

PHY 303L

Lecture # 3

Find the total electric field at P , given x , d , Q

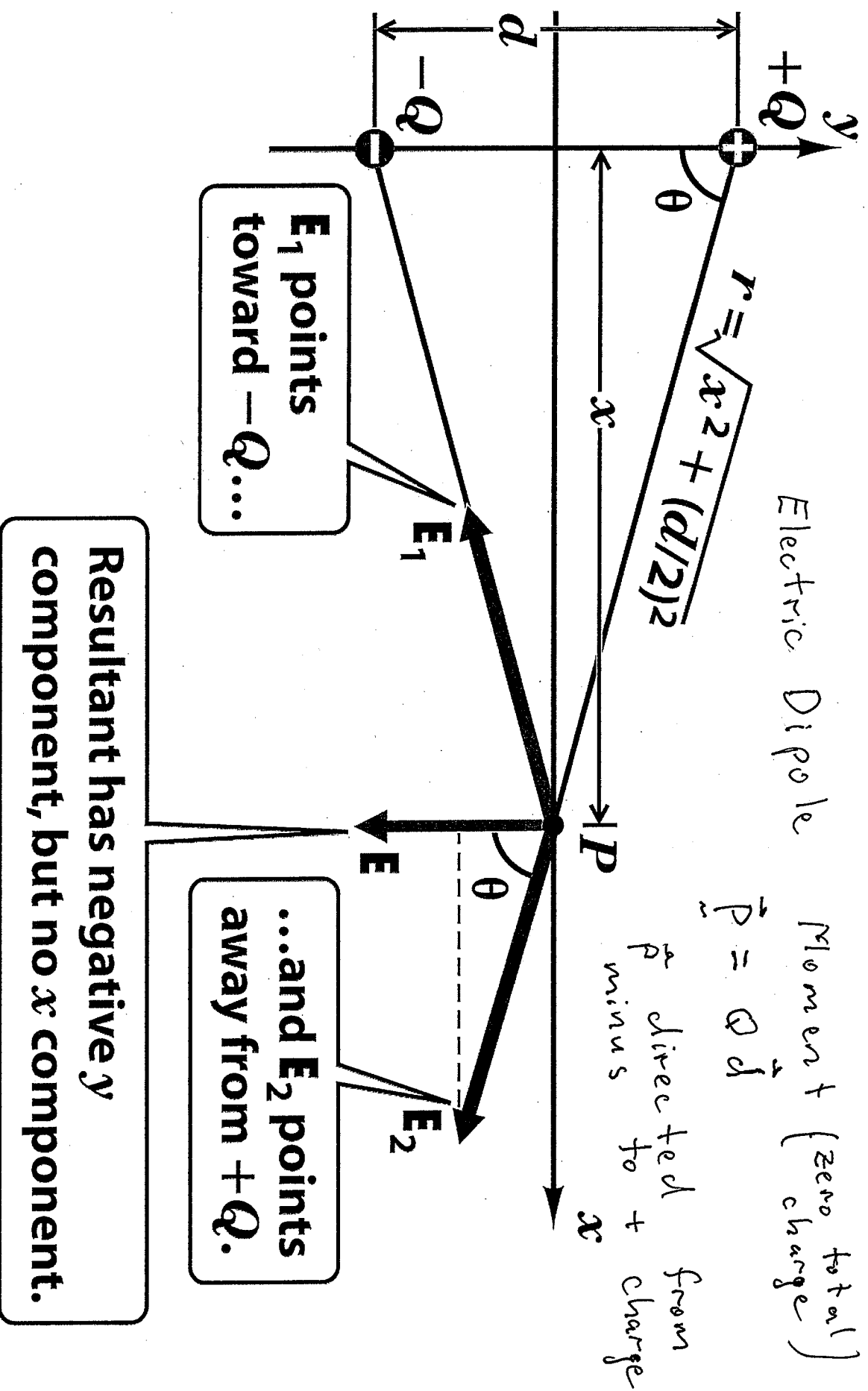


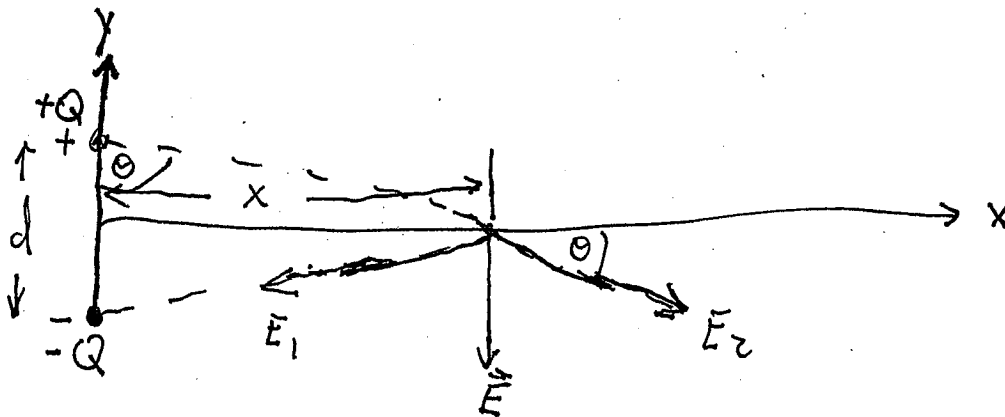
Figure 23-3 Physics for Engineers and Scientists 3/e
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$$\sin \theta = \frac{x}{[x^2 + (d/2)^2]^{1/2}}, \quad \cos \theta = \frac{d/2}{[x^2 + (d/2)^2]^{1/2}}$$

$$|E_z| = |E_1| = \frac{kQ}{r^2} = \frac{kQ}{x^2 + (d/2)^2}$$

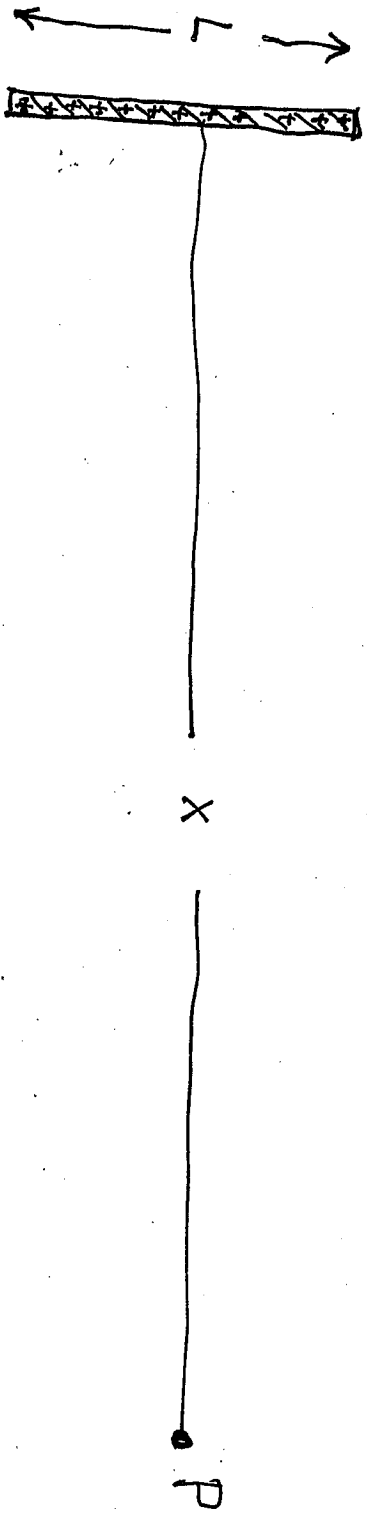
$$\vec{E}_{\text{total}} = -\hat{y} 2 \cos \theta |E_1|$$

$$= -2 \frac{d k Q \hat{y}}{2(x^2 + (d/2)^2)^{3/2}} = -\hat{y} \frac{k \vec{p}}{(x^2 + (d/2)^2)^{3/2}}$$



