Lecture 8

Capacitors

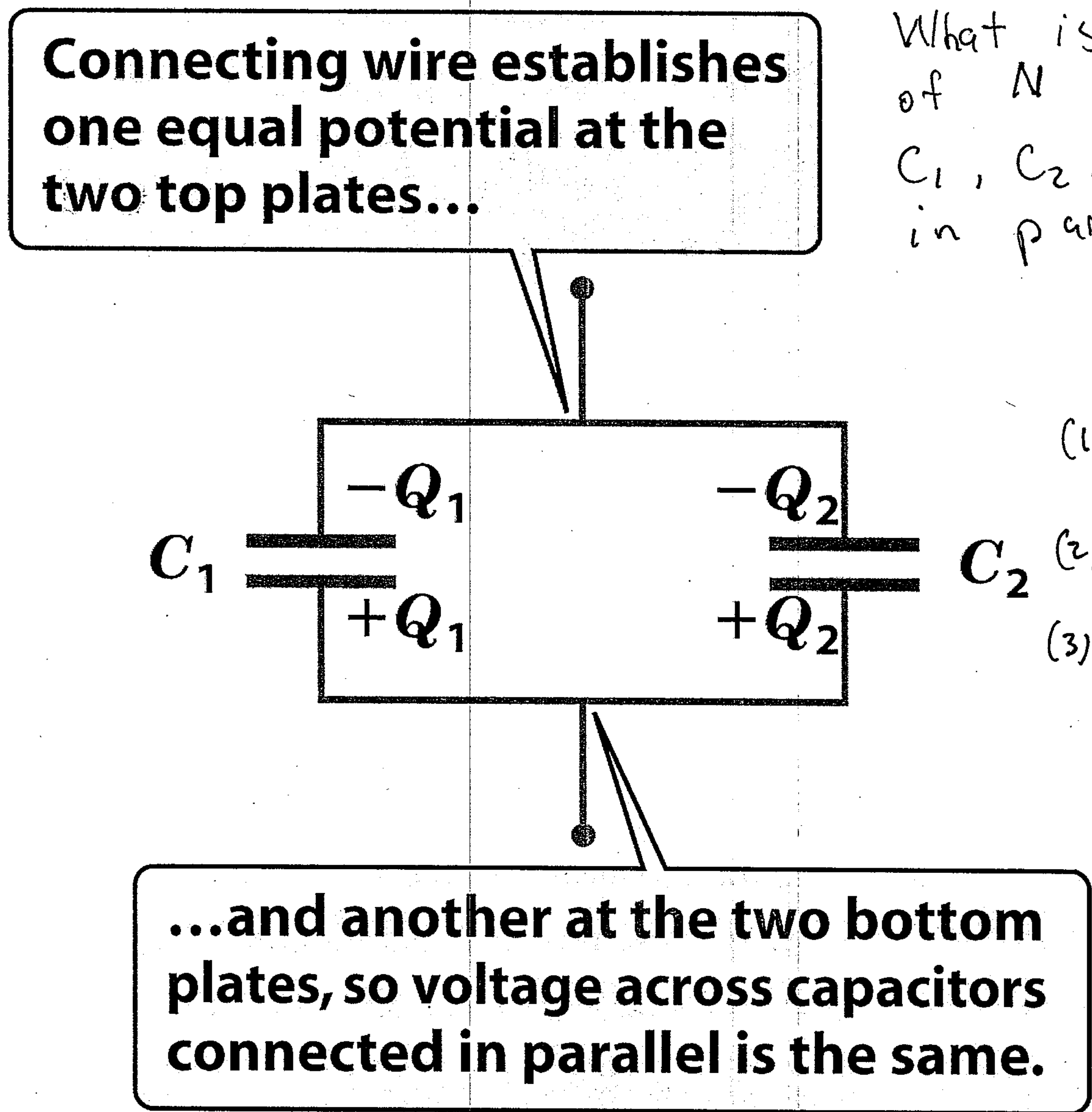


Figure 26-7 Physics for Engineers and Scientists 3/e

What is capacitance of N capacitors $C_1, C_2 \ldots C_N$, connected in parallel

(1) $\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}$

