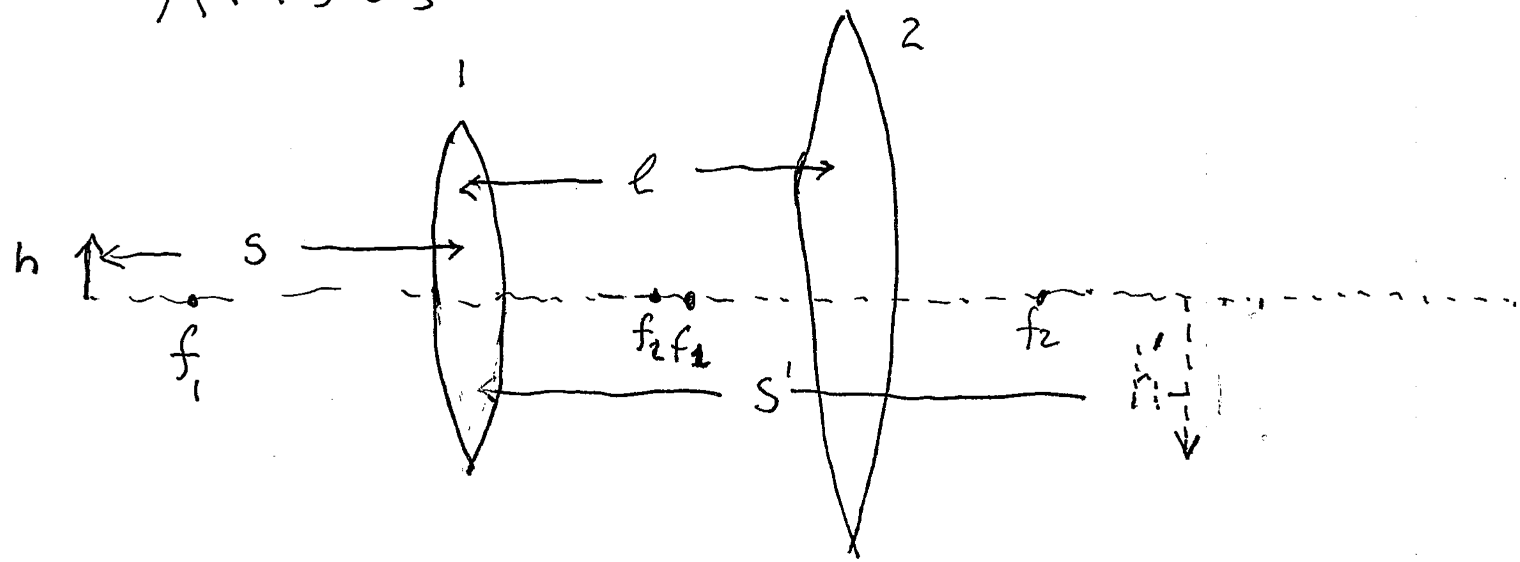
Lecture # 24

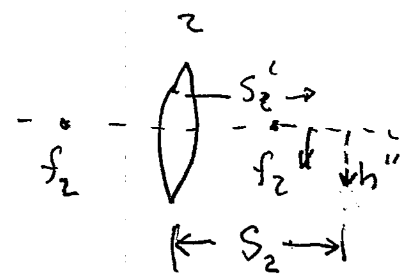
Diffraction

Virtual object

Arises in two lens system



where is the real image?



$$s_2 = -|s' - \ell|$$

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$$

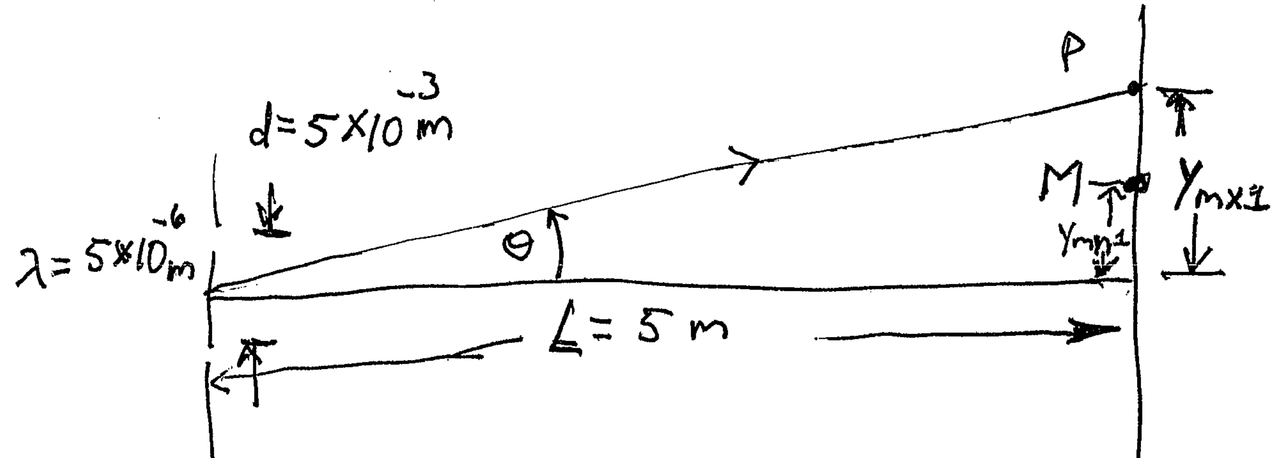
$$s_2 = -(s_1' - \ell)$$

$$\frac{1}{-(s_1' - \ell)} + \frac{1}{s_2'} = \frac{1}{f_2}$$

$$\frac{1}{s_2'} = \frac{1}{f_2} + \frac{1}{s_1' - \ell}$$

$$s_2' = \frac{f_2(s_1' - \ell)}{f_2 + s_1' - \ell}$$

~~$s_2' = s_1 f_1 / (s_1 - f_1)$~~ =



2-Slit Inference

If P is the first bright spot of the interference pattern, and M the first interference minimum. Then

