Lecture # 23

Interference

1. The far point with respect to the human eye is:
(a) the furthest distance a finite size object is from the eye.
(b) the furthest distance a near-sighted person can clearly see without glasses.
(c) the furthest distance that the retina region in the eye is from the eys's lens.

2. To correct for the far point the eye-glasses need to
(a) convert parallel light rays coming from infinity to form a real image at the unaided eye's near point
(b) convert parallel light rays coming from infinity to form a virtual image at the unaided eye's near point
(c) convert parallel light rays (i.e. light from a distant source) going through the lens to form a virtual image that appears to be at the unaided eye's far-point.

How does one determine what the needed focal length of the eyeglasses?

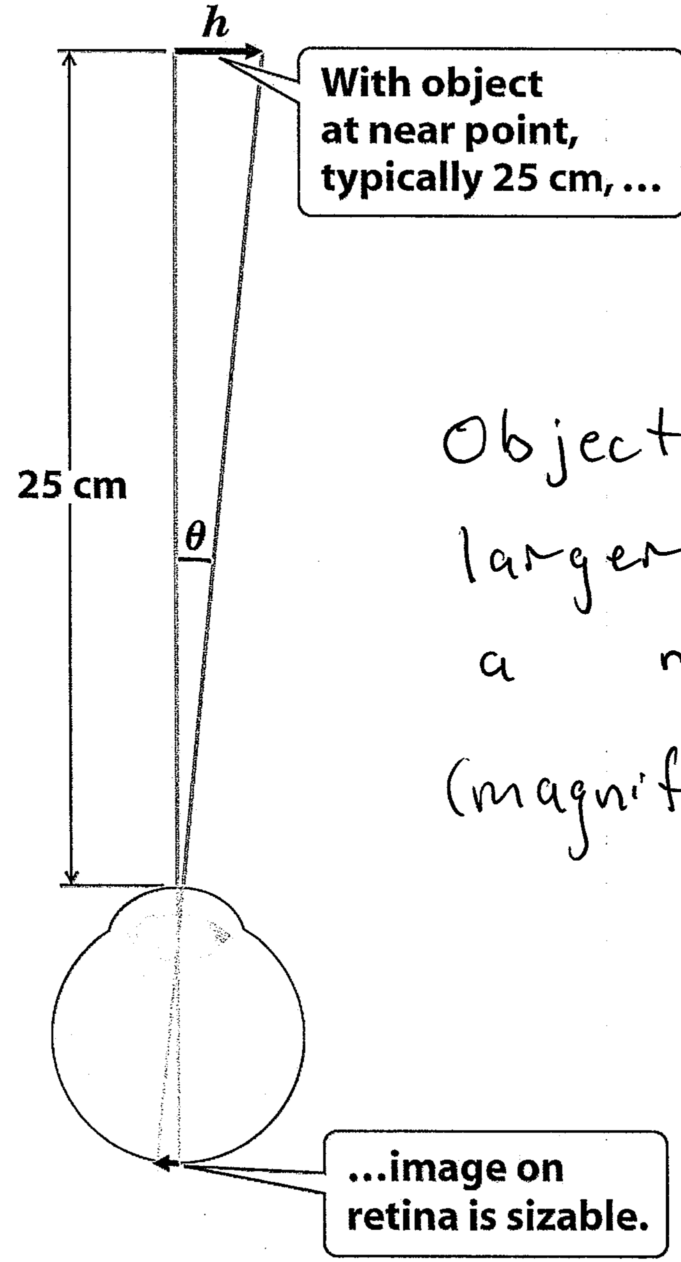
3. A far sighted woman reading a book needs glasses that will:
(a) convert light rays from a source at distance of 25cm from her eye, to appear to be from a virtual image at her more distant near point.
(b) convert light rays from a source at her far point to appear as a virtual image at her near point.
(c) produce a real image at a distance of 25cm from her eye, from a source that is at her more distant near point.

How does one determine what the needed focal length of the eyeglasses?

4. A + A =2A if A is a number
(a) always (b) not all the time

5. A + A = 2A if the first A is an intensity from one laser beam and if the second A is an intensity of a second laser beam with the same frequency as the first beam.
(a) always (b) not all the time

To unaided eye, size (subtended angle) largest and focused, when at near point



$\theta \equiv$ subtended angle

Objects can appear larger when using a magnifier (magnifying glass)

