Lecture 8

Capacitors
Connecting wire establishes one equal potential at the two top plates...

What is capacitance of \( N \) capacitors \( C_1, C_2, ..., C_N \), connected in parallel

\[
(1) \quad \frac{1}{C_{\text{eff}}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_N}
\]

\[
(2) \quad C_{\text{eff}} = C_1 + C_2 + ... + C_N
\]

(3) undetermined

...and another at the two bottom plates, so voltage across capacitors connected in parallel is the same.

Figure 26-7 Physics for Engineers and Scientists 3/e
© 2007 W.W. Norton & Company, Inc.
Charge placed on an outside plate...

...induces opposite charge on an inside plate...

...so capacitors in series have the same magnitude of charge on each plate.

What is capacitance of N capacitors connected in series

1. \( \frac{1}{C_{\text{eff}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N} \)

2. \( C_{\text{eff}} = C_1 + C_2 + \ldots + C_N \)

3. not determined
To determine the net capacitance of a group of capacitors...

**What is Equivalent Capacitance**

(a) 14 μF

(b) 1.5 μF

(c) 0.5 μF

(d) 9 μF

---

...we must first identify any simple combinations, like these two 4.0-μF capacitors in parallel...

\[ C_{eq} = \left[ \frac{1}{C_1 + C_2} + \frac{1}{C_3} \right]^{-1} \]

\[ = \frac{1}{\frac{1}{9} + \frac{1}{6}} = \frac{24}{3 + 7} = \frac{24}{7} \text{ μF} \]

---

Figure 26-11a Physics for Engineers and Scientists 3/e © 2007 W.W. Norton & Company, Inc.
Several Capacitors

\[ E_{\text{in}} = \frac{Q}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0} \]

\[ V = E_{\text{in}} d = \frac{Q}{\varepsilon_0 A} d \]

\[ C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d} \]

\[ C_{\text{slab}} = \frac{\varepsilon_0 A}{d} \]
Cylinder

Grass of Wire

\[ E_r A = \frac{Q}{\varepsilon_0} \]

\[ E_r 2\pi r A = \frac{Q}{\varepsilon_0} \]

\[ E_r = \frac{Q}{2\pi r L \varepsilon_0} \]

\[ V = \int_0^b E_r \, dr = \int_0^b \frac{Q}{2\pi r L \varepsilon_0} \, dr = \frac{Q}{\ln \left( \frac{b}{a} \right)} \]

\[ C = \frac{Q}{\phi} = \frac{Q}{\varepsilon_0 \ln \left( \frac{b}{a} \right)} = \frac{2\pi r L \varepsilon_0}{\ln \left( \frac{b}{a} \right)} \]

\[ C_{cyl} = \frac{2\pi r L \varepsilon_0}{\ln \left( \frac{b}{a} \right)} \]
Capacitance Sphere

\[ \frac{Q}{E_0} = E_r A \]

\[ \frac{Q}{E_0} = E_r 4 \pi r^2 \]

\[ E_r = \frac{Q}{E_0 4 \pi r^2} \]

\[ V = \int_a^b dr E_r = \frac{Q}{4 \pi E_0} \int_a^b \frac{dr}{r^2} \]

\[ V = \frac{Q}{4 \pi E_0} \left( \frac{1}{r} \right) \bigg|_a^b \]

\[ V = \frac{Q}{4 \pi E_0} \left( \frac{1}{a} - \frac{1}{b} \right) \]

\[ C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4 \pi E_0} \left( \frac{1}{a} - \frac{1}{b} \right)} = \frac{4 \pi E_0 a b}{b - a} \]

\[ C_{\text{sphere}} = \frac{4 \pi E_0 a b}{b - a} \]
Proximity Capacitor

Interfacing Area \( \equiv A \)

Gap distance \( \equiv d \)

\( Q_1 \equiv \) charge on top plate, \( Q_2 \equiv \) charge on bottom plate

\[
E_1 = \frac{Q_1}{AE_0}, \quad E_2 = \frac{Q_2}{AE_0}
\]

\[
V = E_1 d = \frac{Q_1 d}{AE_0}, \quad V = E_2 d = \frac{Q_2 d}{AE_0} = \frac{Q d}{2AE_0}
\]

\[
C = \frac{Q}{V} = \frac{Q_1 + Q_2}{V}
\]

\( Q_1 = Q_2 \) \( \Rightarrow \)

\[
C = \frac{\Theta}{\frac{Q d}{2AE_0}} = \frac{2AE_0}{d}
\]
When a dielectric is placed between charged capacitor plates...

