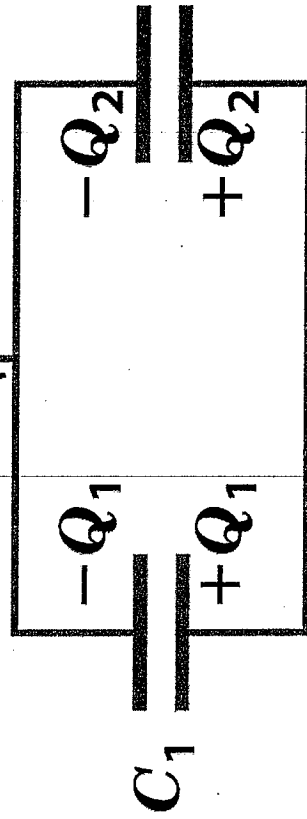


Lecture 8

Capacitors

Connecting wire establishes one equal potential at the two top plates...



...and another at the two bottom plates, so voltage across capacitors connected in parallel is the same.

What is Capacitance of N capacitors C_1, C_2, \dots, C_N , connected in parallel

$$(1) \frac{1}{C_{\text{eff}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

$$(2) C_{\text{eff}} = C_1 + C_2 + \dots + C_N$$

(3) undetermined

Figure 26-7 Physics for Engineers and Scientists 3/e
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Charge placed on an outside plate...

$-Q$

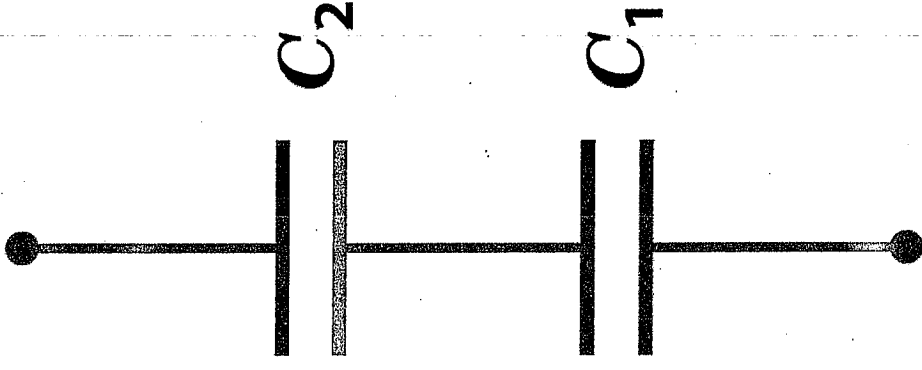
...induces opposite charge on an inside plate...

$+Q$

$-Q$

$+Q$

...so capacitors in series have the same magnitude of charge on each plate.



What is capacitance of N capacitors connected in series

(1) $\frac{1}{C_{\text{eff}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$

(2) $C_{\text{eff}} = C_1 + C_2 + \dots + C_N$

(3) not determined

Figure 26-9 Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.

To determine the net capacitance of a group of capacitors...

