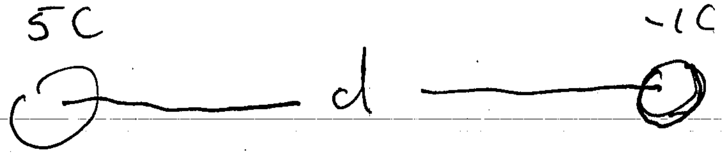
Lecture 8

Review Problems

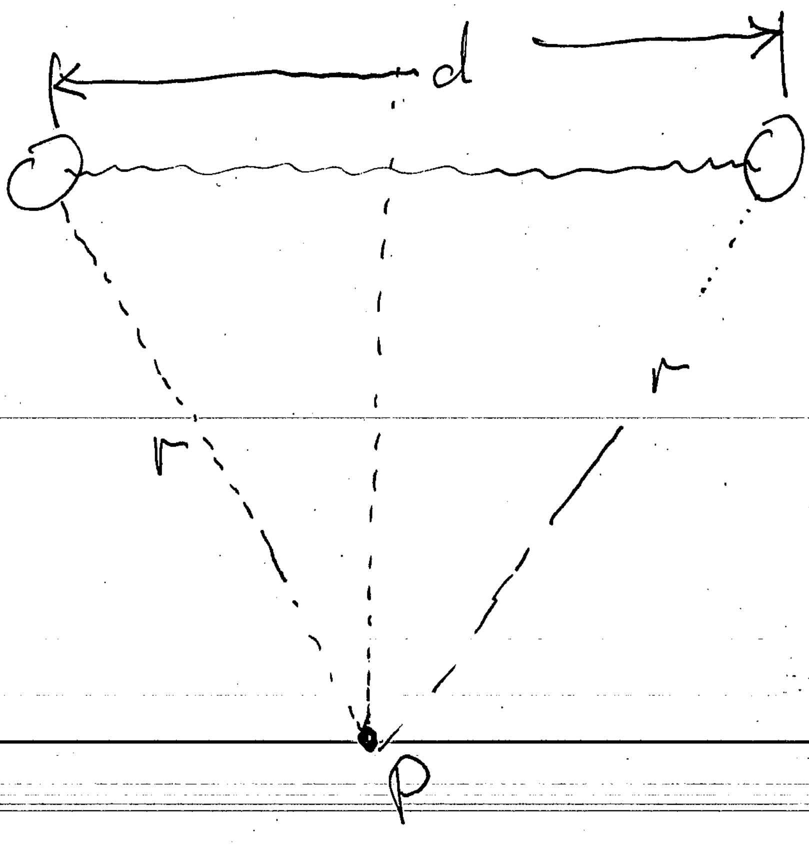
&

Capacitors

Given two identical conducting spheres, a distance d apart, with charge 5 C and -1 C on each of them.



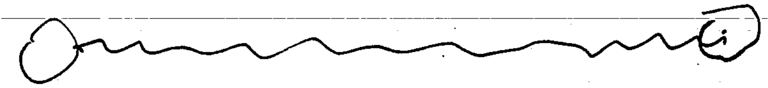
Now connect a thin wire between them. What is the electric field at point P?



This is a two part question:

