Lecture #6

Potentials

Electric Potential

$\Delta V = -\vec{E} \cdot \vec{\Delta S}$ (unit of potential ≡ volt)

Leads to:

$V = -\int_{-\infty}^{\vec{r}} \vec{E} \cdot d\vec{s}$ ≡ i.e. electric potential at point $\vec{r}$, related to electric field

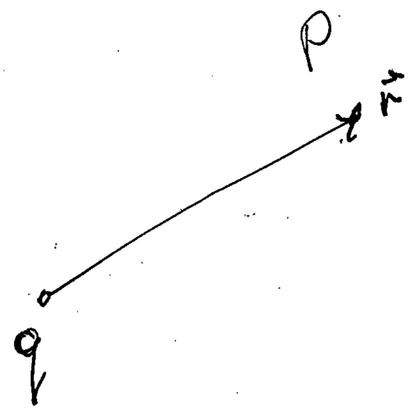
Also leads to:

$$\vec{E} = -\hat{x}\,\frac{\partial V(x,y,z)}{\partial x} - \hat{y}\,\frac{\partial V(x,y,z)}{\partial y} - \hat{z}\,\frac{\partial V(x,y,z)}{\partial z}$$

(this is inverse relation between electric field, and electric potential

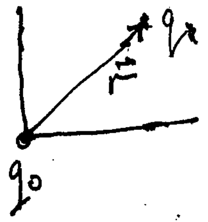
(electric potential frequently called "voltage")

Electric Potential of a point charge q



$$V(r) = \frac{kq}{|\vec{r}|} = \frac{q}{4\pi\epsilon_0 |\vec{r}|}$$

$$V(r) = \frac{kq_0}{r}$$

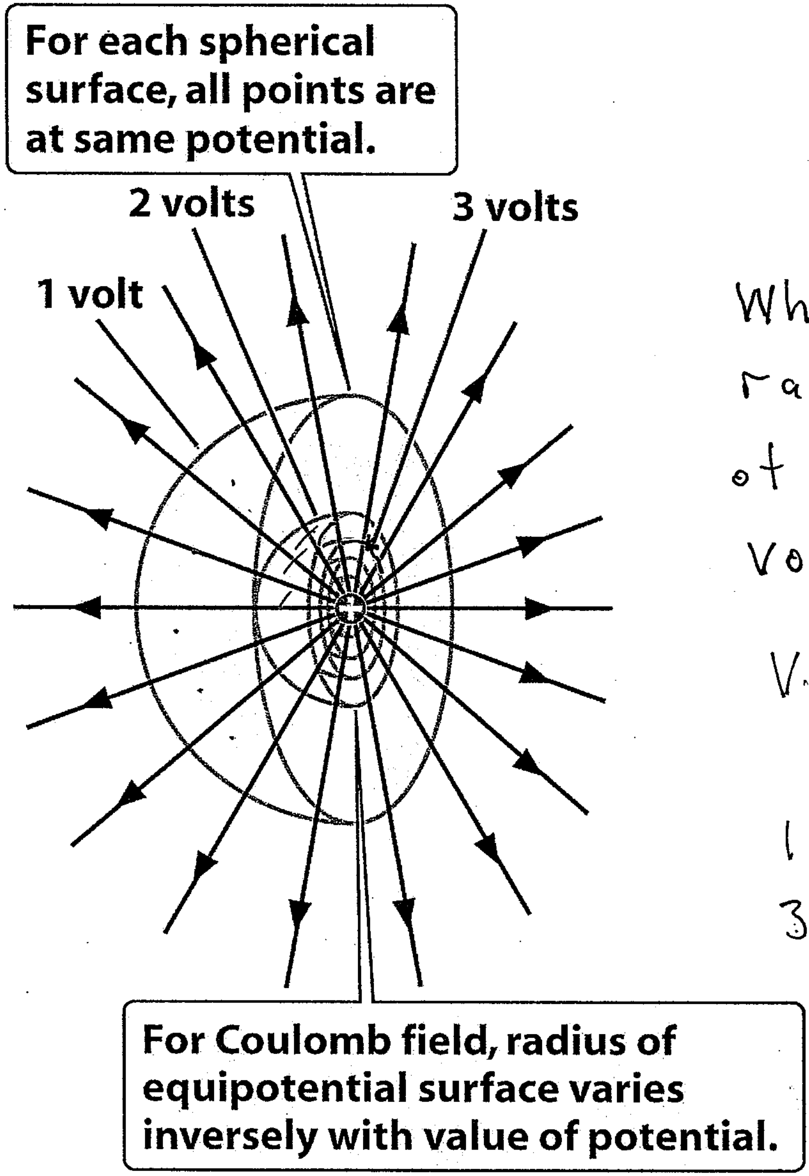


$$E_r = -\frac{\partial V}{\partial r} = -kq_0\frac{\partial}{\partial r}\frac{1}{r} = \frac{kq_0\hat{r}}{r^2}$$

Given a point charge q_0, at the origin, how much work must it take to bring charge q_1 to point r?

Note:

Equipotential surfaces are perpendicular to electric field lines



For each spherical surface, all points are at same potential.

For Coulomb field, radius of equipotential surface varies inversely with value of potential.

Figure 25-24 Physics for Engineers and Scientists 3/e

What is the ratio of radii of the 1 : 2 : 3 volt surface?