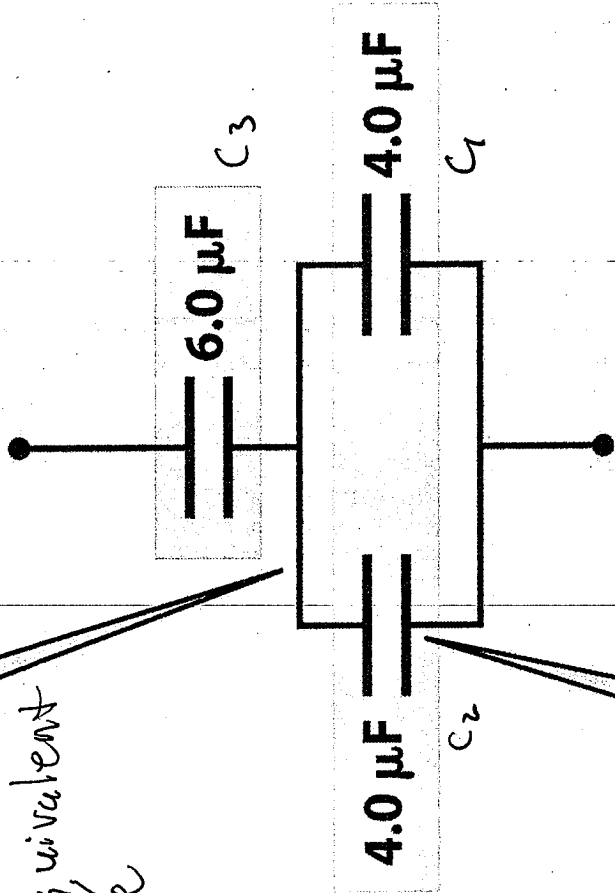
What is equivalent capacitance

(a) 14 μF

(b) **1.5 μF**

(c) ~~18~~ μF

(d) ~~18~~ μF



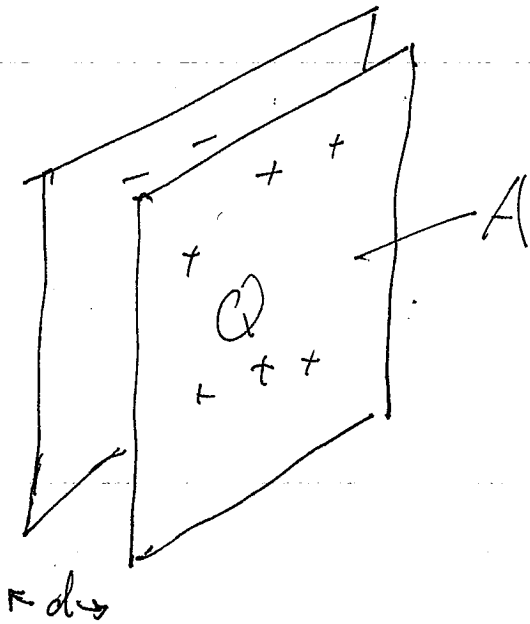
... we must first identify any simple combinations, like these two 4.0- μF capacitors in parallel...

Figure 26-11a Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.

$$C_{\text{eff}} = \left[\frac{1}{C_1 + C_2} \right] + \frac{1}{C_3}$$

$$= \frac{1}{\left(\frac{1}{8} + \frac{1}{6}\right)} + \frac{1}{6} = \frac{24}{3+7} = \frac{24}{7} \mu\text{F}$$

Several Capacitors & their Capacitance



Parallel plate
Capacitor

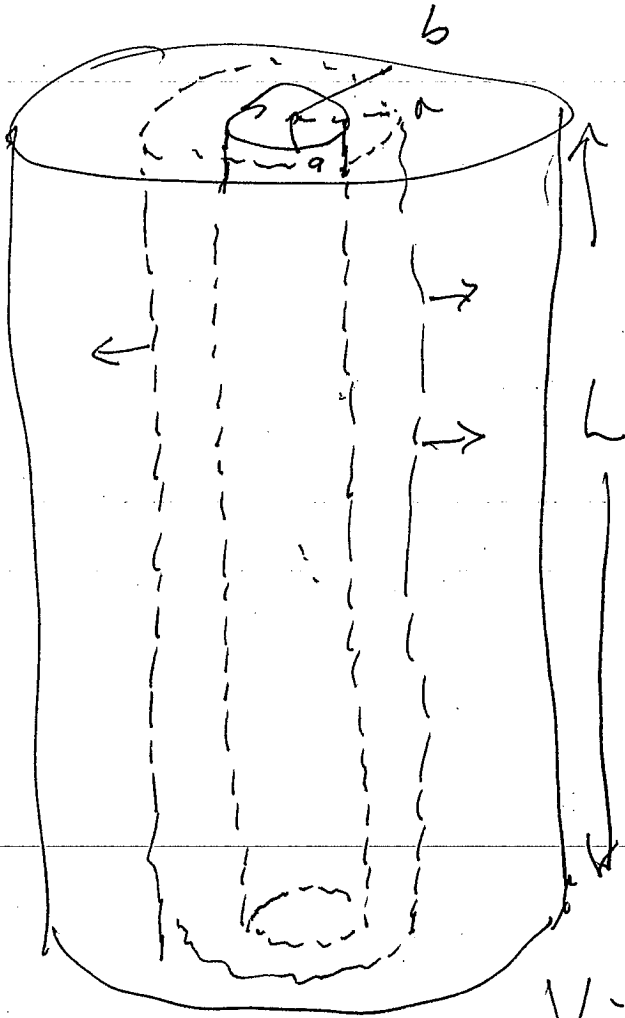
$$E_{in} = \frac{Q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}$$

$$V = E_{in} d = \frac{Q}{\epsilon_0 A} d$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q d}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d}$$

