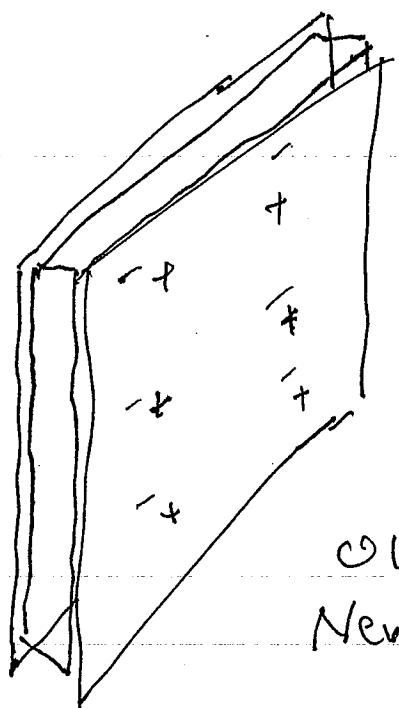


Lecture # 9

Dielectric Surface Charge



Insertion of a dielectric into plates of fixed charge, Q_0 , reduces Voltage, V_0 to V_0/ϵ

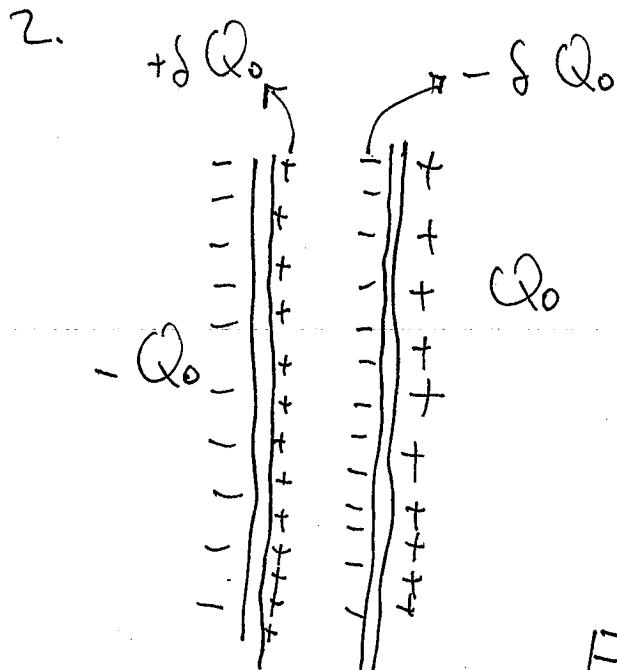
$$\text{Old Capacitance } C_0 = \frac{Q_0}{V_0}$$

New Capacitance, C_d , on dielectric:

$$C_d = \frac{Q_0}{V_0/\epsilon} = \epsilon \frac{Q_0}{V_0} = \epsilon C_0$$

Why does voltage decrease with dielectric insertion?

- (1) Charges move in dielectric to produce partially cancelling electric fields
- (2) Ions are produced in electric field and they flow, to cancel the electric field
- (3) Molecules polarize to form dipoles that produce partially cancelling surface charge



What is δ ?

Voltage across plate
is $V = Ed = \frac{V_0}{\kappa}$

$$E = \frac{(Q_0 - \delta Q_0)}{\epsilon_0 A}$$

$$V = Ed = \frac{Q_0 d}{\epsilon_0 A} (1 - \delta) = \frac{V_0}{\kappa}$$

$$\Rightarrow V_0 = V \kappa$$

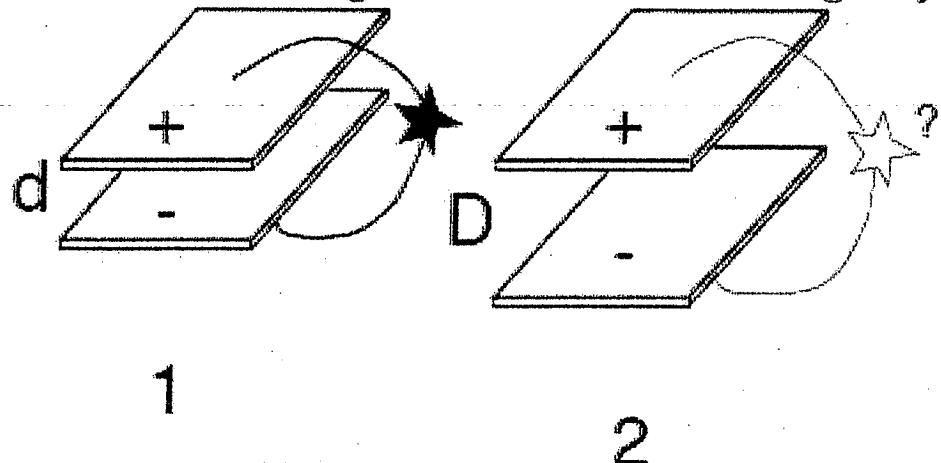
$$\therefore V_0 (1 - \delta) = \frac{V_0}{\kappa}$$