$V(r) = \frac{kq_0}{r}$

1V : 2V : 3V

3 : 3/2 : 1

1. charge q_1

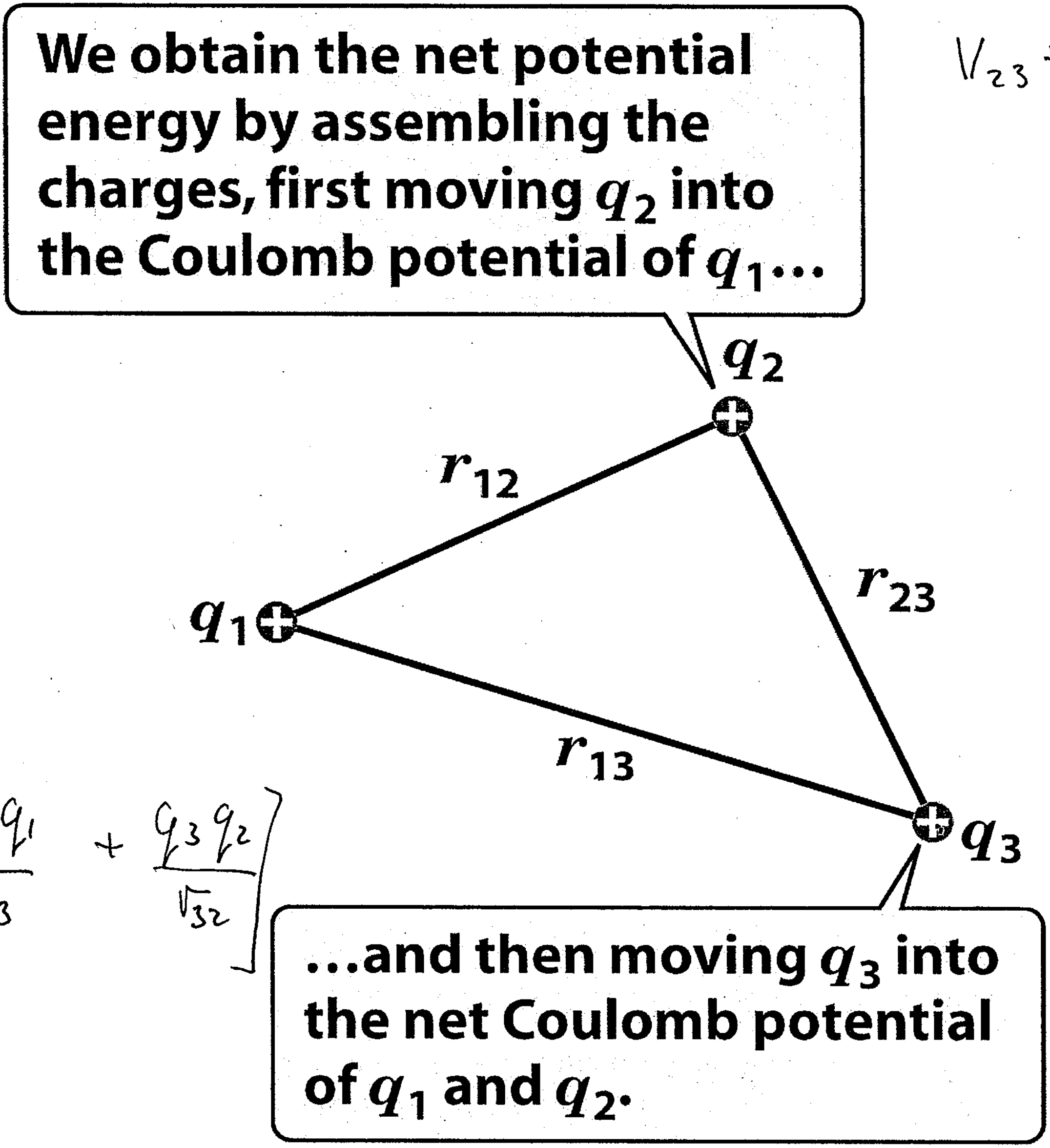
2. bring in q_2

$q_2 V_2 = q_2 \frac{kq_1}{r_{12}}$

3. $\frac{kq_2 q_1}{r_{12}}$

$+ q_3 V_{23}$

$= k\left[\frac{q_2 q_1}{r_{12}} + \frac{q_3 q_1}{r_{13}} + \frac{q_3 q_2}{r_{32}}\right]$


We obtain the net potential energy by assembling the charges, first moving q_2 into the Coulomb potential of q_1…

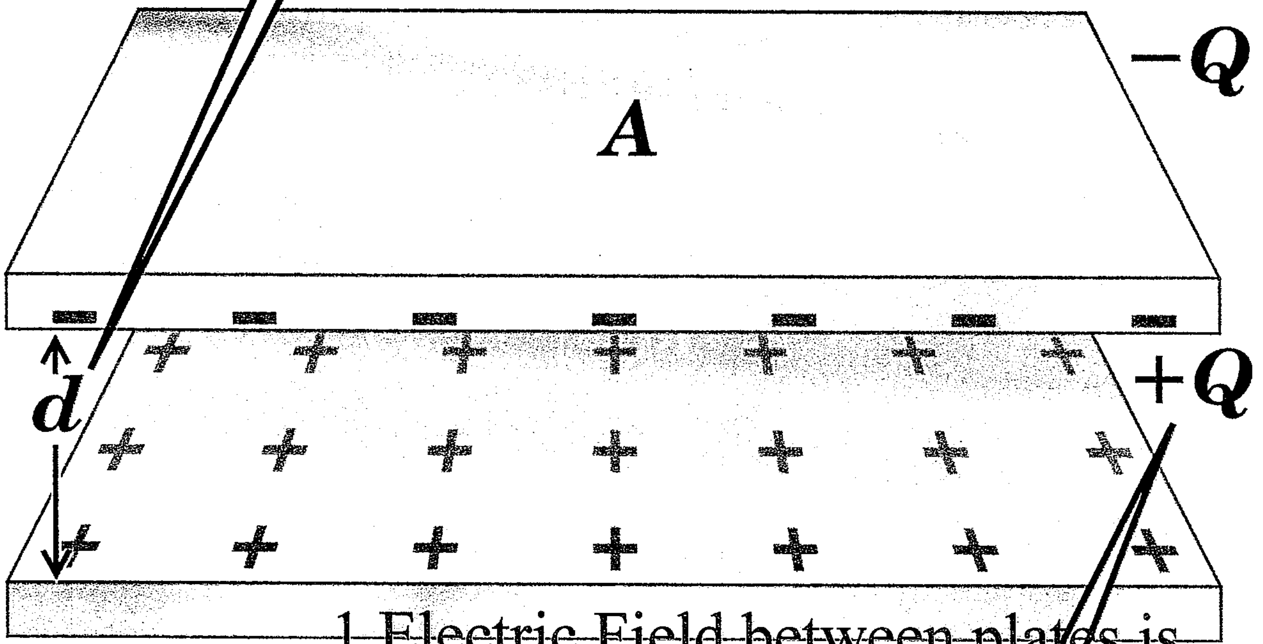
…and then moving q_3 into the net Coulomb potential of q_1 and q_2.

Figure 25-27 Physics for Engineers and Scientists 3/e

$V_{23} = k\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$

$\circ q_4$

For constant electric field the potential difference (voltage) is V=E d



1.Electric Field between plates is

(a) 0; (b) $Q/2A\varepsilon_0$; (c) $Q/A\varepsilon_0$

2. The energy required to bring a small charge q, across conducting plates is

(a) 0; (b) qV; (c) qV/ 2

3. The minimum energy to create this configuration is?

Figure 25-30 Physics for
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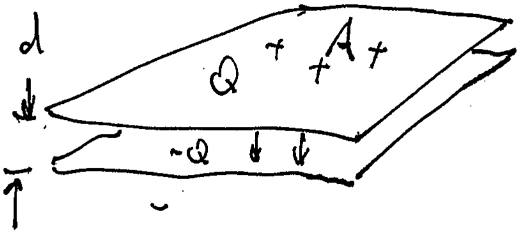
(4a)

Minimun Energy to create charged plates and two ways to look at stored energy



$$E = \frac{Q}{A\epsilon_0}, \qquad V = Ed = \frac{Q}{A\epsilon_0} d$$

$$\Delta W = \Delta Q V = \Delta Q E d = \Delta Q \frac{Q}{A}\frac{d}{\epsilon_0}$$

Minimum Work Done on charged plates

$$W = \int_0^W dW' = \int_0^Q dQ' V(Q')$$

$$V(Q') = \frac{Q'd}{A\epsilon_0} \qquad = \int_0^Q dQ' Q' d / \epsilon_0 A = \frac{Q'^2 d}{2A\epsilon_0}\Bigg|_0^Q$$

$$\boxed{W = \frac{Q^2}{\underset{\uparrow}{2}}\frac{d}{A\epsilon_0}} = \frac{QV}{\underset{\uparrow}{2}}$$