$$C_{slab} = \frac{\epsilon_0 A}{d}$$

Cylinder



Gauss Law
of r

$$E_r A = \frac{Q}{\epsilon_0}$$

$$E_r 2\pi r A = \frac{Q}{\epsilon_0}$$

$$E_r = \frac{Q}{2\pi r L \epsilon_0}$$

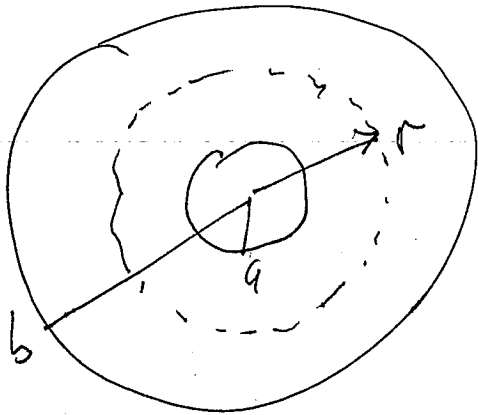
$$V = \int_a^b E_r dr = \frac{Q}{2\pi L \epsilon_0} \int_a^b \frac{dr}{r} = \frac{Q}{2\pi L \epsilon_0} \ln \frac{b}{a}$$

$$V = \frac{Q}{2\pi L \epsilon_0} \ln \left(\frac{b}{a} \right)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi L \epsilon_0} \ln \left(\frac{b}{a} \right)} = \frac{2\pi L \epsilon_0}{\ln \left(\frac{b}{a} \right)}$$

$$C_{\text{cyl}} = \frac{2\pi L \epsilon_0}{\ln \left(\frac{b}{a} \right)}$$

Capacitance Sphere



$$\frac{Q}{\epsilon_0} = E_r A$$
$$= E_r 4\pi r^2$$

$$E_r = \frac{Q}{\epsilon_0 4\pi r^2}$$

$$V = \int_a^b dr E_r = \frac{Q}{4\pi\epsilon_0} \int_a^b \frac{dr}{r^2}$$

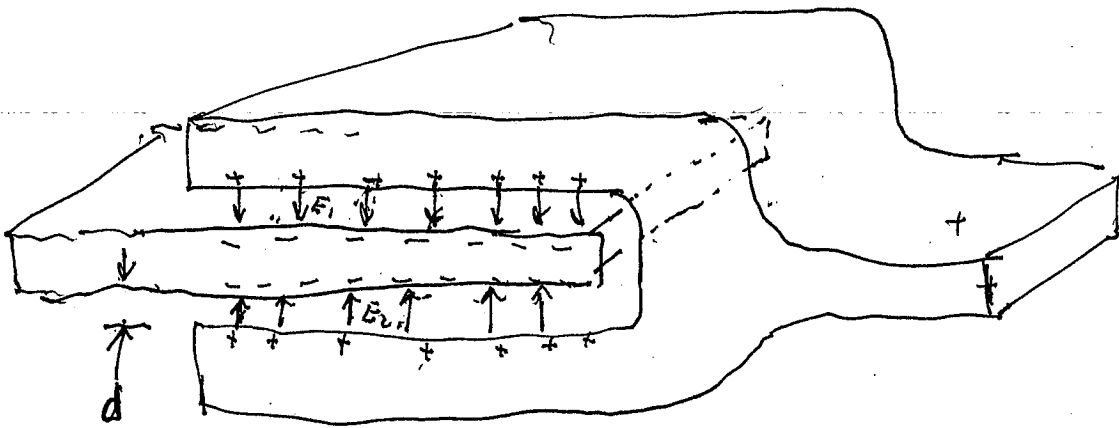
$$= \frac{Q}{4\pi\epsilon_0} \left(-\frac{1}{r}\right) \Big|_a^b$$