$$\therefore \boxed{\delta = 1 - \frac{1}{\kappa}}$$

The magnitude of induced charge is

$$Q_{\text{ind}} = \left(1 - \frac{1}{\kappa}\right) Q_0$$

The two isolated parallel plate capacitors below, one with plate separation d and the other with $D > d$, have the same plate area A and are given the same charge Q .



The energy in the spark produced by discharging the second capacitor is

1. less energetic than the discharge spark of the first capacitor.
2. more energetic than the discharge spark of the first capacitor.
3. the same as the discharge spark of the first capacitor.

Energy stored in Capacitor

$$U_E = \frac{QV}{2} = \frac{Q^2}{2C} = \frac{CV^2}{2}$$

- (1) At fixed charge, smaller capacitors store more energy
- (2) At fixed voltage, large capacitors store more energy
- (3) Batteries & power plants produce fixed voltage, allowing large capacitors to store more energy than smaller capacitors
- (4) The use of molecular polarization allows substantial increase of capacitance for the same device dimensions ($\kappa \approx 10$ fairly typical)
(no dielectric) $C_0 \rightarrow \kappa C_0$ (with dielectric)

- 1) Motion of charges gives rise to currents
- 2) Mechanism in some ways similar to a rain drop falling to the ground

Question

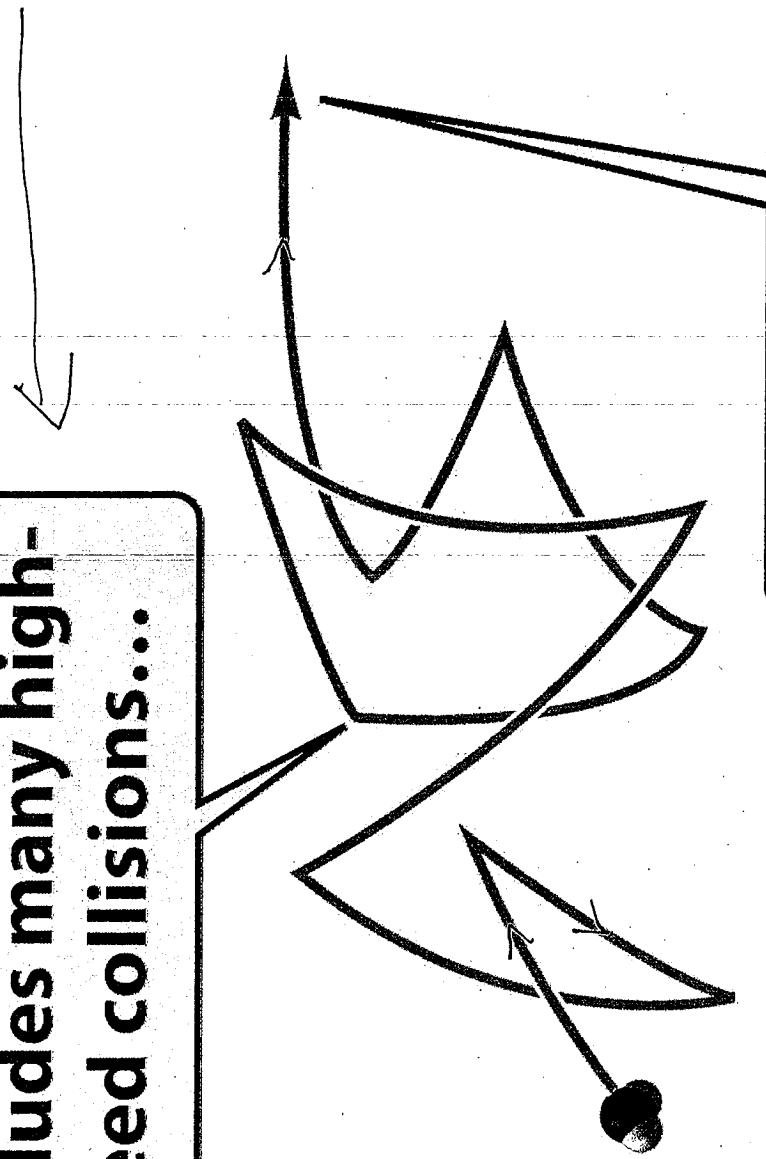
How does a rain-drop fall to the ground

- (1) By continually accelerating
- (2) Falls at constant velocity
- (3) Moves chaotically but on the average moves at constant velocity in direction of gravitational force