Figure 34-60 Physics for Engineers and Scientists 3/e

Magnifier

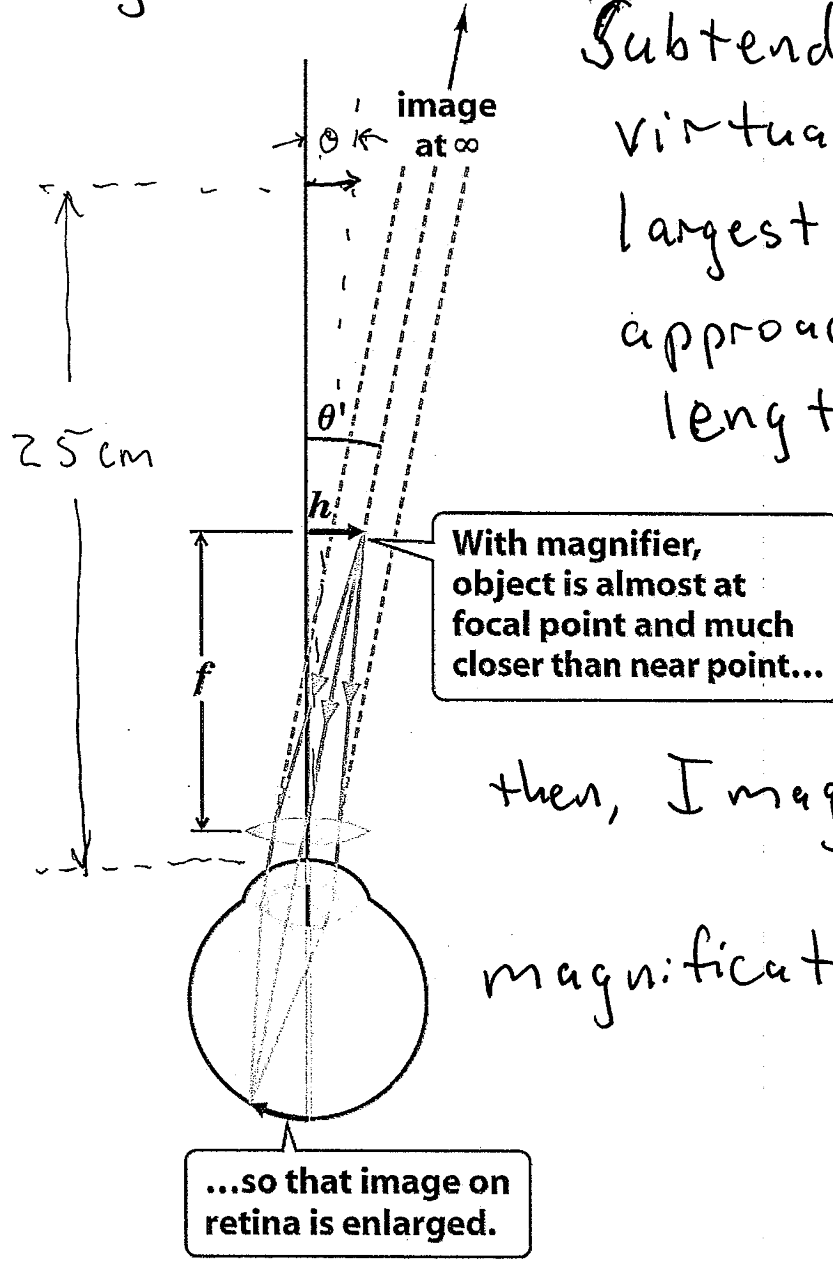


Figure 34-62 Physics for Engineers and Scientists 3/e

Subtended angle, θ', of virtual image (size) largest as source approaches focal length.