$$V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\epsilon_0 ab}{b-a}$$

$$C_{\text{sphere}} = \frac{4\pi\epsilon_0 ab}{b-a}$$

Prong Capacitor



Interfacing Area $\equiv A$

Gap distance $\equiv d$

$Q_t \equiv$ charge on top plate; $Q_b \equiv$ charge on bottom plate

$$E_1 = \frac{Q_t}{A\epsilon_0}, \quad E_2 = \frac{Q_b}{A\epsilon_0}$$

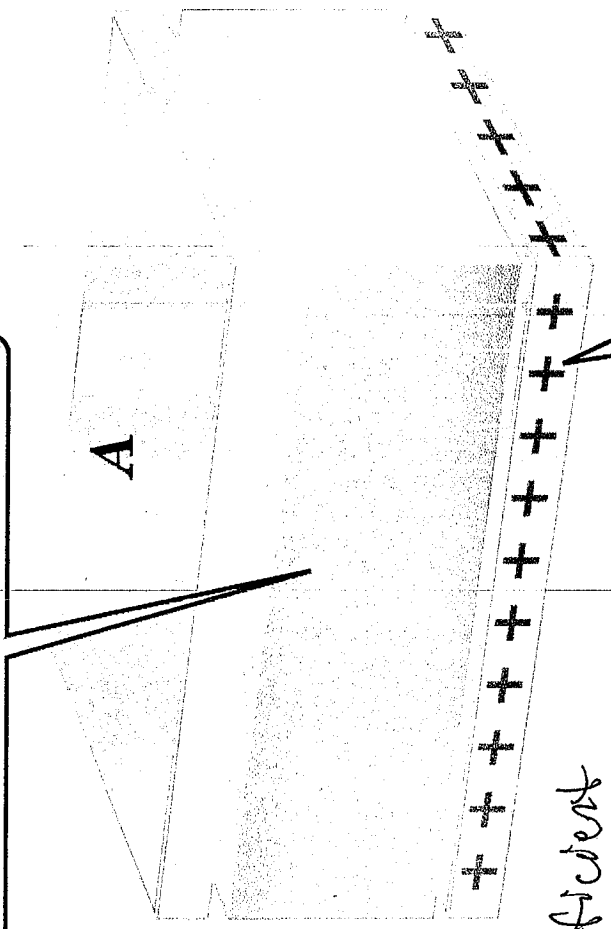
$$V = E_1 d = \frac{Q_t d}{A\epsilon_0}, \quad V = E_2 d = \frac{Q_b d}{A\epsilon_0} = \frac{Q d}{2A\epsilon_0}$$

$$C = \frac{Q}{V} = \frac{Q_t + Q_b}{V}$$

$$Q_1 = Q_2; \quad Q = 2Q_1$$

$$C = \frac{Q}{\frac{Q d}{2A\epsilon_0}} = \frac{2A\epsilon_0}{d}$$

When a dielectric is placed between charged capacitor plates...



dielectric material with dielectric coefficient κ

Dielectric material can increase capacitance significantly

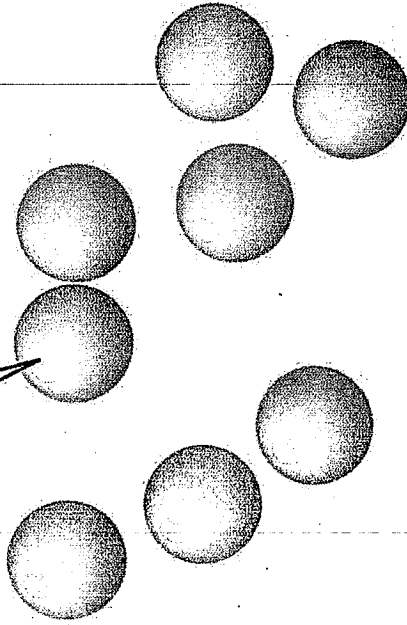
...charges in dielectric will respond to force exerted by electric field of charges on plates.

Figure 26-13 Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.

If for fixed V_0 causes charge Q_0 on plates without dielectric then $Q = \kappa Q_0$ on plates with dielectric; $C \rightarrow \kappa C_0$

(a)

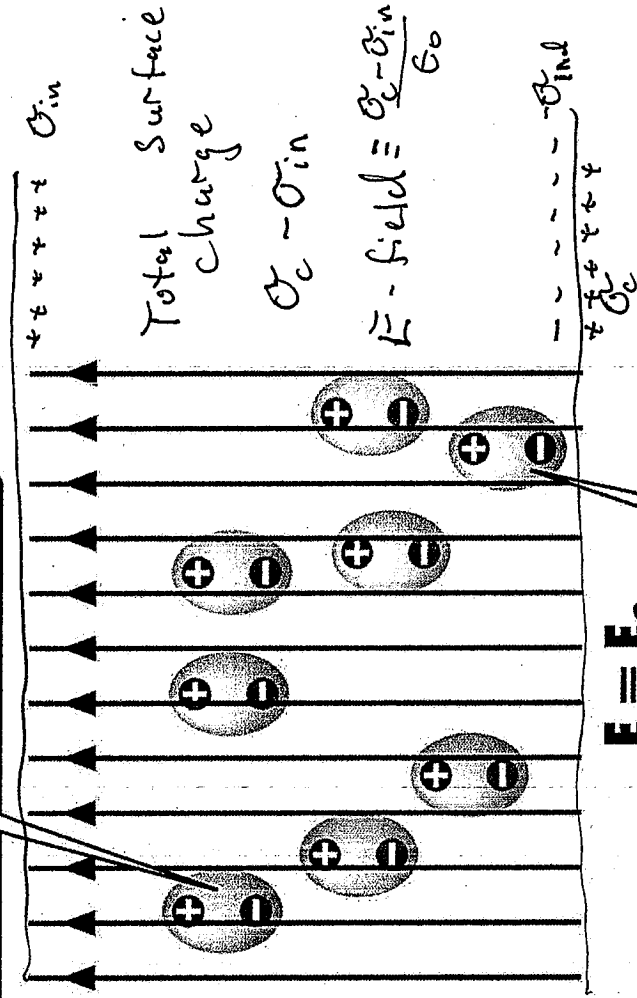
Atoms and many molecules have no electric dipole moment in zero field.



