PHY 303L

Lecture #3

Find the total electric field at P, given x, d, Q

Electric Dipole Moment (zero total charge)

$\vec{p} = Q\vec{d}$

$\hat{p}$ directed from minus to + charge

$\vec{p} \uparrow$

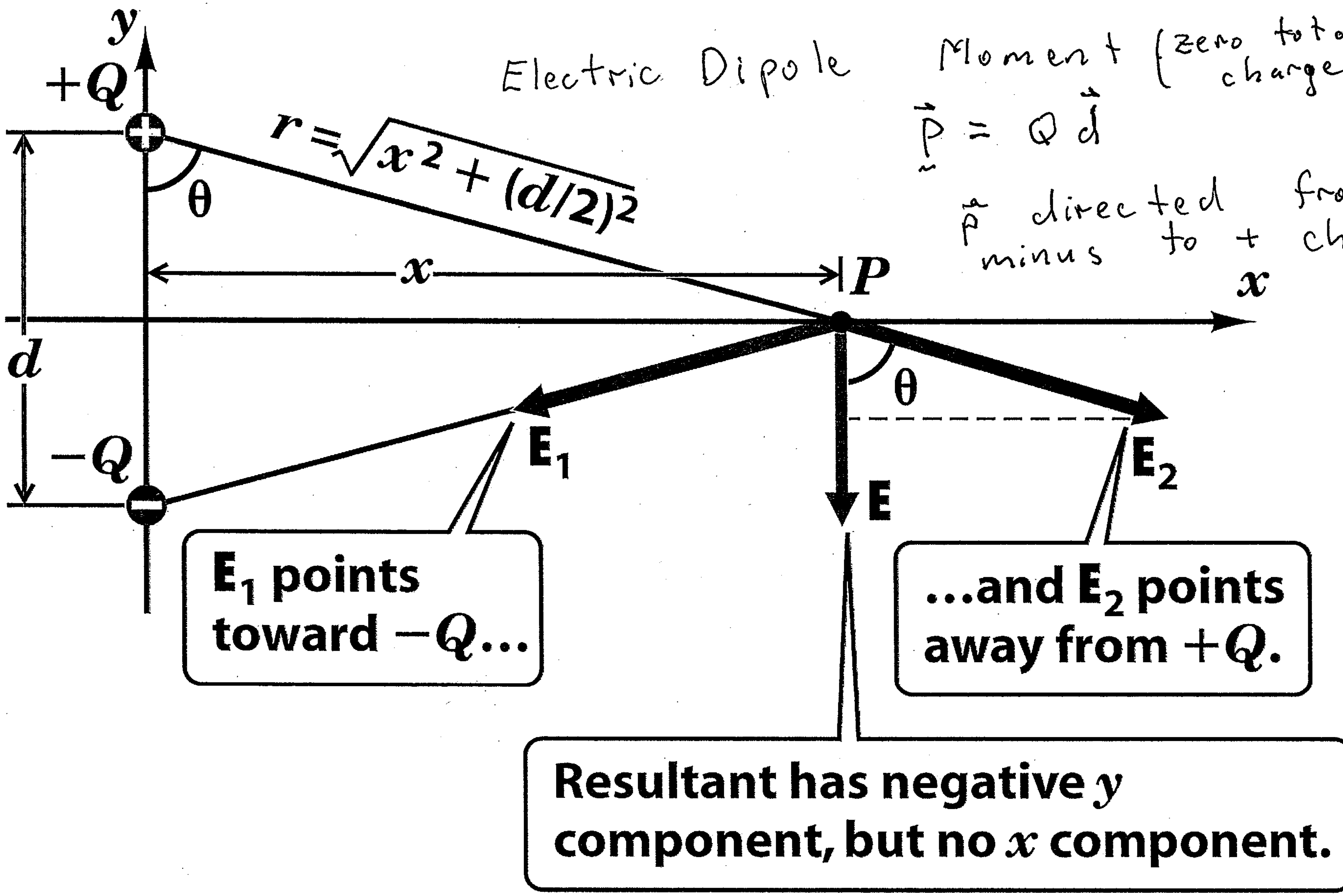


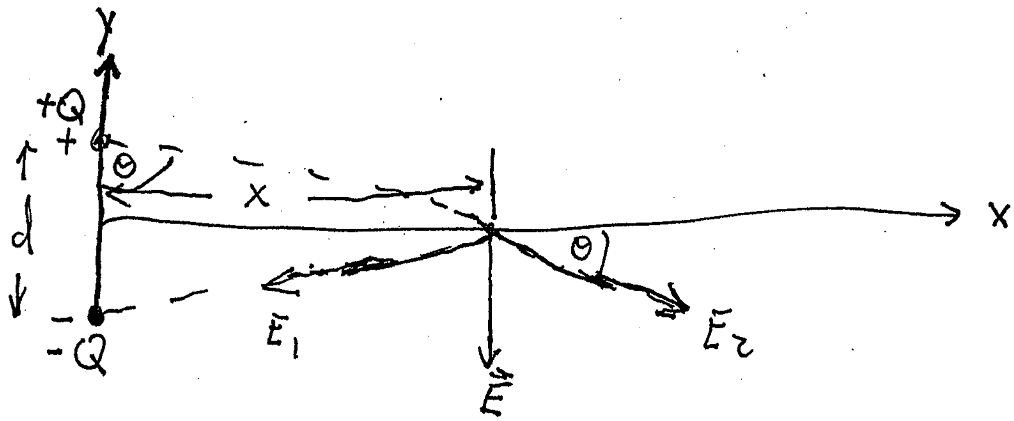
Figure 23-3 Physics for Engineers and Scientists 3/e

$$\sin\theta = \frac{x}{\left[x^2 + (d/2)^2\right]^{1/2}}, \quad \cos\theta = \frac{d/2}{\left[x^2 + (d/2)^2\right]^{1/2}}$$

$$|\underset{\sim}{E}_2| = |\underset{\sim}{E}_1| = \frac{kQ}{r^2} = \frac{kQ}{x^2 + (d/2)^2}$$

$$\underset{\sim}{E}_{total} = -\hat{y}\,2\cos\theta\,|\underset{\sim}{E}_1|$$

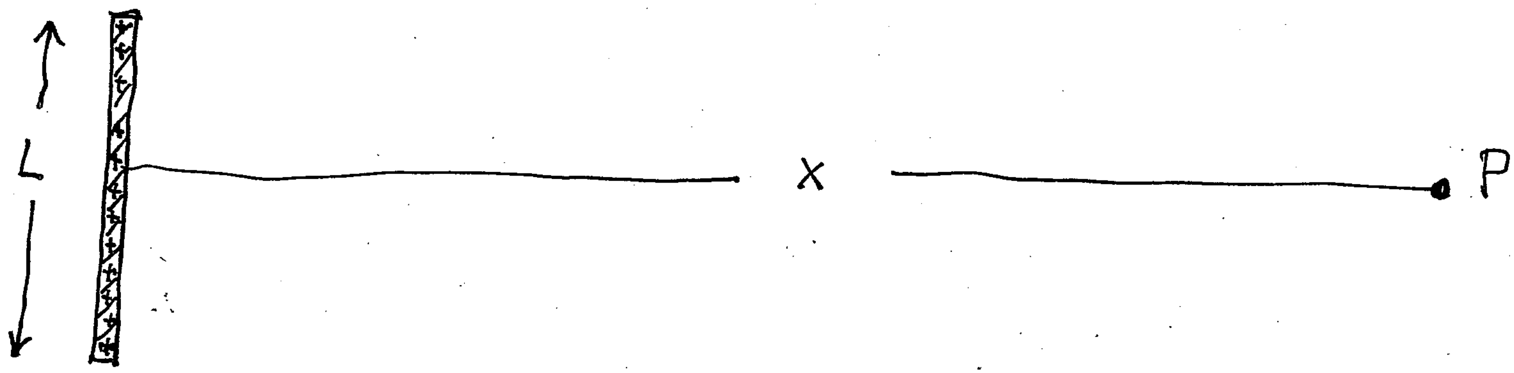
$$= -2\,\frac{d\,k\,Q\,\hat{y}}{2\left(x^2 + (d/2)^2\right)^{3/2}} = -\hat{y}\,\frac{k\vec{p}}{\left(x^2 + (d/2)^2\right)^{3/2}}$$



If $x \gg d$, $\underset{\sim}{E}_{dipole} \rightarrow \dfrac{-\hat{y}\,p\,k}{x^3}$

Consider a uniform charge rod of length L and total charge Q

Which is true at point P:



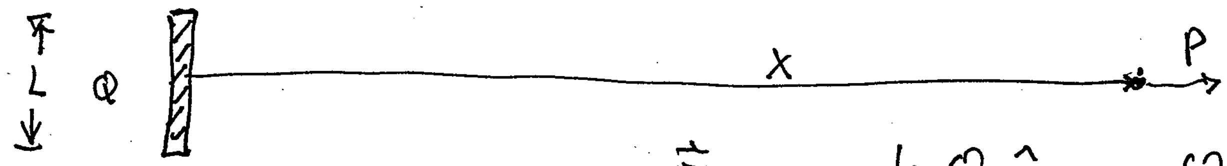
(a) If x is much greater than L, E falls off as $1/x$