$\theta = ?$

(1) (a) 5×10^{-3} rad (b) 1×10^{-3} rad (c) 5×10^{-4} rad

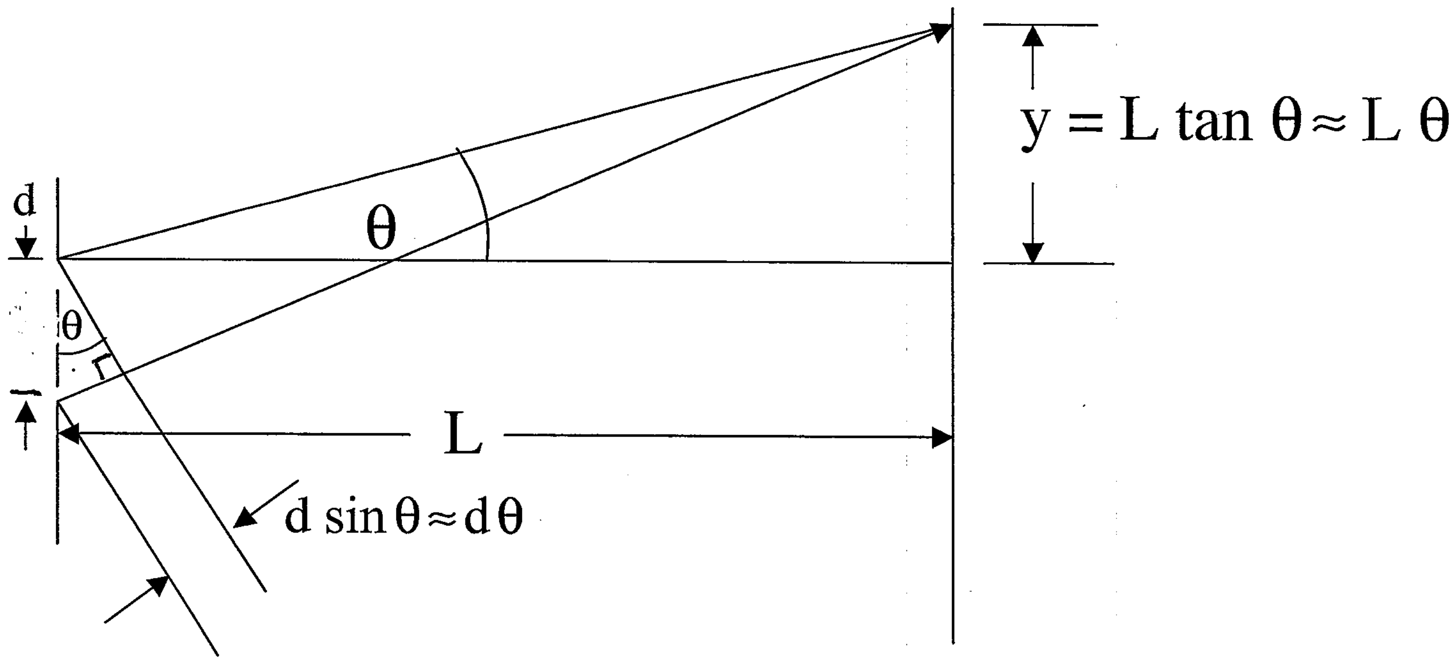
Y_{mx1}, the maximum at point P, is

(2) (a) 5×10^{-3} m (b) 2.5×10^{-3} m (c) 1.0×10^{-3} m

(3) The angular position of the first interference minimum, Y_{mn1} at point M is (same choice as (1))

(4) The screen position of the first interference minimum is (same choice as (2))

TWO-SLIT INTERFERENCE PATTERN



For maxima with small angle $\theta = y/L$

$d\,\theta = n\,\lambda\ (n = 0, \pm 1, \pm 2, \ldots) = d\,y_n\,/L$

For minima with small angle $\theta = y/L$

$d\,\theta = (n+1/2)\,\lambda\ (n = 0, \pm 1, \pm 2, \ldots) = d\,y_{n+1/2}/L$

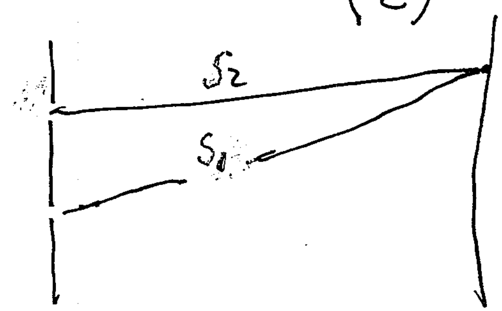
Intensity of 2 - slit interference pattern

Recall when we add two equal amplitude waves of the same phases

$$\mathcal{S} = \overline{\left[\sin\left(\omega t + \frac{\phi}{2}\right) + \sin\left(\omega t - \frac{\phi}{2}\right)\right]^2} = \overline{\left[2\cos\left(\frac{\phi}{2}\right)\sin\omega t\right]^2}$$

$$= 4\cos^2\left(\frac{\phi}{2}\right)\overline{\sin^2\omega t} = 2\cos^2\left(\frac{\phi}{2}\right)$$

when: $\phi = 2\pi\frac{(S_2 - S_1)}{\lambda}$



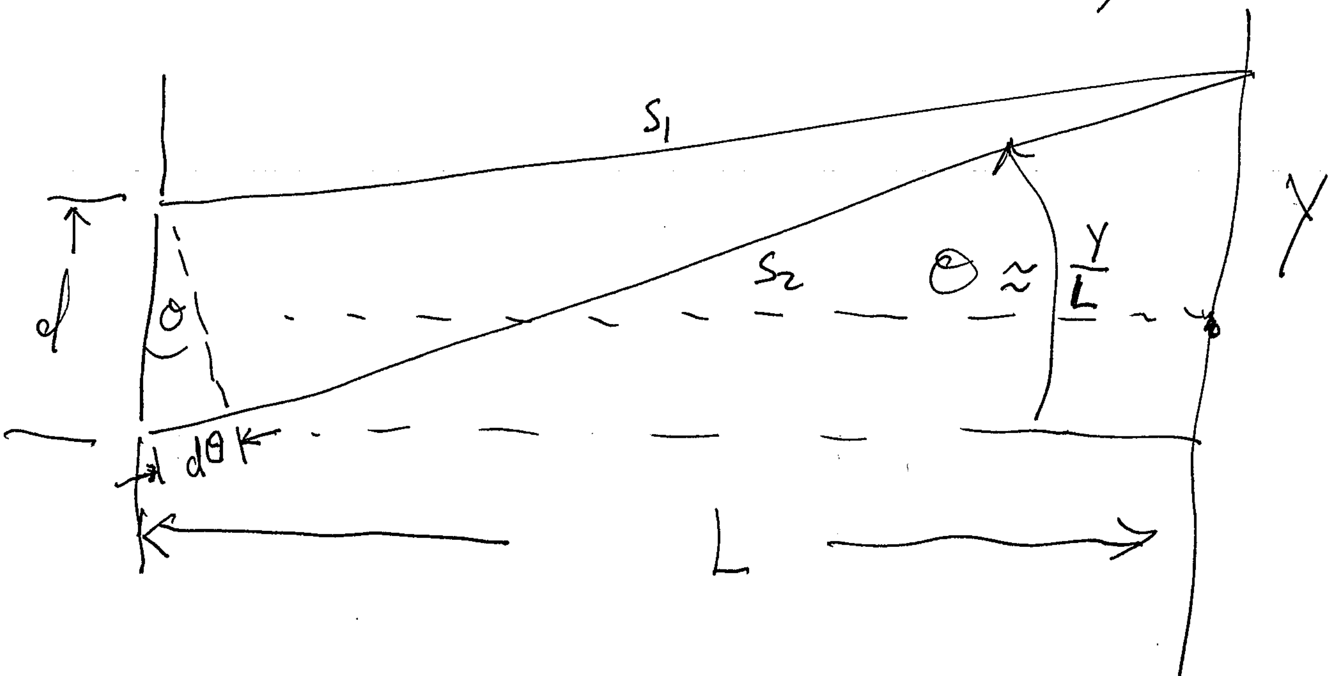
$$\mathcal{S} = \cos^2\left(\frac{\pi(S_2 - S_1)}{\lambda}\right)$$