How does an electron in
a metal move in the
presence of an electric field?

- (1) Moves with continual acceleration
in the direction opposite electric
field direction
- (2) Moves at constant velocity directed
opposite to electric field
- (3) Moves chaotically but on
average moves with constant
velocity opposite to electric field

**Microscopic motion
includes many high-
speed collisions...**



**...but the net
motion is a low-
speed drift.**

Figure 27-6 Physics for Engineers and Scientists 3/e
© 2007 W.W. Norton & Company, Inc.

Resistivity, ρ , (a material's property)
at given temperature

Atomistic model

particle (an electron) has charge q , ($q = -e$)
(for electrons)

Electric force on electron
is balanced by a viscous-force
that is property of material

$$\vec{F}_e = q \vec{E}, \quad \vec{F}_v = m \vec{V}/\tau$$

τ = relaxation time (a material property)

$$\vec{F}_e = \vec{F}_v; \quad \therefore q \vec{E} = \frac{m \vec{V}}{\tau}; \quad m = \text{mass of charge carrier}$$

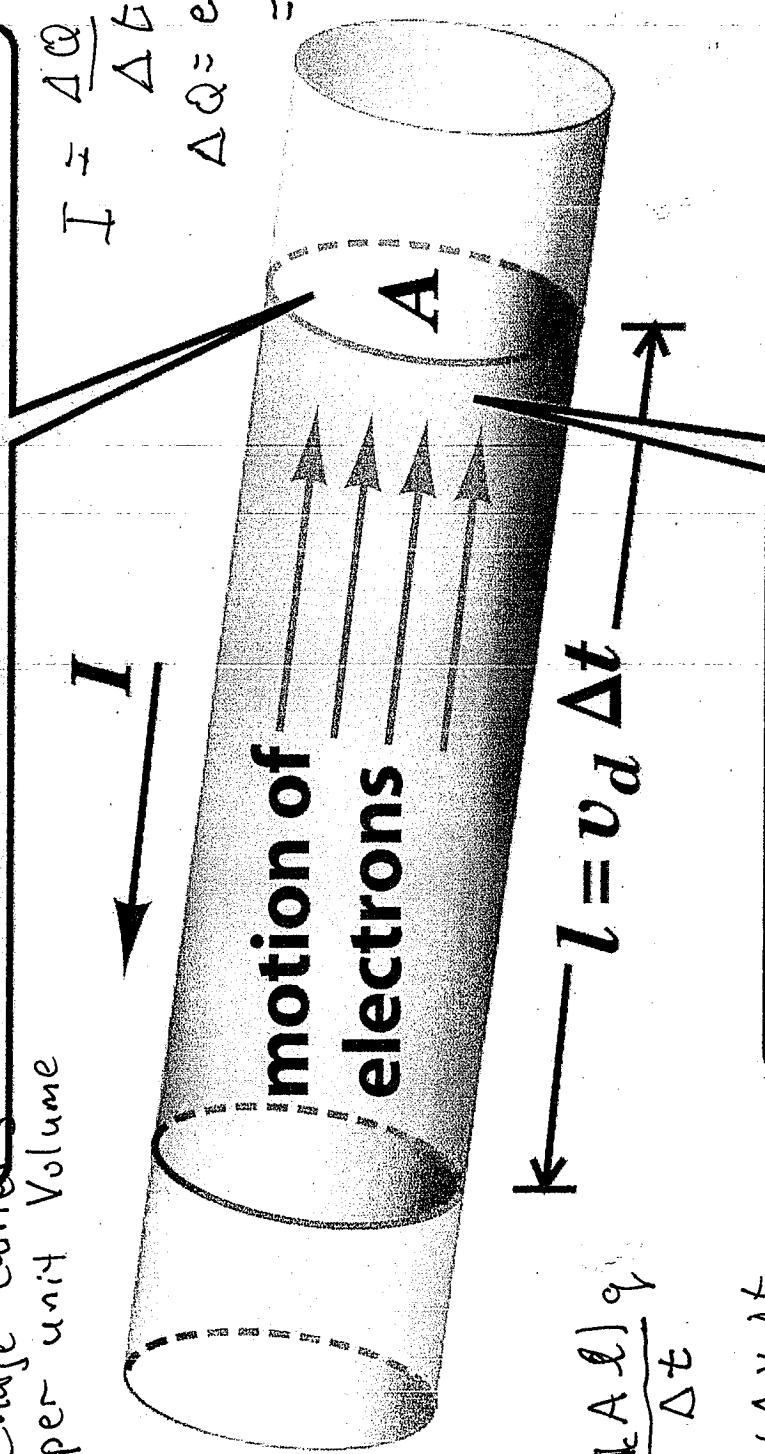
$$\vec{V}_0 = q \frac{\vec{E} \tau}{m} \equiv \vec{V}_0$$