First, what is charge on each conducting sphere after wire touches

5C ⇐ initial charge → -1C

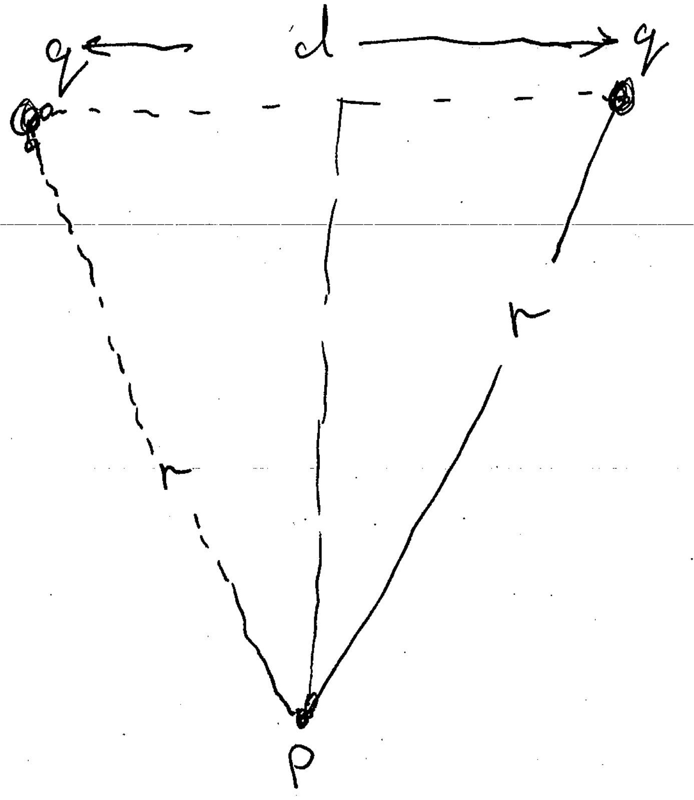


(a) 5C & -1C

(b) 2C & 2C

(c) 6C & 0C

second part find electric field from 2 spheres each with a charge $q = 2$ C



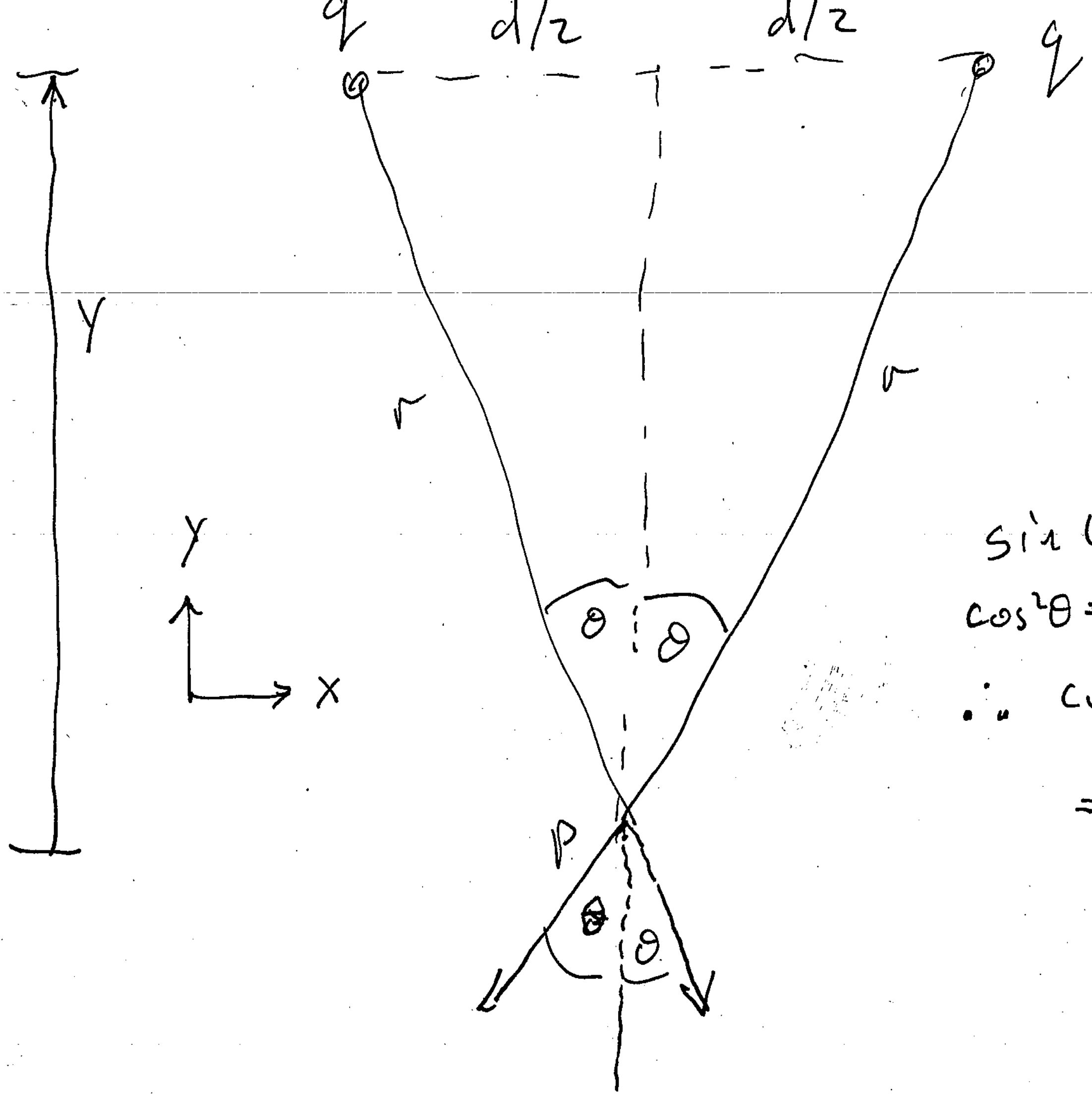
Two methods

(a) direct E-field method

(b) Potential Method

second method faster

Direct Method



$$\sin\theta = \frac{d}{2r}$$

$$\cos^2\theta + \sin^2\theta = 1$$

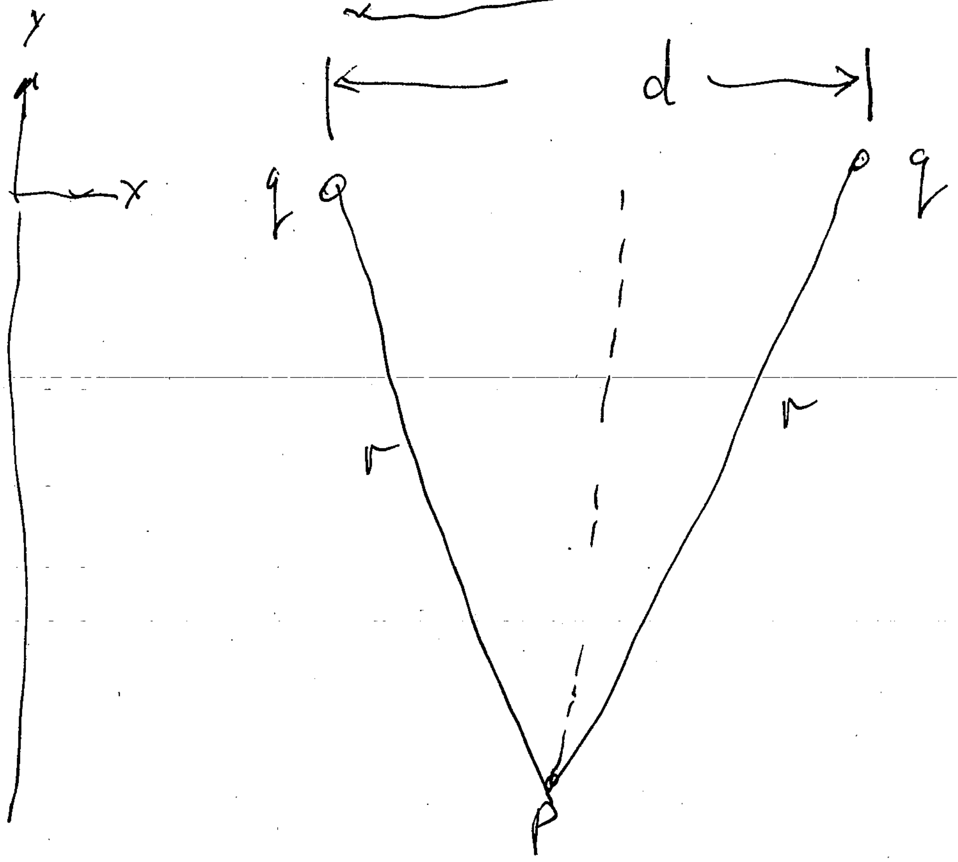
$$\therefore \cos\theta = \left[1 - \frac{d^2}{4r^2}\right]^{1/2}$$

$$= \frac{1}{r}\left[r^2 - \frac{d^2}{4}\right]^{1/2}$$

$$\vec{E} = -\left(\frac{kq}{r^2}\cos\theta + \frac{kq}{r^2}\cos\theta\right)\hat{y}$$

$$= -\frac{2kq}{r^3}\left[r^2 - \frac{d^2}{4}\right]^{1/2}\hat{y} = \frac{-2kq|y|}{\left[y^2 + d^2/4\right]^{3/2}}\hat{y}$$