If $x \gg d$, $\vec{E}_{\text{dipole}} \rightarrow \frac{-\hat{y} p k}{x^3}$

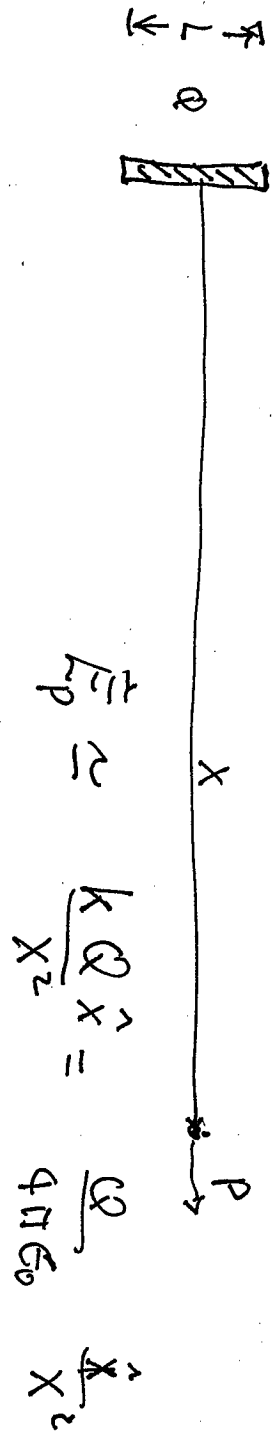
Consider a uniform charge rod of length L and total charge Q which is true at point P



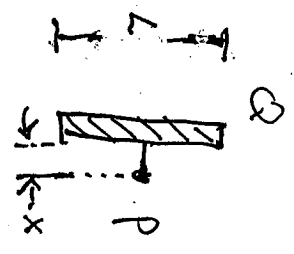
- (a) If x is much greater than L , E falls off as $1/x$
- (b) If x is much greater than L , E falls off as $1/x^2$
- (c) If x is much less than L , E falls off as $1/x^2$
- (d) If x is much less than L , E fall off as $1/x$

- (1) (2) (3) (4) (5)

a a b b a c c a d a a c b a d



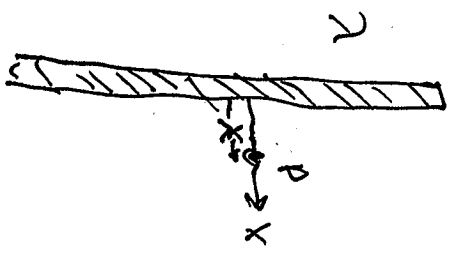
$$E_P \approx \frac{kQx}{x^2} = \frac{Q}{4\pi\epsilon_0} \frac{1}{x^2}$$



$$E_P \approx \frac{2kQ}{Lx} = \frac{Q}{2\pi\epsilon_0 Lx}$$

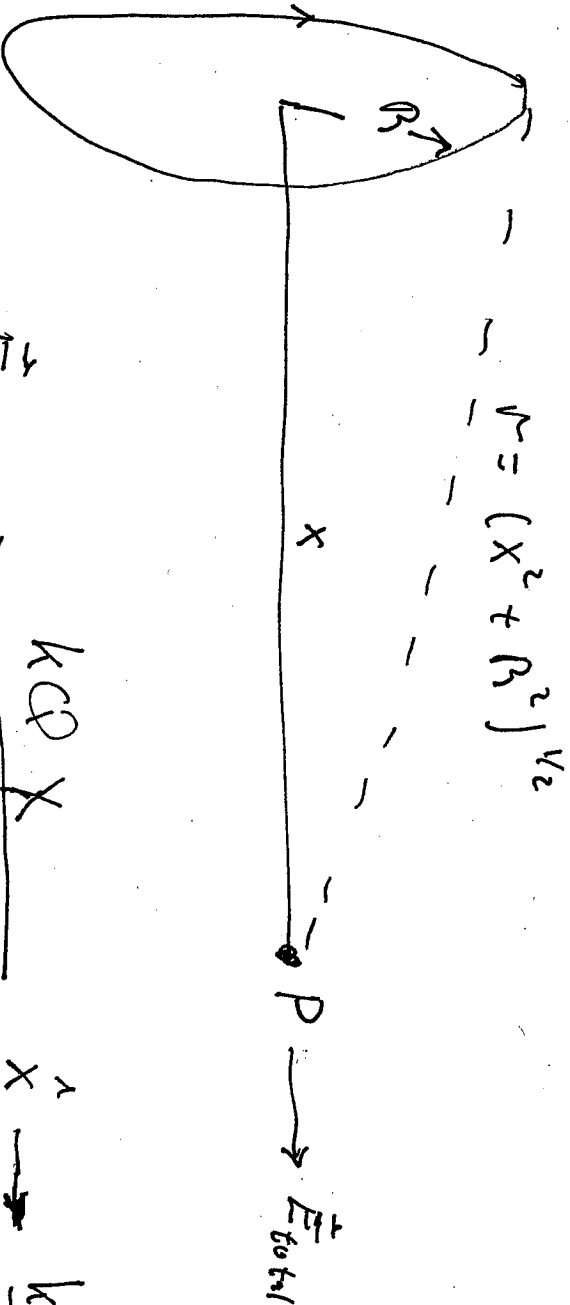
$$= \frac{\lambda}{2\pi\epsilon_0} \frac{1}{x}$$

This is answer for very long rod ($L \rightarrow \infty$), $\lambda = \frac{Q}{L}$ finite



$$E = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{x}$$

Electric field on axis of ring with uniformly distributed charge Q

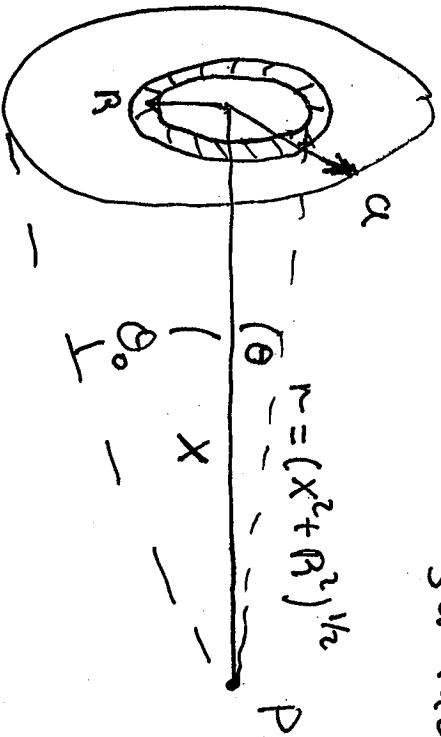


$$E_{total} = \frac{kQx}{(R^2 + x^2)^{3/2}} \quad \hat{x} \quad \frac{kQ}{x^2} \quad \hat{x}$$

$$\frac{Q}{4\pi\epsilon_0} \frac{x}{(R^2 + x^2)^{3/2}}$$

E lectric Field on axis of a disc
with uniformly distributed charge

Surface charge $\sigma = \frac{Q}{A} = \frac{Q}{\pi a^2}$



First we use the field from a ring
charge at radius R

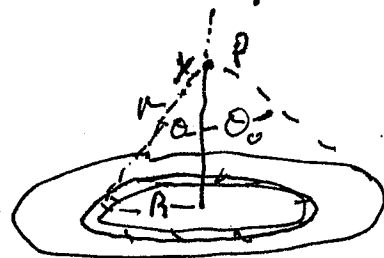
$$\Delta Q = \sigma \Delta A = \sigma 2\pi R dR \Rightarrow \Delta \vec{E} = \hat{x} \frac{\Delta Q k x}{(R^2 + x^2)^{3/2}}$$

Then we sum up (integrate) the contribution
for all the rings to find

$$\vec{E} = \frac{\sigma}{2\epsilon_0} (1 - \cos \theta_0)$$

Appendix Integration over dE_y

$$dE_y = k dQ \frac{\cos \theta}{r^2}$$



$$(dQ = \sigma dA = \sigma 2\pi R dR)$$

$$dE_y = k (\sigma 2\pi R dR) \frac{\cos \theta}{r^2}$$

$$r \cos \theta = y$$

$$\tan \theta = \frac{R}{y}, \therefore \frac{d\theta}{\cos^2 \theta} = \frac{dR}{y}$$

$$\therefore R = y \frac{\sin \theta}{\cos \theta}, \quad dR = y \frac{d\theta}{\cos^2 \theta}, \quad r^2 = \frac{y^2}{\cos^2 \theta}$$