Amount of charge that moves past area A in time Δt ...

N_c = # of charge carriers per unit Volume

$$I = \frac{\Delta Q}{\Delta t}$$

$$\begin{aligned} \Delta Q &\approx e N_c (\ell A) \\ &= e N_c (\ell v_d \Delta t) \\ &= e v_d \tau E \end{aligned}$$



$$I = \frac{(N_c A \ell) q}{\Delta t}$$

$$\begin{aligned} &= \frac{N_c A v_d \ell t}{\Delta t} \quad (\text{use } v_d) \\ &= N_c A v_D \ell \\ &= \underline{e N_c A E} \end{aligned}$$

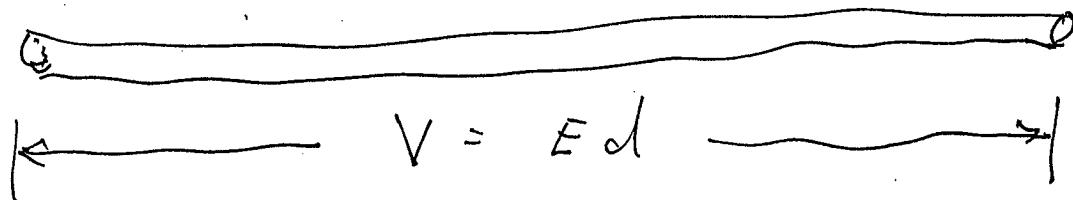
...is the free charge in this volume $A \times l = A v_d \Delta t$

Figure 27-7 Physics for Engineers and Scientists 3/e
© 2007 W.W. Norton & Company, Inc.

$$E = \frac{I}{A} \left(\frac{m}{e \tau N_c} \right) = \frac{I \rho}{A}; \quad \rho = \text{resistivity}$$