(b) If x is much greater than L, E falls off as $1/x^2$

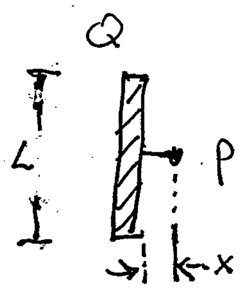
(c) If x is much less than L, E falls off as $1/x^2$

(d) If x is much less than L, E fall off as $1/x$

(1)	(2)	(3)	(4)	(5)
a & b	b & c	c & d	a & c	b & d



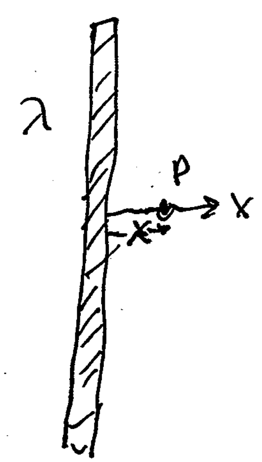
$$\vec{E}_p \simeq \frac{kQ\hat{x}}{x^2} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{x}}{x^2}$$



$$\vec{E}_p \simeq \frac{2kQ}{Lx}\hat{x} = \frac{Q}{2\pi\epsilon_0}\frac{\hat{x}}{Lx}$$

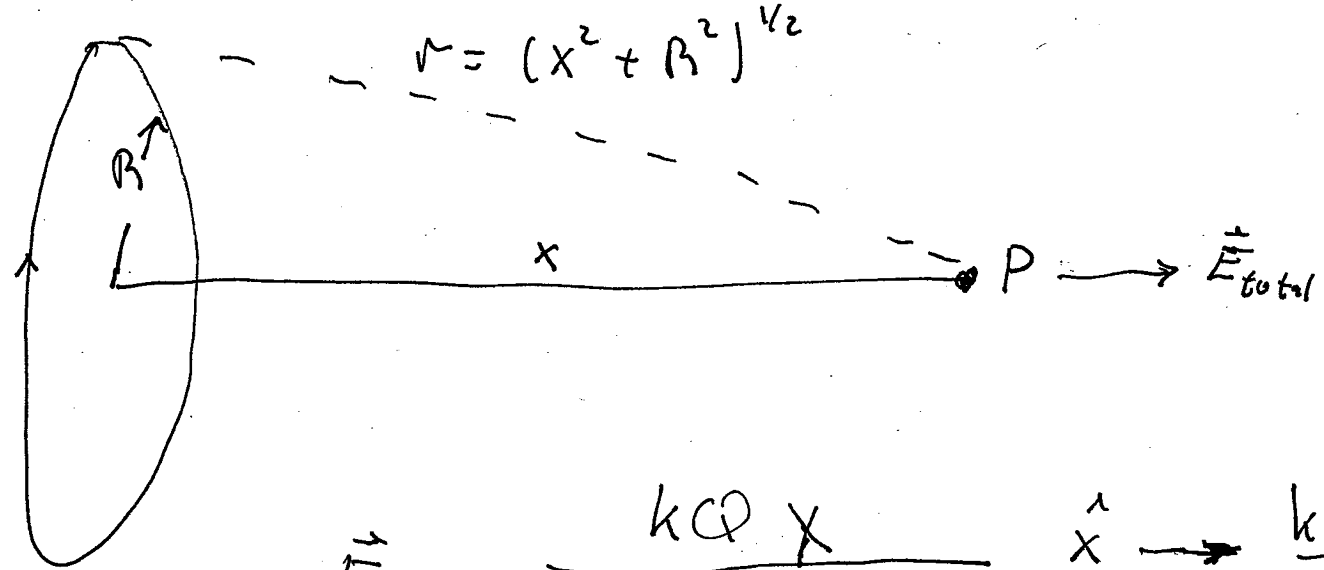
$$= \frac{\lambda}{2\pi\epsilon_0}\frac{\hat{x}}{x}$$

This is answer for very long rod ($L \to \infty$, $\lambda = \frac{Q}{L}$ finite



$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0}\frac{\hat{x}}{x}$$

Electric field on axis of ring with uniformly distributed charge Q

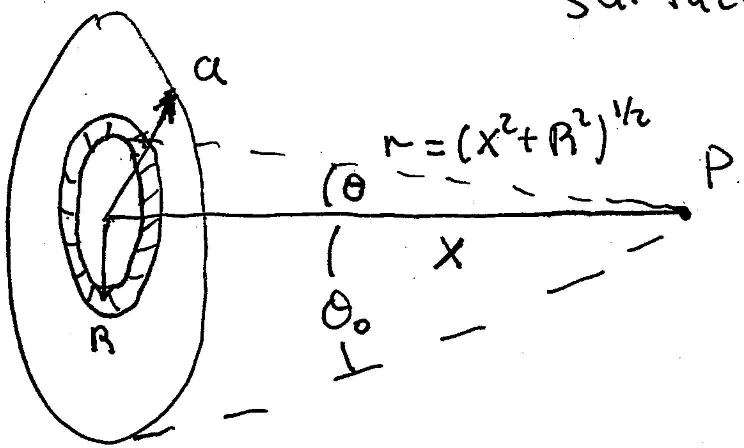


$$\vec{E}_{total} = \frac{kQ\,x}{(R^2 + x^2)^{3/2}}\,\hat{x} \xrightarrow[x \gg R]{} \frac{kQ}{x^2}\,\hat{x}$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{x}{(R^2+x^2)^{3/2}}$$

Electric Field on axis of a disc with uniformly distributed charge

surface charge $\sigma = \frac{Q}{A} = \frac{Q}{\pi a^2}$



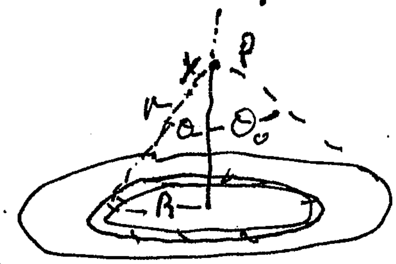
First we use the field from a ring charge at radius R

$$\Delta Q = \sigma \Delta A = \sigma 2\pi R\, dR \Rightarrow \Delta \vec{E} = \hat{x} \frac{\Delta Q k x}{(R^2+x^2)^{3/2}}$$

Then we sum up (integrate) the contribution for all the rings to find

$$\vec{E} = \frac{\sigma}{2\epsilon_0}(1-\cos\theta_0)$$

Appendix — Integration over dE_y



$$dE_y = k\, dQ\, \frac{\cos\theta}{r^2}$$

$(dQ = \sigma\, dA = \sigma\, 2\pi R\, dR)$

$$\boxed{dE_y = k\,(\sigma\, 2\pi R\, dR)\,\frac{\cos\theta}{r^2}}$$

$$r\cos\theta = y$$

$$\tan\theta = \frac{R}{y}, \quad \therefore \quad \frac{d\theta}{\cos^2\theta} = \frac{dR}{y}$$

$$\therefore \quad R = y\,\frac{\sin\theta}{\cos\theta}, \quad dR = y\,\frac{d\theta}{\cos^2\theta}, \quad r^2 = \frac{y^2}{\cos^2\theta}$$

substituting into boxed expression for dE_y

$$dE_y = k\,\sigma\, 2\pi\left(\frac{y\sin\theta}{\cos\theta}\right)\left(\frac{y\,d\theta}{\cos^2\theta}\right)\cos\theta\,\frac{\cos^2\theta}{y^2}$$

(use $k = \frac{1}{4\pi\epsilon_0}$

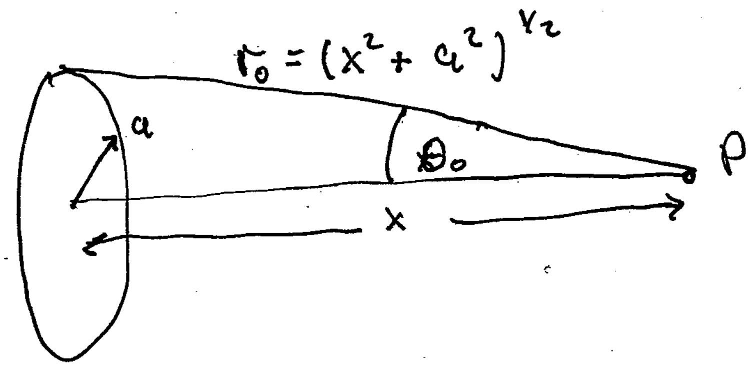
$$= \frac{\sigma}{2\epsilon_0}\, d\theta\, \sin\theta$$

$$\therefore \quad E_y = \int dE_y = \frac{\sigma}{2\epsilon_0}\int_0^{\theta_0} d\theta\,\sin\theta = -\frac{\sigma}{2\epsilon_0}\cos\theta\Big|_0^{\theta_0}$$

$$= \frac{\sigma}{2\epsilon_0}\left(1 - \cos\theta_0\right) \quad (6a$$

(6a)