Substituting into boxed expression for dE_y

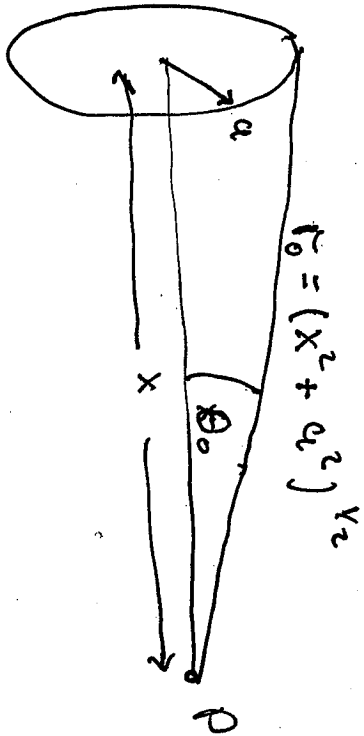
$$dE_y = k \sigma 2\pi \left(\frac{y \sin \theta}{\cos \theta} \right) \left(\frac{y d\theta}{\cos^2 \theta} \right) \cos \theta \frac{\cos^2 \theta}{y^2}$$

$$\text{(use } k = \frac{1}{4\pi \epsilon_0}$$

$$= \frac{\sigma}{2\epsilon_0} d\theta \sin \theta$$

$$\begin{aligned} \therefore E_y = \int dE_y &= \frac{\sigma}{2\epsilon_0} \int_0^{\theta_0} d\theta \sin \theta = \frac{-\sigma}{2\epsilon_0} \cos \theta \Big|_0^{\theta_0} \\ &= \frac{\sigma}{2\epsilon_0} (1 - \cos \theta_0) \quad (6a) \end{aligned}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \left[1 - \cos \theta_0 \right], \quad \sigma = \frac{Q}{\pi a^2}$$



The field at P far from the disc ($x \gg a$) falls of as

- (1) $1/x$
- (2) $1/x^2$
- (3) Constant
- (4) Not determined

The field close to the disc, is constant independent of x if $x \ll a$.

$$\vec{E} \approx \frac{\sigma}{2\epsilon_0} \hat{x} = \frac{Q}{2\pi a^2 \epsilon_0} \hat{x} \quad \left(\begin{array}{l} \text{does not even have} \\ \text{to be on axis} \end{array} \right)$$

(7)

Fields near uniformly charged plates

Electric field of negative sheet points toward the sheet on each side...

...and field due to positive sheet points away.

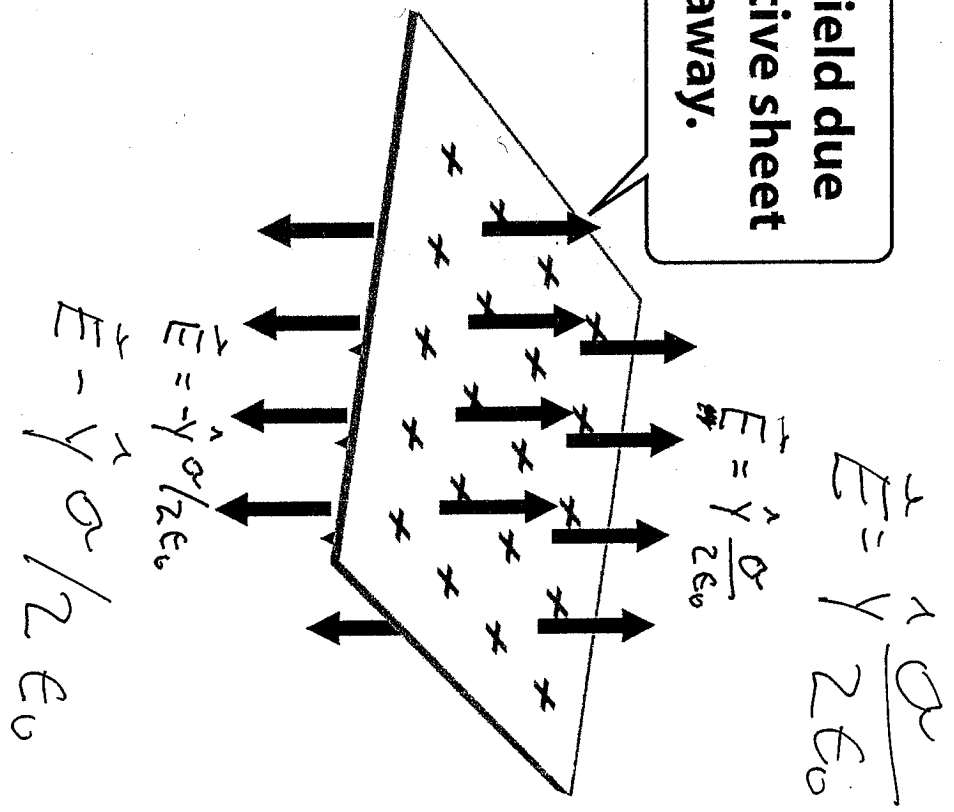
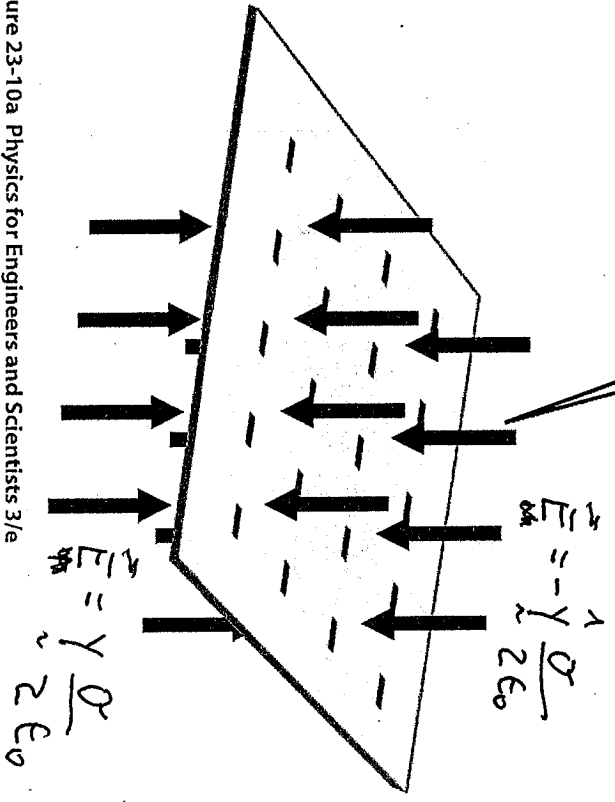
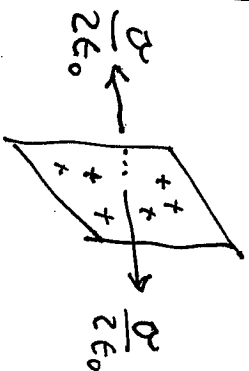
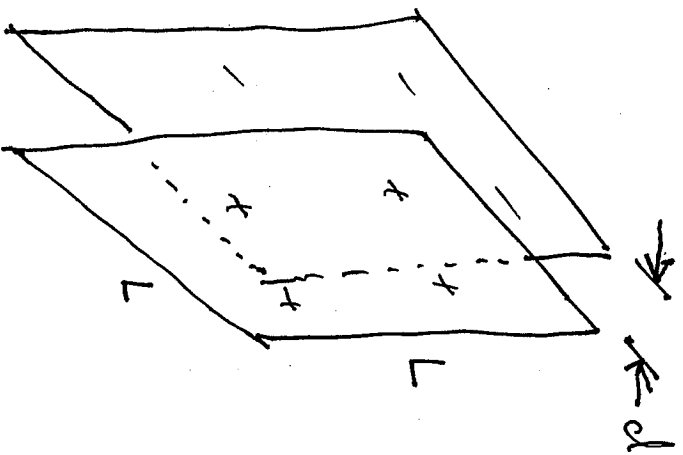