$$E = \frac{I\rho}{A} ; \rho = \frac{m}{e \times N_c}$$

↓
depends on material
property



Voltage across resistor

$$V = Ed = I \left(\frac{\rho d}{A} \right) = IR$$

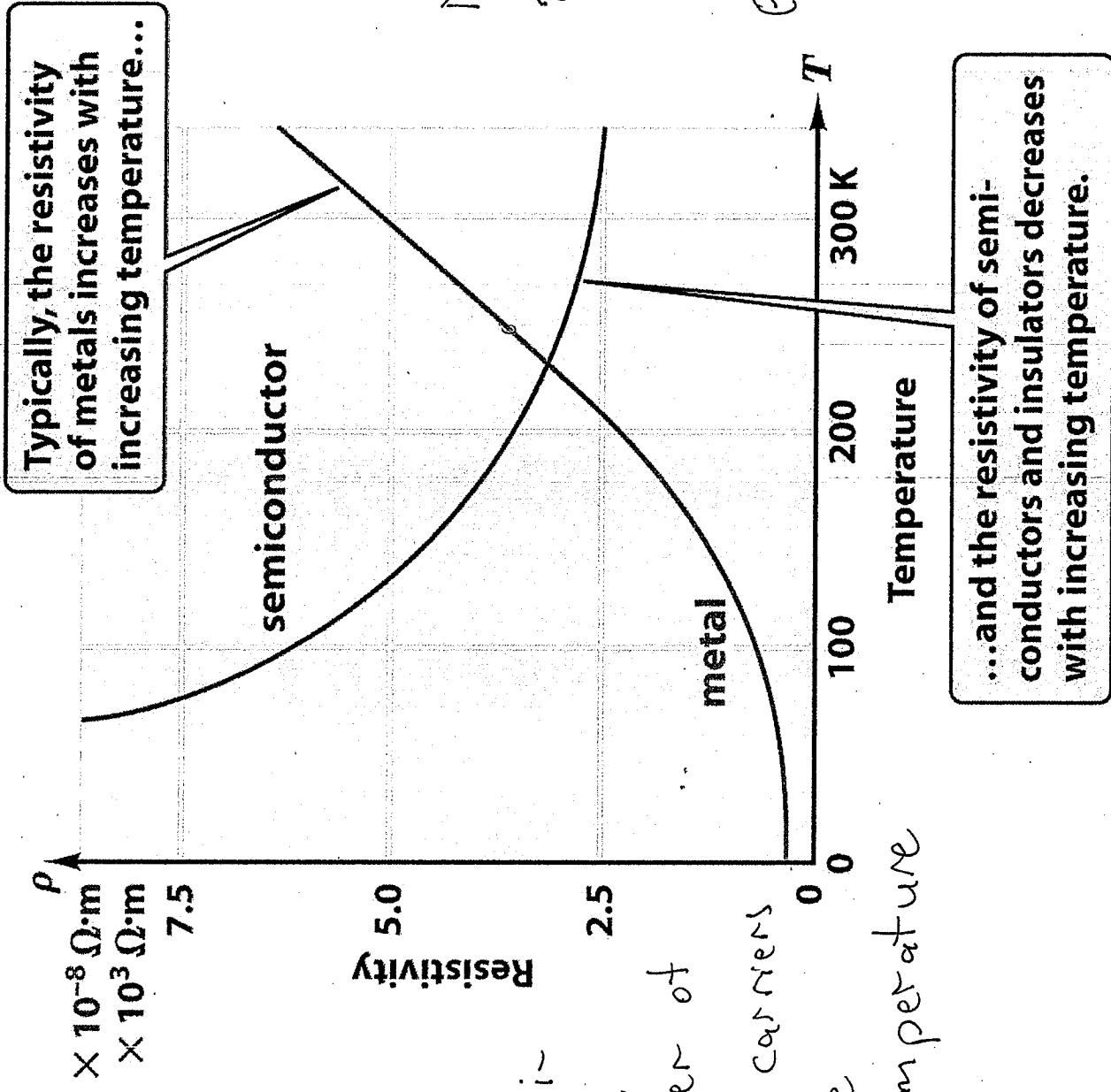
Resistance : $R = \frac{\rho d}{A}$

$$I = \frac{V}{R} \equiv \text{Ohm's Law}$$

Unit of current \equiv Amperes (A)

Unit of Resistance \equiv Ohm (Ω)

Temperature dependence of resistive materials



In semiconductor,
the number of
charge carriers
increase with
temperature

Figure 27-8 Physics for Engineers and Scientists 3/e
© 2007 W.W. Norton & Company, Inc.

Nature can do some strange things

The resistance of a superconductor suddenly drops to zero at a critical temperature.