Potential method



$$V = \frac{kq}{r} + \frac{kq}{r} = 2\frac{kq}{r}$$

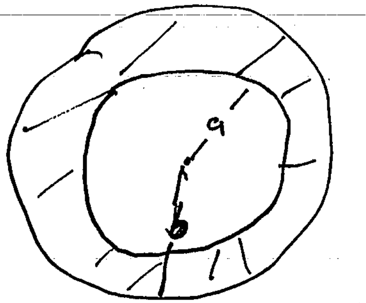
$$= 2\frac{kq}{\left[y^2 + d^2/4\right]^{1/2}}$$

$$E_y = -\frac{\partial V}{\partial y}\hat{y} = \frac{2kq\,y\,\hat{y}}{\left[y^2+d^2/4\right]^{3/2}} = \frac{-2kq\,|y|\,\hat{y}}{\left[y^2+d^2/4\right]^{1/2}}$$

Find the electric fields in symmetric spheres and cyclinders

Uniform charge density in shaded region
Total charge Q

sphere

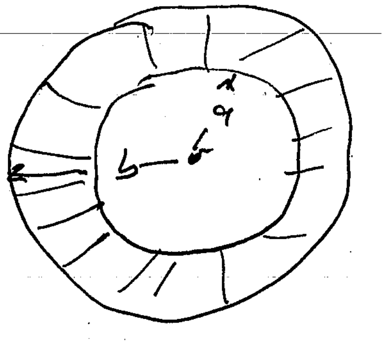


$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4\pi}{3}(b^3 - a^3)}$$

For $r<a$

$E=0$

cylinder (length L)
$L >> b$



$$\rho = \frac{Q}{V} = \frac{Q}{L\pi(b^2 - a^2)}$$

For $r<a$

$E=0$

why?

Would $E(r=0)=0$ if the inner surface was not concentric with the outer surface?

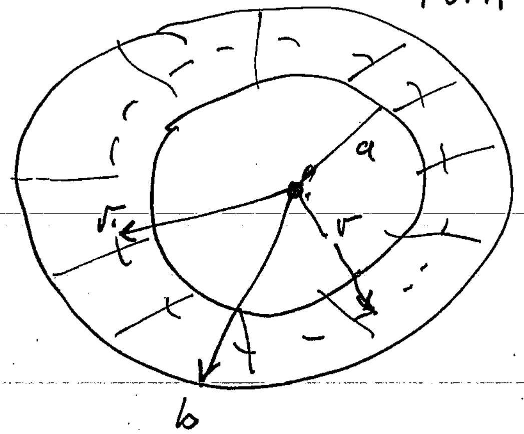
(a) yes

(b) no

Would $E(r=0)=0$ if shaded material were a conductor and (a) and (b) were not concentric? (a) yes (b) no (c)

Back to orignal Problem
what is E field for
$a < r < b$

Total charge Q



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sphere

$$\rho = \frac{3Q}{4\pi(b^3-a^3)}$$

Guass' Law

$$E_r A(r) = q(r)/\epsilon_0$$

$$E_r 4\pi r^2 = \rho V(r)/\epsilon_0$$

$$= \rho \frac{4\pi}{3}(r^3-a^3)/\epsilon_0$$

$$E_r = \frac{\rho(r^3-a^3)}{3\epsilon_0 \; r^2}$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{r^3-a^3}{b^3-a^3}\frac{1}{r^2}$$

cylinder $(b \ll L)$

$$\rho = \frac{Q}{\pi(b^2-a^2)L}$$

Guass' Law

$$E_r A(r) = q(r)/\epsilon_0$$

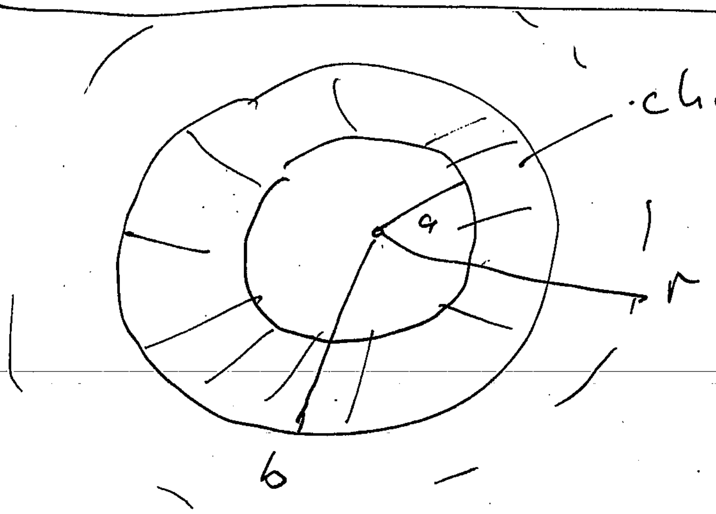
$$E_r 2\pi r L = \rho V(r)/\epsilon_0$$

$$= \rho\pi(r^2-a^2)L/\epsilon_0$$

$$E_r = \frac{\rho}{2\epsilon_0}\frac{r^2-a^2}{r}$$

$$= \frac{Q}{2\pi\epsilon_0}\frac{r^2-a^2}{b^2-a^2}\frac{1}{L}$$

What is E-field for $r > b$



sphere

Guass' Law

$E_r A(r) = Q/\epsilon_0$

$E_r 4\pi r^2 = Q/\epsilon_0$

$E_r = \frac{Q}{4\pi r^2 \epsilon_0}$

Same as point charge

cylinder ($b \ll L$)