(2) $C_{eff} = C_1 + C_2 + \cdots C_N$

(3) undetermined

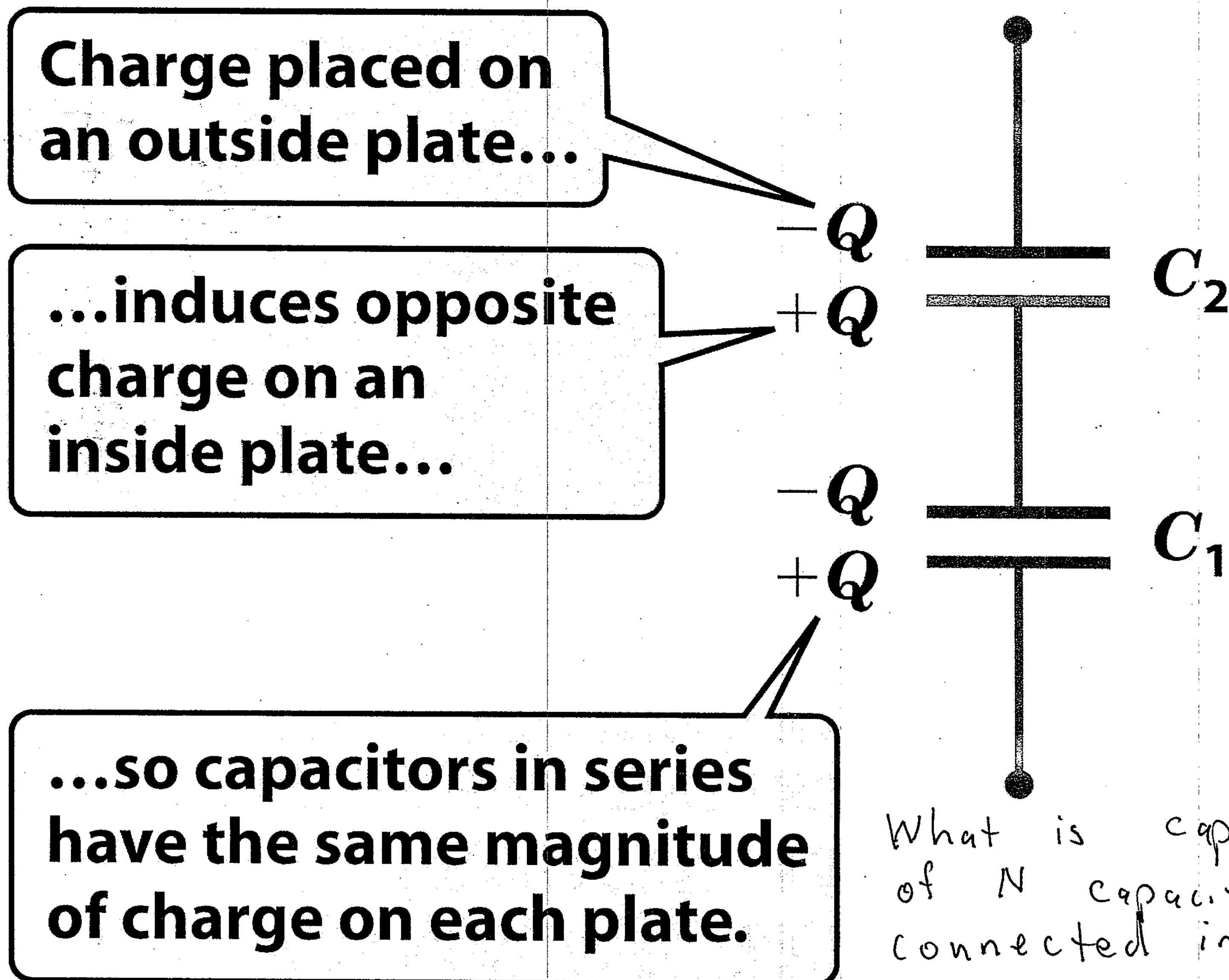


Figure 26-9 Physics for Engineers and Scientists 3/e

What is capacitance of N capacitors connected in series

(1) $\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}$