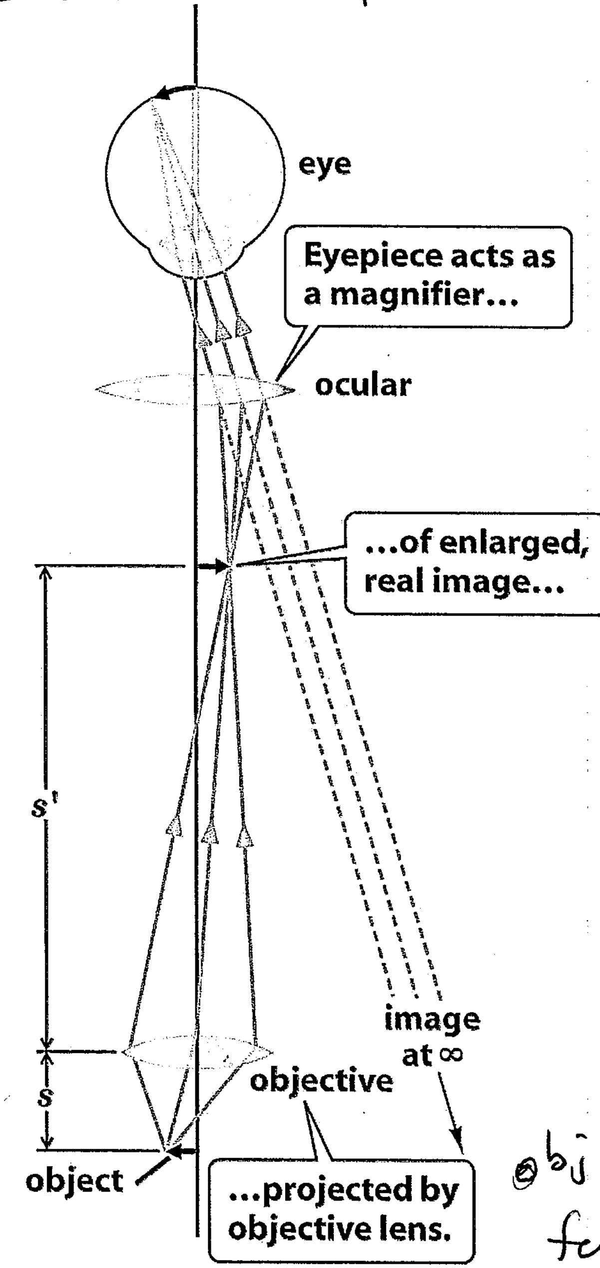
$\theta' = \frac{h}{f}$

$\theta = h/25$

then, Image at ∞

magnification $m \equiv \frac{\theta'}{\theta} = \frac{25}{f}$

Micro scope



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→ further magnifies real image