Guass' Law

$E_r A(r) = Q/\epsilon_0$

$E_r 2\pi r L = \frac{Q}{\epsilon_0}$

$E_r = \frac{Q}{L} \frac{1}{2\pi r \epsilon_0}$

$= \frac{\lambda}{2\pi r \epsilon_0}$

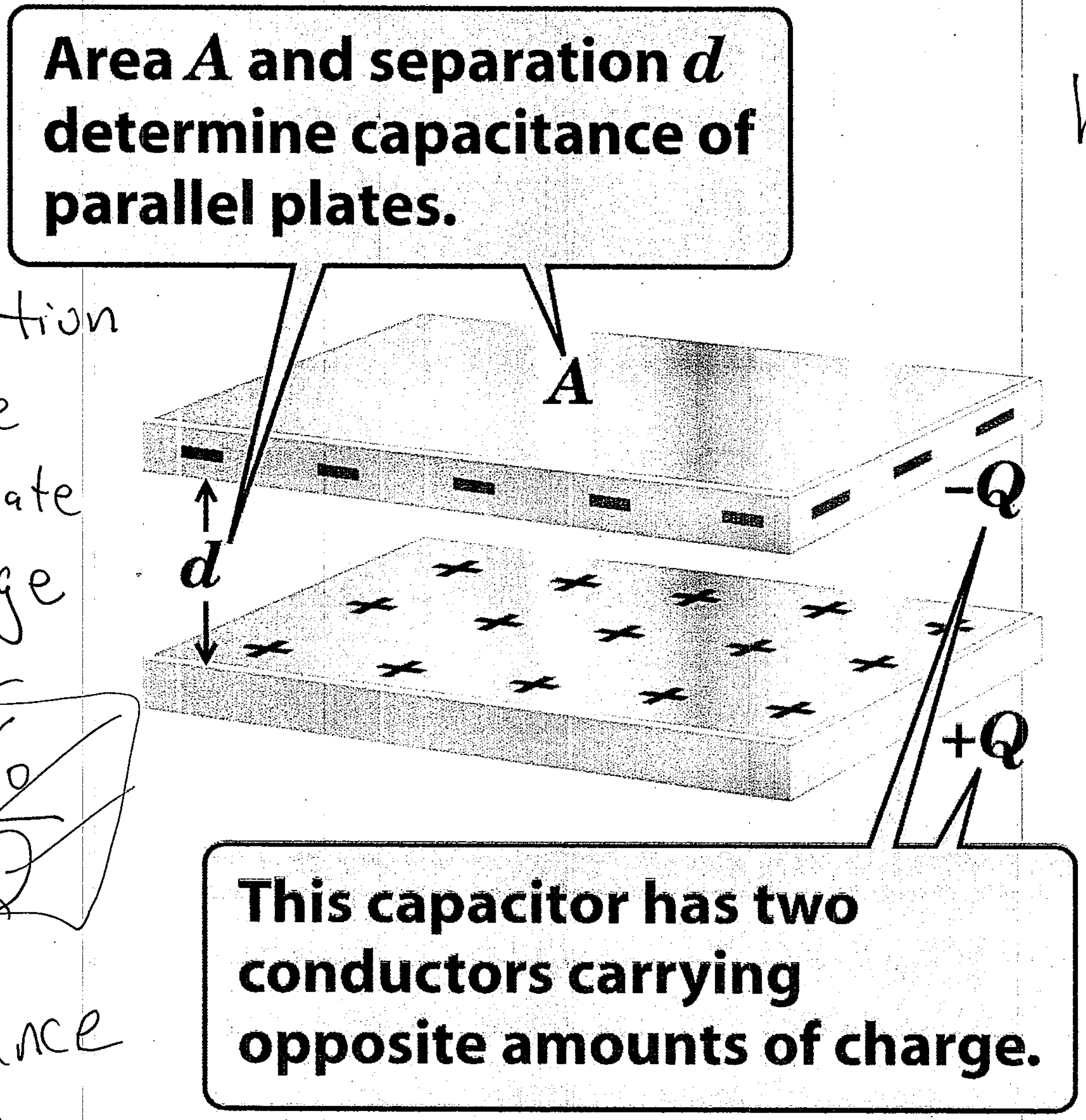
Same as linear charge density on axis

What is E_r is $r \gg L$

(a) $\frac{Q}{4\pi r^2 \epsilon_0}$, (b) $\frac{Q}{L 2\pi r \epsilon_0}$ (c) not enough info.

Find relation of voltage across plate and charge

$$\cancel{\boxed{C = \frac{V_0}{Q}}}$$

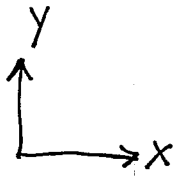


Area A and separation d determine capacitance of parallel plates.

This capacitor has two conductors carrying opposite amounts of charge.