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\left[1 - \cos\theta_0\right], \quad \sigma = \frac{Q}{\pi a^2}$$



The field at P far from the disc ($x \gg a$) falls of as

(1) $1/x$ (2) $1/x^2$ (3) Constant (4) not determined

The field close to the disc, is constant independent of x if $x \ll a$.

$$\vec{E} \simeq \frac{\sigma}{2\epsilon_0}\hat{x} = \frac{Q}{2\pi a^2 \epsilon_0}\hat{x}$$ (does not even have to be on axis)

Fields near uniformly charged plates

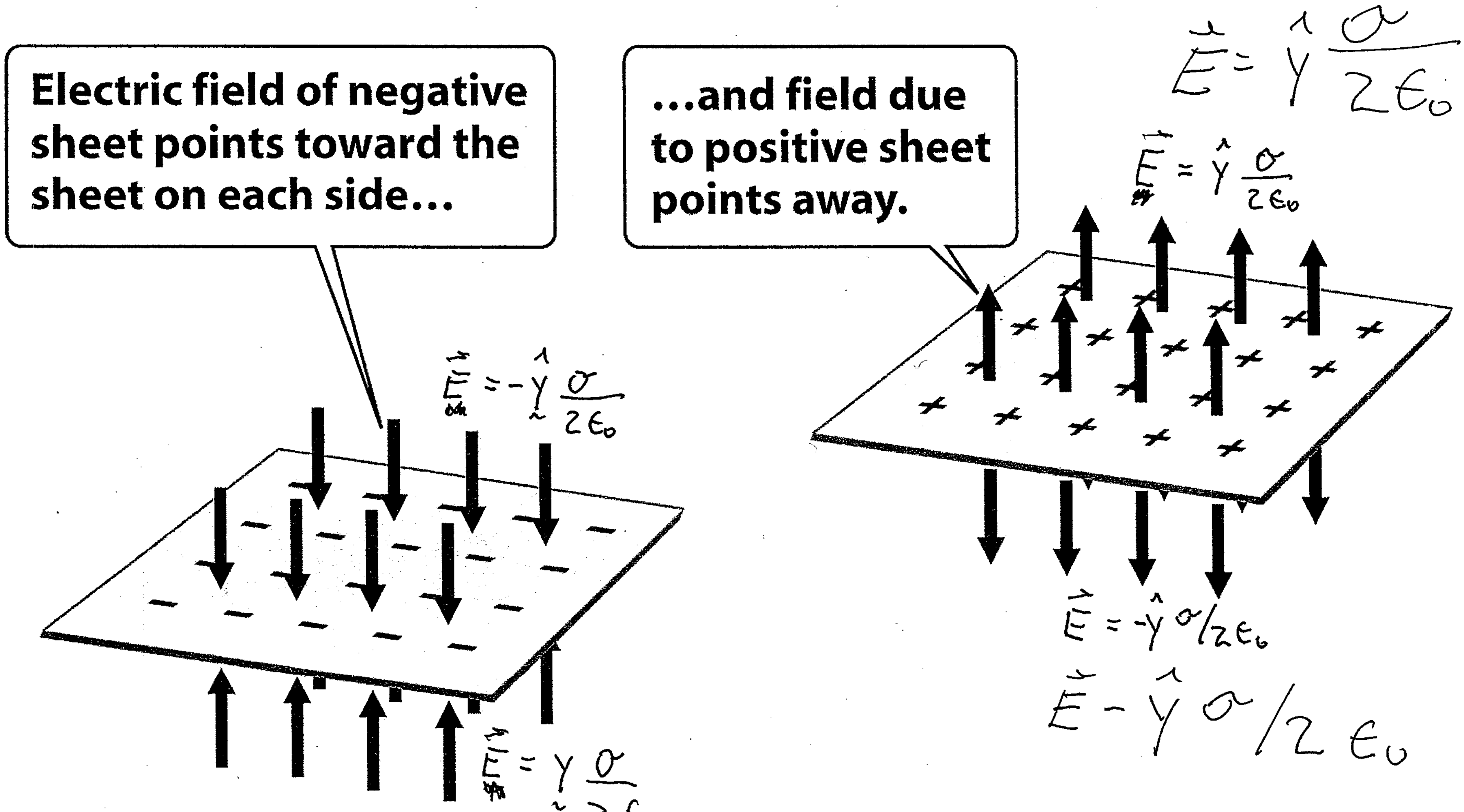
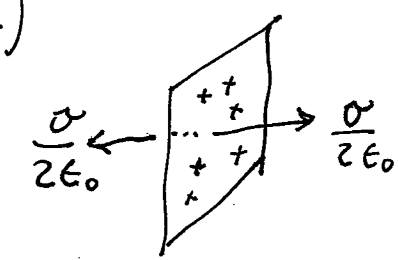
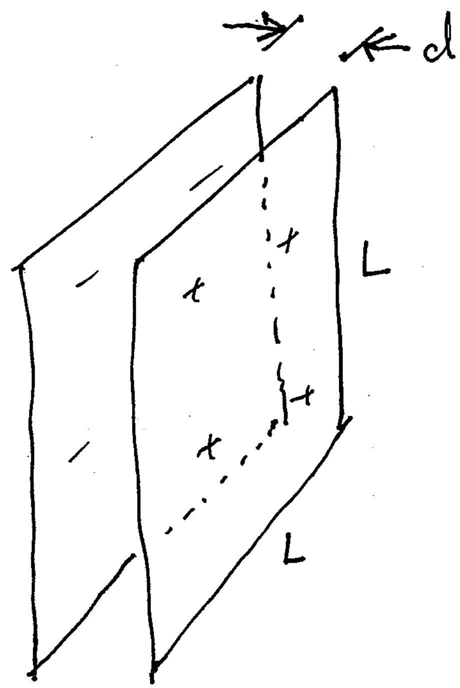


Figure 23-10a Physics for Engineers and Scientists 3/e

What is the field between uniformly, but oppositely charged plates each with equal and opposite surface charge σ. (the plates are larger than their separation)? (or $d \ll L$)





(a) $\frac{\sigma}{\epsilon_0}$ inside, $\pm\frac{\sigma}{2\epsilon_0}$ outside

(b) $-\frac{\sigma}{2\epsilon_0}$ inside, 0 outside

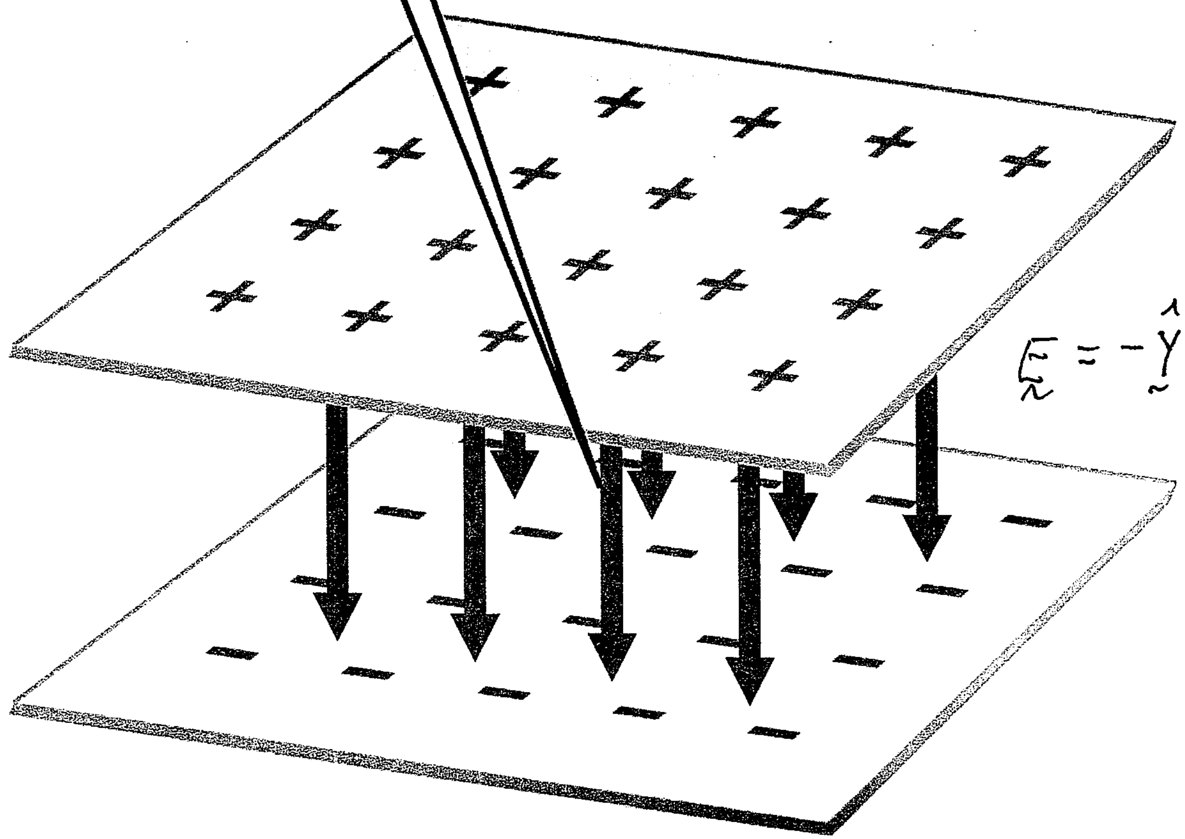
(c) $-\frac{\sigma}{\epsilon_0}$ inside, 0 outside

(d) 0 inside, $\pm\frac{\sigma}{2\epsilon_0}$ outside

(e) $\frac{\sigma}{\epsilon}$ inside, $-\frac{\sigma}{\epsilon_0}$ outside

For both sheets, net field cancels outside and doubles inside.

why?