(2) $C_{eff} = C_1 + C_2 + \cdots C_N$

(3) not determined

What is Equivalent Capacitance

(a) 14 μF

(b) 1.5 μF

(c) ~~18~~ 24/7 μF

~~(d) 18 ... F~~

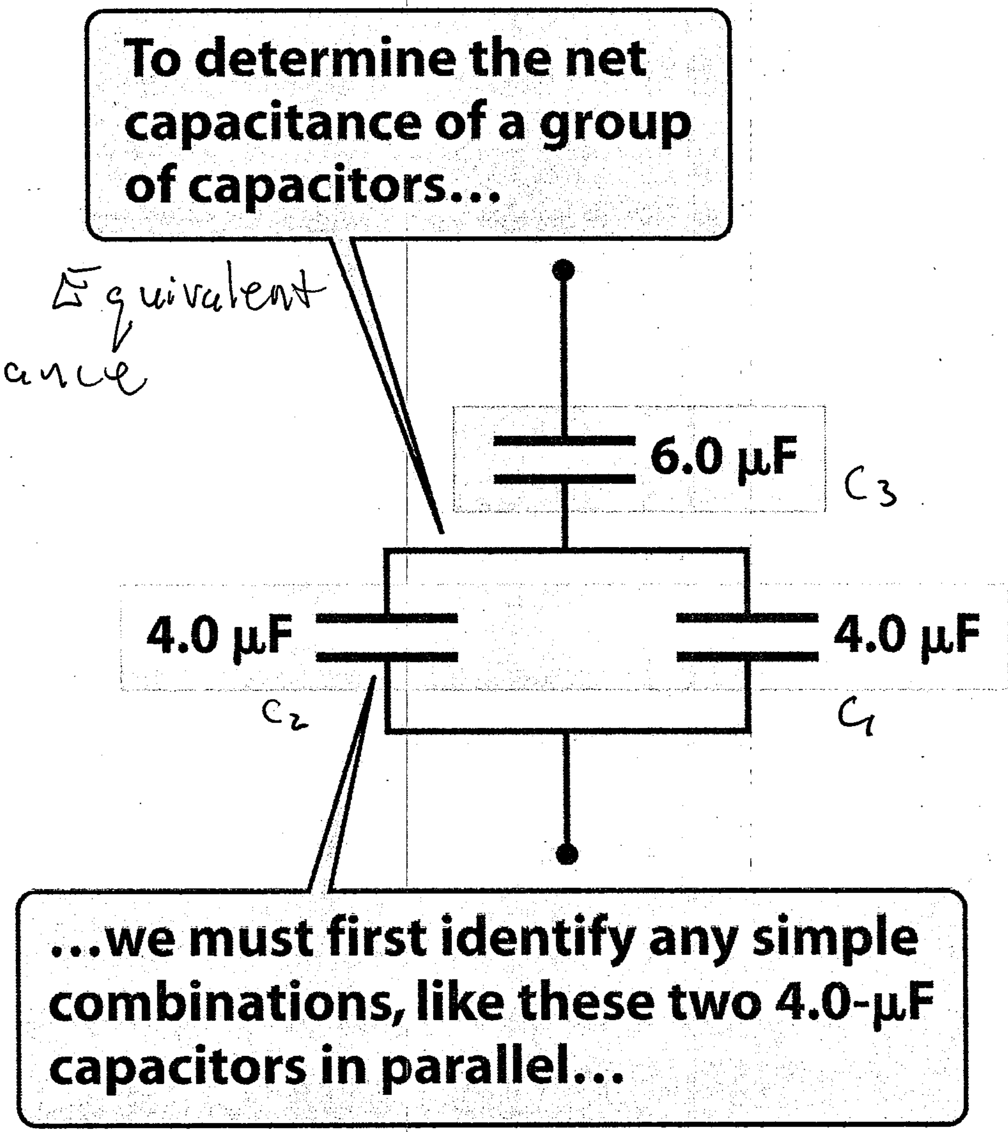
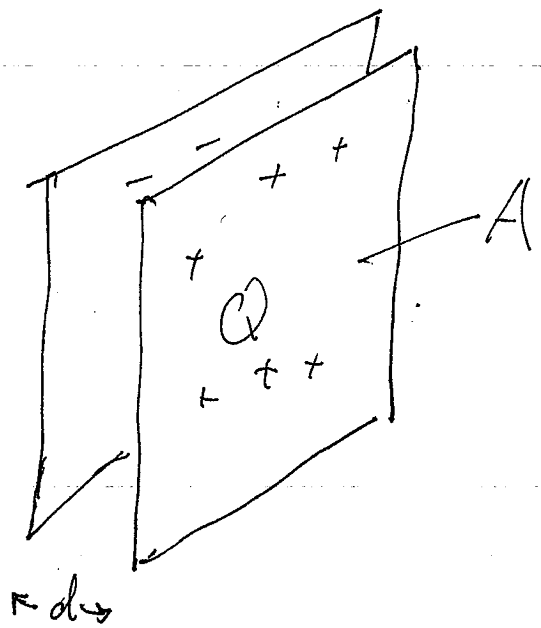