real image is almost at (but slightly smaller) than ocular's focal length

object is slightly further from objective than its focal length

Telescope

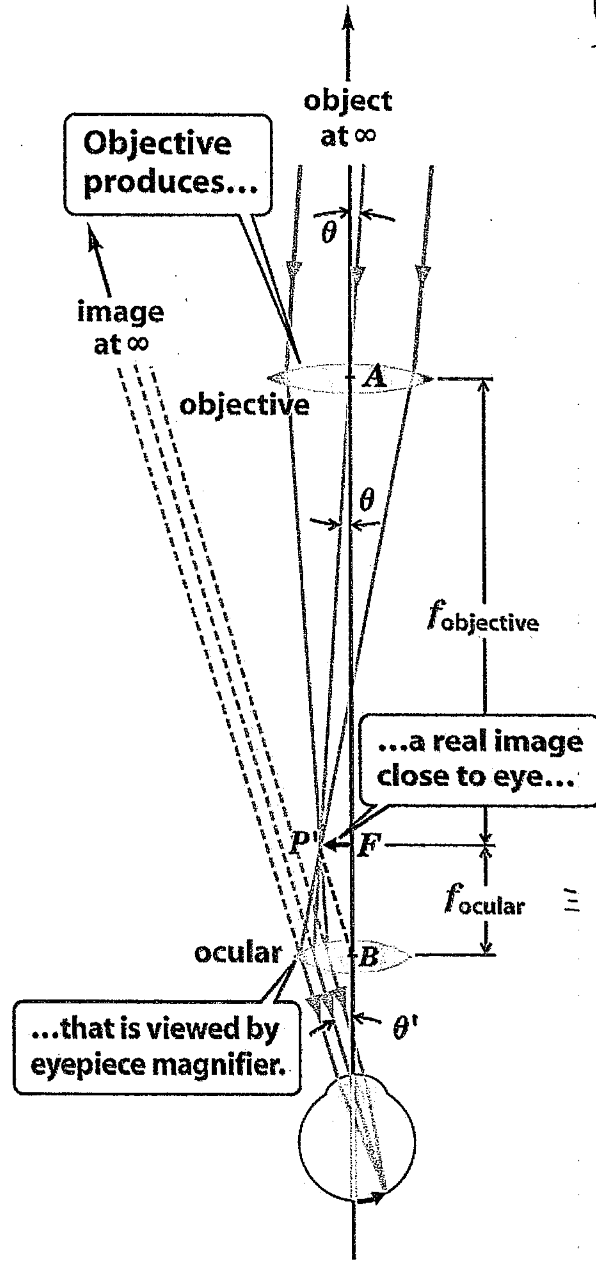


Figure 34-64 Physics for Engineers and Scientists 3/e

$\equiv$ slightly larger than distance to real image

$$m = \frac{\theta'}{\theta}$$

Mirror Telescope

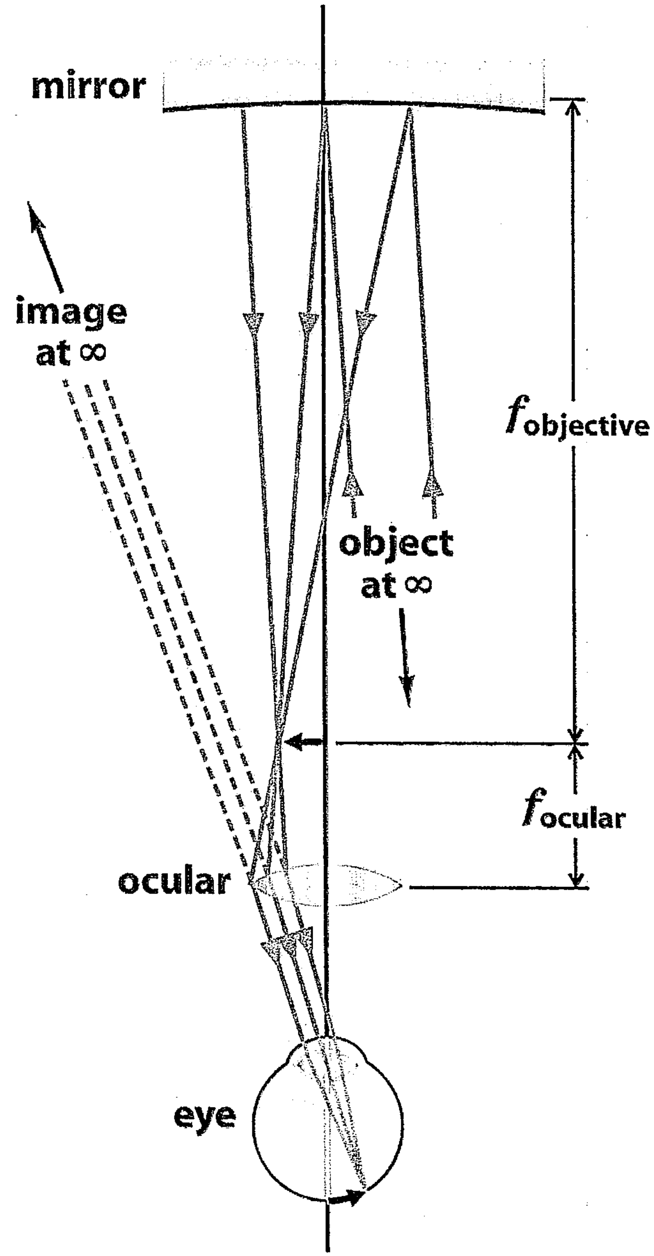


Figure 34-65 Physics for Engineers and Scientists 3/e

Two laser beams of the same frequency each of intensity $I_1 = I$ and $I_2 = I$ can produce any intensity, depending on phase, that is

$$0 \le I_{combined} \le 4I$$

How can this be

at a given position

$$I_1 = \overline{\left(A \sin(\omega t + \theta_1)\right)^2} \equiv A^2 \int_0^T \frac{dt}{T} \sin^2(\omega t + \theta)$$

$$= A^2/2$$

$$I_2 = \overline{\left(A \sin(\omega t + \theta_2)\right)^2} = \frac{A^2}{2}$$

$$I_{combined} = \overline{[A\sin(\omega t+\theta_1) + A\sin(\omega t+\theta_2)]^2}$$

When we combine the two beams we have

$$I_{combined} = A^2 \overline{\left[\sin(\omega t+\theta_1) + \sin(\omega t+\theta_2)\right]^2}$$

How can we add

$\sin(\omega t+\theta_1) + \sin(\omega t+\theta_2)$?

$$\sin(\omega t+\theta_1) = \sin\left(\omega t + \frac{\theta_1+\theta_2}{2} + \frac{\theta_1-\theta_2}{2}\right)$$

$$\sin(\omega t+\theta_2) = \sin\left(\left(\omega t + \frac{\theta_1+\theta_2}{2}\right) - \left(\frac{(\theta_1-\theta_2)}{2}\right)\right)$$

$$= \sin\left(\omega t + \frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right) - \cos\left(\omega t + \frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\theta_1-\theta_2}{2}\right)$$

$$\sin(\omega t+\theta_1) = \sin\left(\omega t + \frac{\theta_1+\theta_2}{2}\right)\cos(\theta_1-\theta_2/2) + \text{〃}$$

Then when we add $\sin(\omega t+\theta_1) + \sin(\omega t+\theta_2)$ we obtain

$$\boxed{2\sin\left(\omega t + \frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$$

Because of how trigonometric functions add:

$$\overline{I_{combined}} = 4 \cos^2\left(\frac{\theta_1-\theta_2}{2}\right) A^2 \overline{\sin^2\left(\omega t+\frac{\theta_1+\theta_2}{2}\right)}$$

$$= 2\cos^2\left(\frac{\theta_1-\theta_2}{2}\right) A^2$$

$$= 4\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)\frac{A^2}{2} = 4I\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)$$

$\therefore$

$$\overline{I_{combined}} = 4I\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)$$

$$= \begin{cases} 0 & \text{if } \frac{\theta_1-\theta_2}{2} = \left(n+\frac{1}{2}\right)\pi \\ 4 & \text{if } \frac{\theta_1-\theta_2}{2} = n\pi \end{cases}$$