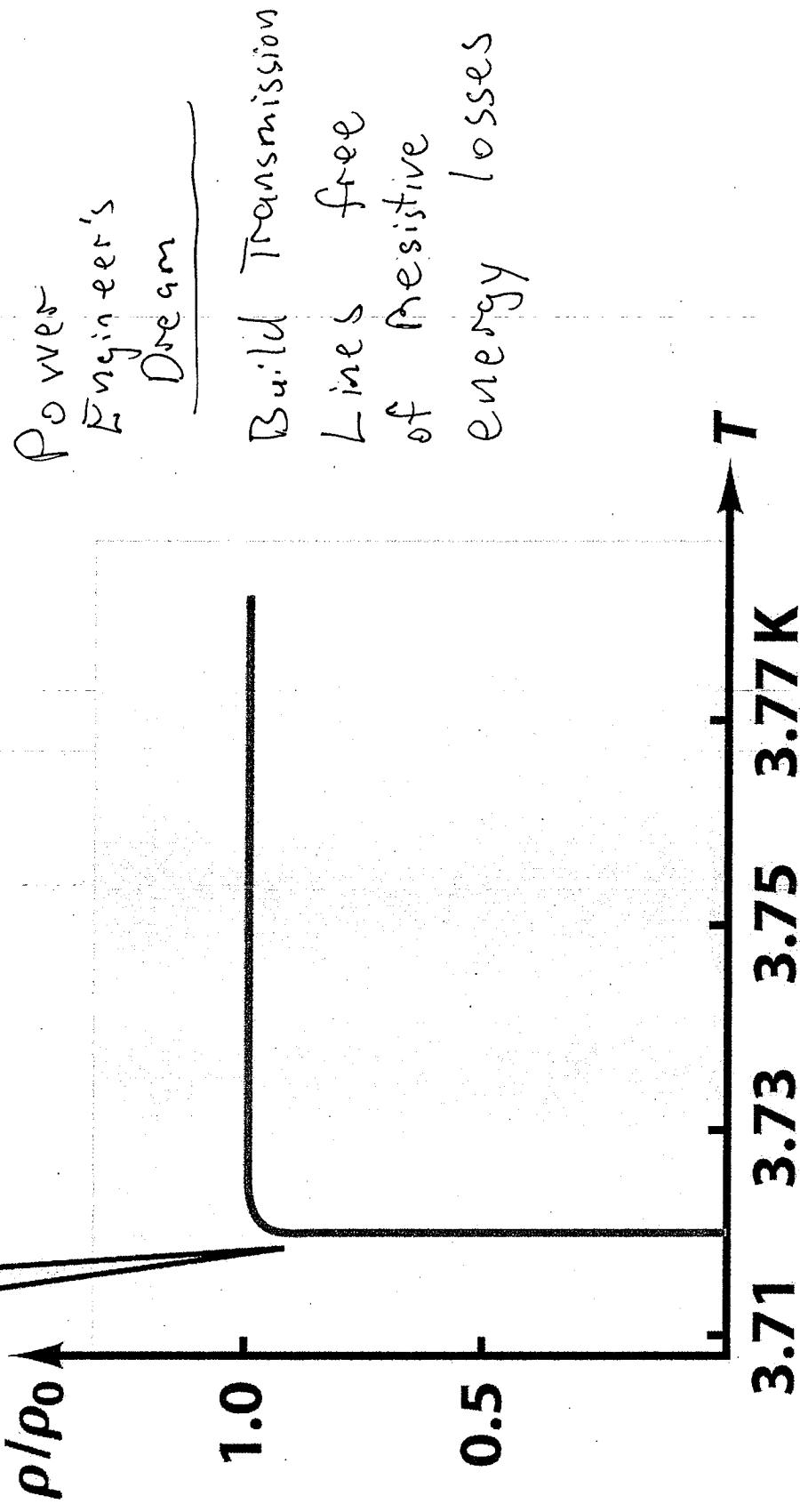


Figure 27-10 Physics for Engineers and Scientists 3/e
© 2007 W.W. Norton & Company, Inc.

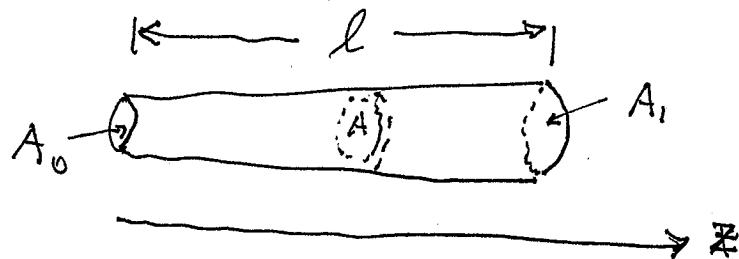
Various currents in nature and
man-made

TABLE 27.1 SOME CURRENTS

Lightning stroke (a)	10^4 A
High-tension power line (b)	10^3
Large transformer (c)	10^3
Large electromagnet	200
Starter motor of automobile (d)	100
Alternator of automobile	30
Fuse blows	30
Defibrillation treatment for heart	20
Air conditioner	12
Hair dryer	10
Ordinary lightbulb	1
Flashlight bulb	0.5
Lethal fibrillation of heart	0.1
Barely perceptible by skin	1×10^{-3}
Electronic calculator (e)	1×10^{-4}
Scanning tunneling microscope	1×10^{-12}