Figure 26-2 Physics for Engineers and Scientists 3/e

V_0



Capacitance

$$C = \frac{Q}{V_0}$$

$$\vec{E} = \frac{Q}{A\epsilon_0}\,\hat{y}$$

Voltage Across plate $\equiv V_0 = E d = \dfrac{Qd}{A\epsilon_0}$

Capacitance $= C \equiv \dfrac{Q}{V_0}$

$$= \frac{Q}{Qd/A\epsilon_0} = \frac{\epsilon_0 A}{d}$$

$C = \dfrac{\epsilon_0 A}{d} \equiv$ capacitance of parallel plate capacitor

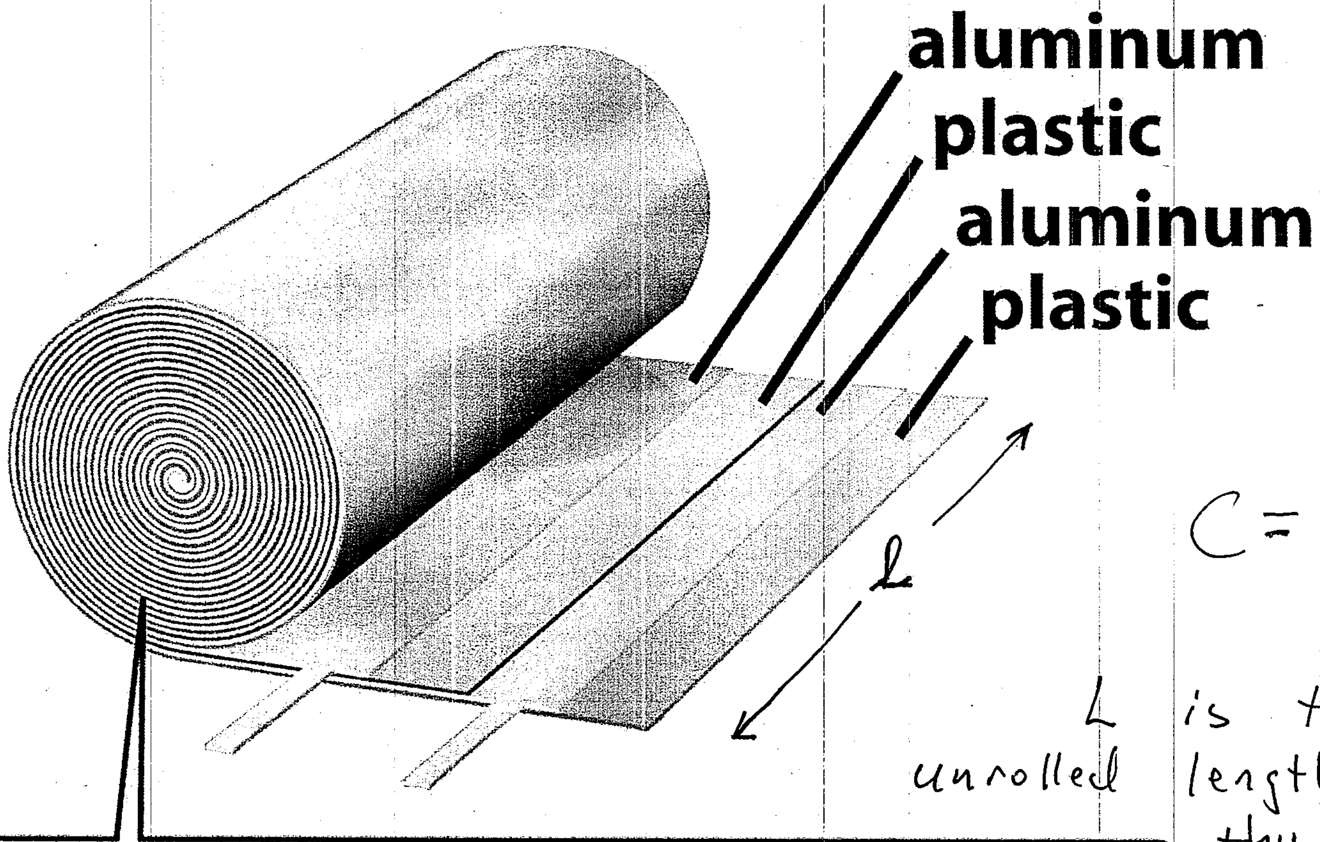
$\equiv$ unit is Farad

(Capacitance is generally a very small number $\mu\mu F \equiv$ picoF)

C for 10 cm × 10 cm ; $d = .2$ cm

$$C \simeq \frac{\epsilon_0 A}{d} \simeq \frac{10^{-11} \times 10^{-2}}{2 \times 10^{-3}} = 5 \times 10^{-11}$$

$$= 50 \text{ pF}$$



To provide large capacitance in a small volume, sheets of large area are rolled up.

