Lecture # 23

Interference
1. The far point with respect to the human eye is:
   (a) the furthest distance a finite size object is from the eye.
   (b) the furthest distance a near-sighted person can clearly see without glasses.
   (c) the furthest distance that the retina region in the eye is from the eye’s lens.

2. To correct for the far point the eye-glasses need to
   (a) convert parallel light rays coming from infinity to form a real image at the
       unaided eye’s near point
   (b) convert parallel light rays coming from infinity to form a virtual image at the
       unaided eye’s near point
   (c) convert parallel light rays (i.e. light from a distant source) going through the lens
       to form a virtual image that appears to be at the unaided eye’s far-point.

How does one determine what the needed focal length of the eyeglasses?

3. A far sighted woman reading a book needs glasses that will:
   (a) convert light rays from a source at distance of 25cm from her eye, to appear to be
       from a virtual image at her more distant near point.
   (b) convert light rays from a source at her far point to appear as a virtual image at her
       near point.
   (c) produce a real image at a distance of 25cm from her eye, from a source that is at
       her more distant near point.

How does one determine what the needed focal length of the eyeglasses?

4. $A + A = 2A$ if $A$ is a number
   (a) always  (b) not all the time

5. $A + A = 2A$ if the first $A$ is an intensity from one laser beam and if the
   second $A$ is an intensity of a second laser beam with the same frequency
   as the first beam.
   (a) always  (b) not all the time
To uncrossed eye, size (subtended angle) largest.

θ = subtended angle

θ = typically 25 cm

25 cm

With object at near point, θ at near point

_objects can appear larger when using a magnifier (glass)

(a magnifier)

Retina is sizable.

...Image on...
Subtended angle, $\theta'$, of virtual image (size) largest as source approaches focal length.

With magnifier, object is almost at focal point and much closer than near point...

$$\theta' = \frac{h}{f}$$

$$\theta = \frac{h}{25}$$

Then, Image magnification

$$M = \frac{\theta'}{\theta} = \frac{25}{f}$$

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Microscope

Eyepiece acts as a magnifier...

...of enlarged, real image...

Real image is almost at (but slightly smaller) than ocular's focal length.

Object is slightly farther from objective than its focal length.

Object

Ocular

Image at $\infty$

Objective

Eye

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Telescope

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\[ m = \frac{\theta'}{\theta} \]

- Objective produces...
- Object at \( \infty \)
- Image at \( \infty \)
- Objective
- Objective focal length \( f_{\text{objective}} \)
- Ocular
- Ocular focal length \( f_{\text{ocular}} \)
- Real image close to eye...
- That is viewed by eyepiece magnifier.
- Slightly larger than distance to real image.
Two laser beams of the same frequency each of intensity $I_1 = I$ and $I_2 = I$ can produce any intensity, depending on phase, that is $0 \leq I_{\text{combined}} \leq 4I$.

How can this be at a given position $\tau$?

$$I_1 = \frac{(A \sin (\omega t + \Theta))^2}{A^2 \int_0^T dt \sin^2 (\omega t + \Theta)} = A^2 / 2$$

$$I_2 = \frac{(A \sin (\omega t + \Theta))^2}{2} = \frac{A^2}{2}$$
\[ I_{\text{combined}} = \left[ A \sin(\omega t + \theta_1) + A \sin(\omega t + \theta_2) \right]^2 \]

When we combine the two beams we have

\[ I_{\text{combined}} = A^2 \left[ \sin(\omega t + \theta_1) + \sin(\omega t + \theta_2) \right]^2 \]

How can we add

\[ \sin(\omega t + \theta_1) + \sin(\omega t + \theta_2) \]?

\[ \sin(\omega t + \theta_1) = \sin\left(\omega t + \frac{\theta_1 + \theta_2}{2} + \frac{\theta_1 - \theta_2}{2}\right) \]

\[ \sin(\omega t + \theta_2) = \sin\left(\omega t + \frac{\theta_1 + \theta_2}{2} - \frac{\theta_1 - \theta_2}{2}\right) \]

\[ = \sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right) \cos\left(\frac{\theta_1 - \theta_2}{2}\right) \]

Then when we add \( \sin(\omega t + \theta_1) + \sin(\omega t + \theta_2) \) we obtain

\[ 2 \sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right) \cos\left(\frac{\theta_1 - \theta_2}{2}\right) \]
Because of how trigonometric functions add:

\[
I_{\text{combined}} = 4 \cos^2 \left( \frac{\theta_1 - \theta_2}{2} \right) A^2 \sin^2 \left( \omega t + \frac{\theta_1 + \theta_2}{2} \right)
\]

\[
= 2 \cos^2 \left( \frac{\theta_1 - \theta_2}{2} \right) A^2
\]

\[
= 4 \cos^2 \left( \frac{\theta_1 - \theta_2}{2} \right) A^2 = 4 I \cos^2 \left( \frac{\theta_1 - \theta_2}{2} \right)
\]

\[
I_{\text{combined}} = 4 I \cos^2 \left( \frac{\theta_1 - \theta_2}{2} \right)
\]

\[
= \begin{cases} 
0 & \text{if } \frac{\theta_1 - \theta_2}{2} = (n + \frac{1}{2}) \pi \\
4 & \text{if } \frac{\theta_1 - \theta_2}{2} = n \pi
\end{cases}
\]

\[n = 0, \pm1, \pm2, \pm3, \ldots\]
Constructive interference

\[ I_{\text{combined}} = 0, \quad \theta_1 - \theta_2 = (2n + 1) \pi \]
\[ n = \text{integer} \]