$\underset{\sim}{E} = -\hat{\underset{\sim}{y}} \frac{\sigma}{\epsilon_0}$

Figure 23-10b Physics for Engineers and Scientists 3/e

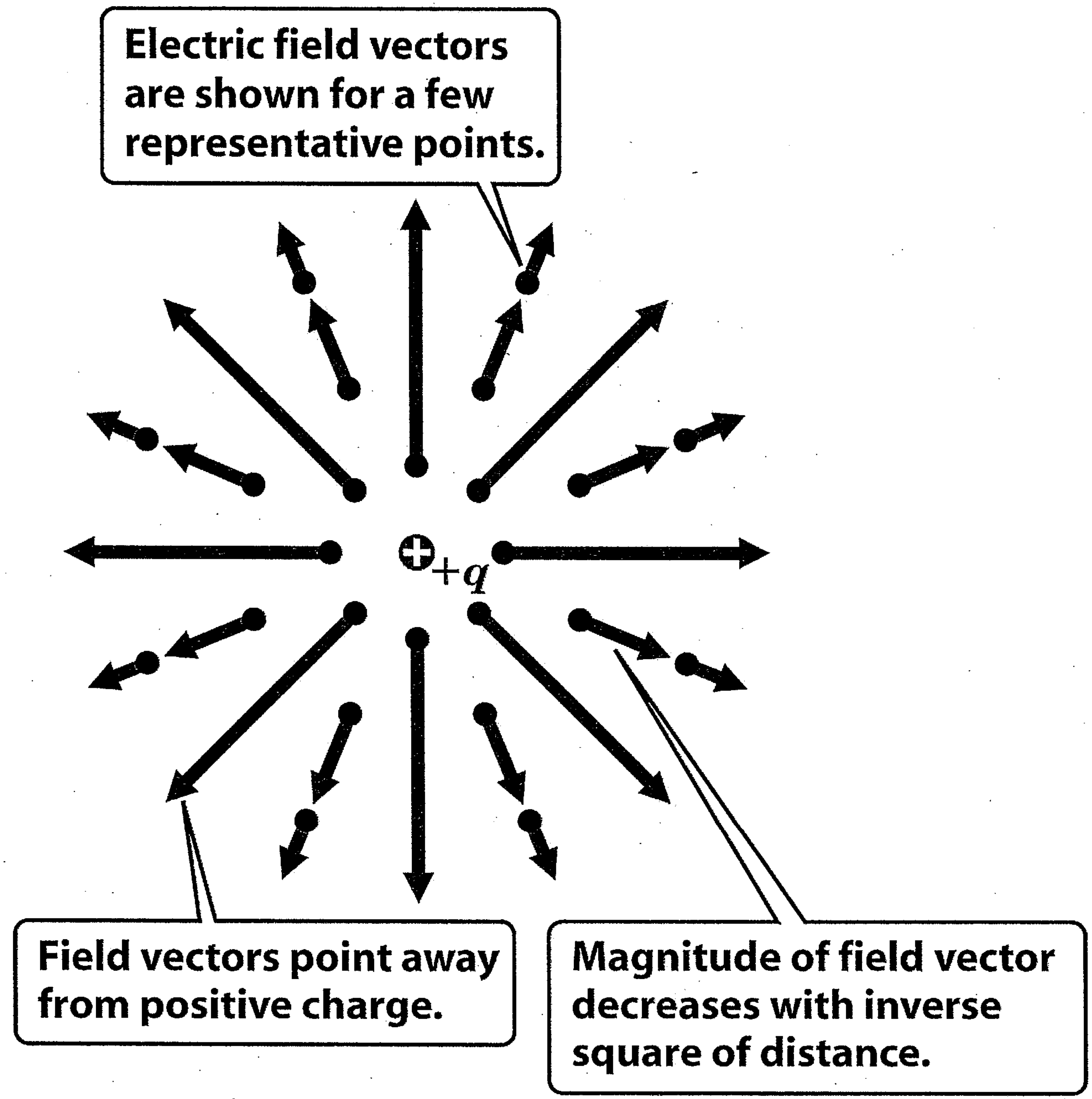


Figure 23-12 Physics for Engineers and Scientists 3/e

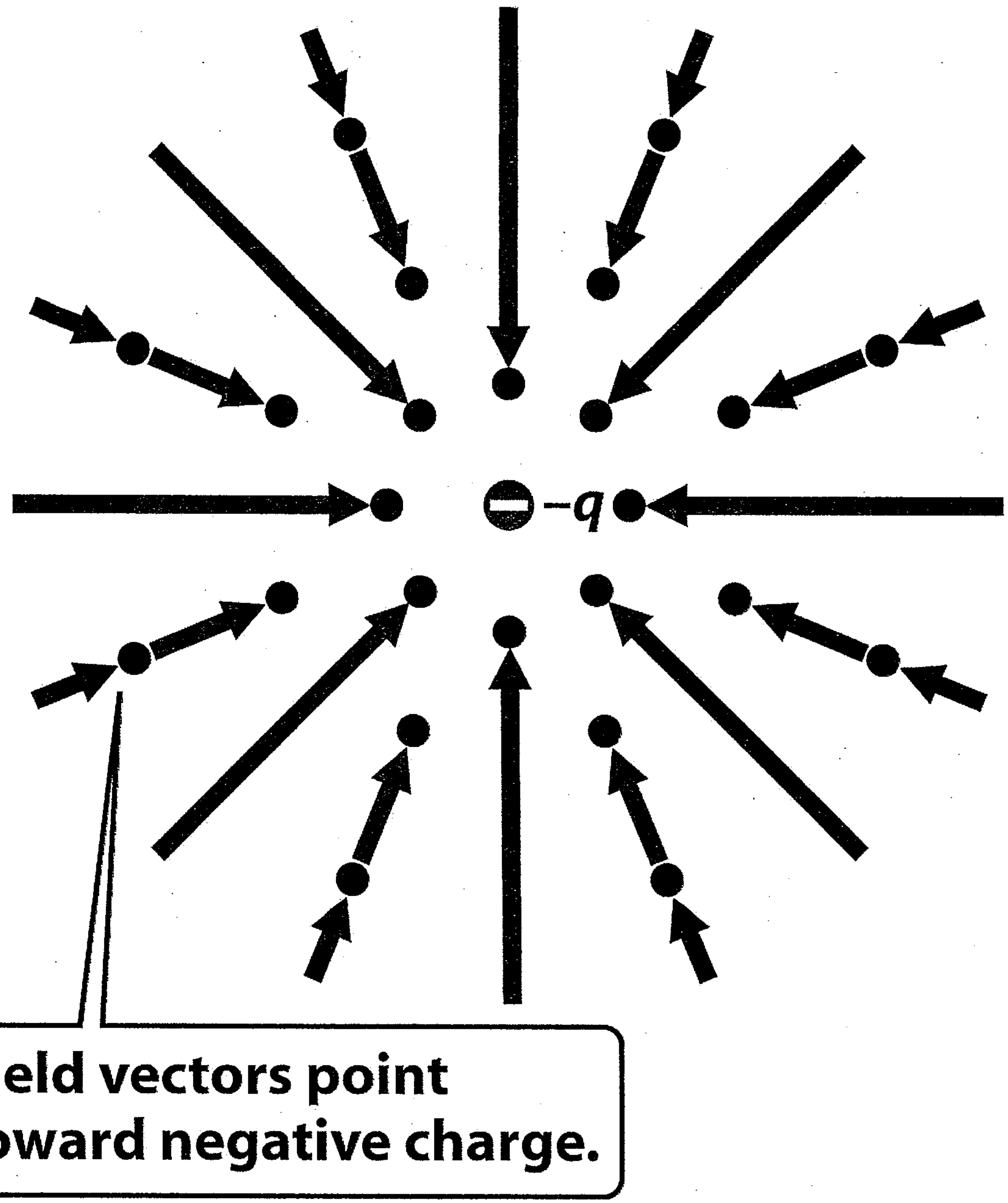


Figure 23-13 Physics for Engineers and Scientists 3/e

