Non-dissipative forces

\[ \mathbf{F} = -\nabla U \]

Work going from point \( a \) to point \( b \) independent of path

\[ \int F \cdot ds = \int F_x \cdot dx + F_y \cdot dy + F_z \cdot dz = \int \mathbf{F} \cdot d\mathbf{s} \]

\[ W = \int_0^1 \mathbf{F} \cdot d\mathbf{s} = -\Delta U(x, y, z) \]

\[ = -(U(x, y, z) - U(x_0, y_0, z_0)) \]

What are non-dissipative forces?

- Gravitational force
- Spring force
- Nuclear force
- Drag force of air

Which forces are non-dissipative?

- (1) & (2)
- (4) (3) & (4)
- (5) (1) & (3)
- (2) & (3)
- (4) & (1)

(over)

(1)
mechanical energy

1. kinetic energy of a point object
2. potential energy of a point object
3. rotational kinetic energy of a finite sized object

non-mechanical energy

1. heat energy (random motion of molecules and random compression, e.g., in a blender) of heated objects (thermodynamics)
2. electrical energy
3. chemical energy (calorie 4.184 J)

Calories (kilo calorie) 1000 calories (energy content of something small)

restricted energy principle

conservation of mechanical energy

under the action of conserved force the total mechanical energy of a point particle is conserved

\[ \frac{1}{2} m v^2 + U(x, y, z) = \frac{1}{2} m v_0^2 + U(x_0, y_0, z_0) \]
\[ U = \frac{1}{2}kx^2 \]

\[ \text{at rest,} \]

If I drop a ball, \( h \) from a height \( h \), how fast will it be when it hits the ground?

\[ U = mgy \quad \text{let ground be} \quad y = 0 \]

\[ mgh = mgy + \frac{1}{2}mv^2 \]

\[ y = 0 \quad v = \sqrt{2gh} \]

\[ \frac{v^2}{2} = gh \]

\[ v^2 = 2gh, \quad v = \sqrt{2gh} \]

It is change of potential energy \( U \) that is important.

\[ U = mgh \]

\[ \text{and potential energy is not a constant,} \]

\[ \text{within a constant} \]

\[ \text{e.g. ask the same question when} \]

\[ U = mg(y + y_0) \quad \text{can we find some curve} \]

\[ mgh + y_0 = mg \left( y_0 + y \right) + \frac{1}{2}mv^2 \]

\[ \Rightarrow mg(y_0 + y) + \frac{1}{2}mv^2 \]

\[ v^2 = 2gh, \quad v = \sqrt{2gh} \]
\[ U = \frac{1}{2} kx^2 + \frac{mv^2}{2} \]

What is the speed of the mass $x = 0$?

\[ \frac{1}{2} m v^2 + \frac{1}{2} kx^2 = \frac{1}{2} kd_l^2 \]

\[ v^2 = \frac{kd_l^2}{m} \]

\[ V = \sqrt{\frac{kd_l}{m} d} \]

Power $\equiv \frac{dW}{dt}$ is the rate of change (on energy on force work is done)

\[ P_{\text{work}} = F \cdot \dot{V} \]

\[ dW = F \cdot d\dot{V} \]

\[ \frac{dW}{dt} = F \cdot \frac{d\dot{V}}{dt} = F \cdot \dot{V} \]
More accurate form of gravitational force

$$F_{12} = \frac{G m_1 m_2}{r^2}$$

attractive force

$$G = 6.67 \times 10^{-11} \text{ mks/m}$$

What is the basic form of these units?

$$\frac{G m_1 m_2}{r^2} = m a$$

$$G = \frac{r^2 a}{m} = \frac{m^2 m}{m} \frac{1}{s^2 \text{ kg}} = \frac{m^3}{s^2 \text{ kg}}$$

or

$$\frac{G m_1 m_2}{r^2} = N$$

$$a = \frac{N m^2}{kg}$$

$$N = \text{kg m m}$$
Period about a celestial body

\[ T = \frac{2\pi R}{v} \]

\[ f = \frac{1}{T} = \frac{v}{2\pi R} \]

\[ \omega = 2\pi f = \frac{v}{R} \]

\[ a = \frac{v^2}{R} \]

\[ \frac{mV^2}{R} = \frac{GM}{R^2} \]

\[ = \frac{v^2}{R^2} = \frac{GM}{R^3} \]

\[ \left( \frac{2\pi}{T} \right)^2 = \frac{GM}{R^3} \]

\[ T = \left( \frac{GM}{R^3} \right)^{\frac{1}{2}} \]

\[ T = \frac{2\pi R^{\frac{3}{2}}}{GM} \]

This is one of Kepler's Laws. Period \( \propto R^{\frac{3}{2}} \)
$g \ \text{related to inverse law}$

Force at edge of sphere
Same as force due to mass concentrated at center, distant $A$ away

\[ F = \frac{G MM_m}{R_E^2} = mg \]

\[ G_e = \frac{GM_E}{R_E^2} \]

\[ G_{mm} = \frac{GM_{mm}}{R_m^2} \]

\[ G_s = \frac{GM_s}{R_s^2} \]
Take $g_E = 10 \text{ m/s}^2$, $g_m = 1.6 \text{ m/s}^2$

The weight of a 150 kg person on the surface of the Earth is about 1000 N

T F

The mass of a 600 kg person on the moon is about 100 kg

T F

The weight of a 1500 kg person on the moon is about 1000 N

T F
Gravitational Potential Energy

\[ PE_g = - \int_{-\infty}^{r} F \cdot dr = G M M \int_{-\infty}^{r} \frac{dr}{r} = -G M M \left( \frac{1}{r} \right) \]

\[ PE_g = -\frac{G M M}{r} \]

How high can a rocket go when blasted off with a velocity \( V_0 \)?

\[ \frac{1}{2} m v_e^2 = \frac{G M_e M}{r_e} = -\frac{G M_e m}{r_{E+th}} + KE_{20} \]

\[ \frac{1}{2} m v_e^2 = G M_e \left( \frac{1}{r_e} - \frac{1}{r_{E+th}} \right) \]
Solve for \( \frac{h}{R_E} \)

\[
\frac{V_i}{2GM_E} = \frac{1}{R_E} - \frac{1}{R_E + h}
\]

\[
\frac{1}{R_E + h} = \frac{1}{R_E} - \frac{V_i}{2GM_E}
\]

\[
R_E + h = \frac{1}{\frac{1}{R_E} - \frac{V_i}{2GM_E}}
\]

\[
\frac{h}{R_E} = \frac{1 - \frac{V_i}{\sqrt{2GM_E}}}{2GM_E}
\]

\[
\frac{h}{R_E} > 0 \quad \text{if} \quad \frac{V_i}{\sqrt{2GM_E}} < 1 \quad \text{or} \quad \frac{V_i}{R_E} < \frac{1}{2GM_E}
\]

\[
\text{so} \quad \text{as} \quad \text{if} \quad V_i \to 2GM_E = V_{esc}
\]
\[ V_e > V_{esc} = \sqrt{\frac{2\,G\,m_e}{R_E}} \]

How fast is a rocket for from Earth?

\[ \frac{1}{2} m V_i^2 - \frac{G\,m_e\,M_e}{R_E} = \frac{1}{2} m V_{\infty}^2 \]

\[ V_{\infty} = V_i^2 - 2 \frac{G\,m_e}{R_E} \]