Constructive interference

\[ I_{\text{combined}} = 4I, \quad \theta_1 - \theta_2 = 2n\pi \]

Now consider combining two waves that come from two different paths.

\[ I_1 = \left[ A \sin \left( \omega t - \frac{s_1}{\lambda} \right) \right]^2 \]
\[ I_2 = \left[ A \sin \left( \omega t + \frac{s_2}{\lambda} \right) \right]^2 \]

\[ I_{12} = \left[ A \sin \left( \omega t - \frac{s_1}{\lambda} \right) + A \sin \left( \omega t - \frac{s_2}{\lambda} \right) \right]^2 \]
\[ = 4A^2 \left[ \sin^2 \left( \omega t - \frac{\pi (s_1 - s_2)}{2\lambda} \right) \right] \cos^2 \left( \pi \frac{s_2 - s_1}{\lambda} \right) \]
Con constructive interference

\[ I_{\text{combined}} = 4I, \quad \Theta_2 - \Theta_1 = 2n \pi \]

as \[ \cos^2(\Theta_2 - \Theta_1) = \cos^2(n \pi) = 1 \]

D destructive interference

\[ I_{\text{combined}} = 0, \quad \Theta_2 - \Theta_1 = (2n+1) \pi \]

as \[ \cos^2(\Theta_2 - \Theta_1) = \cos^2(n \pi + \pi/2) = 0 \]

Now consider combining two waves of the same frequency, initial of the same phase, but taking two different paths, and then combining each alone.

\[ I_1 = \left[ A \sin\left(\frac{wt - \frac{2\pi s_1}{\lambda}}{2}\right) \right]^2 = \frac{A^2}{2} = I_0 \]

\[ I_2 = \left[ A \sin\left(\frac{wt - \frac{2\pi s_2}{\lambda}}{2}\right) \right]^2 = \frac{A^2}{2} = I_0 \]

\[ I_{\text{combined}} = \left[ A \sin\left(\frac{wt - \frac{2\pi s_1}{\lambda}}{2}\right) + A \sin\left(\frac{wt - \frac{2\pi s_2}{\lambda}}{2}\right) \right]^2 \]

\[ = 4A^2 \sin^2\left(\frac{wt + 2\pi (s_1 + s_2)}{\lambda}\right) \cos^2\left(\frac{2\pi (s_1 - s_2)}{\lambda}\right) \]

\[ = 2A^2 \cos^2\left(\frac{2\pi (s_1 - s_2)}{\lambda}\right) = 4I_0 \cos^2\left(\frac{2\pi (s_2 - s_1)}{\lambda}\right) \]
Young's Interference Experiment

2-slit experiment

Interference at point \( P \)
is determined by path difference.

At point \( P \):

\[
I_n = 4I \cos^2 \left( \frac{s_2 - s_1}{\lambda} \right)
\]

For constructive interference:
\[ s_2 - s_1 = n\lambda \]

For destructive interference:
\[ s_2 - s_1 = (n + \frac{1}{2})\lambda \]

\( n = 0, \pm 1, \pm 2 \)
Finding Path Difference

For a faraway point $P$, rays from slits are nearly parallel.

$S_2 - S_1 = d \sin \theta \\ \approx d \theta$

(as $\sin \theta \approx \theta$ if $\theta \ll 1$)  
$\theta$ in radians
$tan \theta \approx \theta$

$\theta \ll 1$

$d \sin \theta$ is path difference to $P$.

$\theta$ is angular position of point $P$ with respect to midline.

Constructive interference  
$n \lambda = d \sin \theta$

$n = 0, \pm 1, \pm 2, \ldots$

Destructive interference  
$d \sin \theta = (n + \frac{1}{2}) \lambda$

Figure 35-15  Physics for Engineers and Scientists 3/e  
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Prellection of "fringes" to screen position

Constructive:
\[ \sin \theta_n = \frac{n \lambda}{d}, \quad \tan \theta_n = \frac{Y_{\text{max}, n}}{L} \]
for small angles

\[ \sin \theta_n \sim \tan \theta_n \sim \theta_n = \frac{n \lambda}{d} = \frac{Y_{\text{max}, n}}{L} \]

Destructive:
\[ \sin \theta_n = \frac{(n + \frac{1}{2}) \lambda}{d}, \quad \tan \theta_n = \frac{Y_{\text{min}, n}}{L} \]
for small angles

\[ \sin \theta_n \approx \frac{(n + \frac{1}{2}) \lambda}{d}, \quad \tan \theta_n \approx \theta_n = \frac{(n + \frac{1}{2}) \lambda}{d} = \frac{Y_{\text{min}, n}}{L} \]
Realistic

Young's interference pattern
(2-slit pattern)