Figure 26-11a Physics for Engineers and Scientists 3/e

$$C_{eff} = \left[\left(\frac{1}{C_1 + C_2}\right) + \frac{1}{C_3}\right]^{-1}$$

$$= 1\Big/\left(\frac{1}{8} + \frac{1}{6}\right) = \frac{24}{3+7} = \frac{24}{7}\ \mu F$$

Several Capacitors & their Capacitance



Parallel plate Capacitor

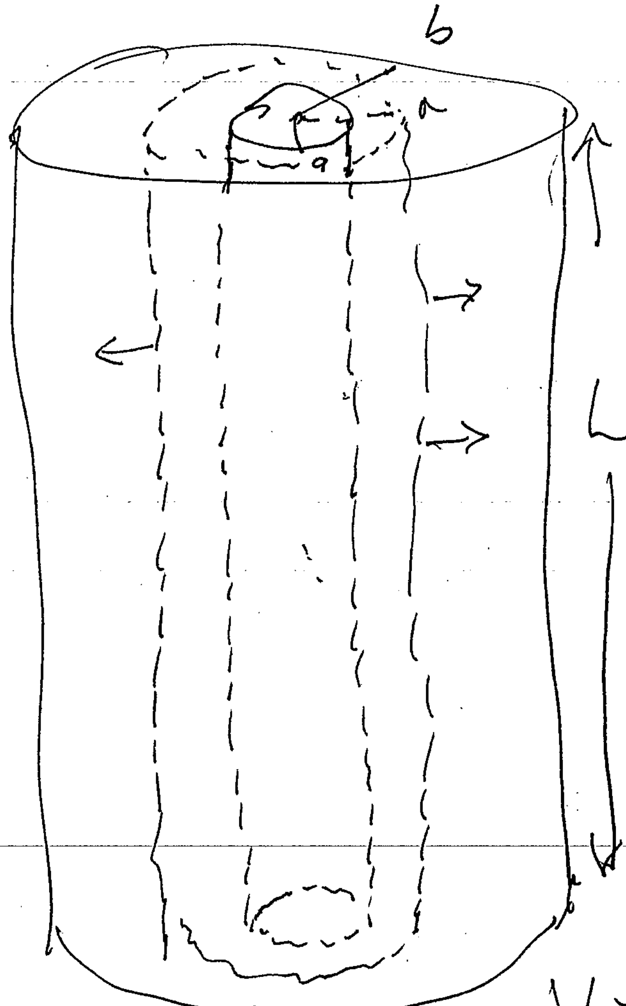
$$E_{in} = \frac{Q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