If Brightness of pattern is

$$I = I_0\cos^2\left(\frac{\pi(S_2 - S_1)}{\lambda}\right)$$

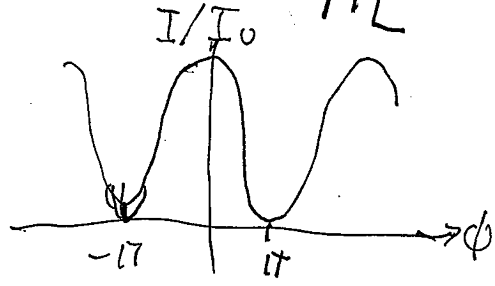
2-slit Interference pattern

$$I = I_0 \cos^2\left(\frac{\pi (s_2 - s_1)}{\lambda}\right)$$



$$S_2 - S_1 = d\sin\theta \simeq d\theta \simeq dy/L$$

as $\theta \simeq y/L$



$$I = \cos^2\left(\frac{\pi (S_2 - S_1)}{\lambda}\right) I_0$$

$$= \cos^2\left(\frac{\pi d y}{\lambda L}\right) I_0$$

$$= I_0 \cos^2\left(\frac{\pi d}{\lambda}\theta\right)$$

More slits sharpens maxima

on screen

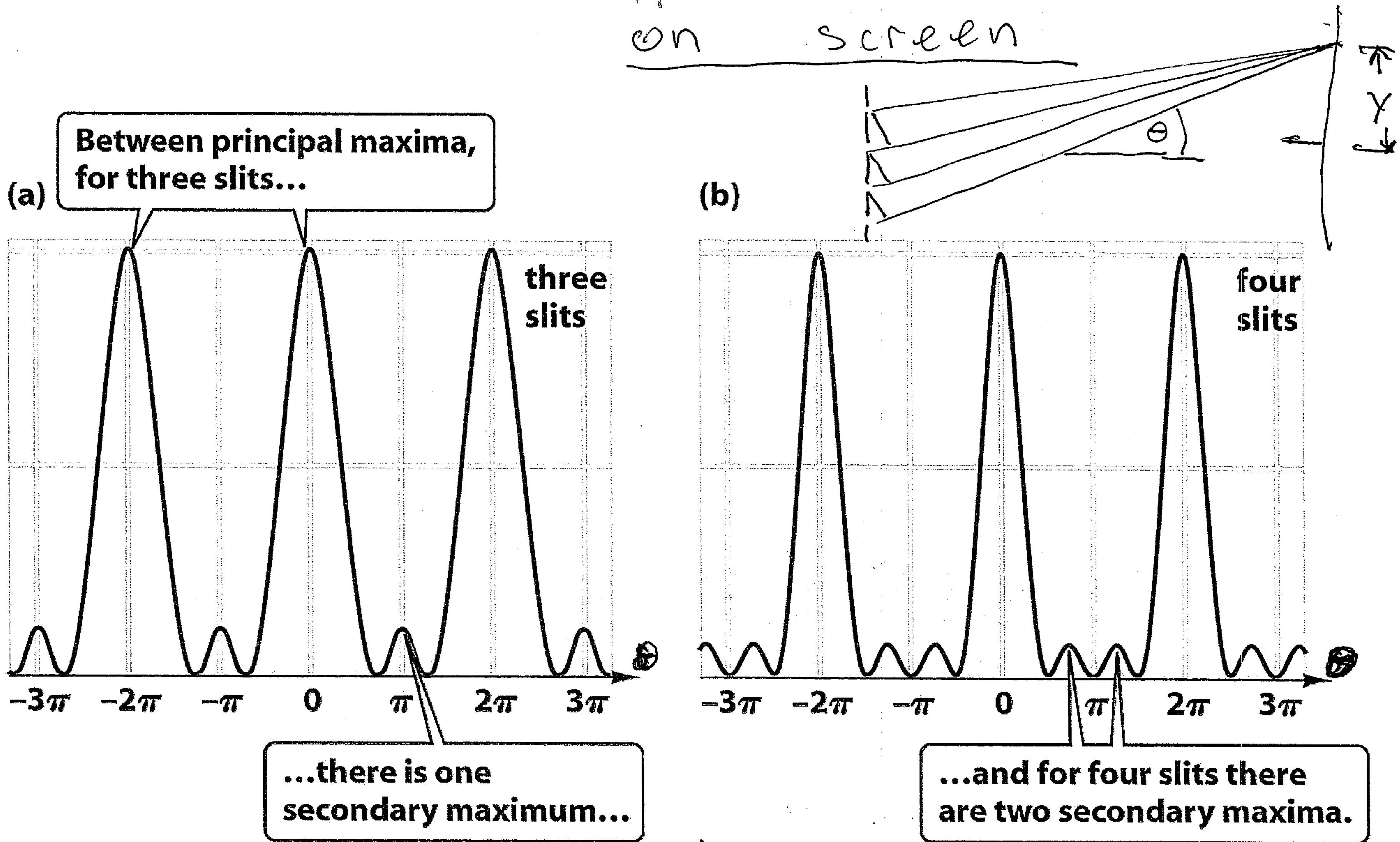


Figure 35-27 Physics for Engineers and Scientists 3/e

$\theta = 2\pi \frac{dy}{\lambda L}$; peaks where $dy/\lambda L = m$ $(0, \pm 1, \pm 2 \ldots)$

Diffraction grating (many slots ~1000), and intensity extremely focused around $y = \frac{\lambda L}{d} m$

Figure 35-1 Physics for Engineers and Scientists 3/e
Peter Aprahamian/Photo Researchers, Inc.

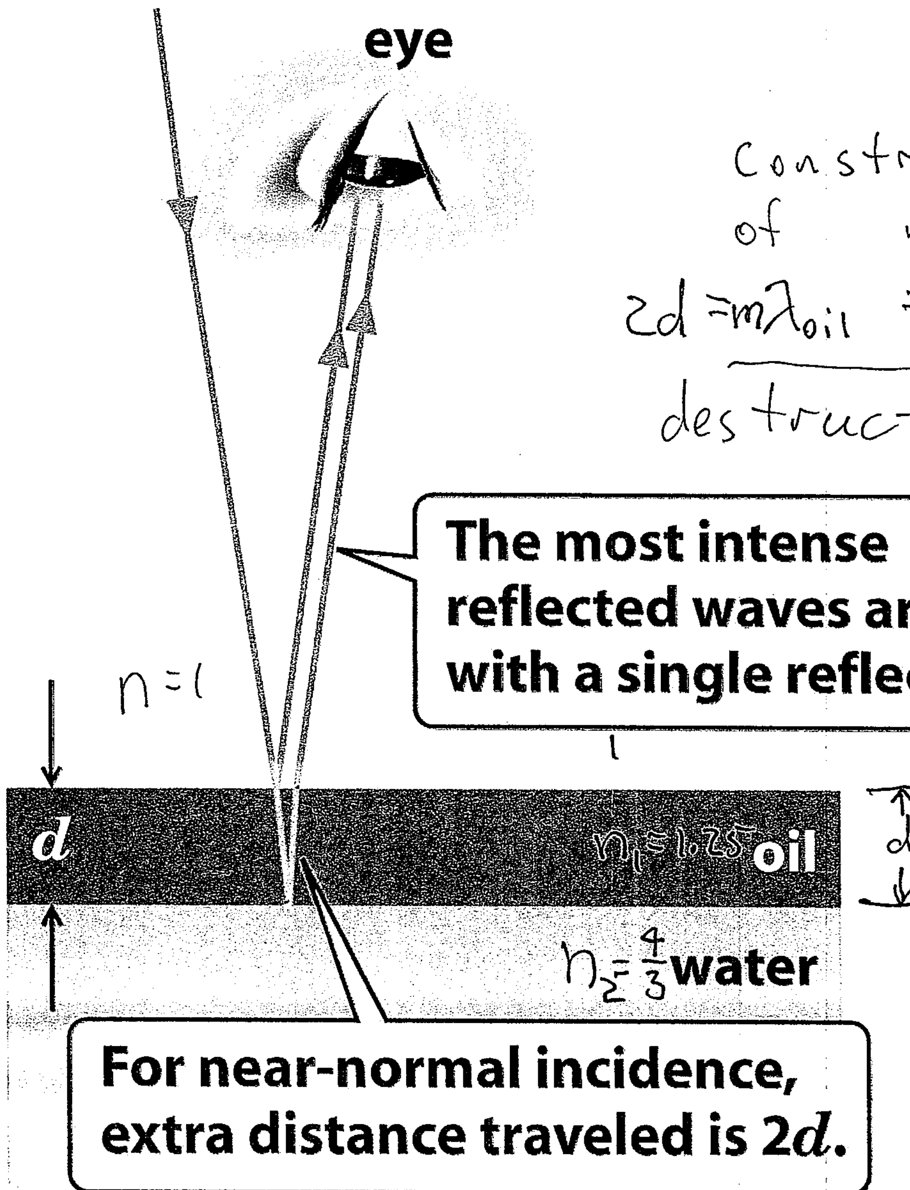


Figure 35-3 Physics for Engineers and Scientists 3/e

Constructive interference of reflected wave

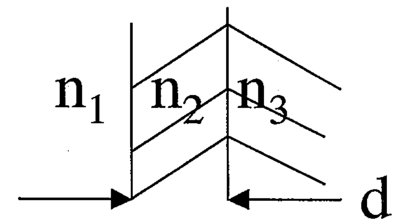
$2d = m\lambda_{oil} = m\lambda_{air}/n_{oil}$, $m = 1, 2, \ldots$

destructive interf of reflected wave.