Figure 26-3b Physics for Engineers and Scientists 3/e

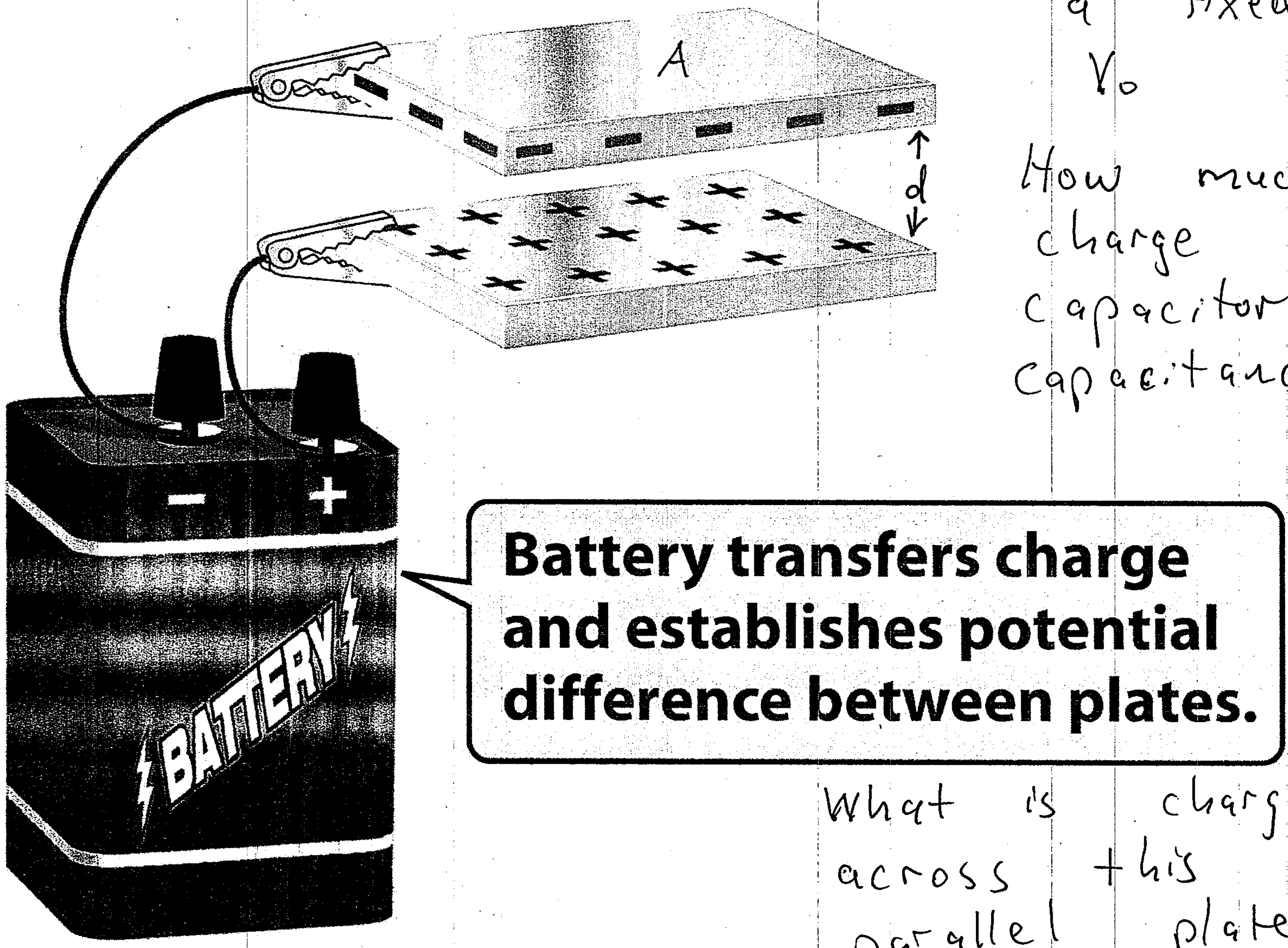
$$C = \frac{L\ell}{d}\epsilon_0$$

L is the unrolled length of this capacitor

$Q = C V_0$

Battery Supplies a fixed Voltage V_0

How much charge across capacitor of capacitance C?



Battery transfers charge and establishes potential difference between plates.

Figure 26-4 Physics for Engineers and Scientists 3/e
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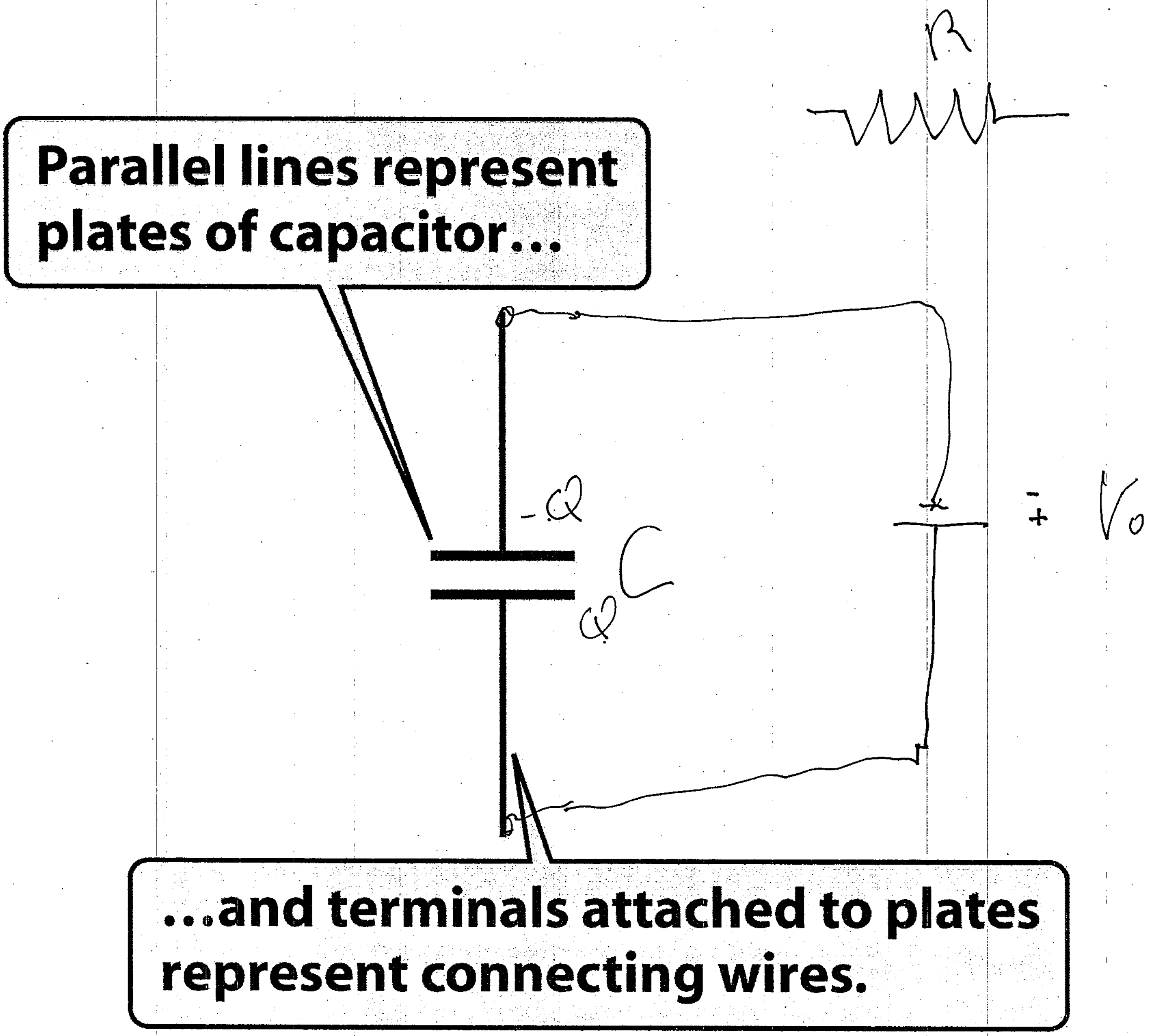
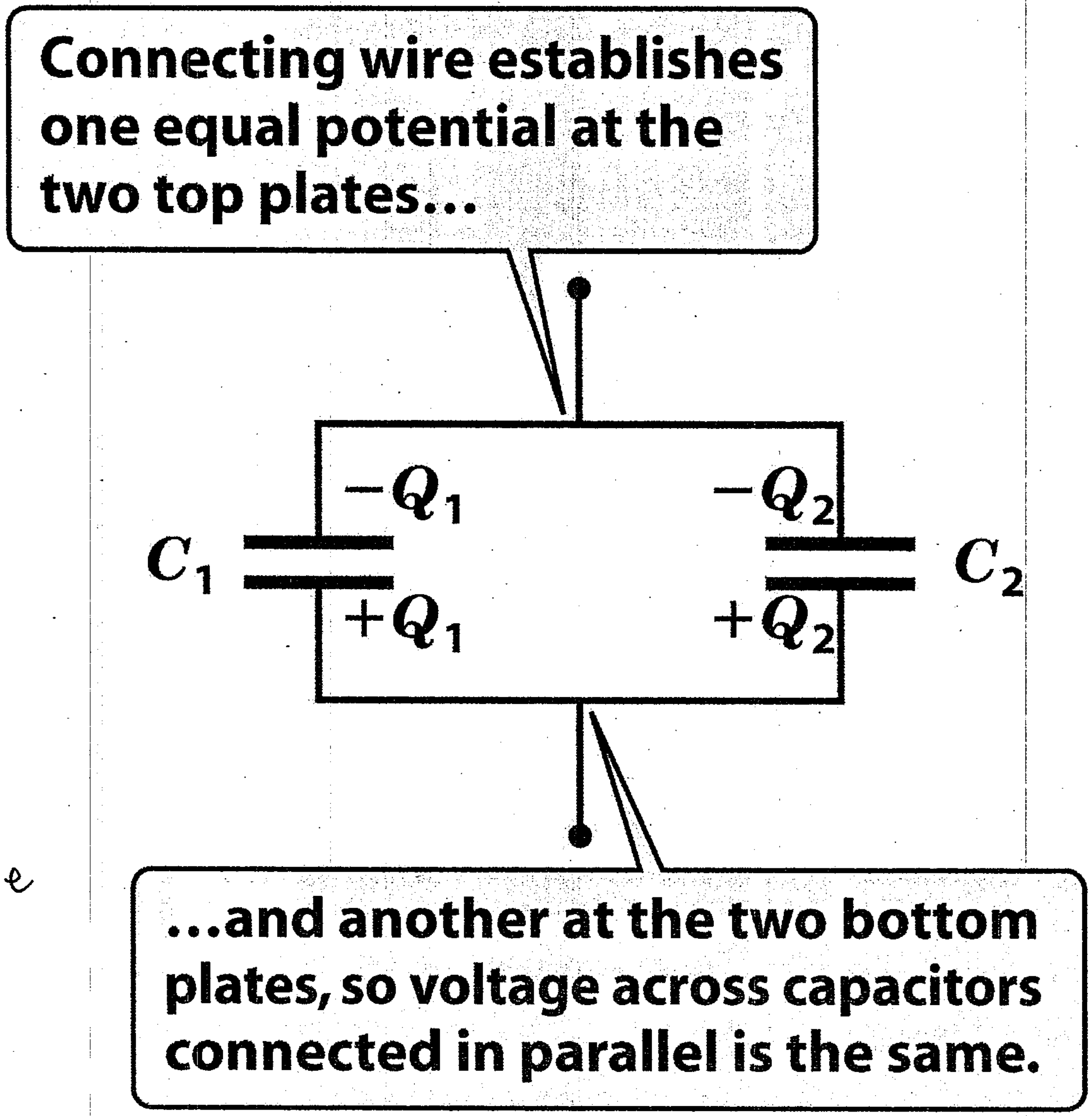
What is charge across this parallel plate capacitor?



