Lecture #19

AC Currents

1). Consider an AC 10 ampere peak current going through a 10 ohm resistor. The power dissipated is

(a) 10^3 watts

(b) 5×10^2 watts

(c) 100 watts

(d) 50 watts

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(2) The units of the quantity $\omega^2 LC$ is

(a) ohms

(b) ohm^2

(c) dimensionless

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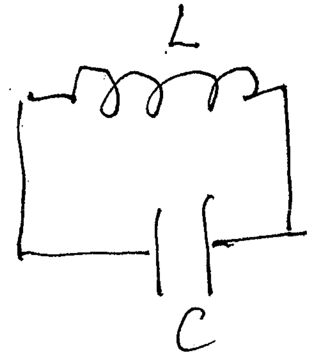
(3) The units of the quantity L/C is

(a) ohms

(b) ohm^2

(c) dimensionless

(4) An LC circuit with $L = 10^{4}\,H$ and a capacitance of $10^{-8}\,F$ has a period, T, of oscillation of approximately



(Hertz is 1 cycle/sec)

(a) 10^{2} Hertz (b) 10^{-2} Hz

(c) 16 Hz (d) 1600 Hz

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(5) In the above example the impedance of the inductor is

(a) $2\pi\,10^{6}\,\Omega$ (b) $10^{6}\,\Omega$

(c) $\frac{10^{6}}{2\pi}\,\Omega$

(6) The impedance of the capacitor is (choices above)

(7) An RMS voltage of 10.0V, with a frequency of $\frac{10^2}{2\pi}$ Hz, is applied across a 100 H inductor. The power dissipated is

(a) 1.0 watts (b) 0.5 watts (c) 0

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(8) The mean energy stored in the inductor in the previous problem is,

(a) 10^{-2} J (b) 0 J (c) 5×10^{-3} J

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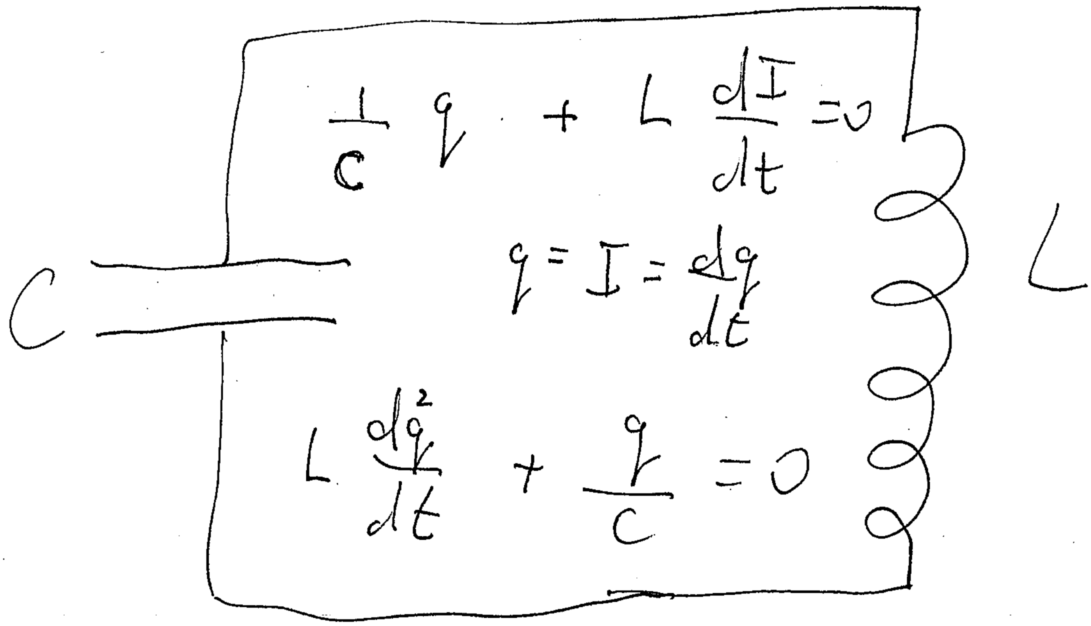
(9) An oscillator at $\frac{10^3}{2\pi}$ Hz charges a capacitor of 10^{-9} F to 10^{-3} C RMS. The peak current through the capacitor is