$d\lambda = (m + 1/2)\lambda_{oil}$

THIN-FILM INTERFERENCE

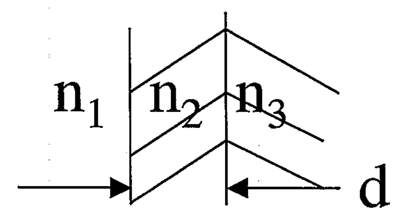
When $1 = n_1 < n_2 < n_3$



$m = 1, 2, \ldots$ $2d = m\lambda/n_2$ = Maxima: $2d = \lambda_1/n_2,\ 2\lambda_1/n_2,\ 3\lambda_1/n_2 \ldots.$ enhanced reflection

$m = 0, 1, 2, \ldots$ $2d = (m+1/2)\lambda/n_2$ = Minima: $2d = \lambda_1/2n_2,\ 3\lambda_1/2n_2,\ 5\lambda_1/2n_2 \ldots$ → enhanced transmission

When $1 = n_1 < n_2 > n_3$

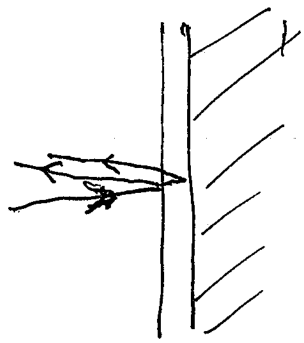


$m = 0, 1, 2$ $2d = (m+1/2)\lambda/n_2$ — Maxima: $2d = \lambda_1/2n_2,\ 3\lambda_1/2n_2,\ 5\lambda_1/2n_2 \ldots$

$m = 1, 2, \ldots$ $2d = m\lambda/n_1$ Minima: $2d = \lambda_1/n_2,\ 2\lambda_1/n_2,\ 3\lambda_1/n_2 \ldots.$

Some application of thin films

Thin Films used to decrease reflection of camera lens

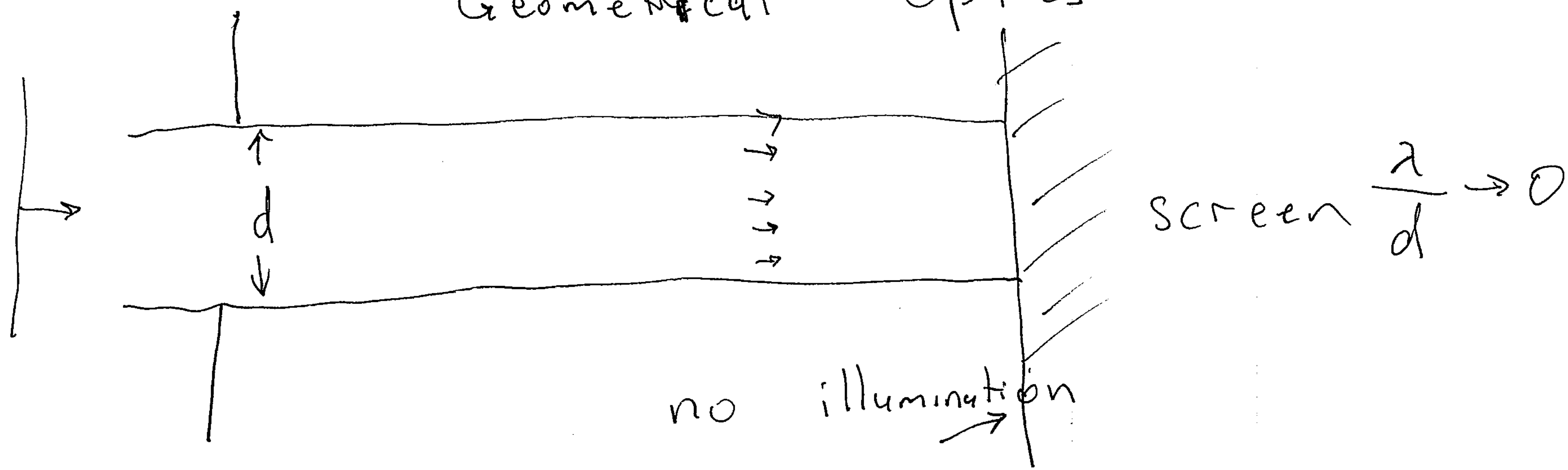


Reflected wave undergoes destructive interference

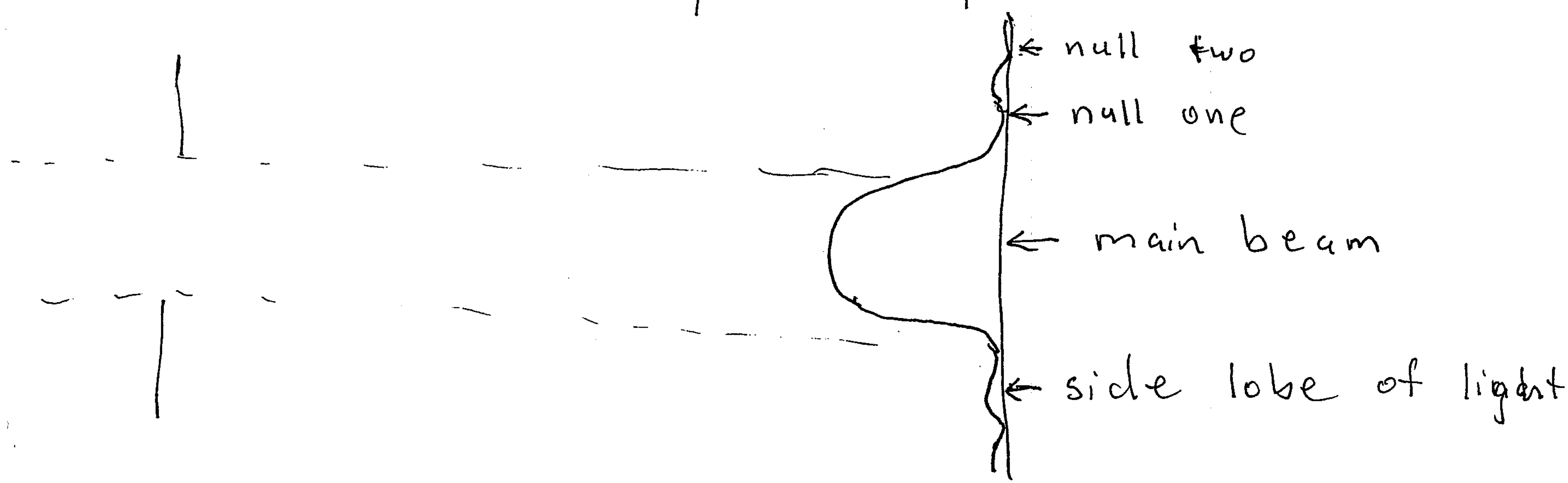
To improve thermal insulation home windows are tinted with thin film for reflected wave to constructively interfere, and less heat enters home