$\boxed{\mathcal{V} \equiv \text{volume}}$

$$W = \frac{Q^2 d}{2\epsilon_0 A} = \frac{\epsilon_0 A d}{2}\left(\frac{Q}{A\epsilon_0}\right)^2 = \frac{\epsilon_0}{2}\mathcal{V} E^2$$

$\frac{W}{\mathcal{V}}$ = Energy ~~Density~~/Volume $= \frac{\epsilon_0 E^2}{2}$

= Energy Density

very fundamental relationship; Energy in field ⑤

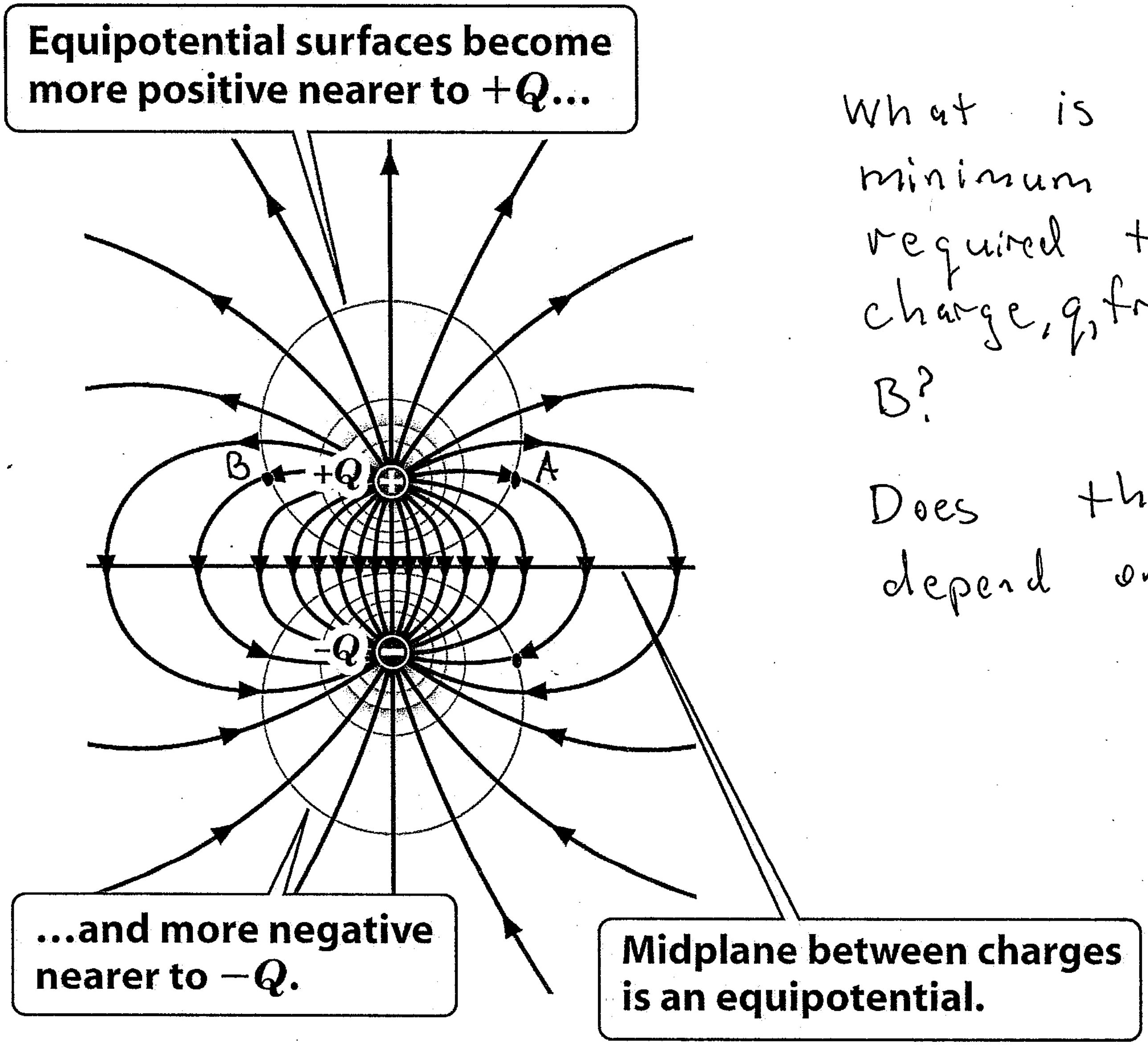


Figure 25-25 Physics for Engineers and Scientists 3/e
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What is the minimum work required to take charge, q, from A to B?

Does this work depend on path?