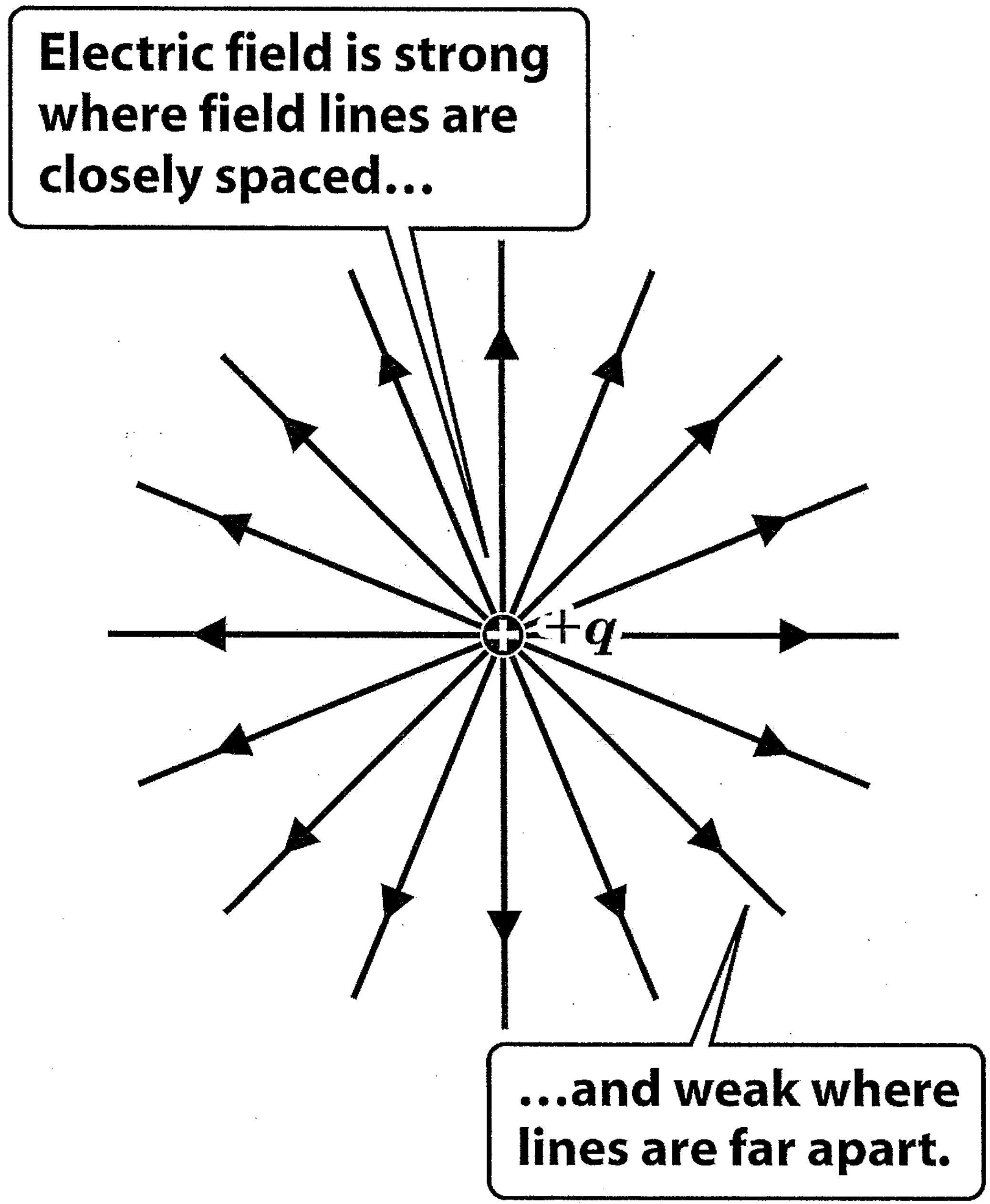


Figure 23-14 Physics for Engineers and Scientists 3/e

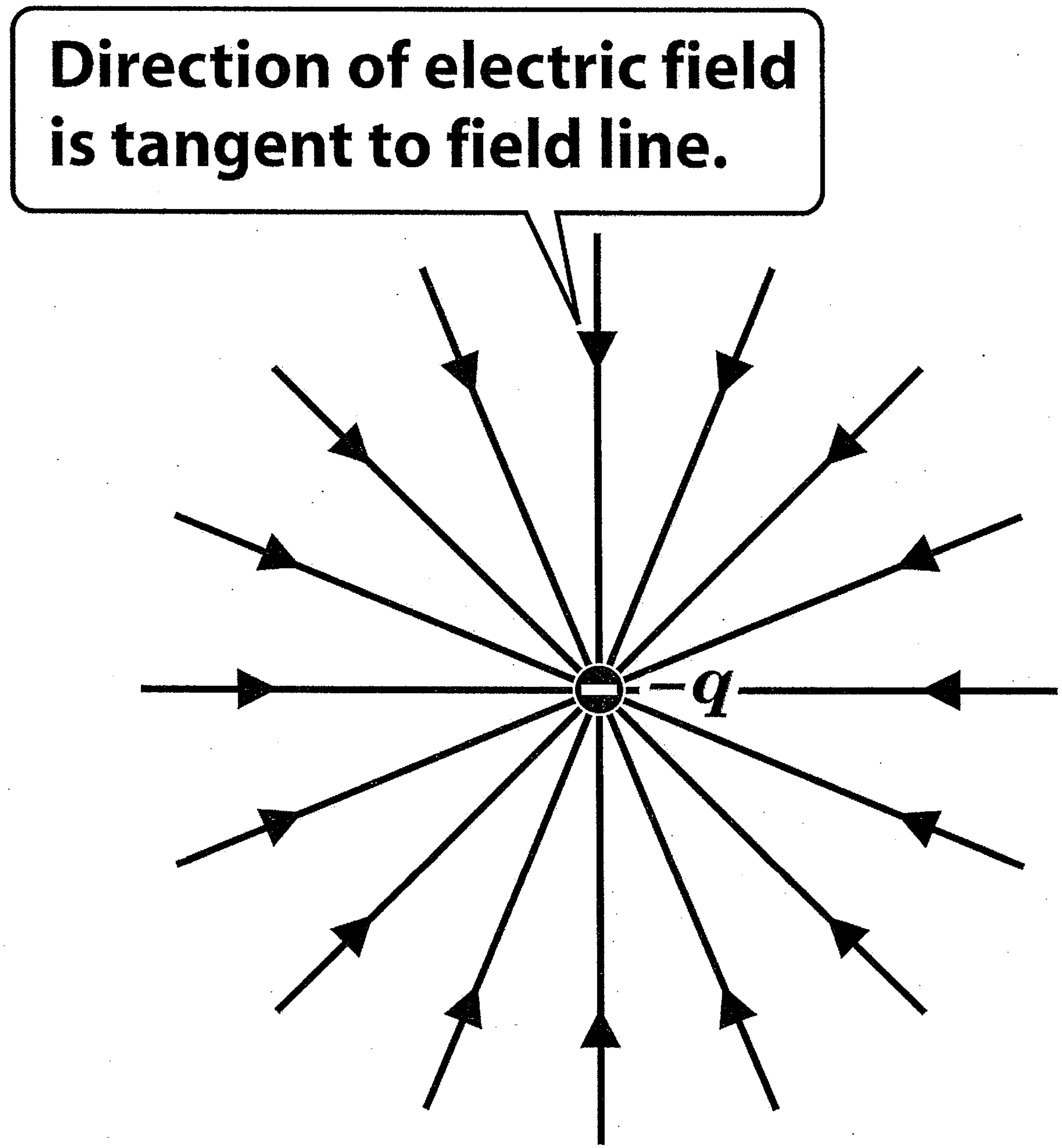


Figure 23-15 Physics for Engineers and Scientists 3/e

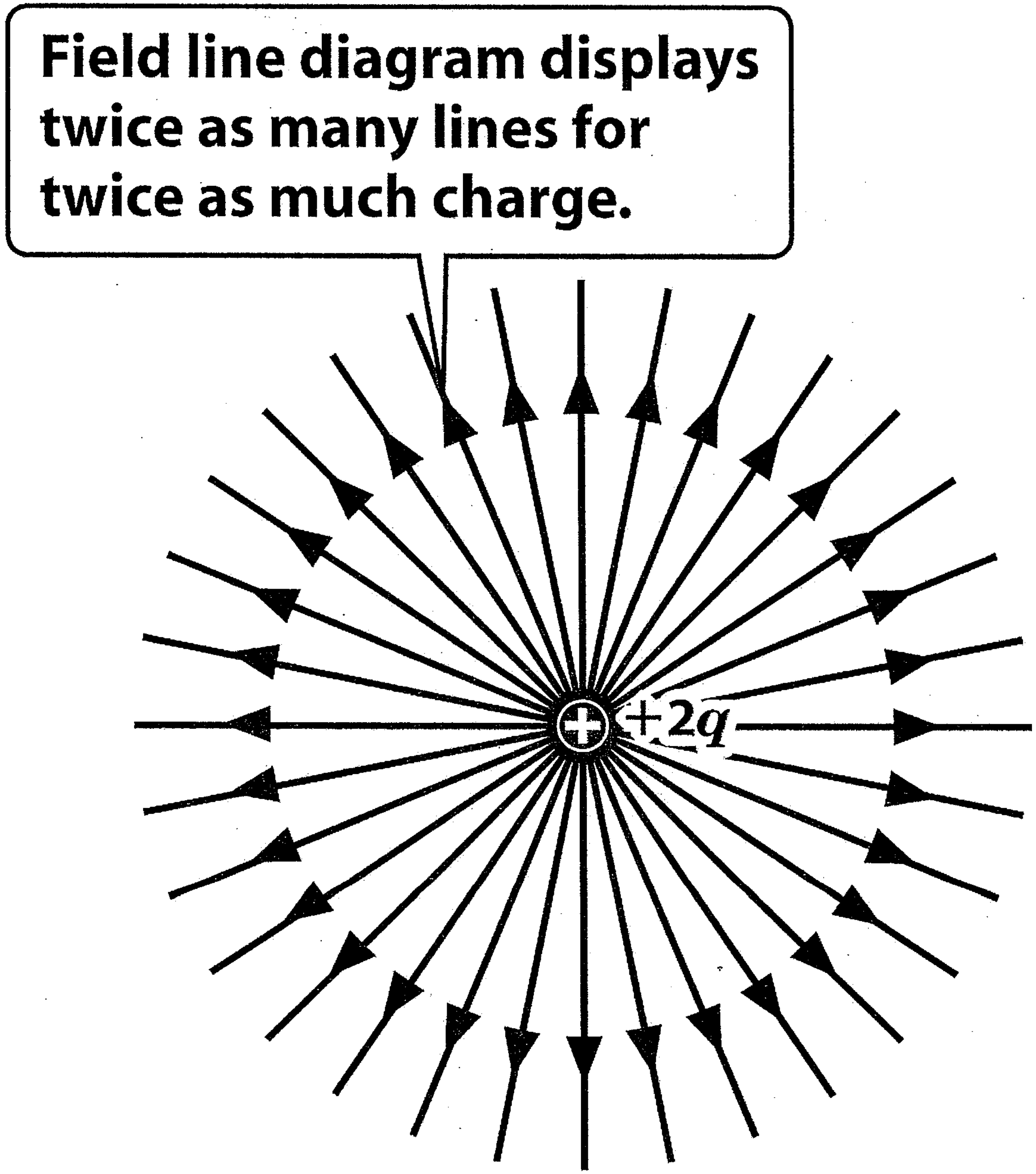


Figure 23-16 Physics for Engineers and Scientists 3/e

Dipoles have zero net charge