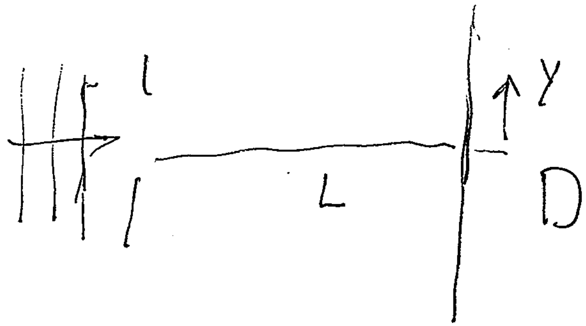
Diffraction

Geometrical Optics of a beam



Physical Optics





Diffraction Pattern

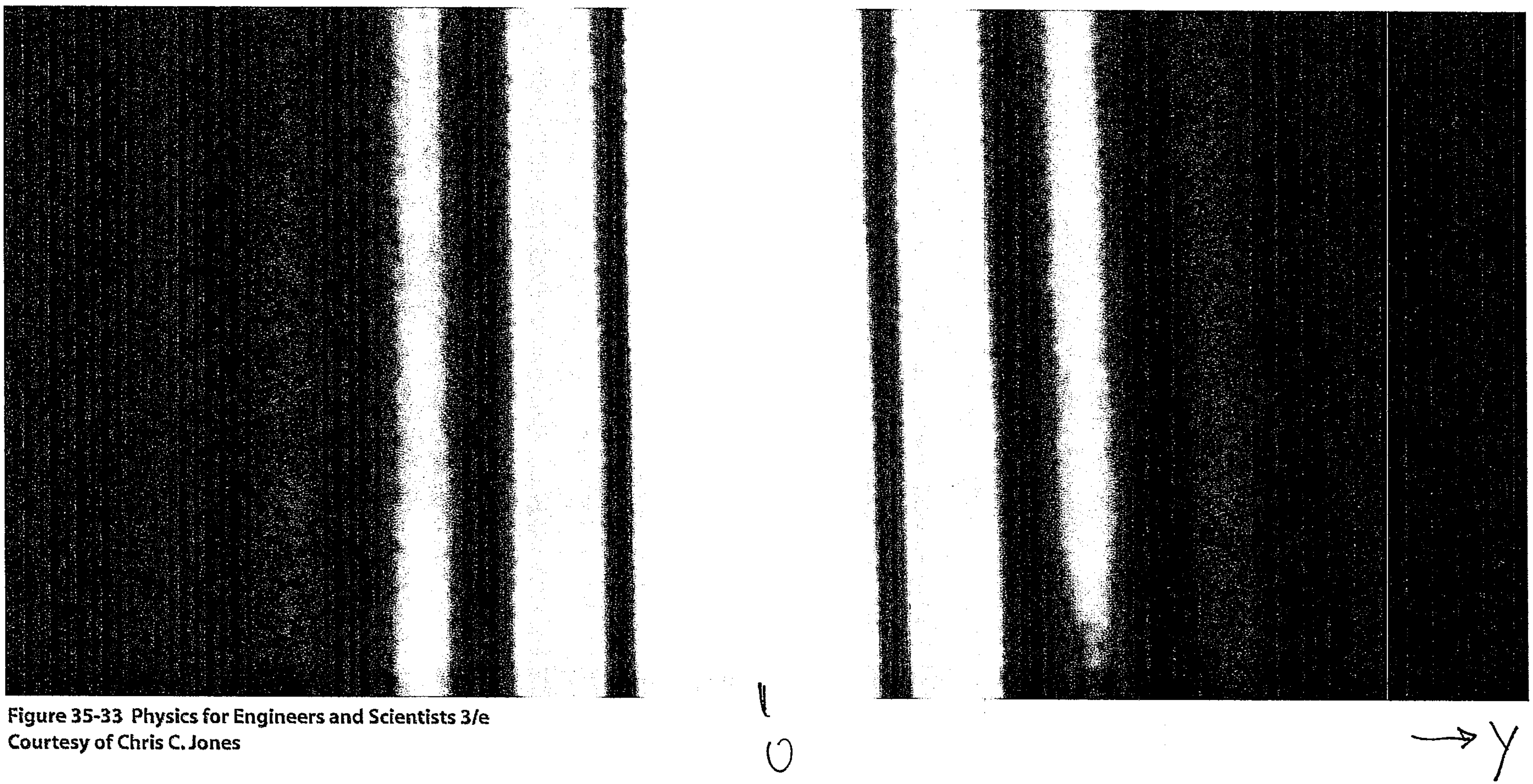


Figure 35-33 Physics for Engineers and Scientists 3/e
Courtesy of Chris C. Jones

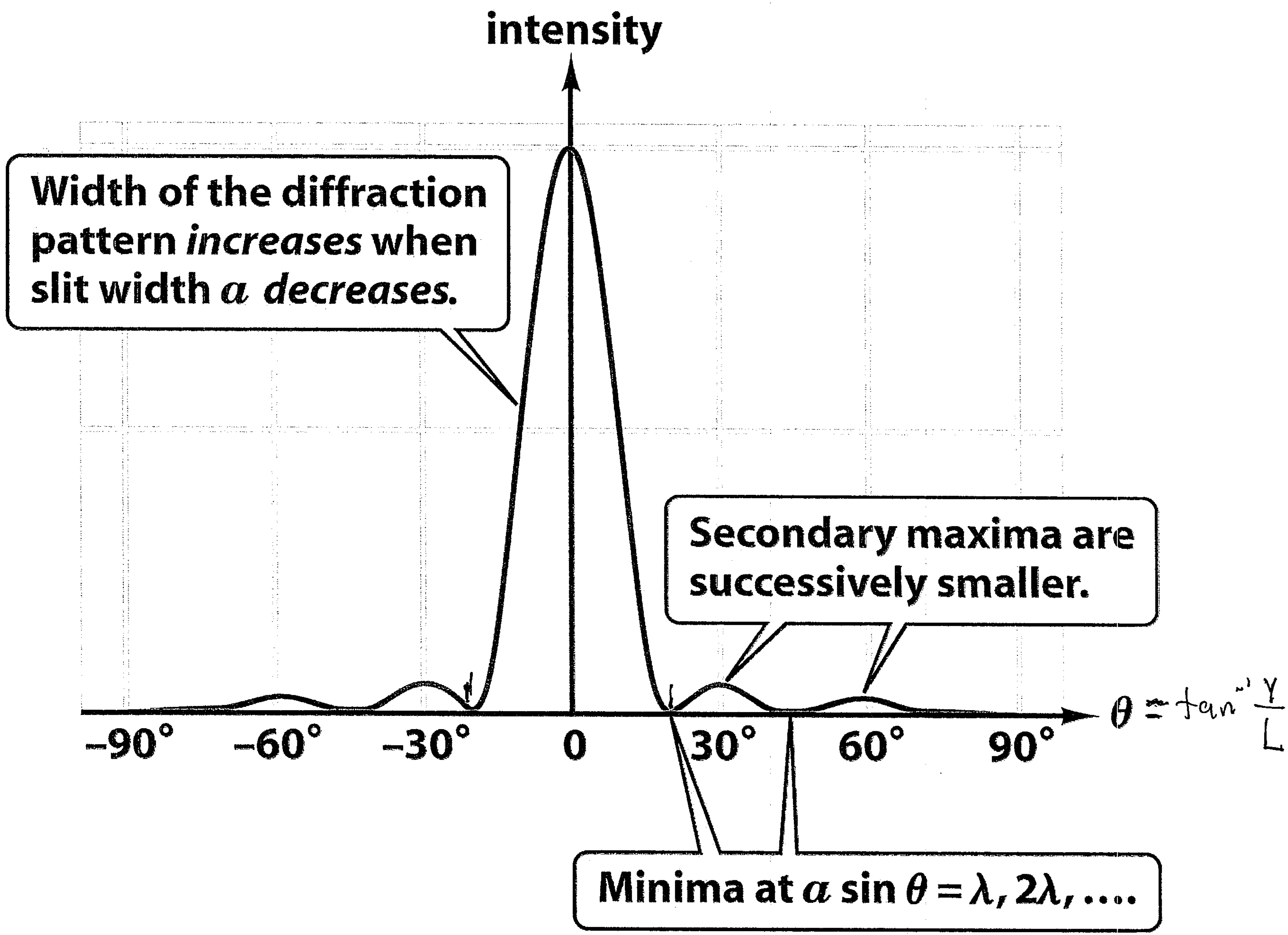


Figure 35-37 Physics for Engineers and Scientists 3/e