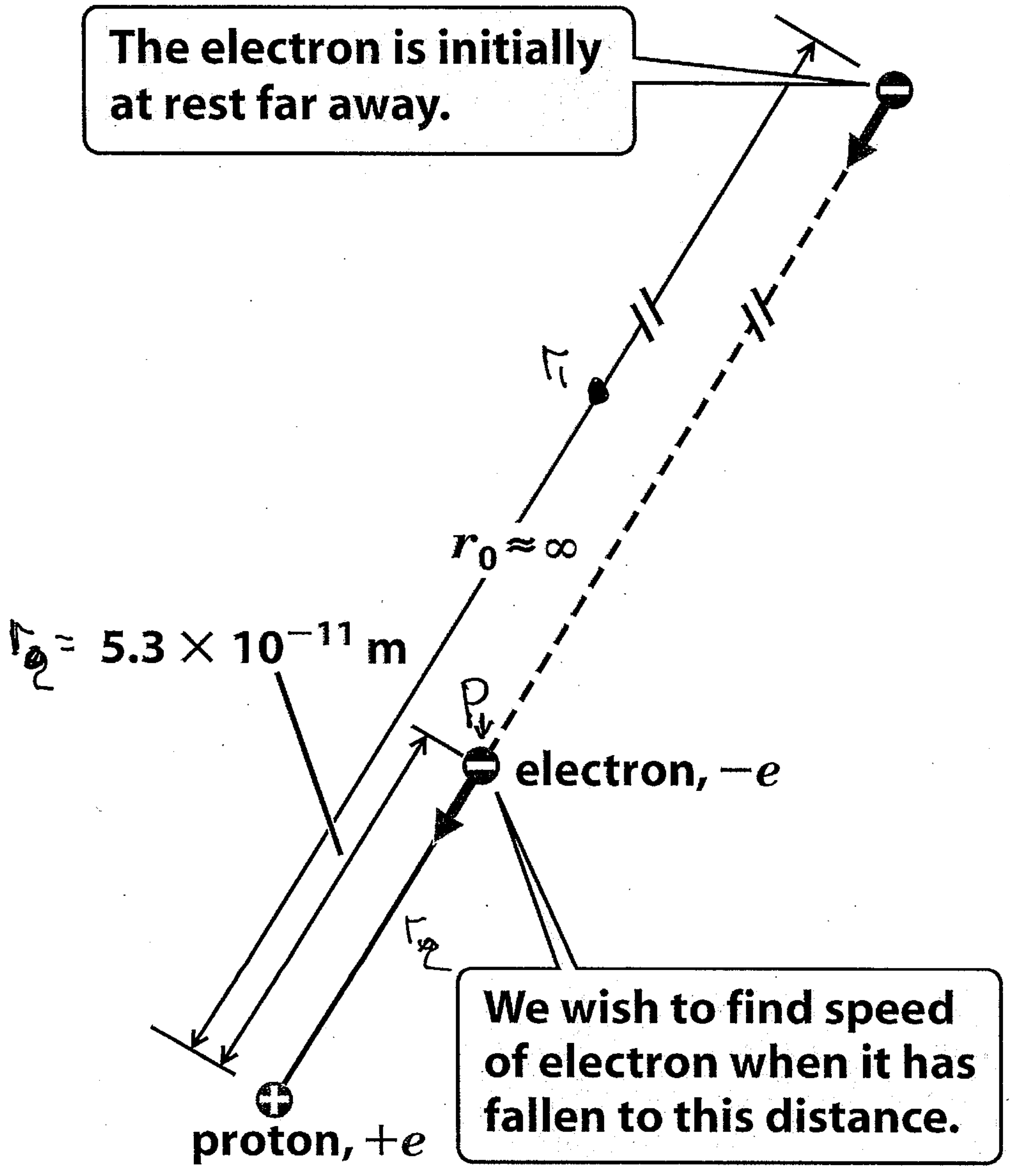


Figure 25-11 Physics for Engineers and Scientists 3/e

Find electron's speed at point P, if, (a) electron is at rest at infinity (b) electron is at rest at r_1

(7)

How will the potential vary
on each conductor?

a. Since there are many
Conductors, the potential will
vary on each conductor

b. Each conductor will have its
unique constant potential

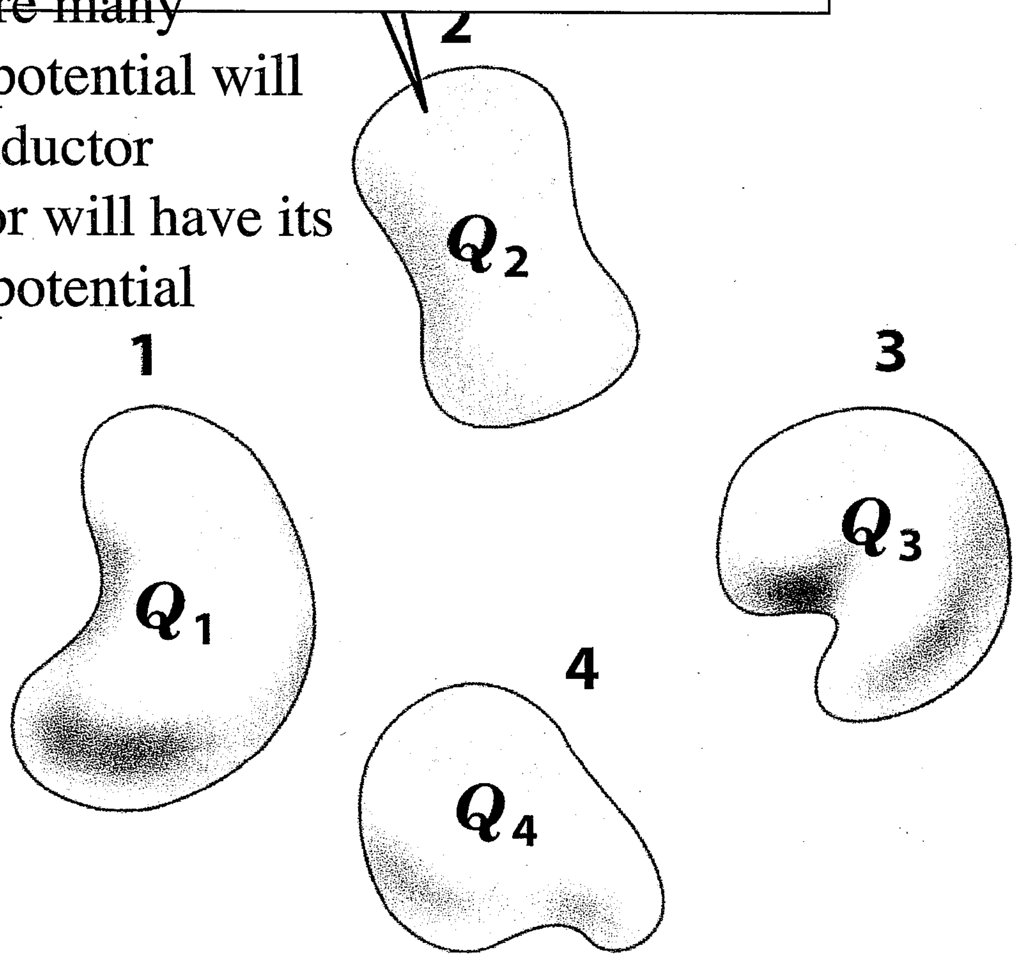
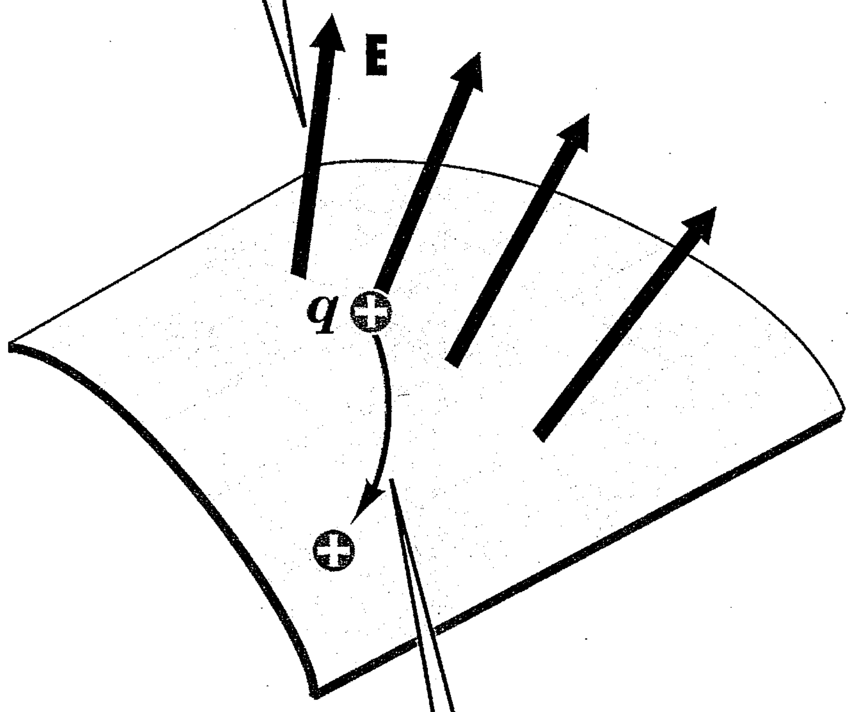


Figure 25-29 Physics for Engineers and Scientists 3/e



Electric field is everywhere perpendicular to any equipotential surface.