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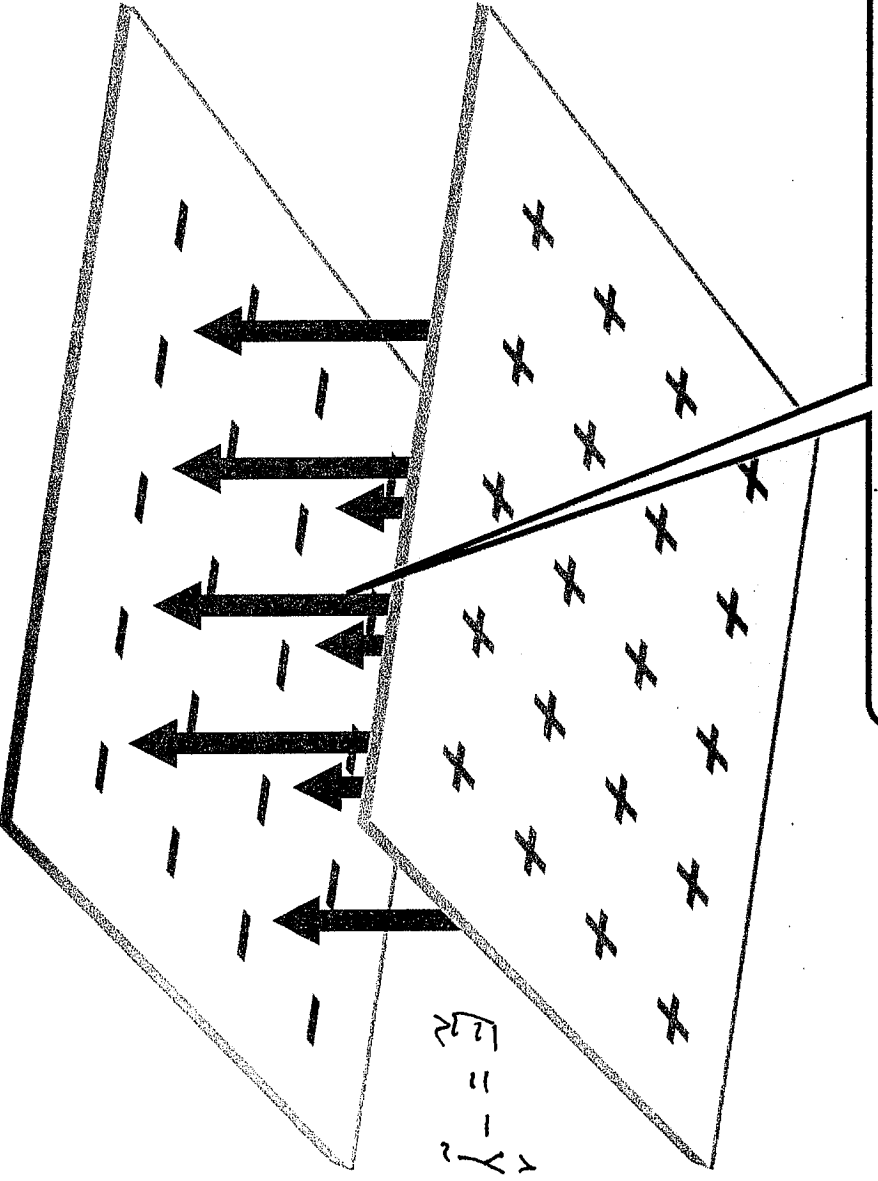
What is the field between uniformly, but oppositely charged plates each with equal and opposite surface charge σ . (the plates are larger than their separation)? (for $d \ll L$)



- (a) $\frac{\sigma}{\epsilon_0}$ inside, $\frac{\pm\sigma}{2\epsilon_0}$ outside
- (b) $\frac{-\sigma}{2\epsilon_0}$ inside, 0 outside
- (c) $-\frac{\sigma}{\epsilon_0}$ inside, 0 outside
- (d) 0 inside, $\pm \frac{\sigma}{2\epsilon_0}$ outside
- (e) $\frac{\sigma}{\epsilon}$ inside, $-\frac{\sigma}{\epsilon_0}$ outside

For both sheets, net field cancels outside and doubles inside.

why?



$$\vec{E} = -\hat{y} \frac{\sigma}{\epsilon_0}$$

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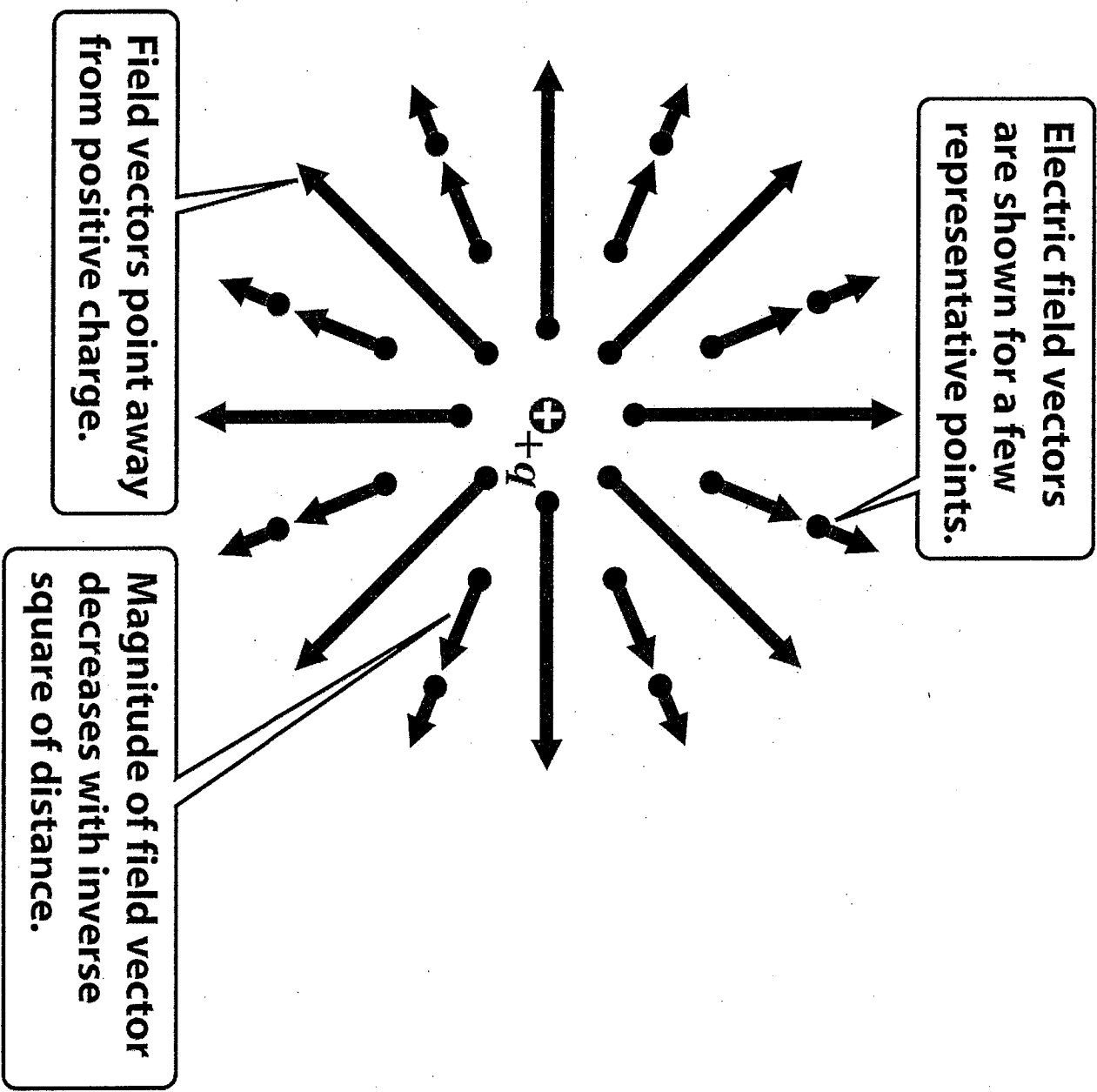