If $\theta \ll 1$; $a\theta = \lambda, 2\lambda, 3\lambda$

Diffraction of a Slit

$a \equiv$ slit width:

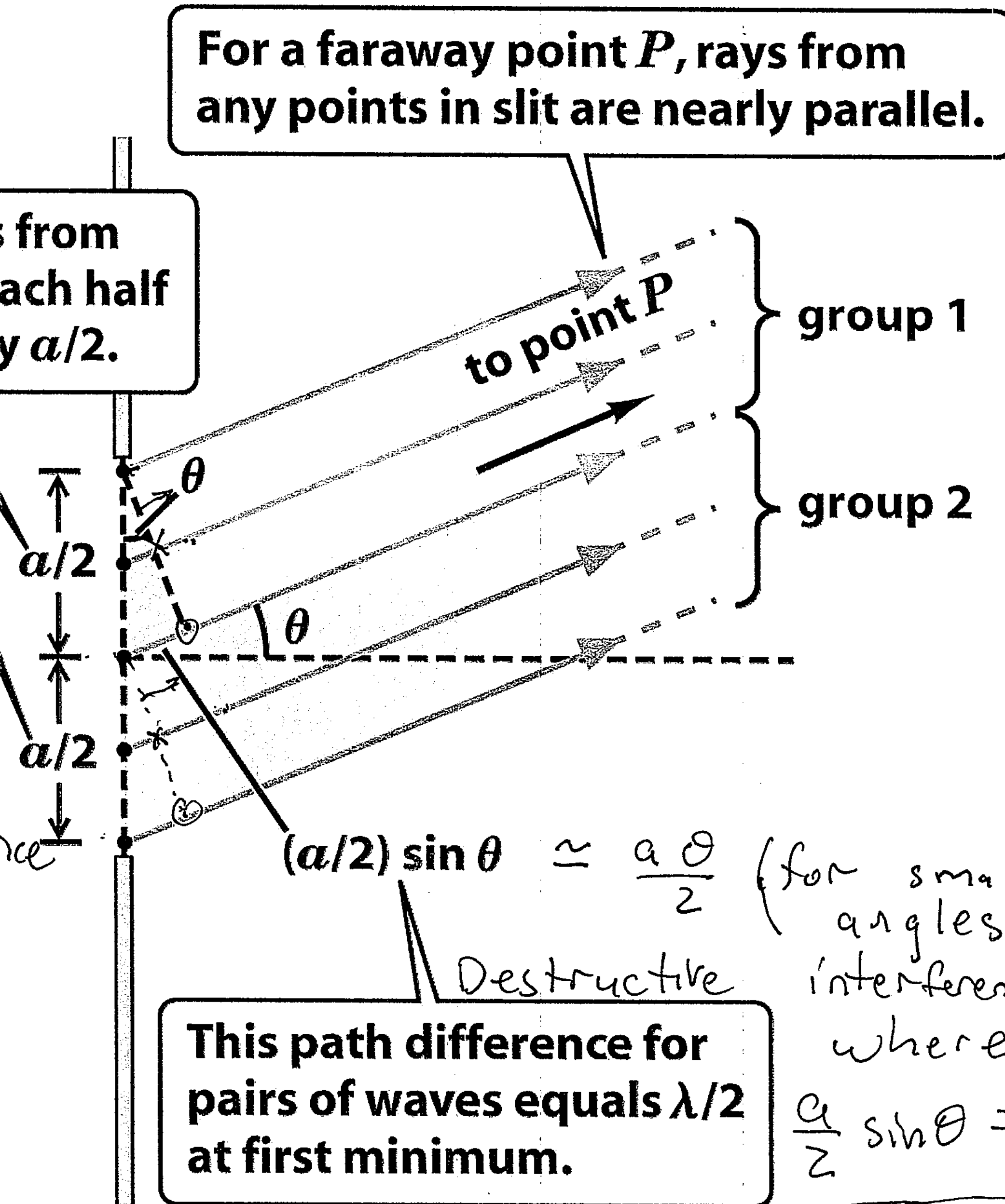


Figure 35-31 Physics for Engineers and Scientists 3/e

$(a/2)\sin\theta \simeq \frac{a\theta}{2}$ (for small angles)

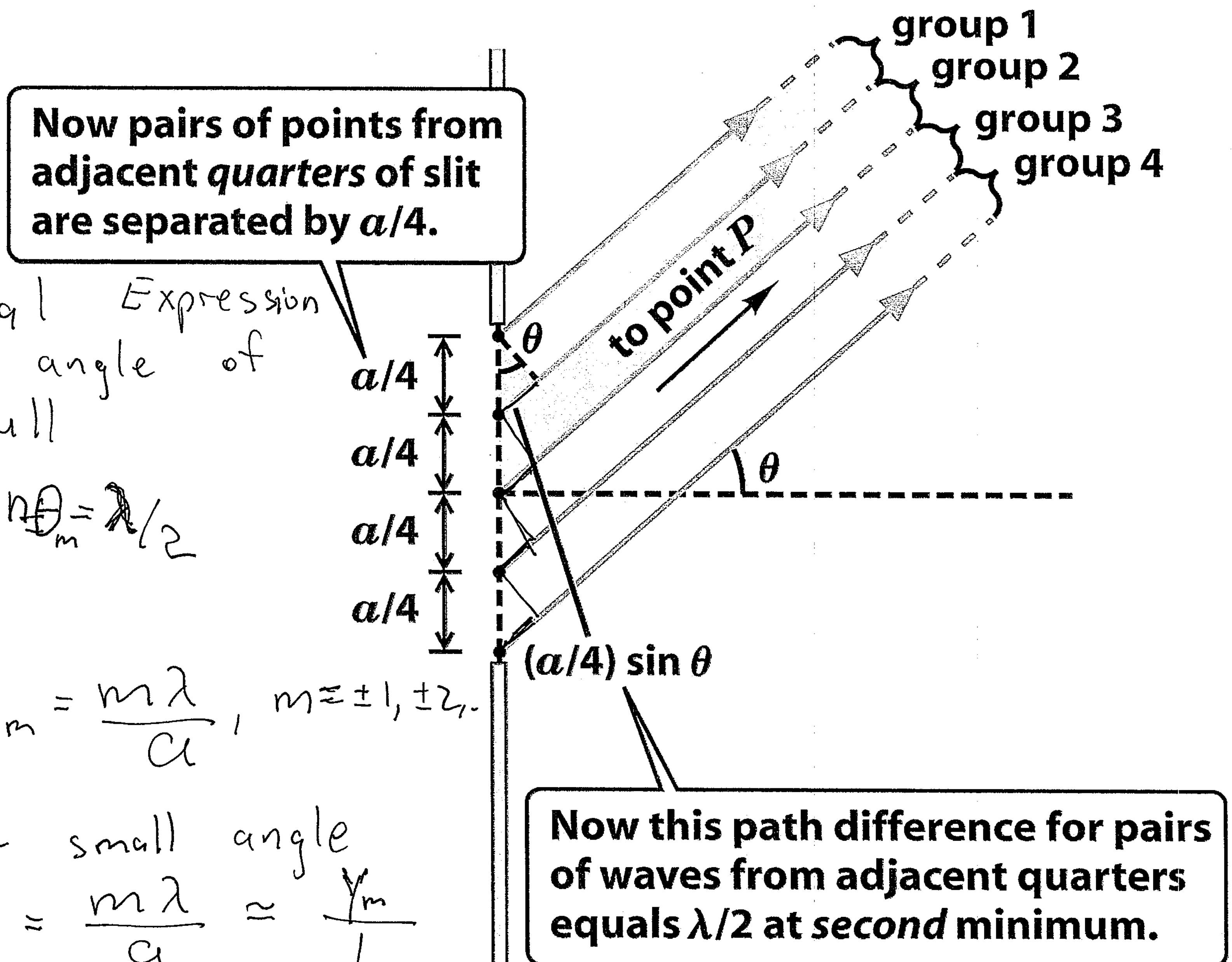
Destructive interference where $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$