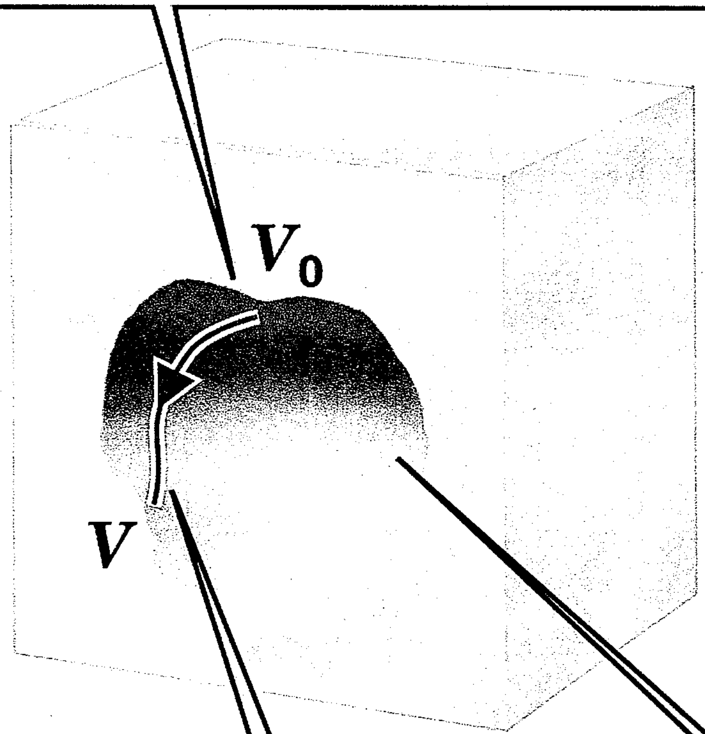
It takes no energy to move a charge along a surface of constant potential.

Figure 25-26 Physics for Engineers and Scientists 3/e

What is the surface charge on the conductor in terms of the electric field?

$\cancel{\sigma = \frac{E}{\epsilon_0}}$

$E = \frac{\sigma}{\epsilon_0}$



For empty cavity, any field line would have to begin and end at surface.

Path parallel to field line would imply a potential difference: impossible!

Electric field is zero everywhere in cavity.

Figure 25-19 Physics for Engineers and Scientists 3/e

Electric field is zero inside a conducting cavity.

What is the potential difference between sphere 1 and sphere 2, when there is a conducting wire touching both conducting spheres?

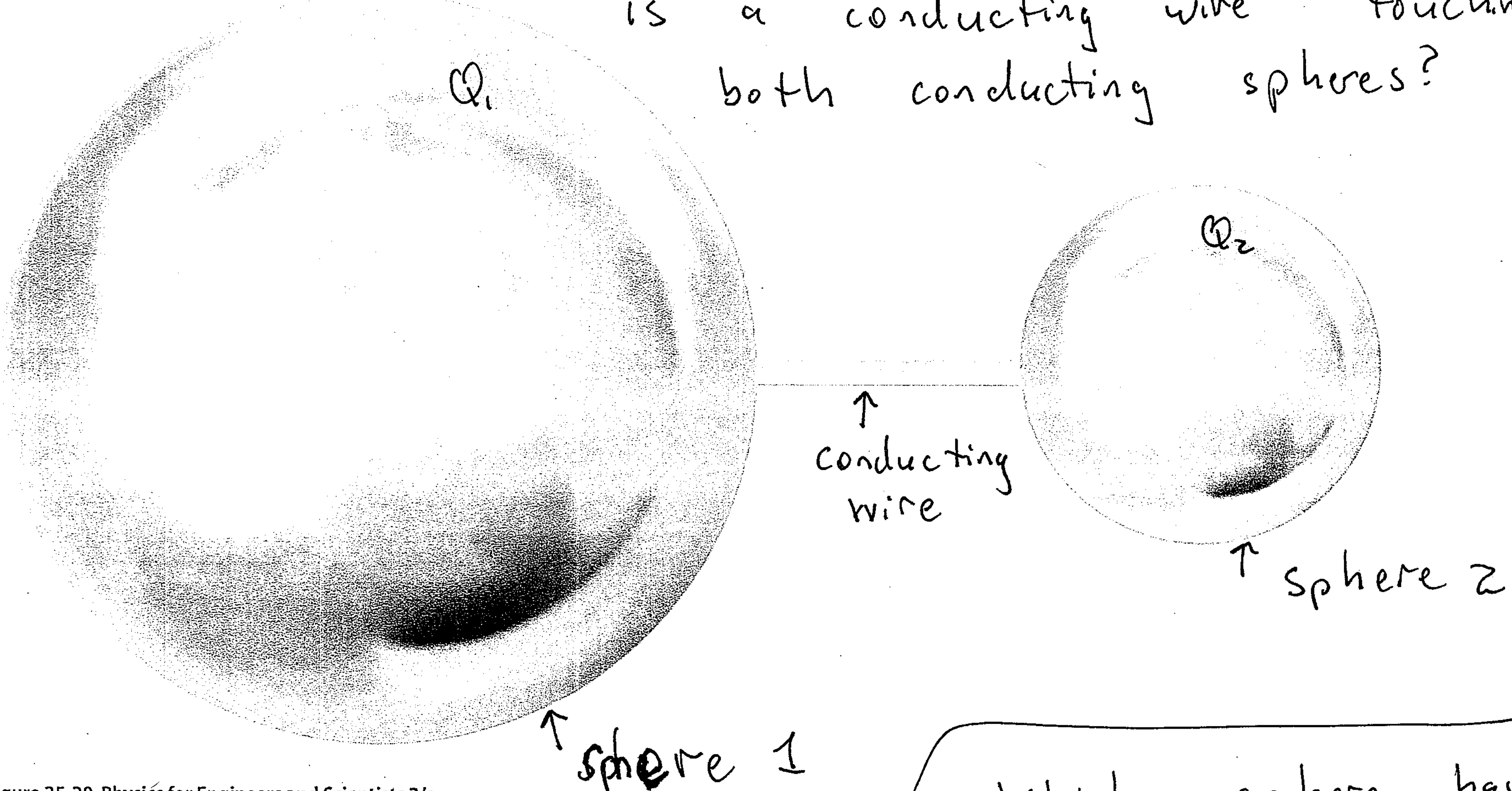
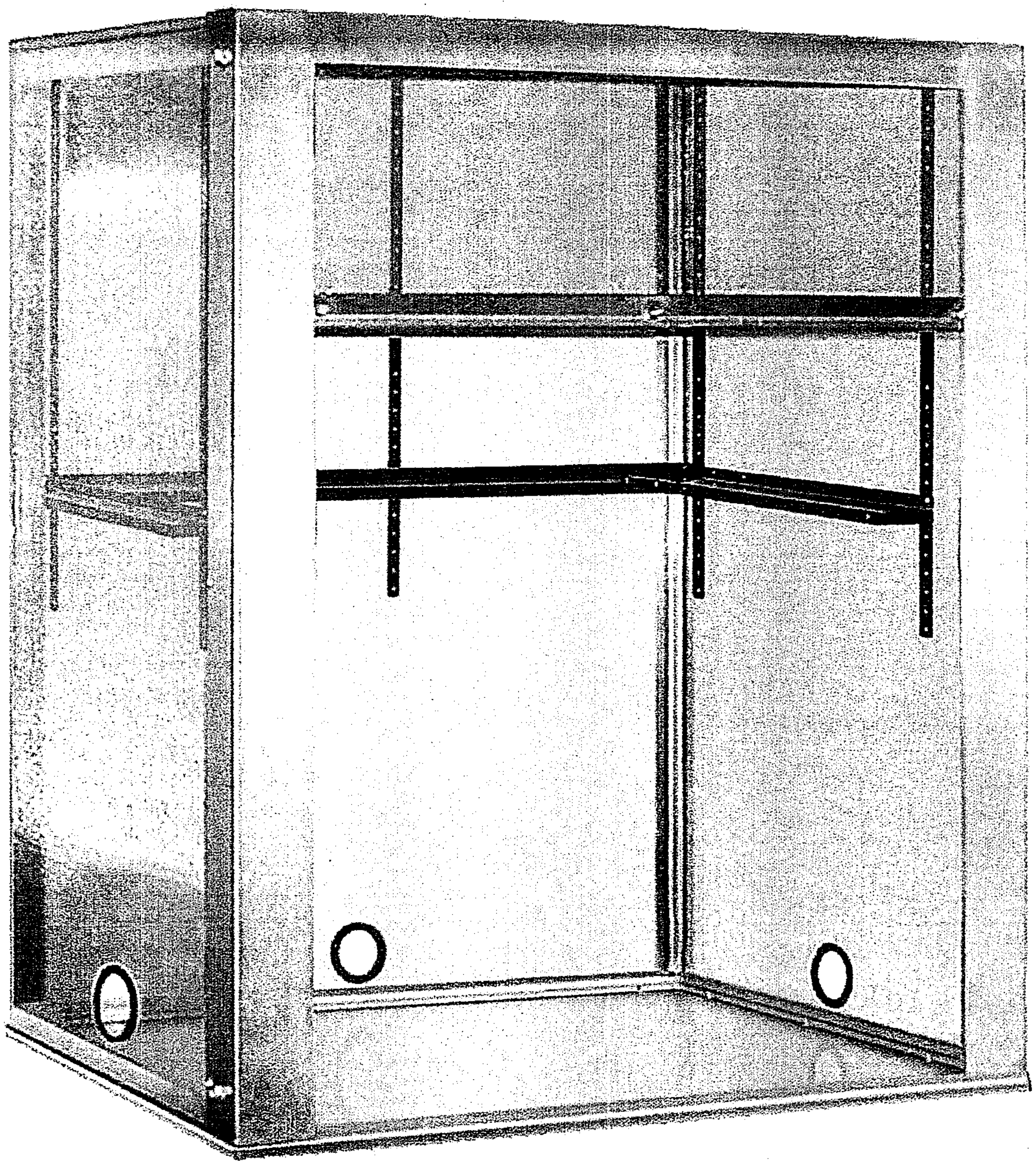