$$V = E_{in} d = \frac{Q}{\epsilon_0 A} d$$

$$C = \frac{Q}{V} = \frac{Q}{dQ/\epsilon_0 A} = \frac{\epsilon_0 A}{d}$$

$$\boxed{C_{slab} = \frac{\epsilon_0 A}{d}}$$

Cylinder



Gauss Law of r

$$E_r A = \frac{Q}{\epsilon_0}$$

$$E_r 2\pi r L = \frac{Q}{\epsilon_0}$$

$$E_r = \frac{Q}{2\pi r L \epsilon_0}$$

$$V = \int_a^b E_r dr = \frac{Q}{2\pi L \epsilon_0} \int_a^b \frac{dr}{r}$$

$$\int_a^b \frac{dr}{r} = \ln r \Big|_a^b$$

$$V = \frac{Q}{2\pi L \epsilon_0} \ln(b/a)$$

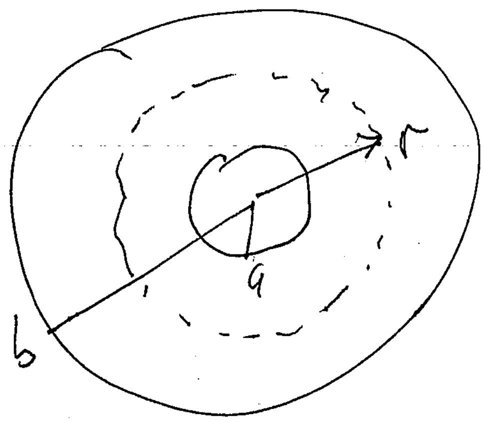
$$C = \frac{Q}{V} = \frac{Q}{Q \ln(b/a)} 2\pi L \epsilon_0 = \frac{2\pi L \epsilon_0}{\ln(b/a)}$$

$$C_{cyl} = \frac{2\pi L \epsilon_0}{\ln(b/a)}$$

1b)

8

Capacitance Sphere



$$\frac{Q}{\epsilon_0} = E_r A$$

$$= E_r 4\pi r^2$$

$$E_r = \frac{Q}{\epsilon_0 4\pi r^2}$$

$$V = \int_a^b dr E_r = \frac{Q}{4\pi\epsilon_0} \int_a^b \frac{dr}{r^2}$$

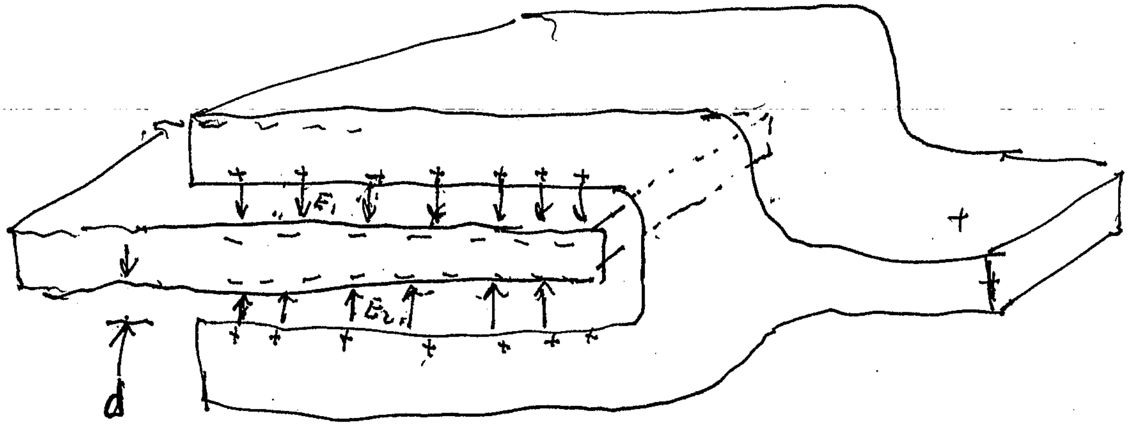
$$= \frac{Q}{4\pi\epsilon_0} \left(-\frac{1}{r}\right)\Big|_a^b$$

$$V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{V} = \frac{Q}{Q} \frac{4\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon_0 ab}{b-a}$$

$$C_{sphere} = \frac{4\pi\epsilon_0 ab}{b-a}$$

Prong Capacitor



Interfacing Area $\equiv A$

Gap distance $\equiv d$

$Q_t \equiv$ charge on top plate; $Q_b \equiv$ charge on bottom plate

$$E_1 = \frac{Q_t}{A\epsilon_0} \quad , \quad E_2 = \frac{Q_b}{A\epsilon_0}$$

$$V = E_1 d = \frac{Q_t d}{A\epsilon_0} \quad , \quad V = E_2 d = \frac{Q_b d}{A\epsilon_0} = \frac{Q}{2}\frac{d}{A\epsilon_0}$$