Find resistance through resistive object with non-uniform cross section

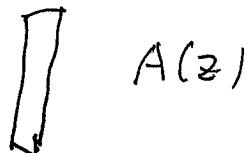
resistivity ρ



$$A(z) = A_0 + (A_1 - A_0) z/l$$

Current, I , is the same anywhere in object

$$I \rightarrow / \leftarrow dz$$



Voltage drop across incremental slab; $R(z) = \rho dz/A(z)$

$$\Delta V = I \frac{\rho dz}{A(z)} = \frac{I \rho dz}{A_0 + (A_1 - A_0) z/l}$$

$$\frac{dV}{dz} = \frac{I \rho}{(A_1 - A_0)} \left[\frac{A_0}{A_1 - A_0} + \frac{z}{l} \right] = E$$

$$V = \int_0^l \frac{dV}{dz} dz = \frac{I \rho}{(A_1 - A_0)} \int_0^l \frac{dz}{\frac{A_0}{A_1 - A_0} + \frac{z}{l}} = \frac{I \rho l}{A_1 - A_0} \left(\ln \left(\frac{A_0}{A_1 - A_0} + \frac{z}{l} \right) \right)_0^l$$

$$= \frac{I \rho l}{A_1 - A_0} \ln \left(\frac{A_1}{A_0} \right) \equiv IR ; R = \frac{\rho l}{A_1 - A_0} \ln \left(\frac{A_1}{A_0} \right)$$

**First stripes represent digits:
yellow = 4 and violet = 7 ...**

**...and third stripe represents
power of ten: orange = 3,
so $R = 47 \times 10^3 \Omega$.**

**Remaining stripe indicates
tolerance: silver = $\pm 10\%$.**

B	0	1	2	3	4	5	6	7	8	9	Gray	W	G	A	Ag	Re
Br	O	R	O	Y	G	Br	V	Y	W	W	Gold	Ag	Ag	Ag	Ag	Re

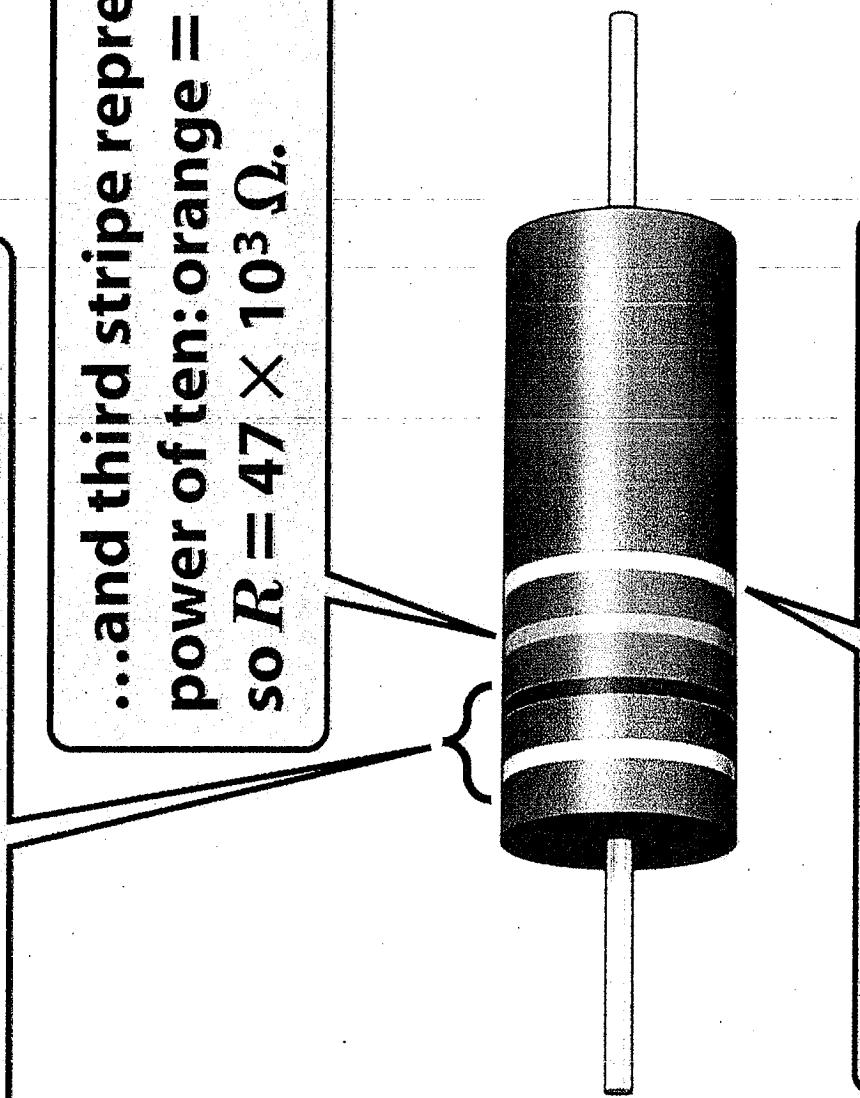


Figure 27-13 Physics for Engineers and Scientists 3/e
© 2007 W.W. Norton & Company, Inc.