Figure 25-20 Physics for Engineers and Scientists 3/e

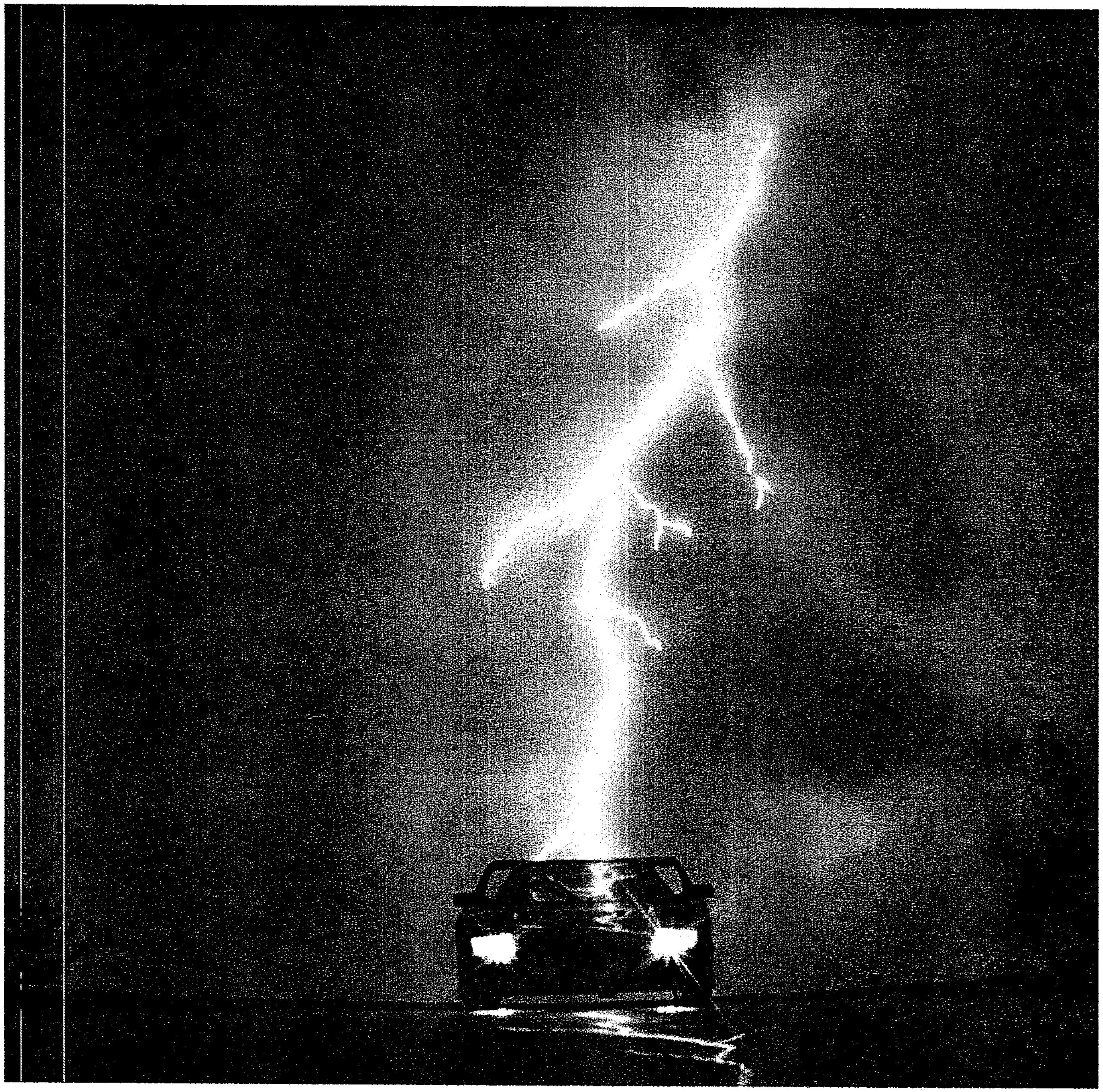
Which sphere has more charge?

(a) sphere 1 ; (b) sphere 2

(c) indeterminate



Physics in Practice 25-1 Physics for Engineers and Scientists 3/e



Physics in Practice 25-3 Physics for Engineers and Scientists 3/e
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Are the
car
occupants
in danger?