For dipoles all field lines begin at + charge and end on minus charge

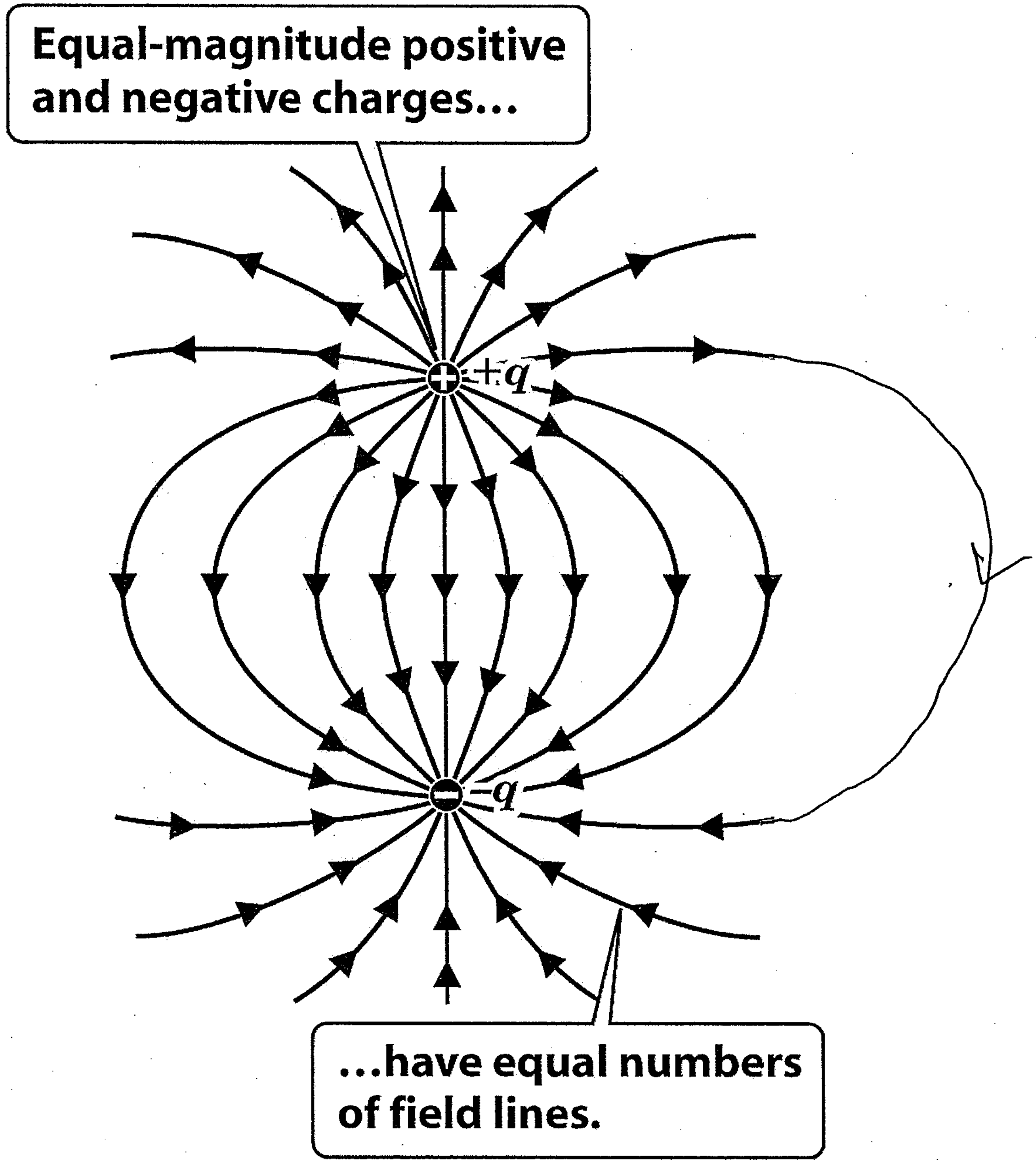


Figure 23-17 Physics for Engineers and Scientists 3/e

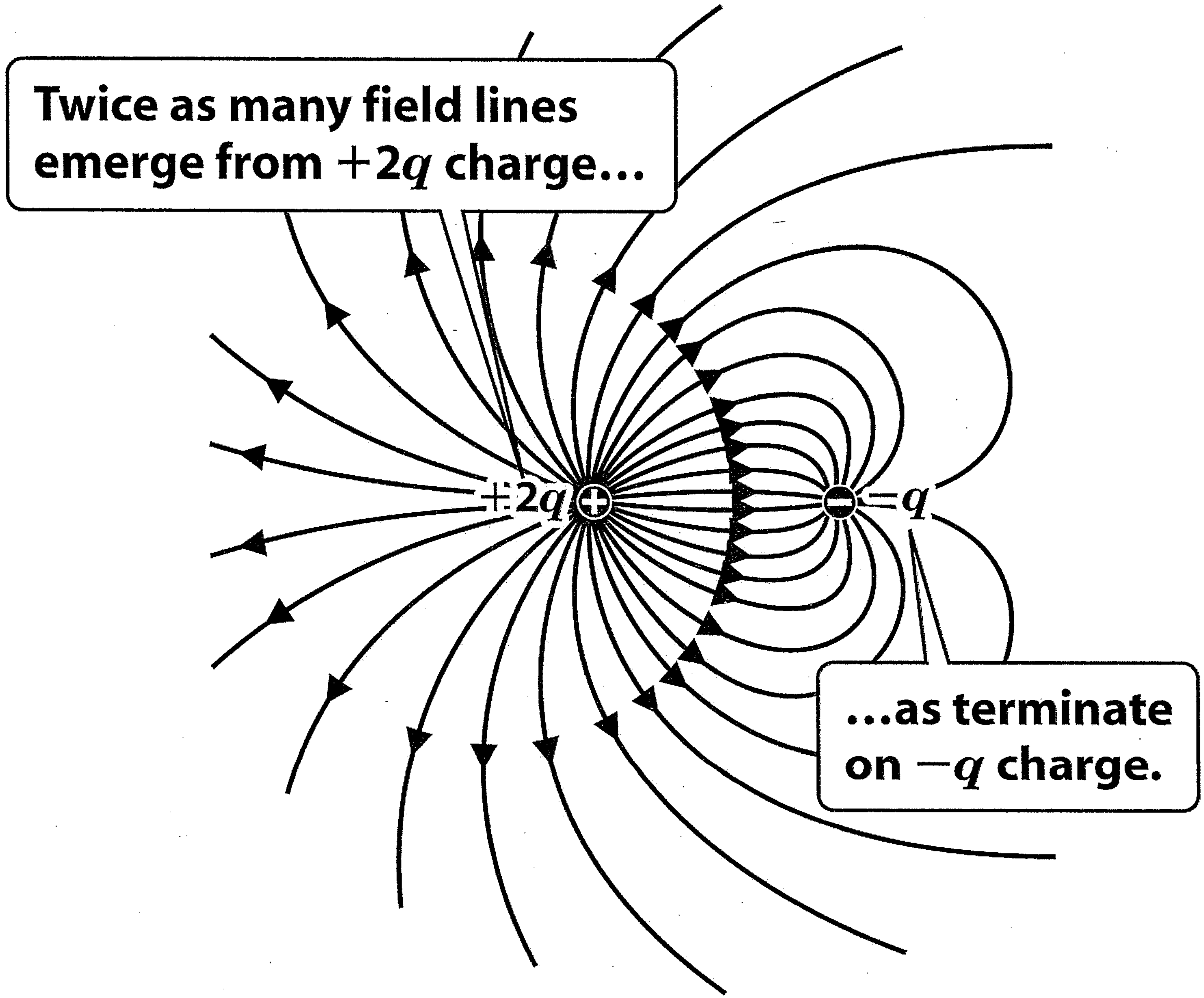


Figure 23-19 Physics for Engineers and Scientists 3/e

Field lines from positive charge here go off to infinity.

