{b}}{mc}
\]

\[
\omega_c = \frac{eB}{mc}
\]

Gyration for positive charge particle, as \( \mathbf{B} \)-field.

\[
\mathbf{u}_t = \mathbf{u}_0 \left[ \cos (\phi - \omega_c t) \hat{x} + \sin (\phi - \omega_c t) \hat{y} \right]
\]

\[
\mathbf{p}_t (t) = \mathbf{p}_0 \left[ \sin (\phi - \omega_c t) \hat{x} + \cos (\phi - \omega_c t) \hat{y} \right]
\]

\[
\mathbf{u}_{tt} = \mathbf{u} \cdot \mathbf{b} = \text{constant} ; \quad \mathbf{z} = \mathbf{z}_0 + \mathbf{u}_{tt} t
\]