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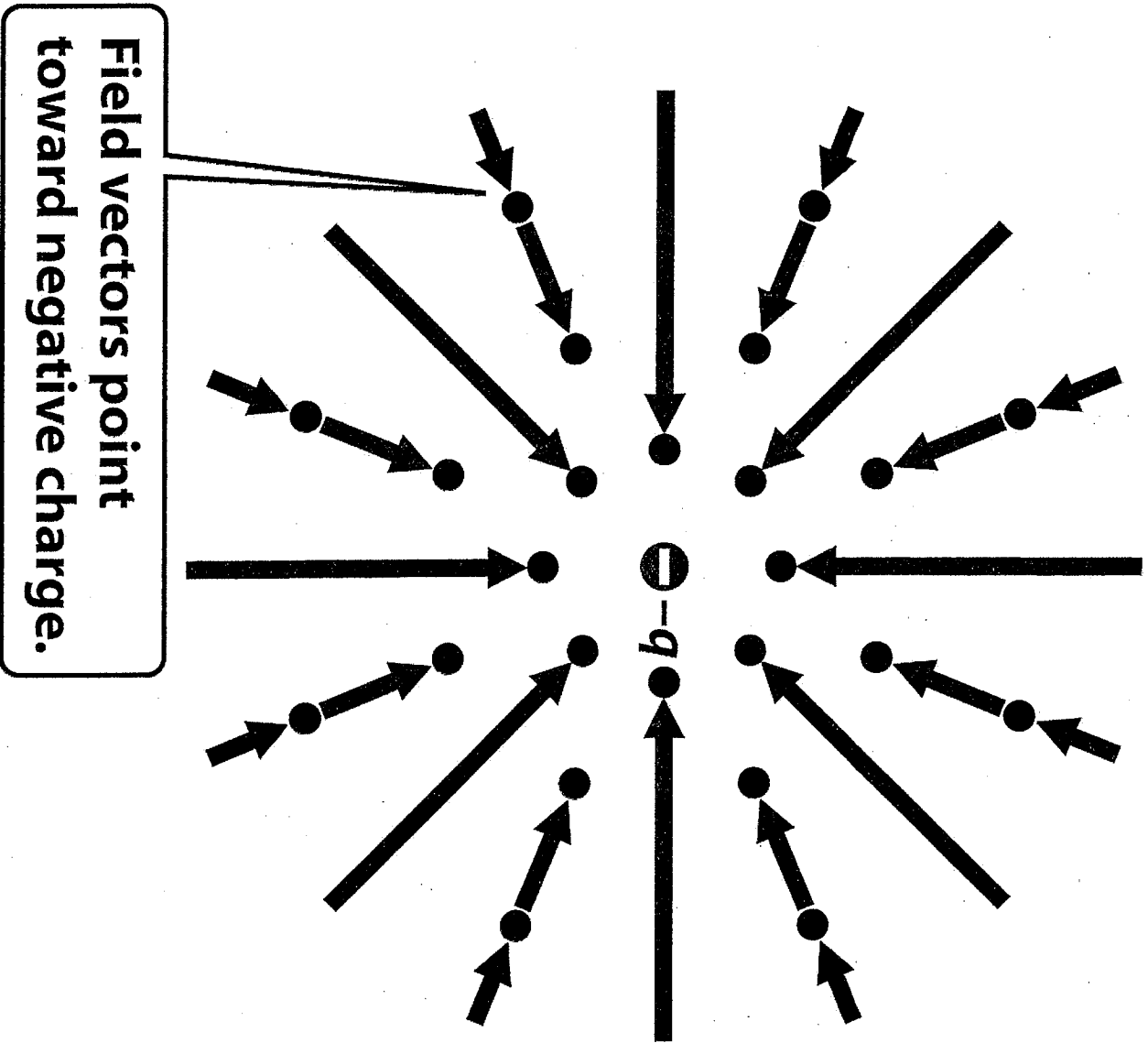
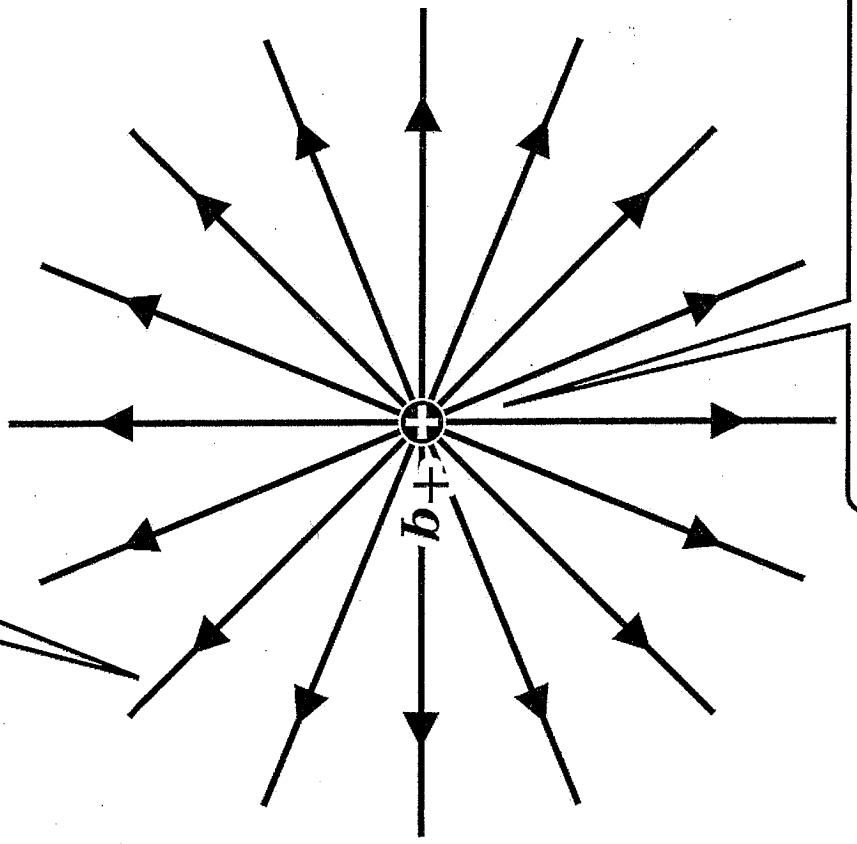


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Electric field is strong where field lines are closely spaced...



...and weak where lines are far apart.

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**Direction of electric field
is tangent to field line.**

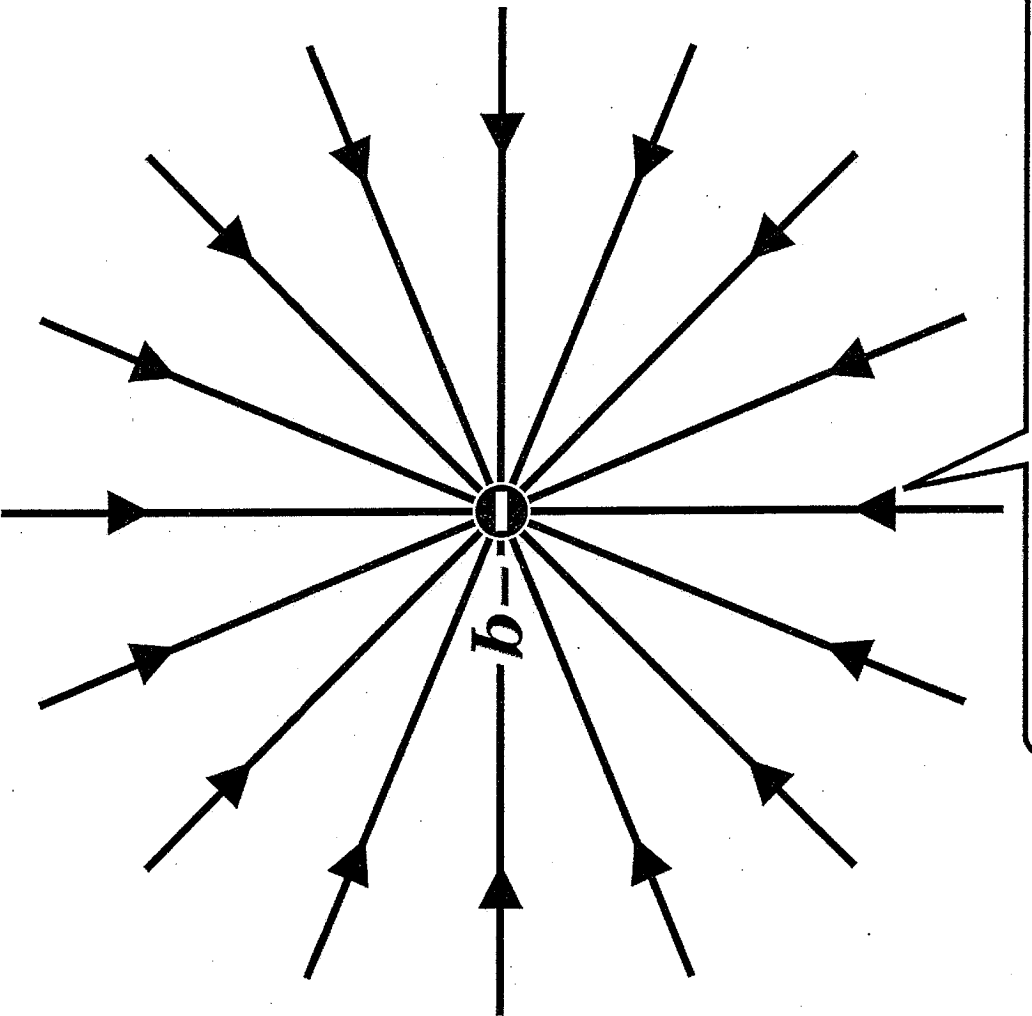


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**Field line diagram displays
twice as many lines for
twice as much charge.**