$$C = \frac{Q}{V} = \frac{Q_t + Q_b}{V}$$

$Q_1 = Q_2$; $Q = 2Q_1$

$$C = \frac{Q}{\frac{Q}{2}\frac{d}{A\epsilon_0}} = \frac{2A\epsilon_0}{d}$$

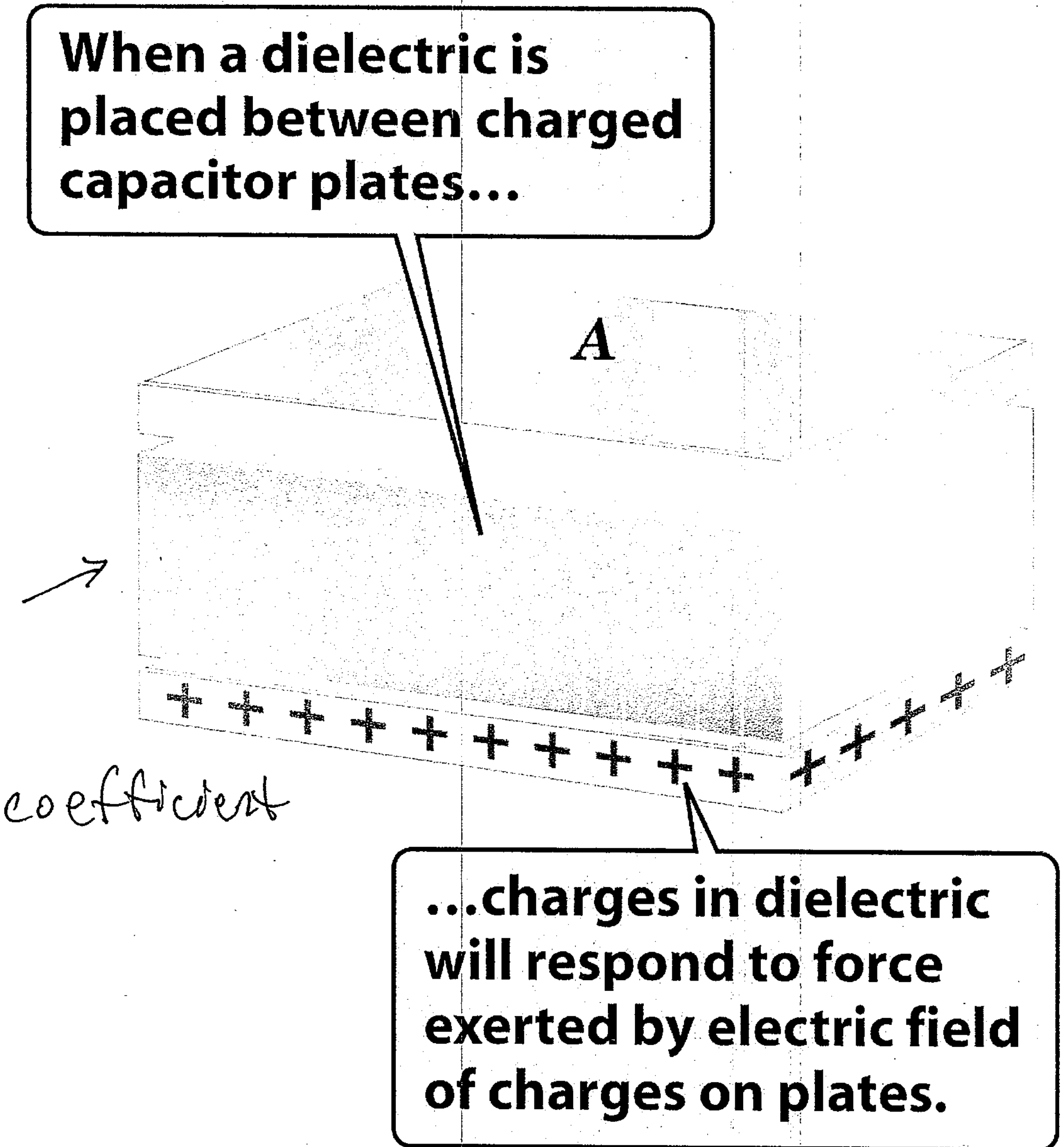


Figure 26-13 Physics for Engineers and Scientists 3/e

dielectric material with dielectric coefficient χ

Dielectric material can increase capacitance significantly

If for fixed V_0 causes charge Q_0 on plates without dielectric then $Q = \chi Q_0$ on plates with dielectric; $C \to \chi C_0$