$E = 0$

(b)

In an electric field, electrons and nuclei remain bound together...



$E = E_0$

...but move slightly in opposite directions; a dipole moment is induced.

Figure 26-14. Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.

$C_0 \equiv$ capacitance without dielectric
 $\kappa C_0 \equiv$ capacitance with dielectric

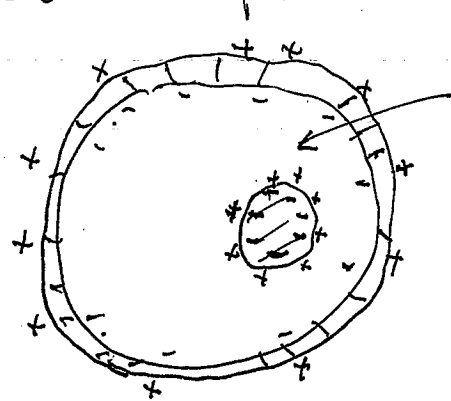
Relation between dielectric constant and induced surface charge

$$Q_{ind} = Q \left(1 - \frac{1}{\kappa} \right)$$

explanation: $Q \equiv$ charge on conducting plate

$Q_{ind} \equiv$ surface charge on dielectric surface

$C_0 \equiv$ capacitance without dielectric material



filled with dielectric material
 $\kappa C_0 \equiv$ capacitance with dielec...

$$Q = \kappa C_0 V$$

Total charge on outer plate is ~~is~~

$$Q = \kappa C_0 V \quad \text{solve for } V \quad Q - Q_{ind} = C_0 V \quad \left(\begin{array}{l} \text{Using both} \\ \text{conductor} \\ \text{and} \\ \text{dielectric} \\ \text{charge} \end{array} \right)$$

$$V = \frac{Q}{\kappa C_0} = \frac{Q - Q_{ind}}{C_0}$$

$$\therefore Q_{ind} = Q - \frac{Q}{\kappa}$$

$$Q_{ind} = Q \left(1 - \frac{1}{\kappa} \right)$$

Relation between κ
and induced charge, Q_{ind}

With dielectric $C = \kappa C_0$, where
 C_0 is capacitance with no dielectric

$$V = \frac{Q}{\kappa C_0}$$

But same voltage would appear
without dielectric material if
one takes the full charge,

$$V = \frac{Q - Q_{ind}}{C_0}$$

$$\therefore V = \frac{Q}{\kappa C_0} = \frac{Q - Q_{ind}}{C_0}$$

$$\therefore Q_{ind} = Q \left(1 - \frac{1}{\kappa} \right)$$

$$\boxed{\frac{Q_{ind}}{Q} = 1 - \frac{1}{\kappa}}$$

$$\text{or } \kappa = \frac{1}{1 - Q_{ind}/Q}$$

Very large κ allows Q_{ind} to be
nearly equal to Q

Energy of a capacitor

Energy of capacitor is work it takes bring about charge distribution:

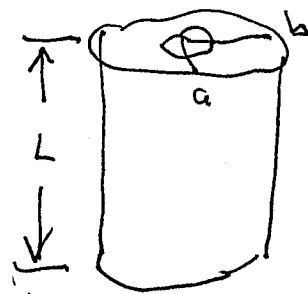
$$U = \frac{1}{2} QV \quad (\text{recall})$$

$$CV = Q; \quad V = \frac{Q}{C}; \quad Q = CV$$

$$U = \frac{1}{2} QV = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

Take capacitance of cylinder

$$C_{\text{cyl}} = \frac{2\pi L \epsilon_0}{\ln(b/a)}$$



Compare U with

$$dV = 2\pi r dr L$$

$$\epsilon_0 \int_{a \rightarrow b} dV \frac{E^2}{2} = U_E$$
$$2\pi r E = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi \epsilon_0 r}$$

$$U_E = \frac{\epsilon_0 Q^2}{(2\pi)^2 \epsilon_0^2} \int_a^b \frac{2\pi r dr L}{2 r^2} = \frac{Q^2 L}{4\pi \epsilon_0} \ln(b/a) = \frac{1}{2} \frac{Q^2}{C_{\text{cyl}}}$$