Find electric field on axis from electric potential expression

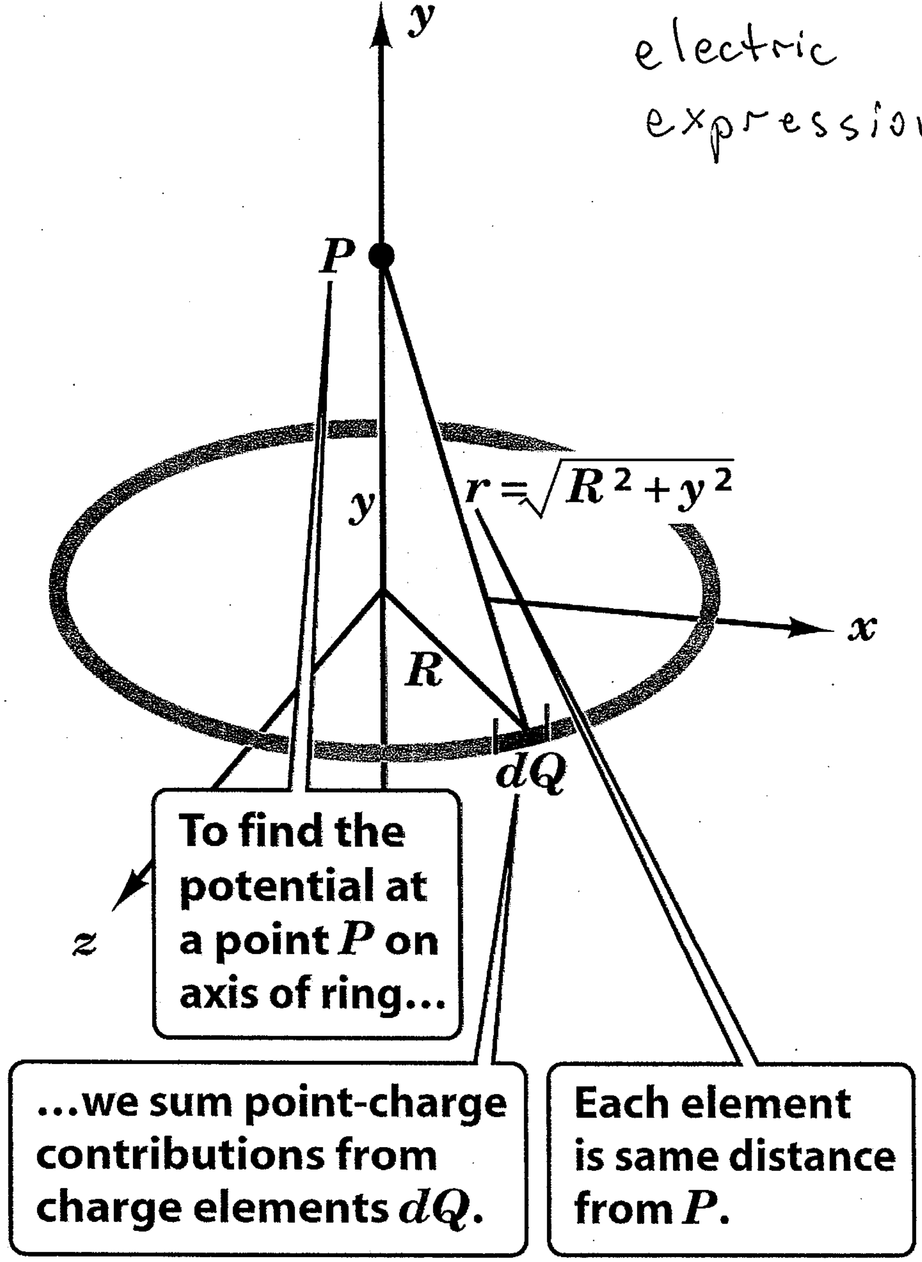
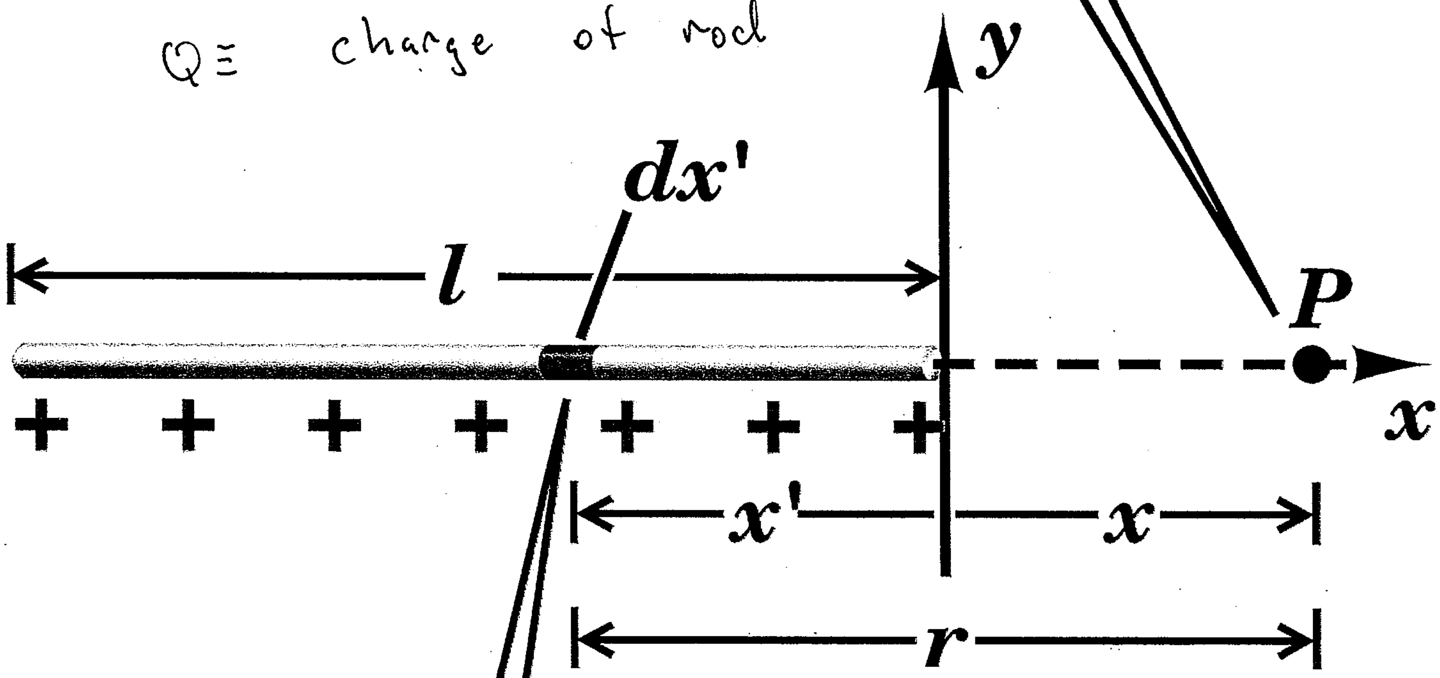


Figure 25-22 Physics for Engineers and Scientists 3/e

We find the potential at a point P on the positive x axis…



Q ≡ charge of rod

…by summing contributions at various $r = x - x'$ along rod.