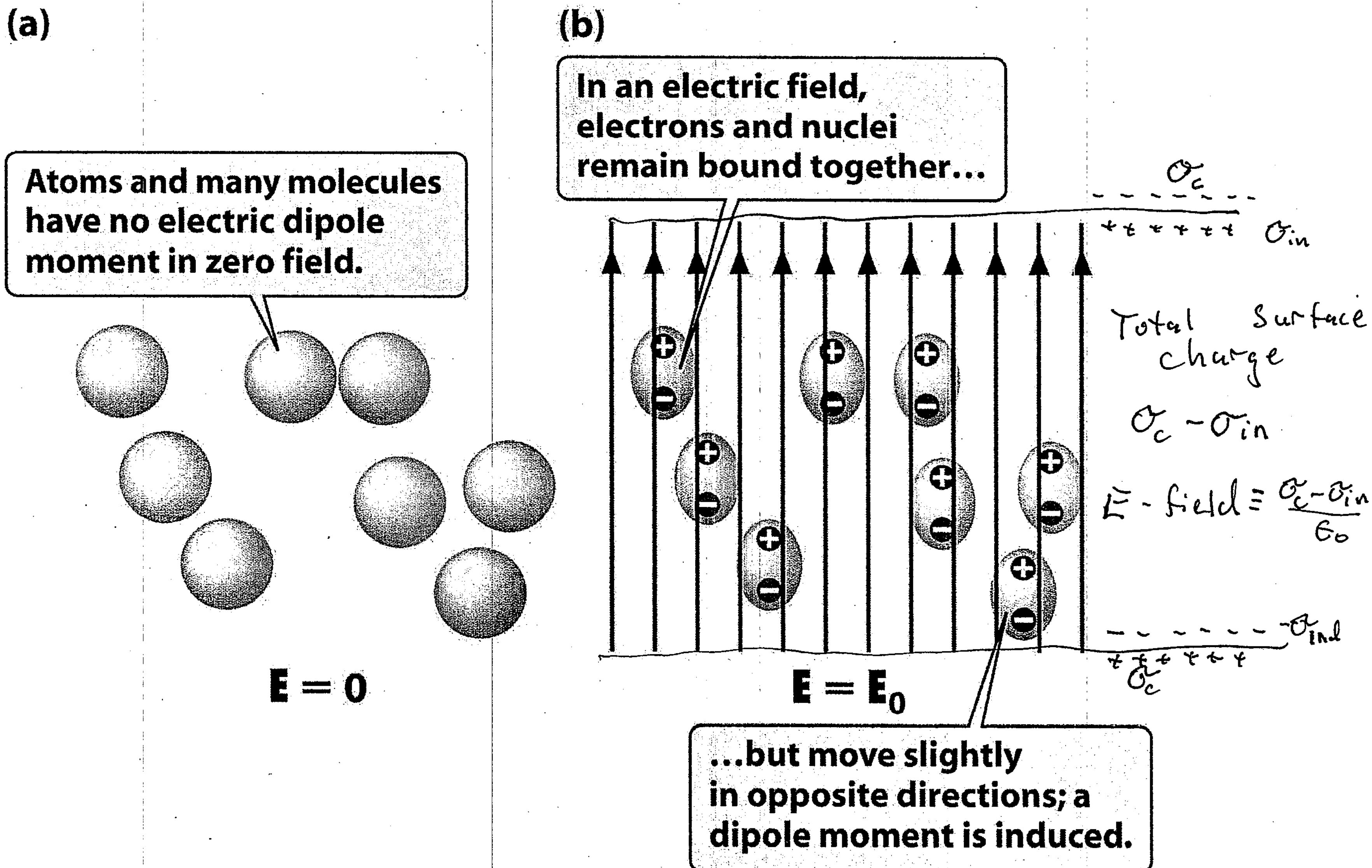


Figure 26-14 Physics for Engineers and Scientists 3/e

$C_0 \equiv$ capacitance without dielectric

$\kappa C_0 \equiv$ capacitance with dielectric

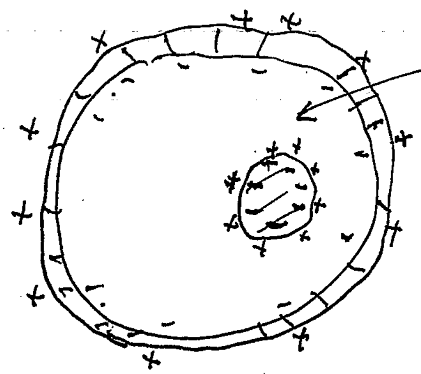
Relation between dielectric constant and induced surface charge

$$Q_{ind} = Q\left(1 - \frac{1}{\kappa}\right)$$

explanation: $Q \equiv$ charge on conducting plate

$Q_{ind} \equiv$ surface charge on dielectric surface

$C_0 \equiv$ capacitance without dielectric material



filled with dielectric material

$\kappa C_0 \equiv$ capacitance with diel...

$Q = \kappa C_0 V$

Total charge on outer plate is

$Q = \kappa C_0 V$; $Q - Q_{ind} = C_0 V$ → (Using both conductor and dielectric charge)

solve for V

$$V = \frac{Q}{\kappa C_0} = \frac{Q - Q_{ind}}{C_0}$$

$$\therefore \quad Q_{ind} = Q - \frac{Q}{\kappa}$$

$$\boxed{Q_{ind} = Q\left(1 - \frac{1}{\kappa}\right)}$$

Relation between κ and induced charge, Q_{ind}

With dielectric $C = \kappa C_0$, where C_0 is capacitance with no dielectric

$$V = \frac{Q}{\kappa C_0}$$