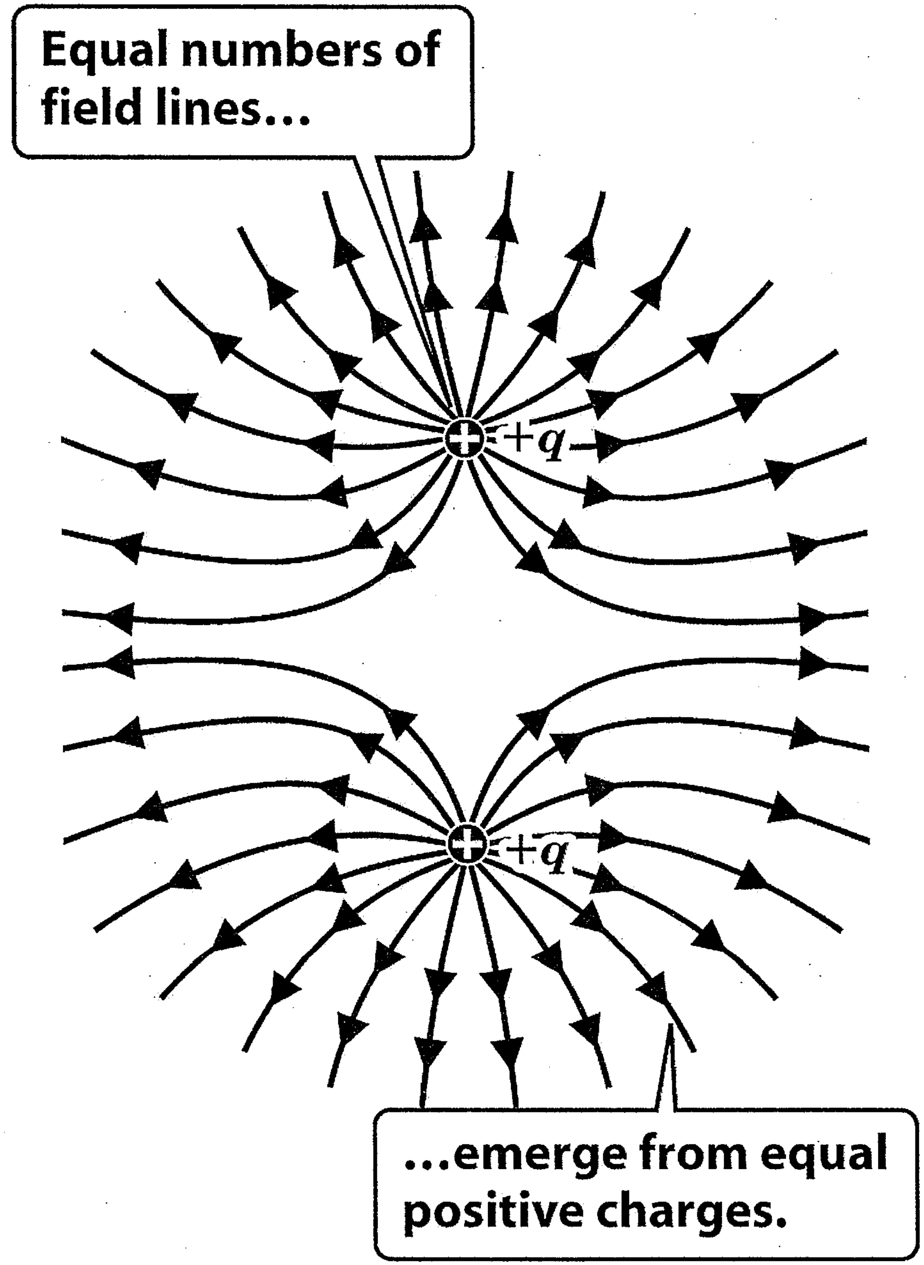


Figure 23-18 Physics for Engineers and Scientists 3/e

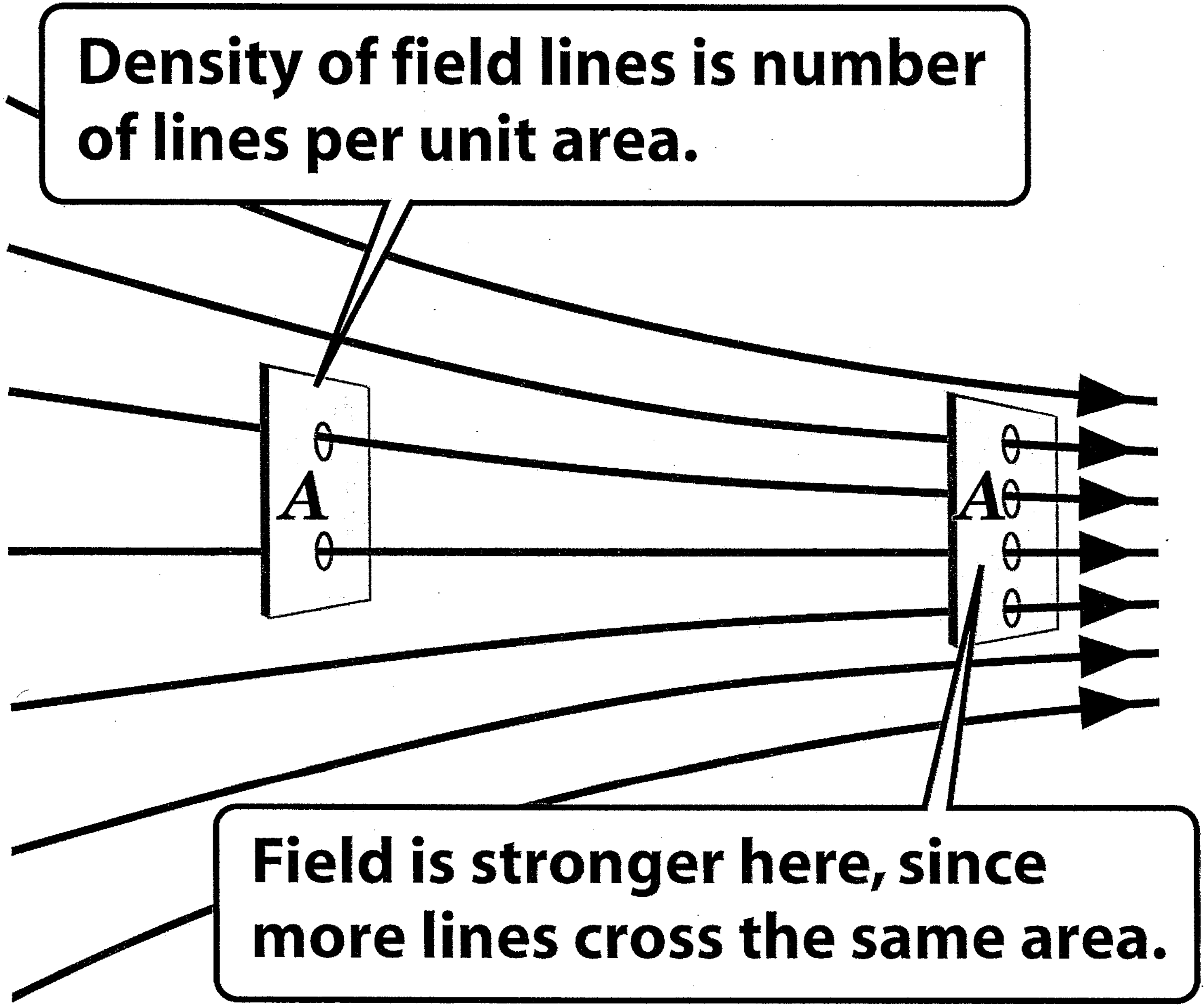


Figure 23-20 Physics for Engineers and Scientists 3/e

Motion in a uniform electric field

$$v_x = v_{ox} + a_x t \quad ; \quad v_y = v_{oy} + a_y t$$

$$\vec{a} = \vec{F}/m = q\vec{E}/m$$

$$x = x_0 + v_{ox} t + a_{ox} t^2/2 \quad ; \quad y = y_0 + v_{oy} t + a_{oy} t^2/2$$