Parallel lines represent plates of capacitor…

…and terminals attached to plates represent connecting wires.

Figure 26-6 Physics for Engineers and Scientists 3/e



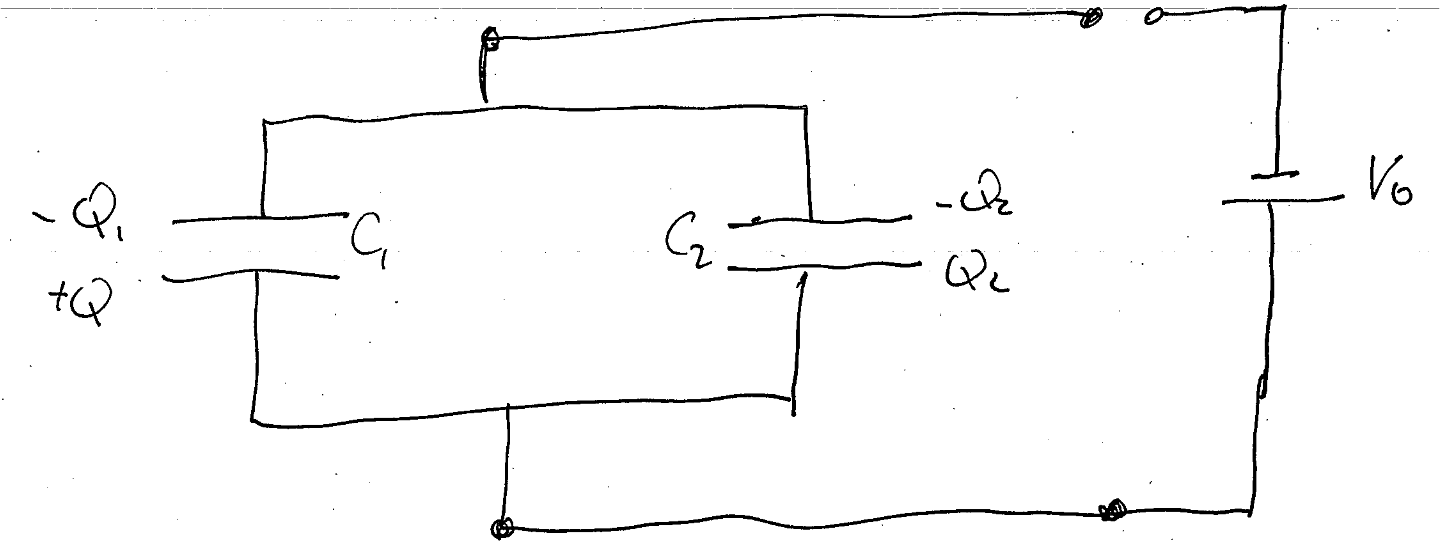
Connecting wire establishes one equal potential at the two top plates…

…and another at the two bottom plates, so voltage across capacitors connected in parallel is the same.

Figure 26-7 Physics for Engineers and Scientists 3/e

Find effective capacitance of two capacitors connected in parallel.