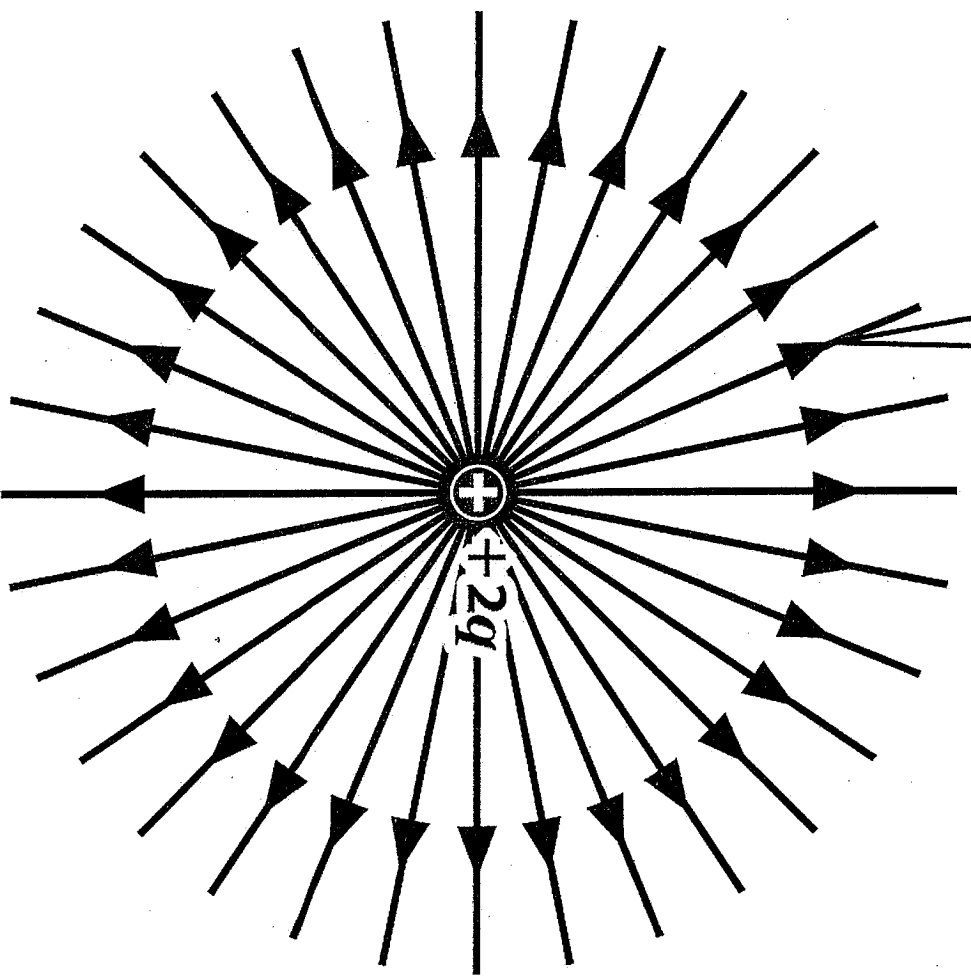


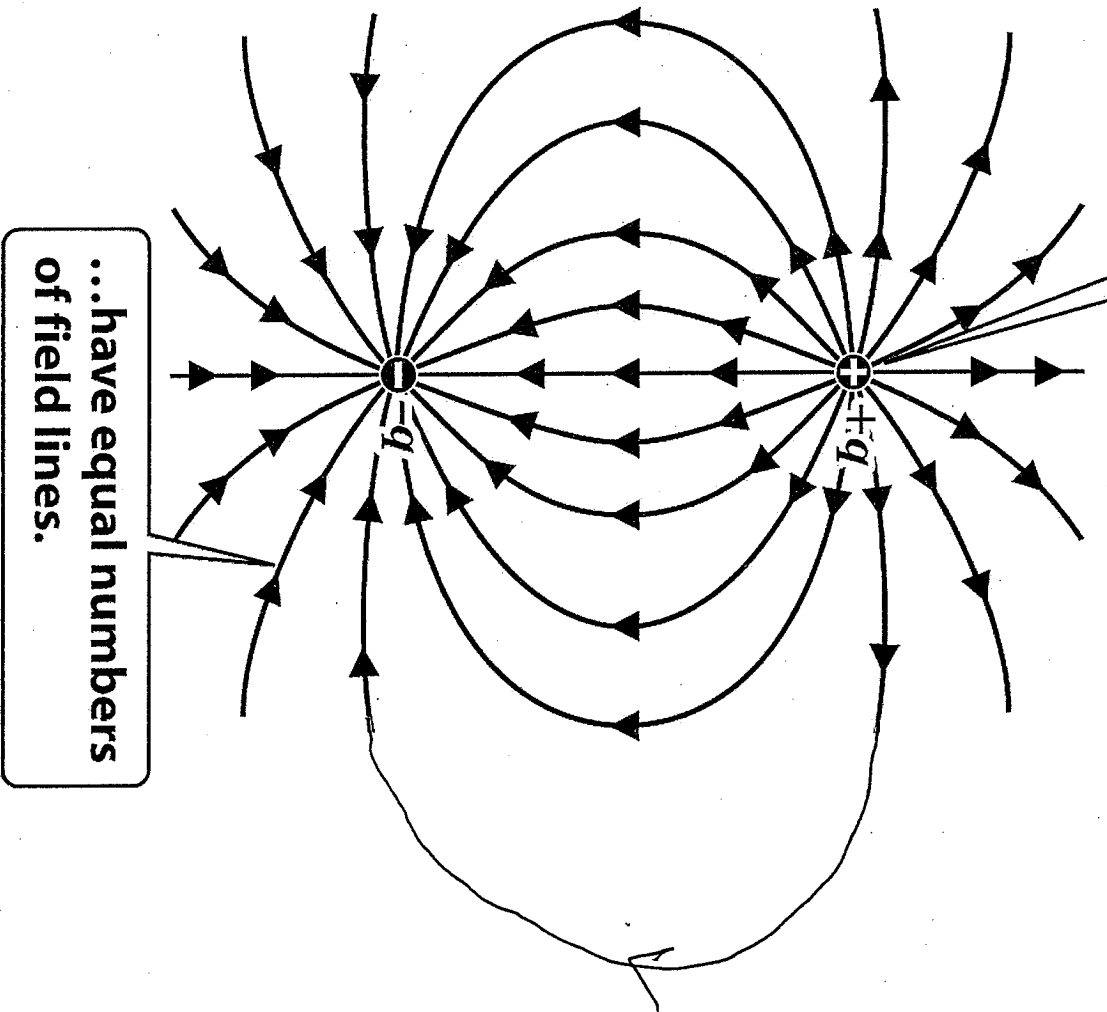
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Dipoles have zero net charge

For dipoles

all field lines begin at + charge and end on minus charge

Equal-magnitude positive and negative charges...



...have equal numbers of field lines.