For small angles, interference null at

$$\frac{a\theta}{2} = \frac{\lambda}{2}$$

or

or for small angle $\theta \simeq \frac{\lambda}{a}$ for null



General Expression of angle of null

$$\frac{a}{2m} \sin\theta_m = \lambda/2$$

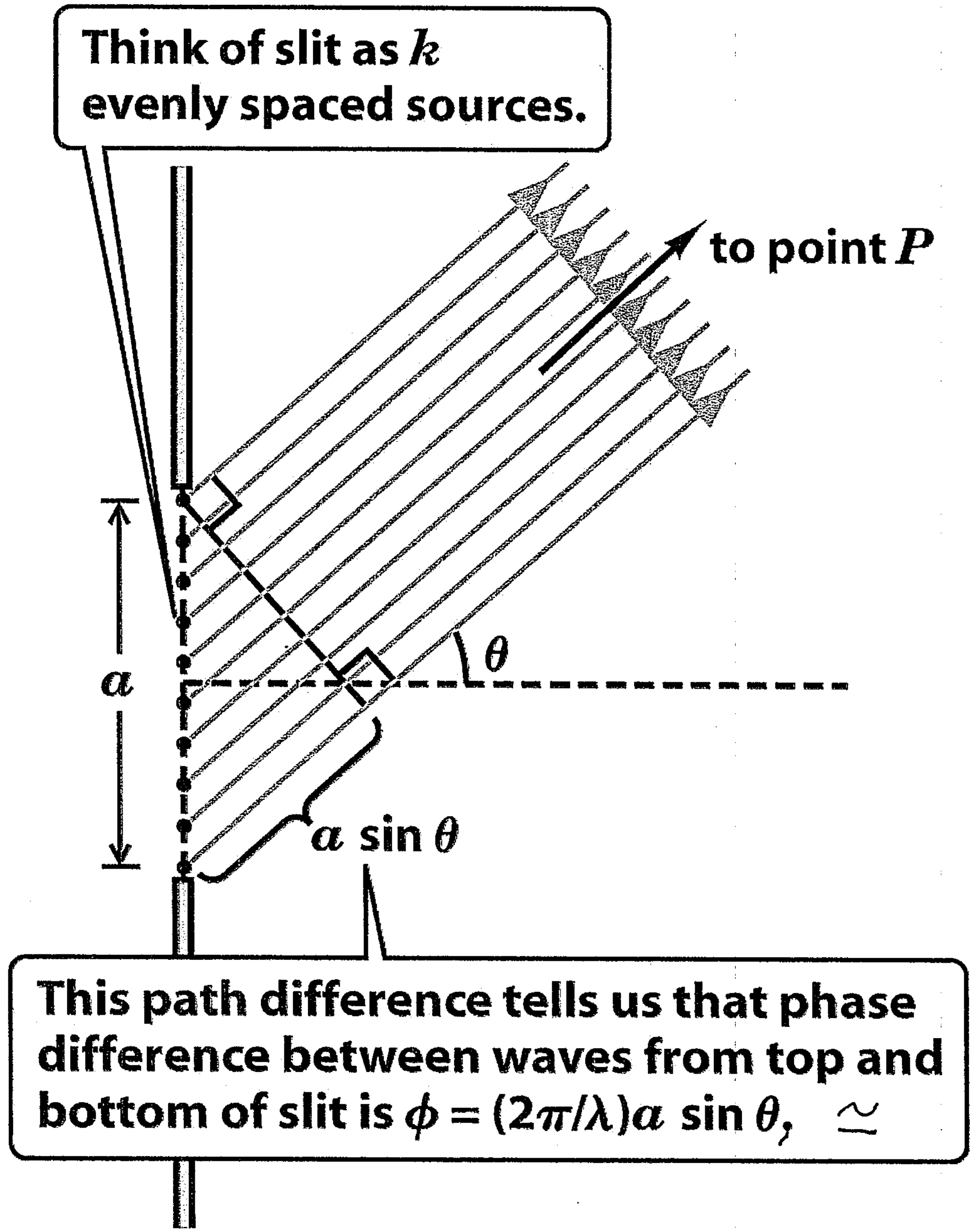
$$\sin\theta_m = \frac{m\lambda}{a}, \quad m = \pm 1, \pm 2, \ldots$$