Capacitance of two capacitors, C_1 and C_2 in parallel



$$C_{eff} = \frac{Q_{total}}{V_0} = \frac{Q_1 + Q_2}{V_0}$$

$$= \frac{C_1 \cancel{V_0} + C_2 \cancel{V_0}}{\cancel{V_0}}$$

$$= C_1 + C_2$$

$$\boxed{C_{eff} = C_1 + C_2}$$

≡ total capacitance for two capacitors in parallel

Charge placed on an outside plate…

…induces opposite charge on an inside plate…

…so capacitors in series have the same magnitude of charge on each plate.

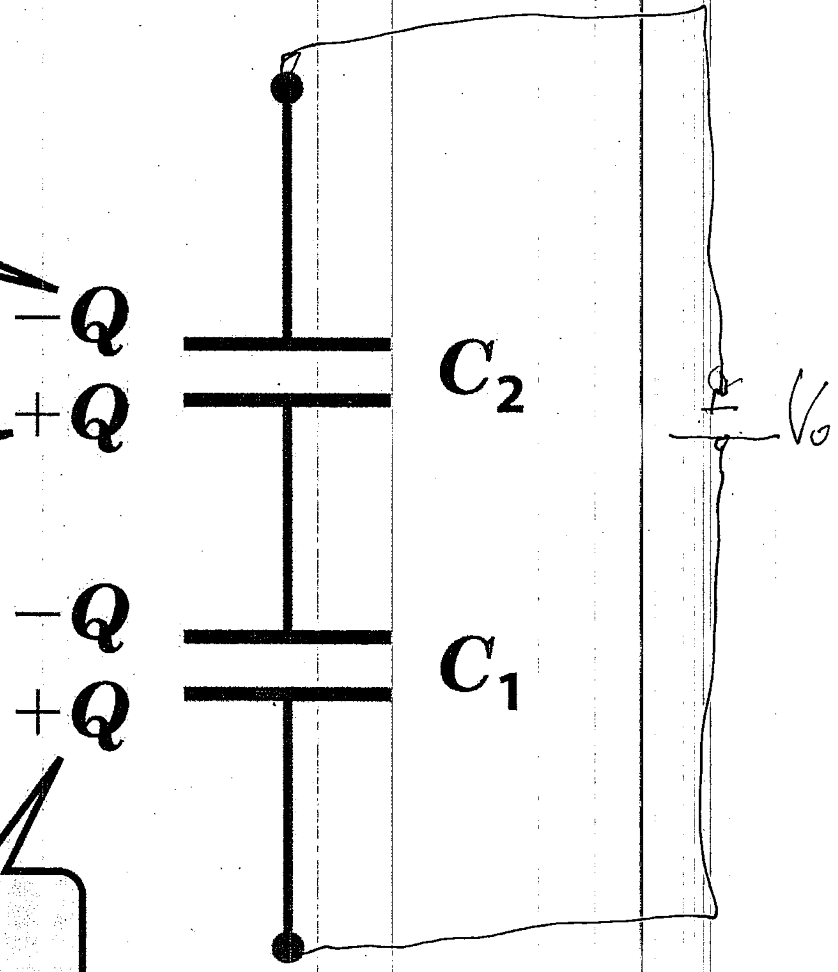
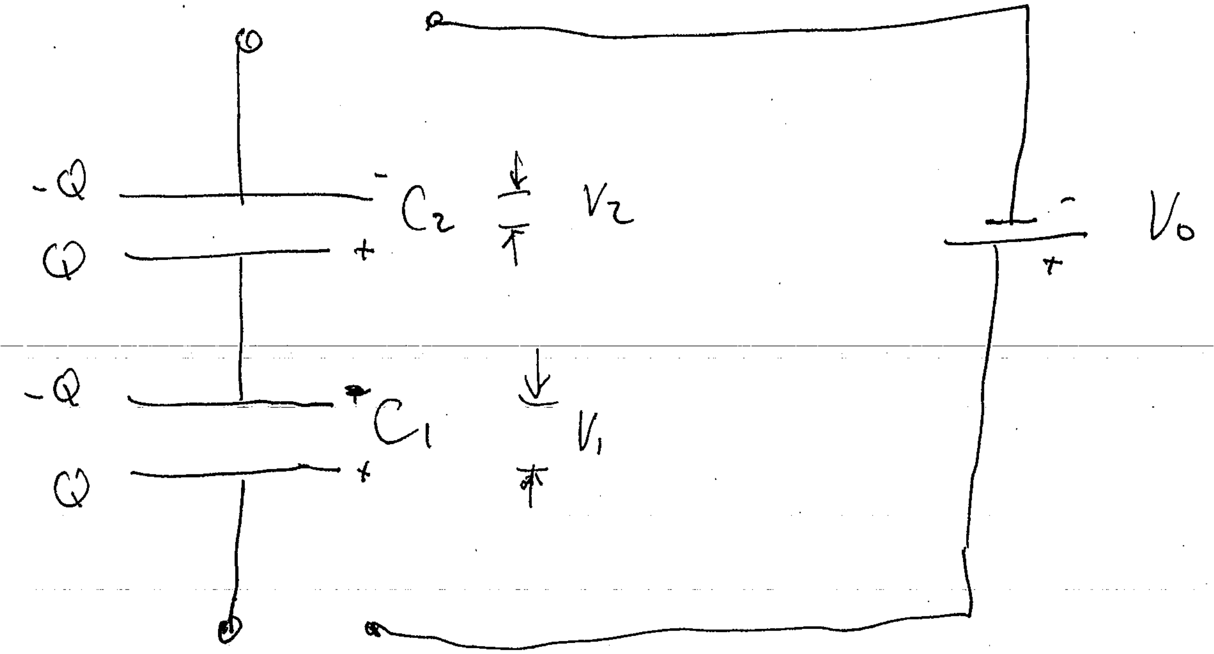


Figure 26-9 Physics for Engineers and Scientists 3/e

Capacitors in Series



$$Q = C_{eff} V_0$$

$$\frac{Q}{C_{eff}} = V_0 = V_1 + V_2$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$$

effective capacitance of capacitors linked in series