Figure 25-15 Physics for Engineers and Scientists 3/e

Notes on charged Ring

$$V(P) = \sum \frac{dQ\,k}{r}\,, \qquad r = R^2 + y^2$$

$$= \frac{Q\,k}{(R^2+y^2)^{1/2}}$$

$$E_y = -\frac{\partial V}{\partial y} = \frac{Q\,k\,y}{(R^2+y^2)^{3/2}}$$

Notes on charge rod

$$dV = \frac{k\,dx'\,\lambda}{(x+x')}\,, \quad \lambda = \frac{Q}{\ell}$$

$$V = \lambda k \int_0^{\ell} \frac{dx'}{x+x'}$$

$$= \lambda k \ln(x+x')\Big|_0^{\ell}$$

$$= \lambda k \left[\ln(x+\ell) - \ln(x)\right]$$

$$= \frac{Q}{\ell} k \ln\left(\frac{x+\ell}{x}\right) = \frac{Q}{\ell} k \left(1+\frac{\ell}{x}\right)$$

$$\xrightarrow[\ell/x \ll 1]{} \frac{Q}{\ell} k \frac{\ell}{x} = \frac{Qk}{x} \qquad \left(\text{potential of a point charge}\right)$$

$$\xrightarrow[x \to 0]{} \frac{Q}{\ell} k \ln(\ell/x)$$

Electric Field along axis of rod

$$\underset{\sim}{E} = -\hat{x}\,\frac{\partial V}{\partial x}$$

$$= -x\,\frac{Q}{\ell}\,k\,\frac{\partial}{\partial x}\ln\left((x+\ell)/x\right)$$

$$= -\frac{Qk}{\ell}\left[\frac{1}{x+\ell} - \frac{1}{x}\right]$$

$$= +\frac{Qk}{\ell}\left[\frac{1}{x} - \frac{1}{x+\ell}\right]$$

$$\xrightarrow[x>>1]{} \frac{Qk}{\ell}\left[\frac{1}{x} - \frac{1}{x(1+\frac{\ell}{x})}\right]$$

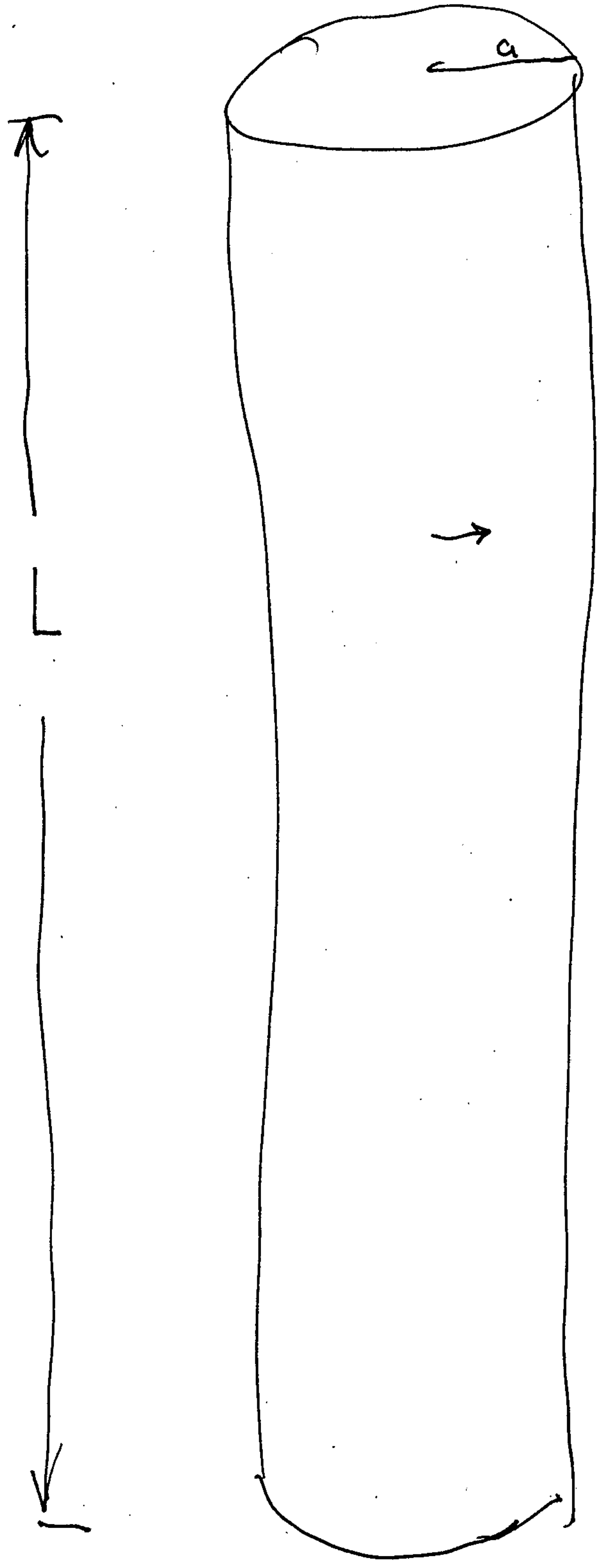
$$\frac{Qk}{\ell}\left[\frac{1}{x} - \frac{1}{x} + \frac{\ell}{x^2}\right]$$

$$= \frac{Qk}{x^2}$$ (point charge result)

$$E \xrightarrow[x<<1]{} \frac{Qk}{\ell}\left[\frac{1}{\underset{\uparrow}{x}} - \frac{1}{\ell}\right]$$

field gets very large

(principle of lightning rod)



Uniform cylindrical charge. Total Charge Q uniformly distributed for $r \leq a$

What is potential, relative to the axis at radius r,

if $r < a$

$r > a$

$$\lambda = \frac{Q}{L}$$

Notes on charged cylinder

$$E\,2\pi r L = \frac{Q(r)}{\epsilon_0} = \frac{\rho\,\mathbf{V}(r)}{\epsilon_0}$$

$$\rho = \frac{Q}{L\pi a^2}$$

$$\mathbf{V}(r) = \pi a^2 L r^2$$

$$\rho\,\mathbf{V}(r) = \frac{Q}{L}\,L\,\frac{r^2}{a^2} = Q\frac{r^2}{a^2}$$

$$E\,2\pi r L = \frac{Q}{\epsilon_0}\,\frac{r^2}{a^2}$$

$$E = \frac{\lambda}{2\pi\epsilon_0}\,\frac{r}{a^2}$$

$$V = -\int_0^r E\,dr = \frac{-\lambda}{2\pi\epsilon_0}\,\frac{r^2}{a^2}\,\frac{1}{2}$$

$$V(a) = -\frac{\lambda}{4\pi\epsilon_0}\,\frac{r^2}{a^2}$$

$r > b$

$$E\,2\pi r L = \frac{Q}{\epsilon_0}\,,\quad E = \frac{Q}{\epsilon_0\,2\pi r L} = \frac{\lambda}{2\pi r\,\epsilon_0}$$

$$V = -\int_a^r E\,dr + V(a) = \frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{a}\right) - \frac{\lambda}{4\pi\epsilon_0}$$

$$V(r) = -\frac{\lambda}{2\pi\epsilon_0}\left[\ln\left(\frac{r}{a}\right) + \frac{1}{2}\right]$$