For small angle

$$\theta_m \approx \frac{m\lambda}{a} \simeq \frac{y_m}{L}$$

Figure 35-32 Physics for Engineers and Scientists 3/e

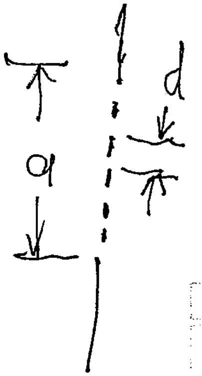
Combined interference and diffraction



$\left(\frac{2\pi}{\lambda}\right) a\, \theta$, if $\theta \ll 1$

Figure 35-35 Physics for Engineers and Scientists 3/e

Combined interference and diffraction



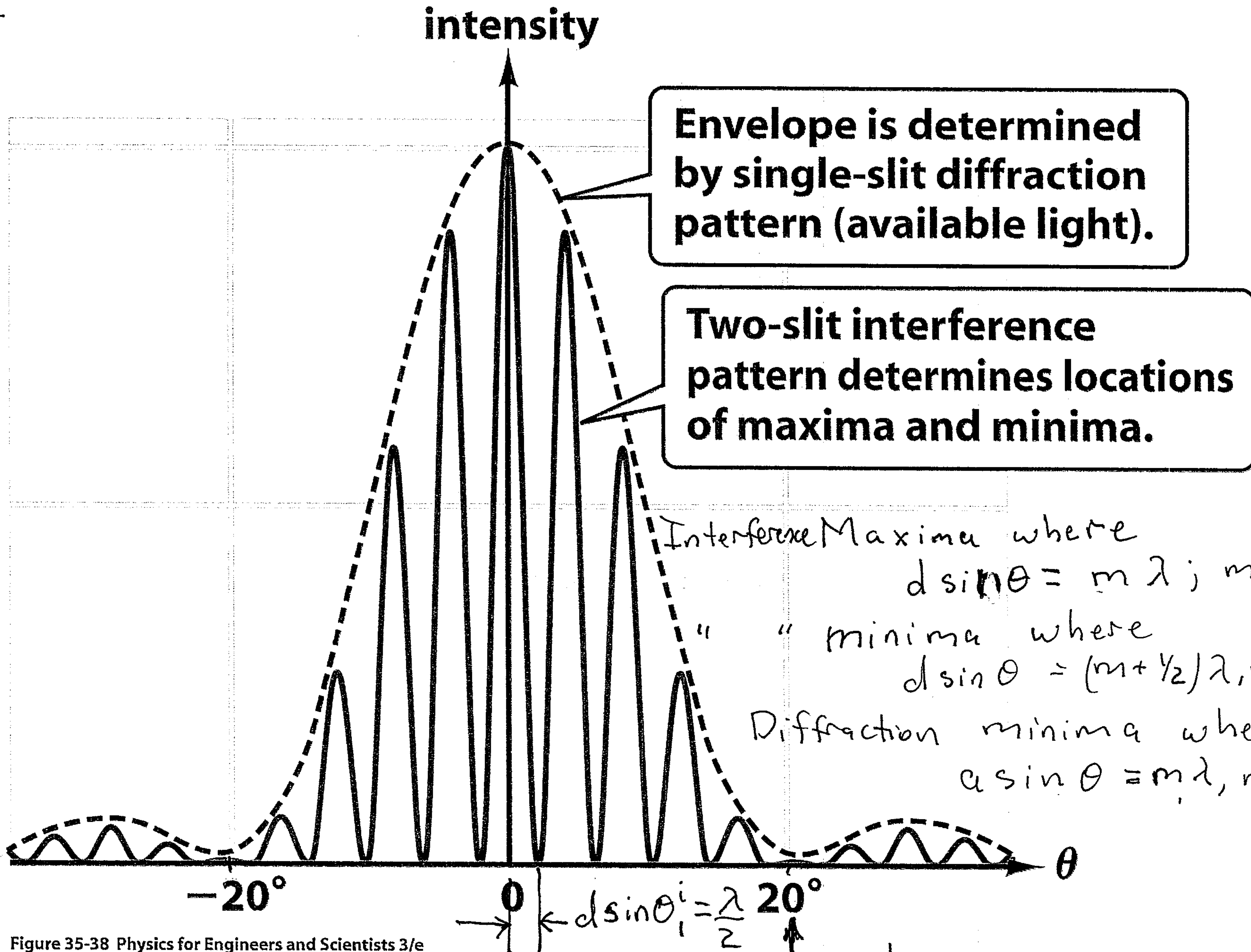


Figure 35-38 Physics for Engineers and Scientists 3/e

Interference Maxima where
$d \sin\theta = m\lambda$; $m = 0, \pm 1, \ldots$

" " minima where
$d \sin\theta = (m + 1/2)\lambda$, $m = 0, \pm 1, \ldots$

Diffraction minima where
$a \sin\theta = m\lambda$, $m = 0, \pm 1, \ldots$

$d \sin\theta_1^i = \frac{\lambda}{2}$

$a \sin\theta_1^d = \lambda$

Diffraction is intrinsic limit to resolution even with perfect geometrical optics

Rayleigh criterion

$\theta < 1.22 \frac{\lambda}{a}$

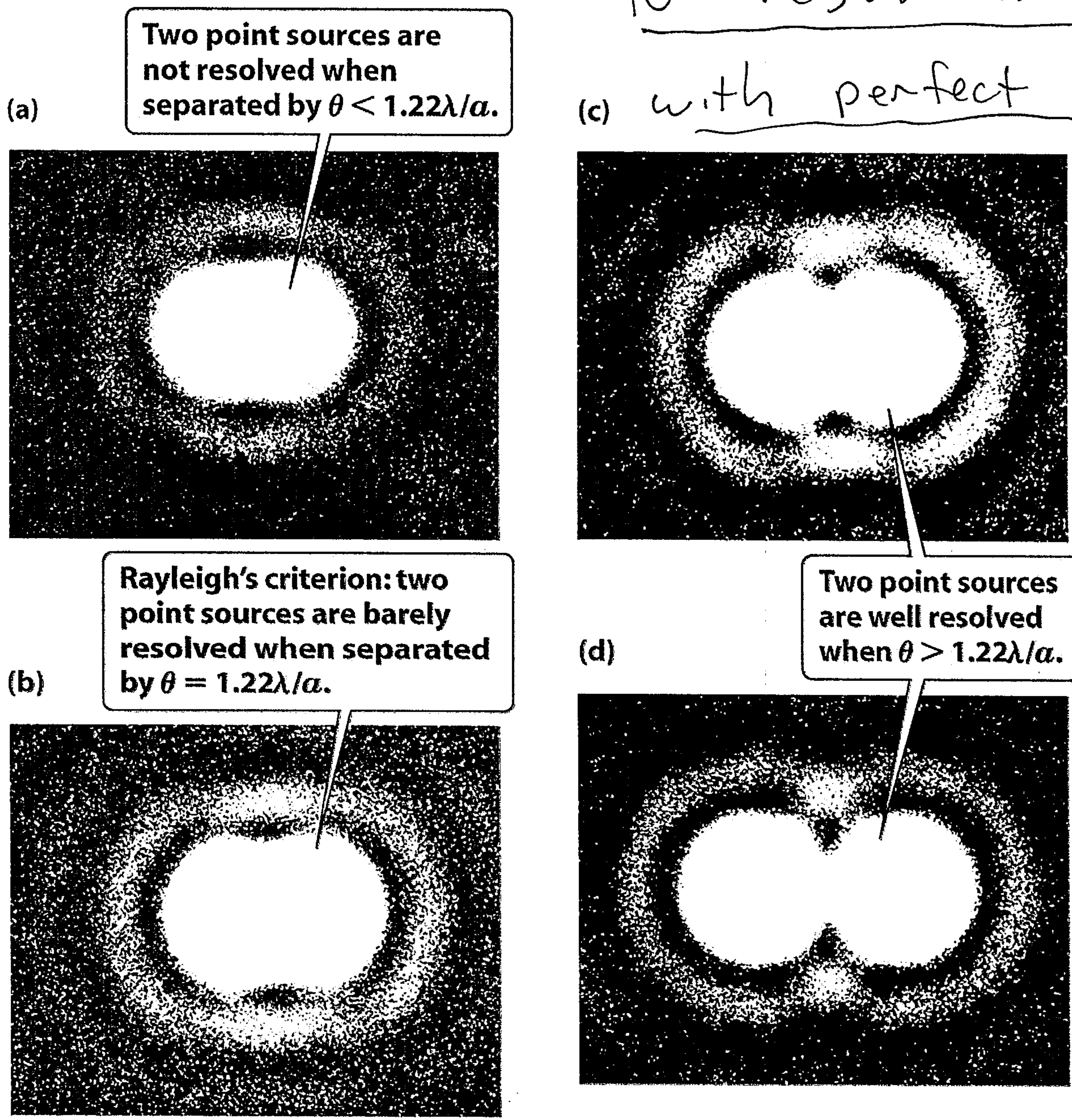


Figure 35-40 Physics for Engineers and Scientists 3/e
Courtesy of Chris C. Jones