...charges in dielectric will respond to force exerted by electric field of charges on plates.

Dielectric material can increase capacitance significantly.

If for fixed $V_0$ causes charge $Q$ on plates without dielectric, then $Q = xQ_0$ on plates with dielectric $C \rightarrow xC_0$. 

Figure 26-13 Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.
(a) 
Atoms and many molecules have no electric dipole moment in zero field.

(b) 
In an electric field, electrons and nuclei remain bound together...

...but move slightly in opposite directions; a dipole moment is induced.

\[ E = 0 \]

\[ E = E_0 \]

\[ \frac{C_0}{\kappa C_0} = \text{capacitance without dielectric} \]

\[ \text{Capacitance with dielectric} \]
Relation between dielectric constant and induced surface charge

\[ Q_{\text{ind}} = Q (1 - \frac{1}{\varepsilon}) \]

Explanation:
- \( Q \) = charge on conducting plate
- \( Q_{\text{ind}} \) = surface charge on dielectric surface
- \( C_0 \) = capacitance without dielectric material filled with dielectric material
- \( \varepsilon \) = capacitance with dielectric material
- \( Q = \varepsilon C_0 V \)

Total charge on outer plate is

\[ Q = \varepsilon C_0 V \]

\[ Q - Q_{\text{ind}} = \frac{Q}{C_0} \]

Solve for \( V \)

\[ V = \frac{Q}{\varepsilon C_0} = \frac{Q - Q_{\text{ind}}}{C_0} \]

\[ Q_{\text{ind}} = Q - \frac{Q}{\varepsilon} \]

\[ Q_{\text{ind}} = Q (1 - \frac{1}{\varepsilon}) \]
Relation between $x$ and induced charge $Q_{\text{ind}}$

With dielectric $C = xC_0$, where $C_0$ is capacitance with no dielectric

$$V = \frac{Q}{xC_0}$$

But same voltage would appear without dielectric material if one takes the full charge, $Q = Q_{\text{ind}}$

$$V = \frac{Q_{\text{ind}}}{C_0}$$

$$\therefore V = \frac{Q}{xC_0} = \frac{Q_{\text{ind}}}{C_0}$$

$$\therefore Q_{\text{ind}} = Q \left(1 - \frac{1}{x}\right)$$

$$\frac{Q_{\text{ind}}}{Q} = 1 - \frac{1}{x}$$

or $x = \frac{1}{1 - Q_{\text{ind}}/Q}$

Very large $x$ allows $Q_{\text{ind}}$ to be nearly equal to $Q$
Energy of a Capacitor

Energy of capacitor is work it takes bringing about charge distribution:

\[ U = \frac{1}{2} QV \text{ (recall)} \]

\[ C \times V = Q \text{; } V = \frac{Q}{C} \text{; } Q = CV \]

\[ U = \frac{1}{2} QV = \frac{Q^2}{2C} = \frac{1}{2} CV^2 \]

Take capacitance of cylinder

\[ C_{cyl} = \frac{2\pi L \varepsilon_0}{\ln(b/a)} \]

Compare \( U \) with

\[ C \int dV \frac{E^2}{2} = U_E \]

\[ dV = 2\pi r dr L \]

\[ 2\pi r E = \frac{Q}{\varepsilon_0} \]

\[ E = \frac{Q}{2\pi \varepsilon_0 r} \]

\[ U_E = \frac{\varepsilon_0}{(2\pi)^2 \varepsilon_0} \int_{\frac{a}{2}}^{b} 2\pi r dr L = \frac{Q^2 L}{4\pi \varepsilon_0} \ln \left( \frac{b}{a} \right) = \frac{1}{2} \frac{Q^2}{C_{cyl}} \]