$n = 0, \pm 1, \pm 2$

Constructive interference

$\overline{I_{combined}} = 0$, $\theta_1 - \theta_2 = (2n+1)\pi$

$n \equiv$ integer

Constructive interference

$\overline{I_{combined}} = 4I$, $\theta_1 - \theta_2 = 2n\pi$

Now consider combining two waves fro that come from two different paths

$$\overline{I_1 = \left[A \sin\left(\omega t - \frac{s_1}{\lambda}\right)\right]^2}$$

$$I_2 = \left[A \sin\left(\omega t + \frac{s_2}{\lambda}\right)\right]^2$$

$$I_{12} = \overline{\left[A \sin\left(\omega t - \frac{2\pi s_1}{\lambda}\right) + A \sin\left(\omega t - \frac{s_2 2\pi}{\lambda}\right)\right]^2}$$

$$= 4A^2 \left[\sin^2\left(\omega t - \frac{2\pi(s_1 - s_2)}{2\lambda}\right)\right] \cos^2\left(\pi \frac{s_2 - s_1}{\lambda}\right)$$

construtive interference

$$I_{combined} = 4I, \quad \theta_2 - \theta_1 = 2n\pi$$

as $\cos^2(\theta_2 - \theta_1) = \cos^2(n\pi) = 1$

destructive interfence

$$I_{combined} = 0, \quad \theta_2 - \theta_1 = (2n+1)\pi$$

as $\cos^2(\theta_1 - \theta_1) = \cos^2(n\pi + \pi/2) = 0$

Now consider combining two wave of the same frequency, initial of the same phase, but taking two different paths, and then combining

each alone

$$I_1 = \overline{\left[A\sin\left(\omega t - \frac{2\pi s_1}{\lambda}\right)\right]^2} = \frac{A^2}{2} \equiv I_0$$

$$I_2 = \overline{\left[A\sin\left(\omega t - \frac{2\pi s_2}{\lambda}\right)\right]^2} = \frac{A^2}{2} \equiv I_0$$

$$I_{combined} = \overline{\left[A\sin\left(\omega t - \frac{2\pi s_1}{\lambda}\right) + A\sin\left(\omega t - \frac{2\pi s_2}{\lambda}\right)\right]^2}$$

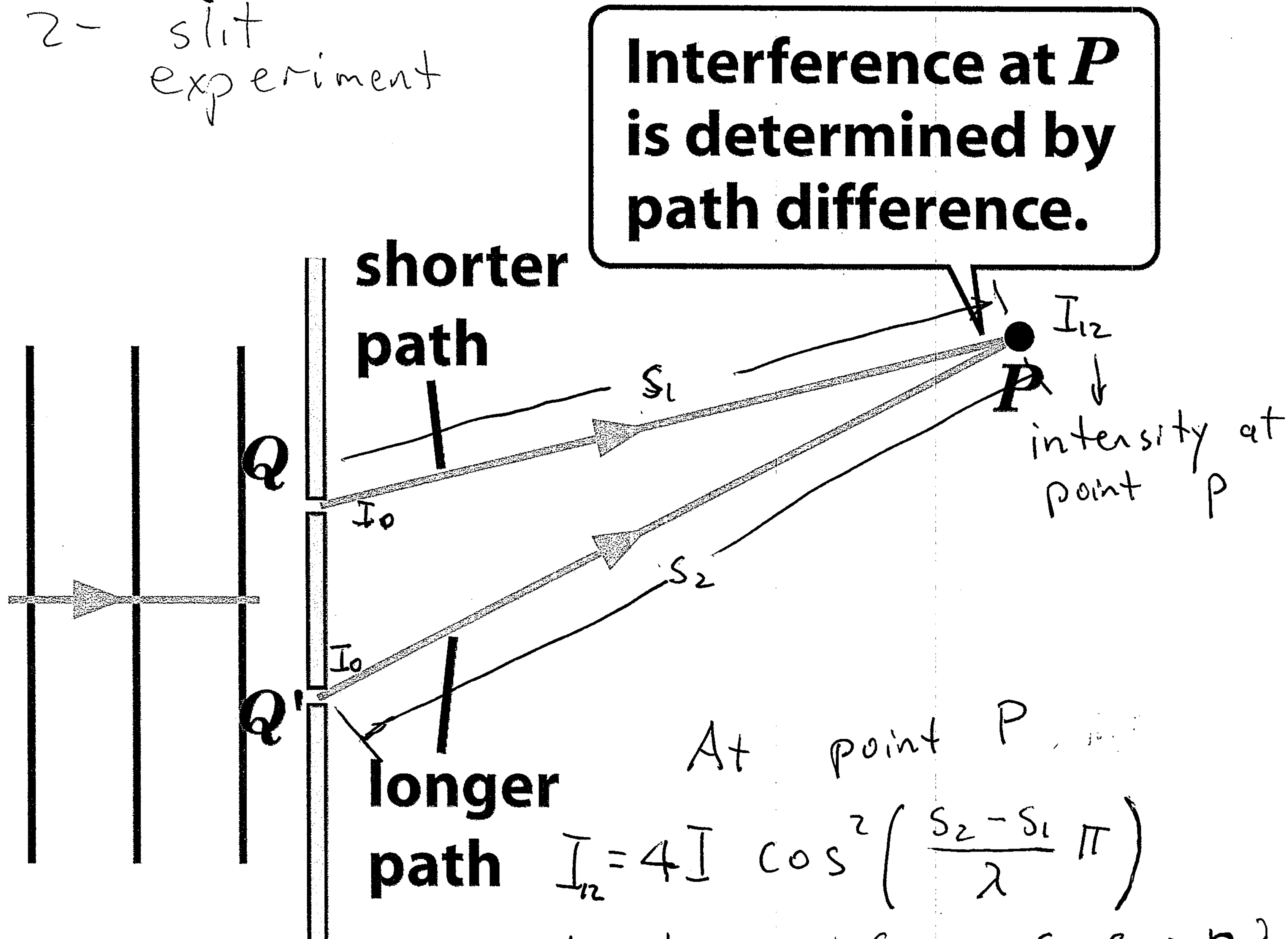
$$= 4A^2 \overline{\sin^2\left(\omega t + \frac{2\pi(s_1+s_2)}{\lambda}\right)} \cos^2\left(\frac{2\pi}{\lambda}(s_1 - s_2)\right)$$