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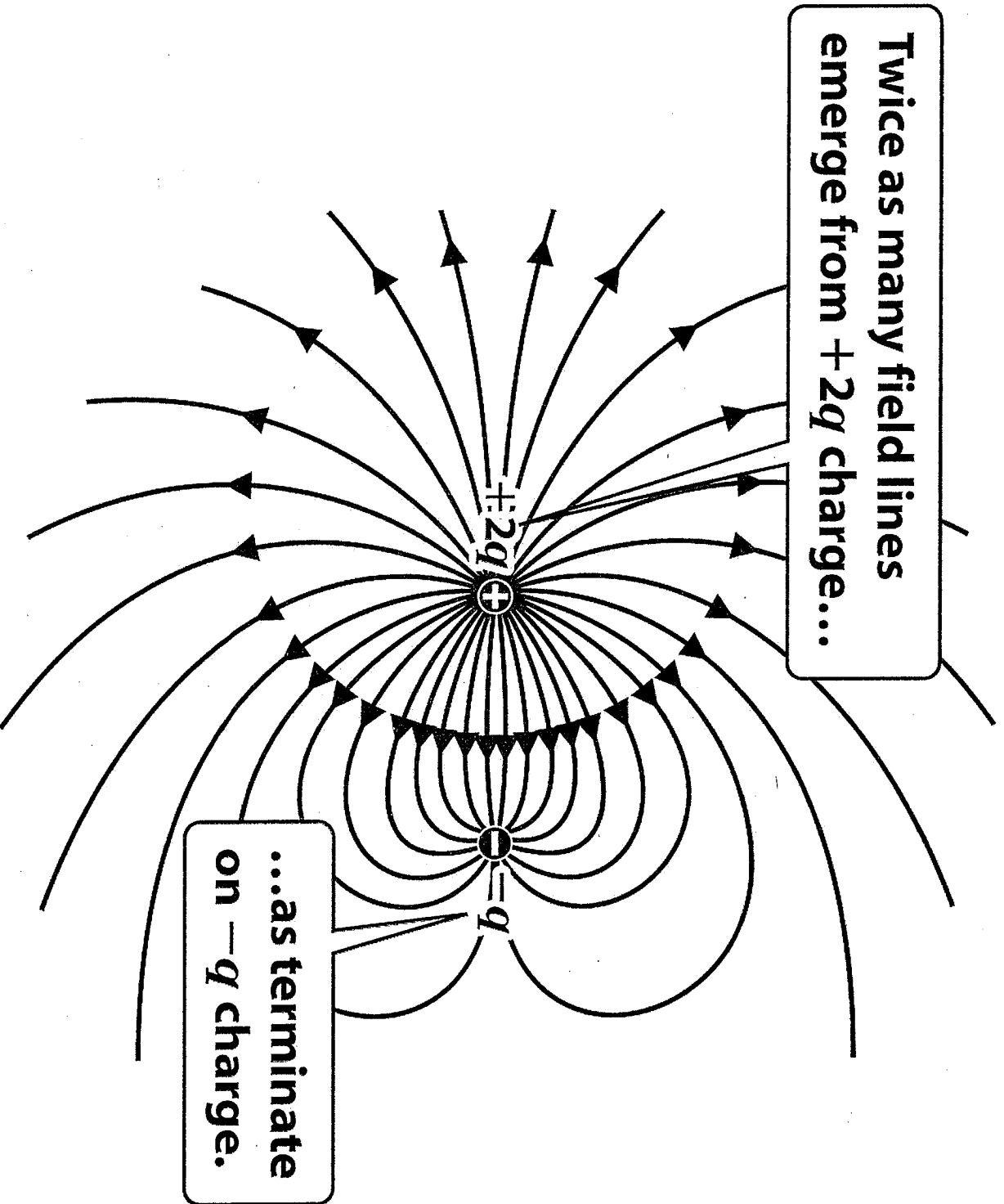


Figure 23-19 Physics for Engineers and Scientists 3/e
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Field lines
from positive
charge here
go off to
infinity

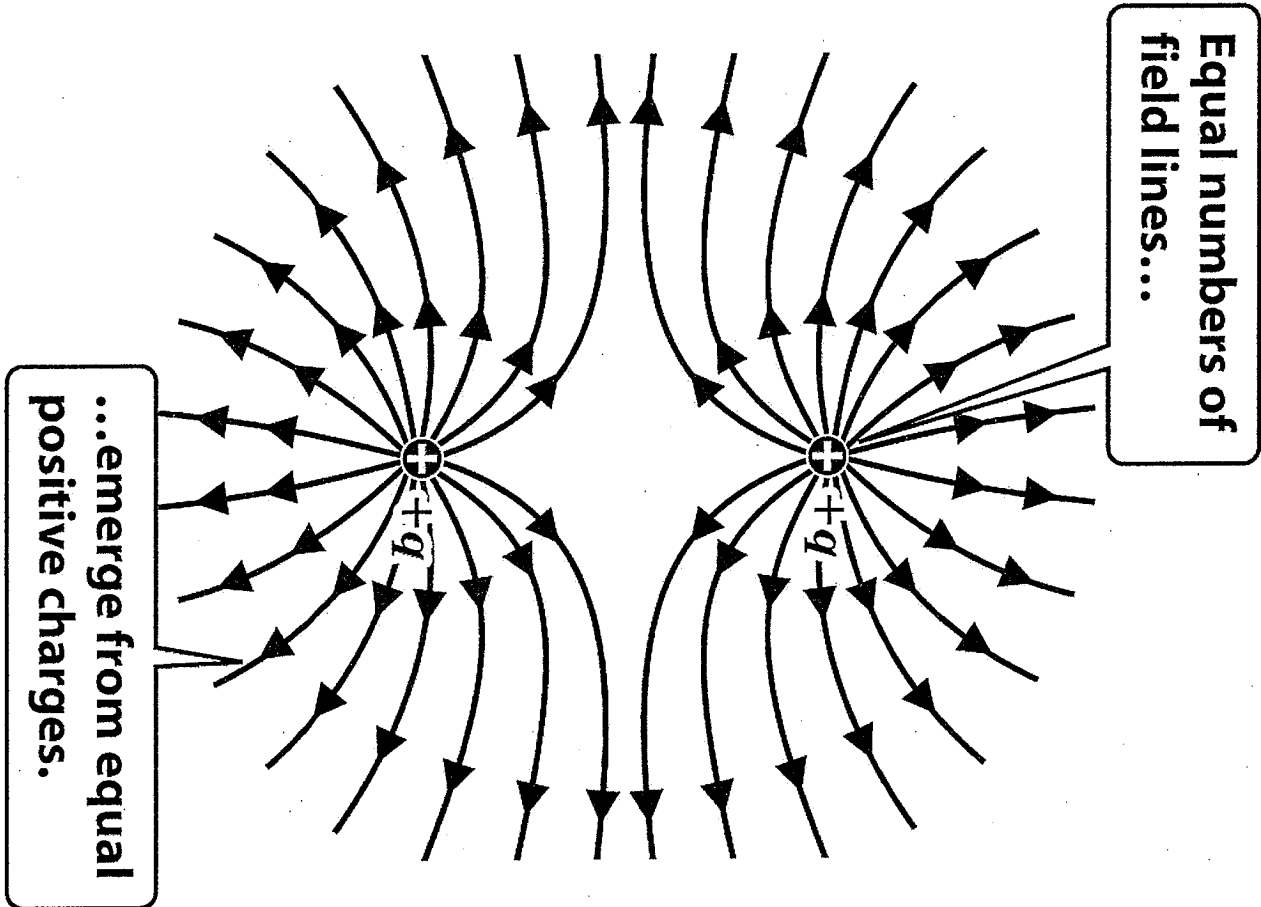
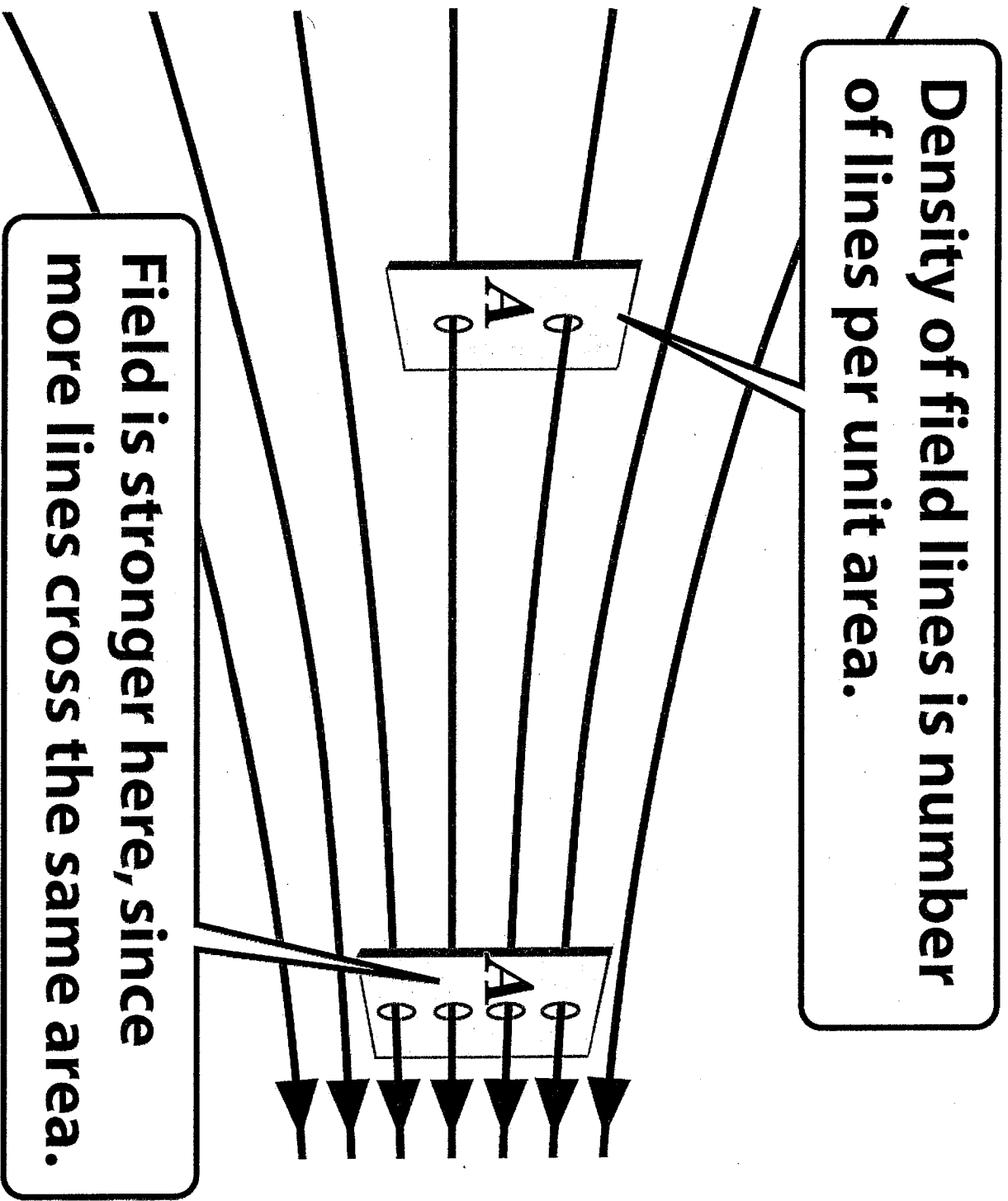