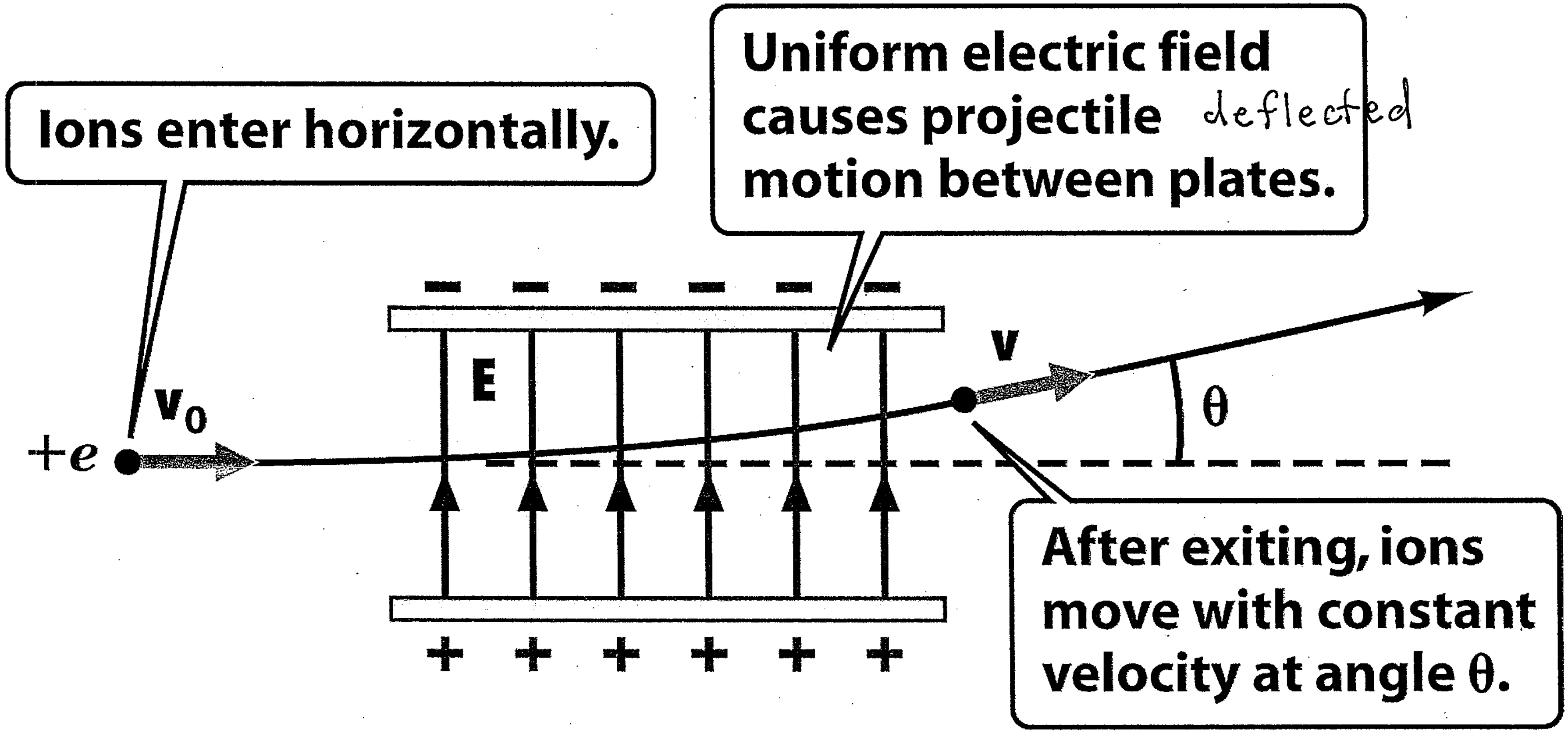


Figure 23-23 Physics for Engineers and Scientists 3/e

Particle trajeitory between plates is a parabola

$$x = x_0 + V_0 t$$

$$t = \frac{x - x_0}{V_0}$$

$$y = y_0 + a_{0y} t^2/2$$

$$= y_0 + \frac{eE}{2m} \frac{(x - x_0)^2}{V_0^2}$$

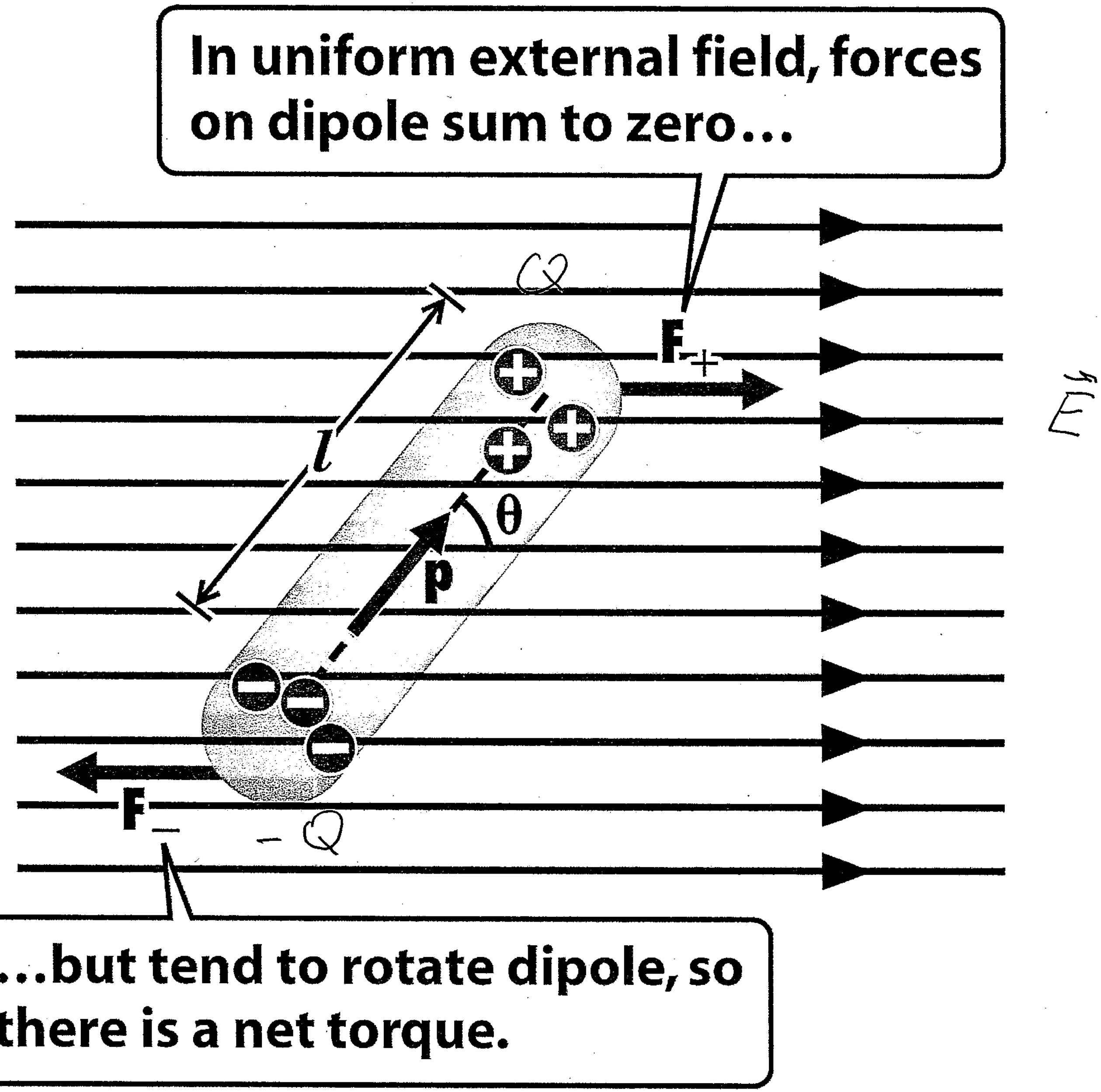
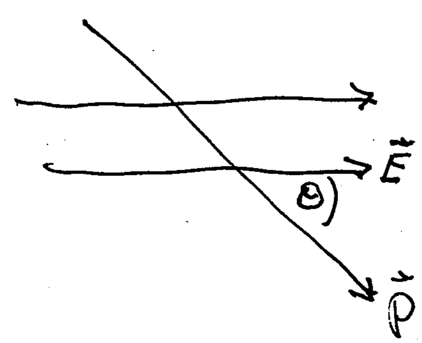


Figure 23-25 Physics for Engineers and Scientists 3/e

Torque on dipole will attempt to make it rotate



$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$|\vec{\tau}| = pE \sin\theta$$

(here torque in counter-clockwise (+) direction)

Work ΔW done by an external torque to move $\Delta\theta$

$$\Delta W = -\tau \Delta\theta = -pE \sin\theta \Delta\theta = pE \Delta \cos\theta$$

$U \equiv$ Potential Energy $\equiv$ − Work Done by external torque

$$U = -pE\cos\theta$$

(dipoles like to align along electric field lines (lowest energy))