$$= 2A^2 \cos^2\left(\frac{2\pi}{\lambda}(s_1 + s_2)\right) = 4I_0 \cos^2\left(\frac{2\pi(s_2 - s_1)}{\lambda}\right)$$

(09)

Young's Interference Experiment

2- slit experiment



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At point P,

$$I_{12} = 4I \cos^2\left(\frac{S_2 - S_1}{\lambda}\pi\right)$$

constructive intef. $\equiv$ $S_2 - S_1 = n\lambda$

destructive interf. $\equiv$ $S_2 - S_1 = (n + 1/2)\lambda$

$n = 0, \pm 1, \pm 2$

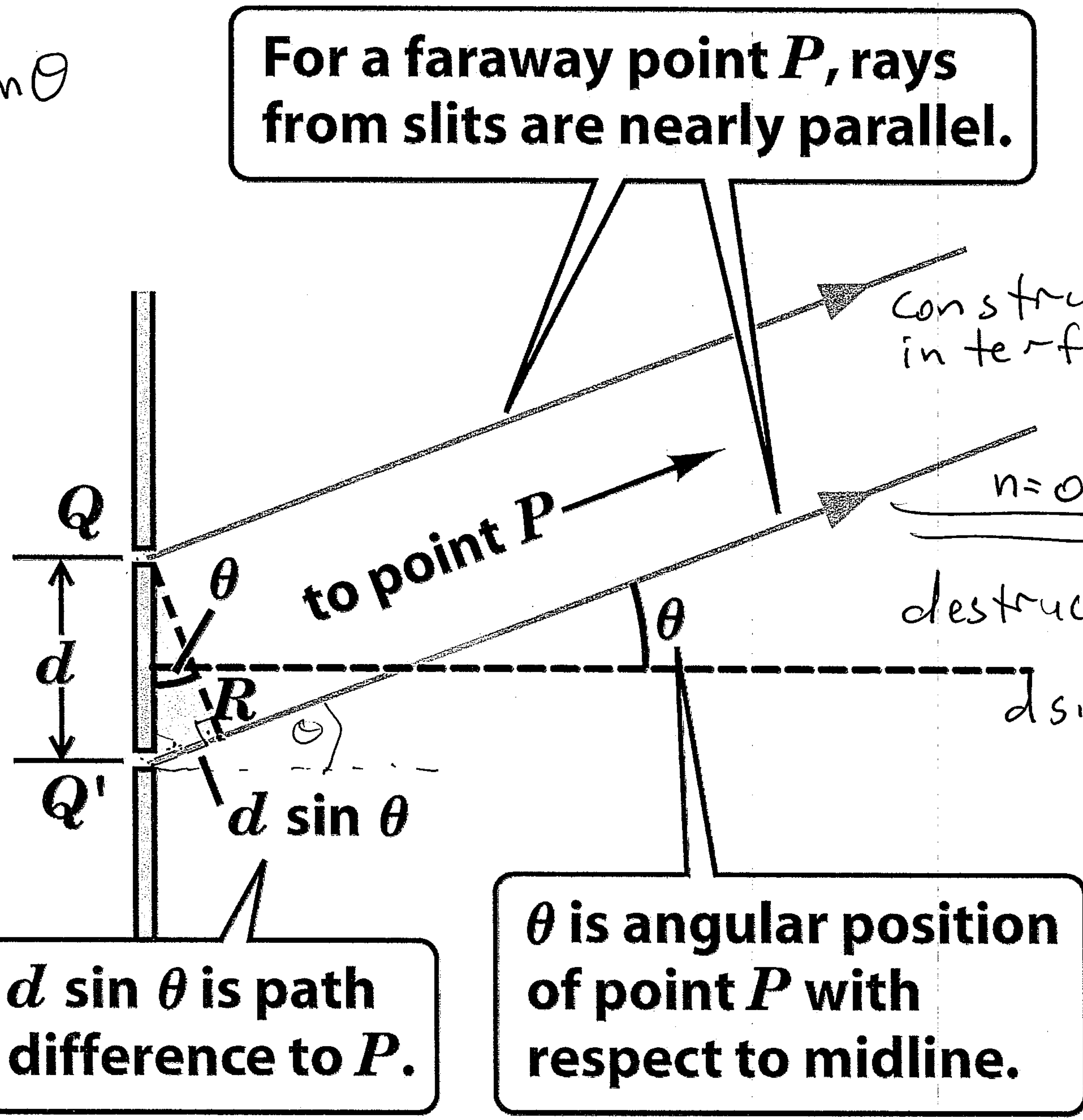
(10b)