But same voltage would appear without dielectric material if one takes the full charge, $Q - Q_{ind}$

$$V = \frac{Q - Q_{ind}}{C_0}$$

$$\therefore V = \frac{Q}{\kappa C_0} = \frac{Q - Q_{ind}}{C_0}$$

$$\therefore Q_{ind} = Q\left(1 - \frac{1}{\kappa}\right)$$

$$\boxed{\frac{Q_{ind}}{Q} = 1 - 1/\kappa}$$

or $$\kappa = \frac{1}{1 - Q_{ind}/Q}$$

Very large κ allows Q_{ind} to be nearly equal to Q

Energy of a Capacitor

Energy of Capacitor is work it takes bring about charge distribution:

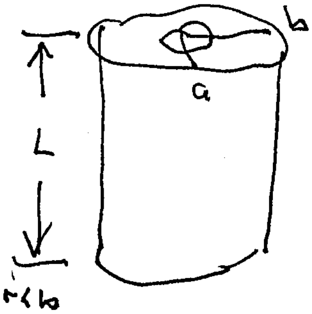
$U = \frac{1}{2} QV$ (recall)

$CV = Q$; $V = \frac{Q}{C}$; $Q = CV$

$$U = \frac{1}{2} QV = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

Take capacitance of cylinder

$$C_{cyl} = \frac{2\pi L \epsilon_0}{\ln(b/a)}$$



Compare U with $\epsilon_0 \int_{r>a}^{r<b} dV \frac{E^2}{2} \equiv U_E$

$dV = 2\pi r\, dr\, L$, $2\pi r E = \frac{Q}{\epsilon_0}$

$E = \frac{Q}{2\pi \epsilon_0 r}$

$$U_E = \frac{\epsilon_0 Q^2}{(2\pi)^2 \epsilon_0^2} \int_a^b \frac{2\pi r\, dr\, L}{2\, r^2} = \frac{Q^2 L}{4\pi\epsilon_0} \ln(b/a) = \frac{1}{2} \frac{Q^2}{C_{cyl}}$$