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Density of field lines is number of lines per unit area.

Field is stronger here, since more lines cross the same area.

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Motion in a uniform electric field

$$v_x = v_{0x} + a_x t ; \quad v_y = v_{0y} + a_y t$$

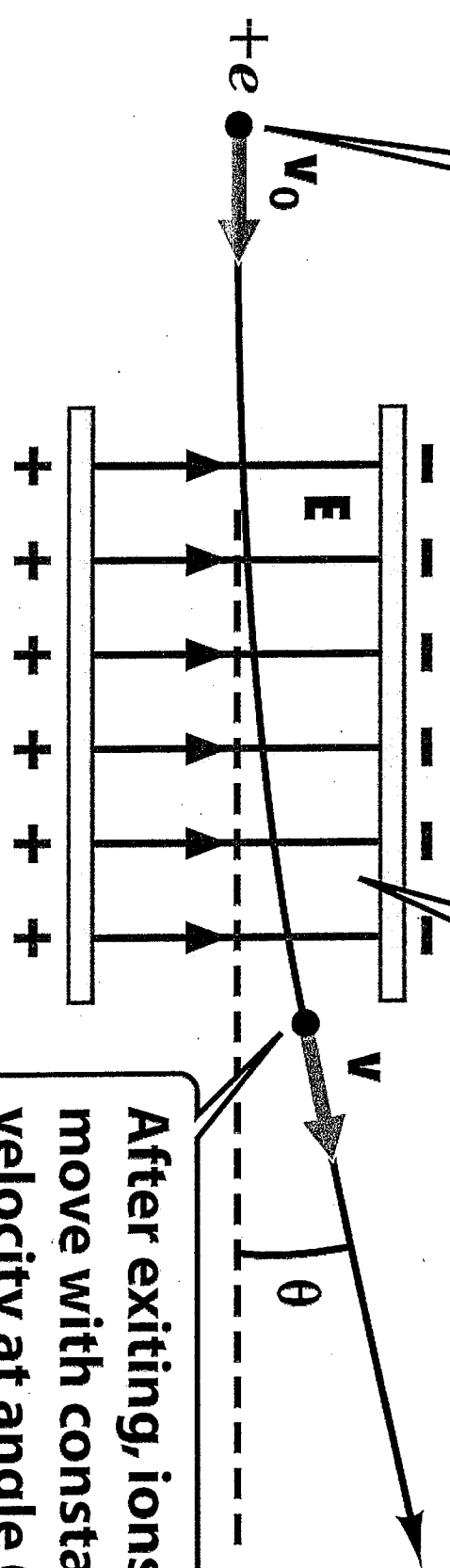
$$\vec{a} = \vec{F}/m = q\vec{E}/m$$

$$x = x_0 + v_{0x}t + a_{0x}t^2/2 ; \quad y = y_0 + v_{0y}t + a_{0y}t^2/2$$

Ions enter horizontally.

Uniform electric field causes projectile deflected motion between plates.

After exiting, ions move with constant velocity at angle θ .



Particle trajectory between plates is a parabola

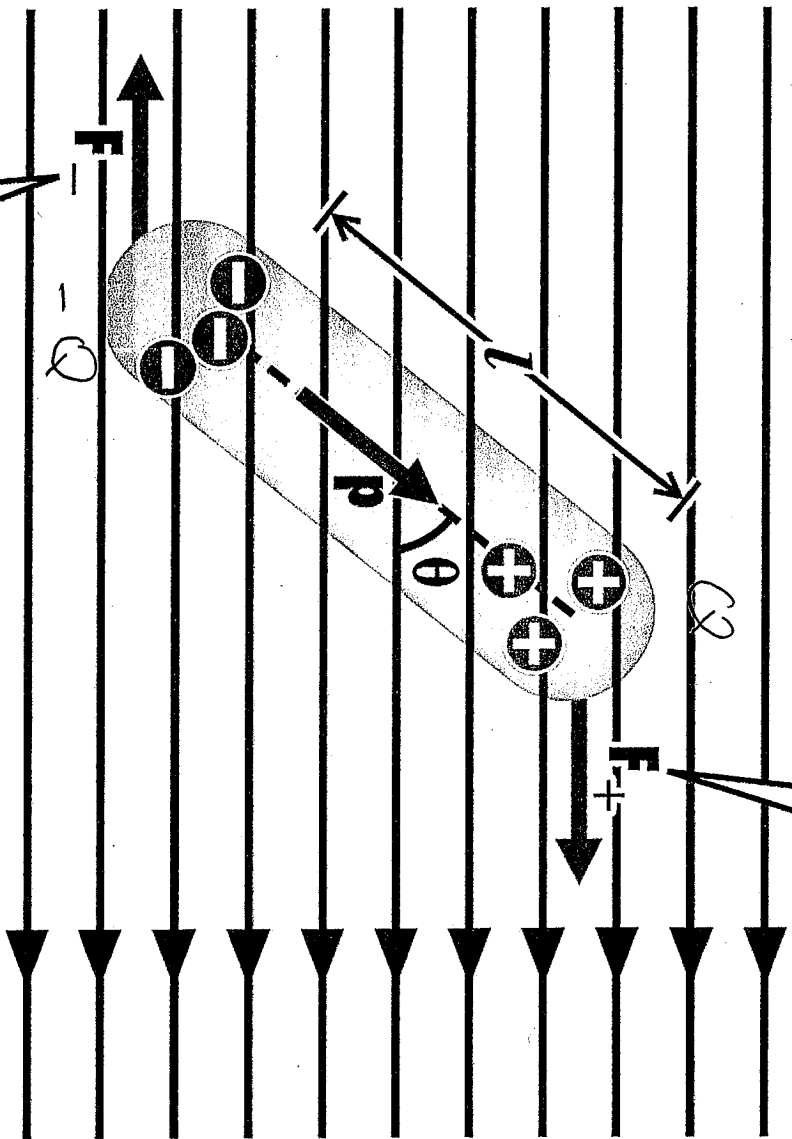
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$$X = X_0 + V_0 t$$

$$t = \frac{X - X_0}{V_0}$$

$$y = y_0 + a_0 y t^{1/2} \\ = y_0 + \frac{e E}{2m} \frac{(X - X_0)^2}{V_0^2}$$

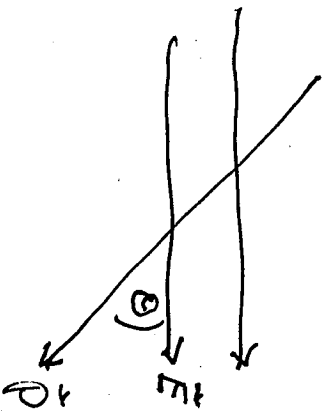
In uniform external field, forces on dipole sum to zero...



...but tend to rotate dipole, so there is a net torque.

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Torque on dipole will attempt to make it rotate



$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$|\vec{\tau}| = pE \sin \theta$$

(here torque in counter-clockwise direction)

Work ΔW done by an external torque to move $\Delta \theta$

$$\Delta W = -\tau \Delta \theta = -pE \sin \theta \Delta \theta = pE \Delta \cos \theta$$

$U \equiv$ Potential Energy

\equiv - Work Done by external torque

$$U = -pE \cos \theta$$

(dipoles like to align along electric field lines (lowest energy))