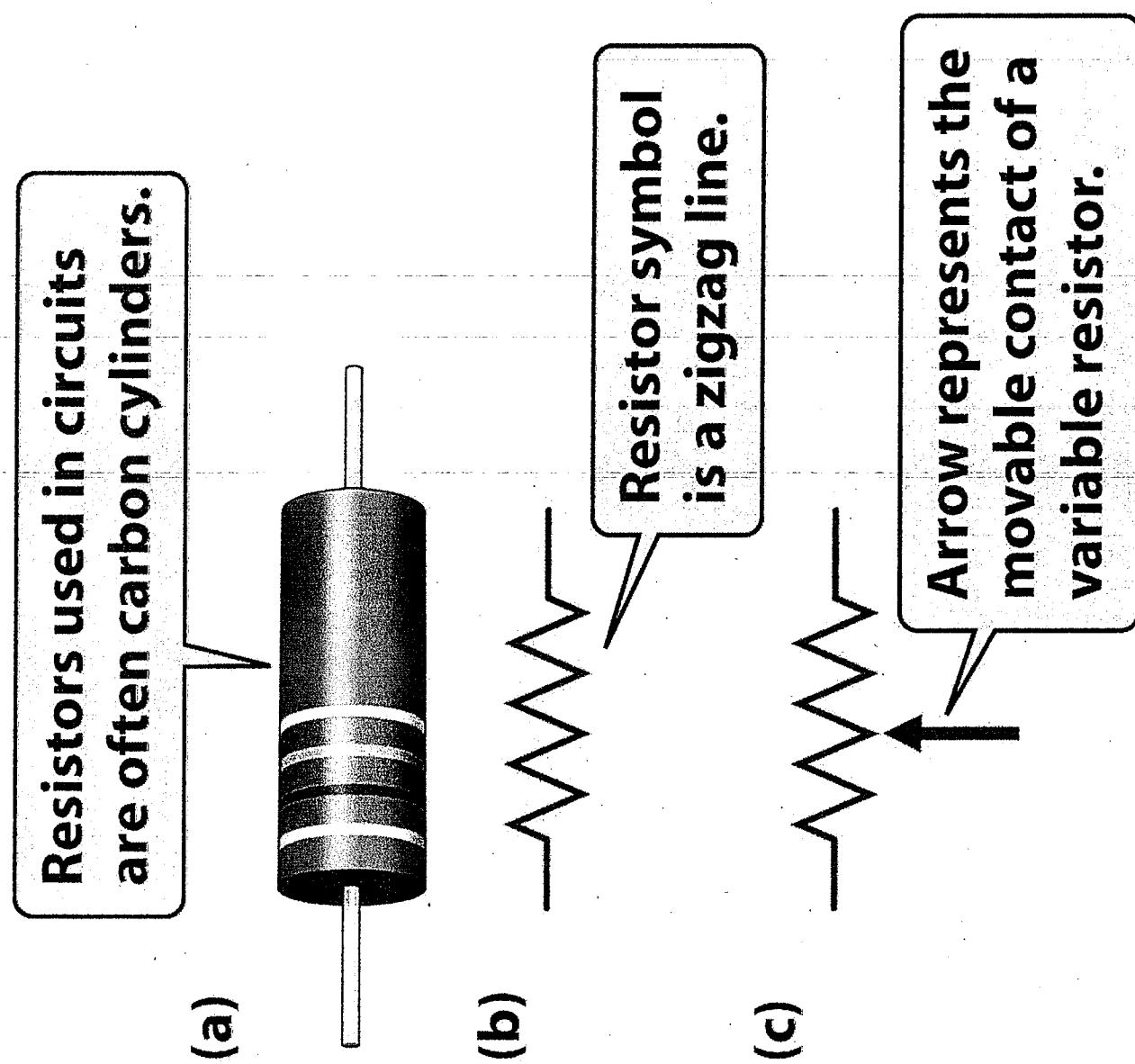


Figure 27-12 Physics for Engineers and Scientists 3/e
© 2007 W.W. Norton & Company, Inc.