Finding Path difference

$s_2 - s_1 = d\sin\theta$
$\simeq d\theta$

(as $\sin\theta \simeq \theta$
if $\theta \ll 1$
θ in radians)

$\tan\theta \simeq \theta$
$\theta \ll 1$



constructive interference

$d\sin\theta = n\lambda$

$n = 0, \pm 1, \pm 2, \ldots$

destructive interference

$d\sin\lambda = (n + \frac{1}{2})\lambda$

Relation of "fringes" to screen position

Constructive: $\begin{cases} \sin\theta_n = n\lambda/d\,, \quad \tan\theta_n = \dfrac{y_{max,n}}{L} \end{cases}$

for small angles

$$\sin\theta_n \sim \tan\theta_n \sim \theta_n = \frac{n\lambda}{d} = \frac{y_{max,n}}{L}$$

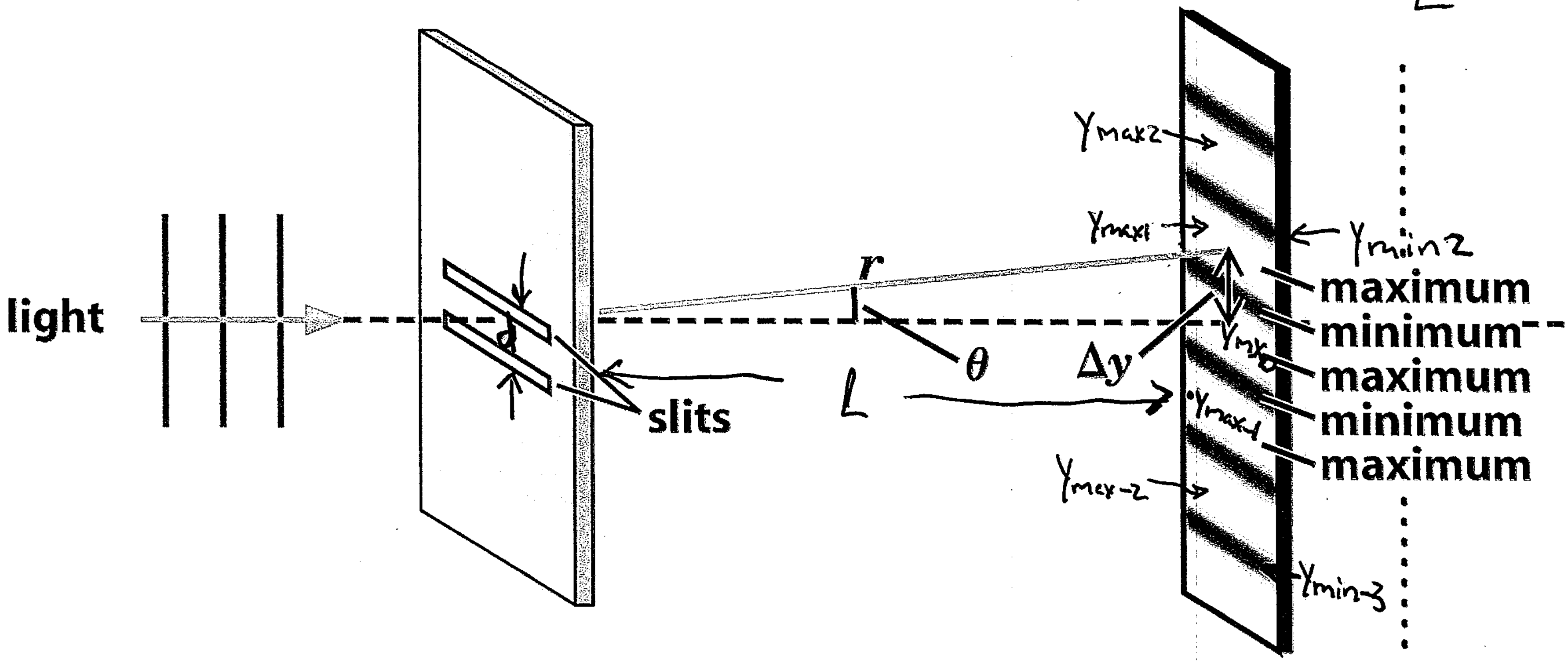


Figure 35-17b Physics for Engineers and Scientists 3/e

Destructive: $\sin\theta_n = (n+1/2)\dfrac{\lambda}{d}$, $\tan\theta_n = \dfrac{y_{min,n}}{L}$

for small angles

$$\sin\theta_n \simeq (n+1/2)\frac{\lambda}{d} \Big\| \tan\theta_n \sim \theta_n = \frac{(n+1/2)\lambda}{d} = \frac{y_{min,n}}{L}$$

Realistic

Young's intereference pattern

(2 - slit pattern)

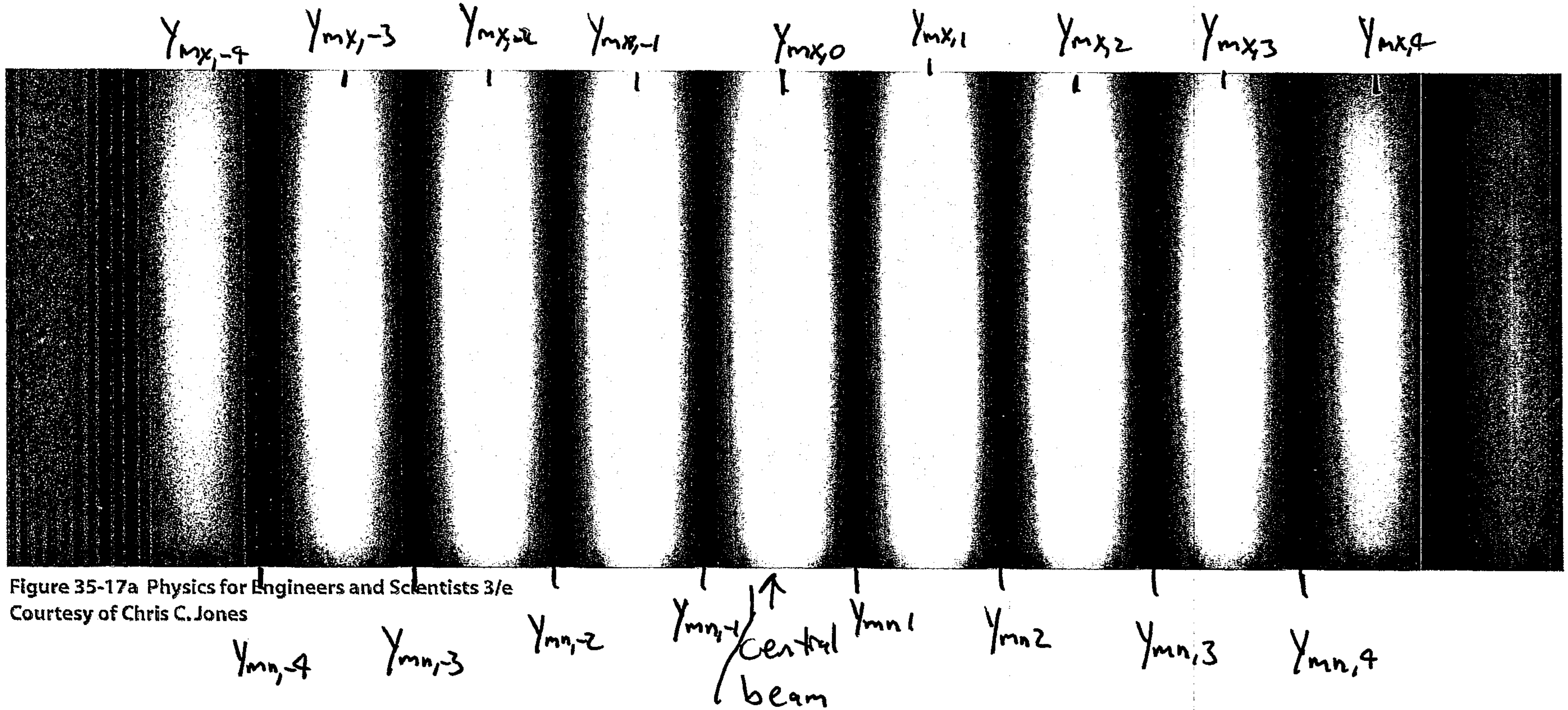


Figure 35-17a Physics for Engineers and Scientists 3/e
Courtesy of Chris C. Jones