(a) 1A (b) 10^{-9} A (c) $\sqrt{2}$ A (d) $\sqrt{2} \times 10^{-9}$ A

L - C circuit



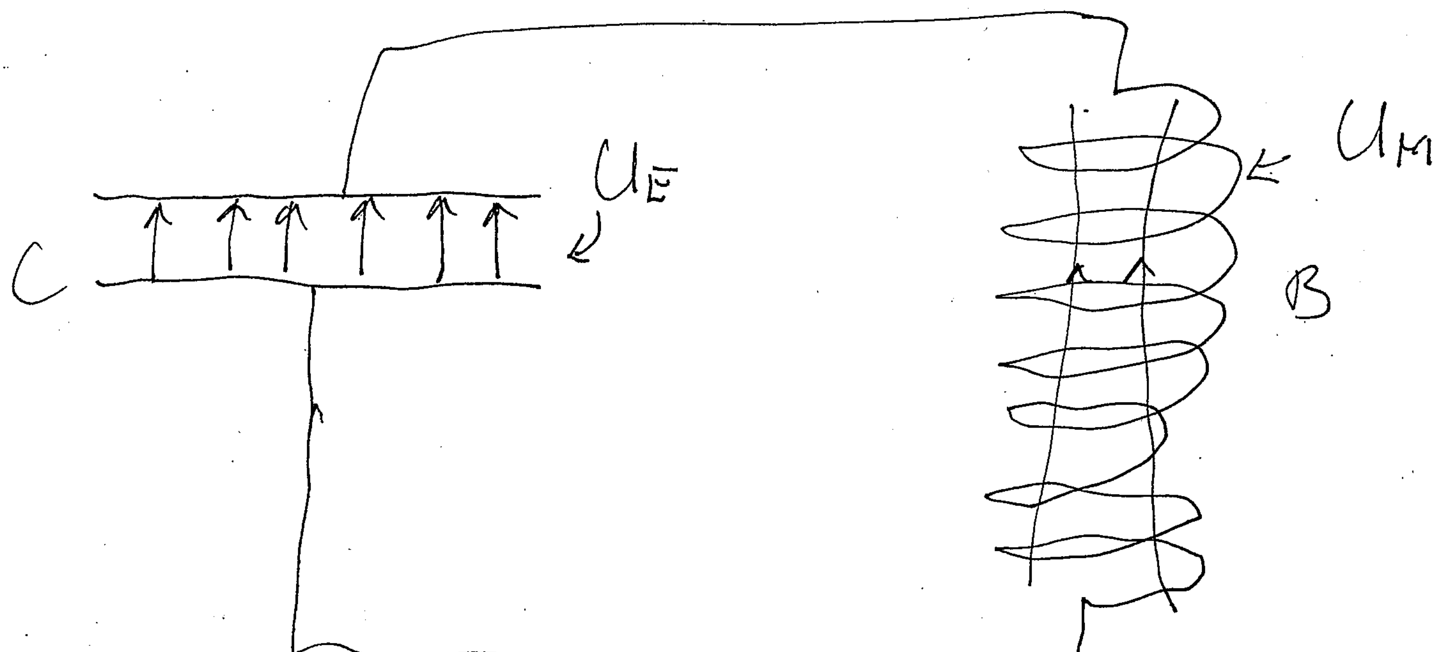
$$\frac{1}{C} q + L \frac{dI}{dt} = 0$$

$$\dot{q} = I = \frac{dq}{dt}$$

$$L \frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

$$q = q_0 \sin(\omega_0 t - \phi)$$

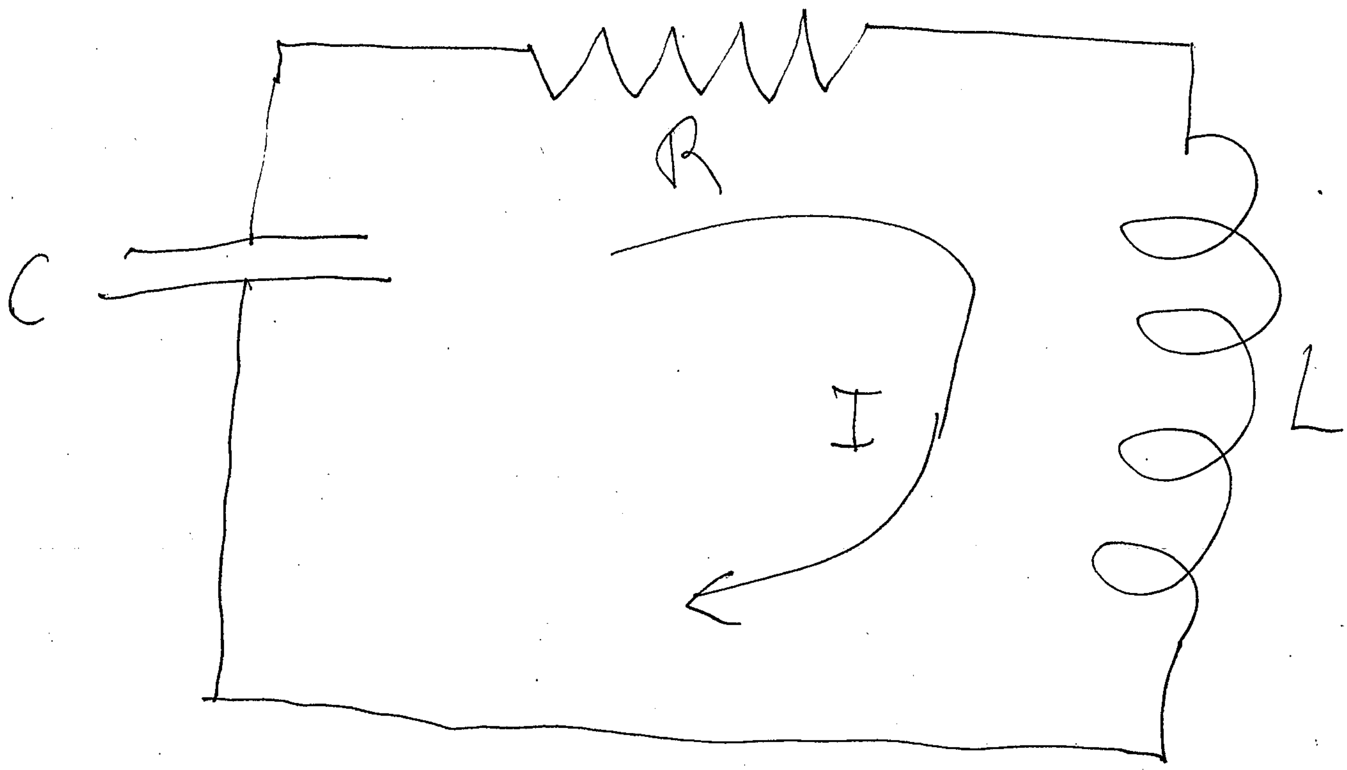
$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$U_E = \epsilon_0 E^2(t)/2 \equiv$ electric energy

$U_M = B^2(t)/2\mu_0 \equiv$ magnetic energy

L R C circuit



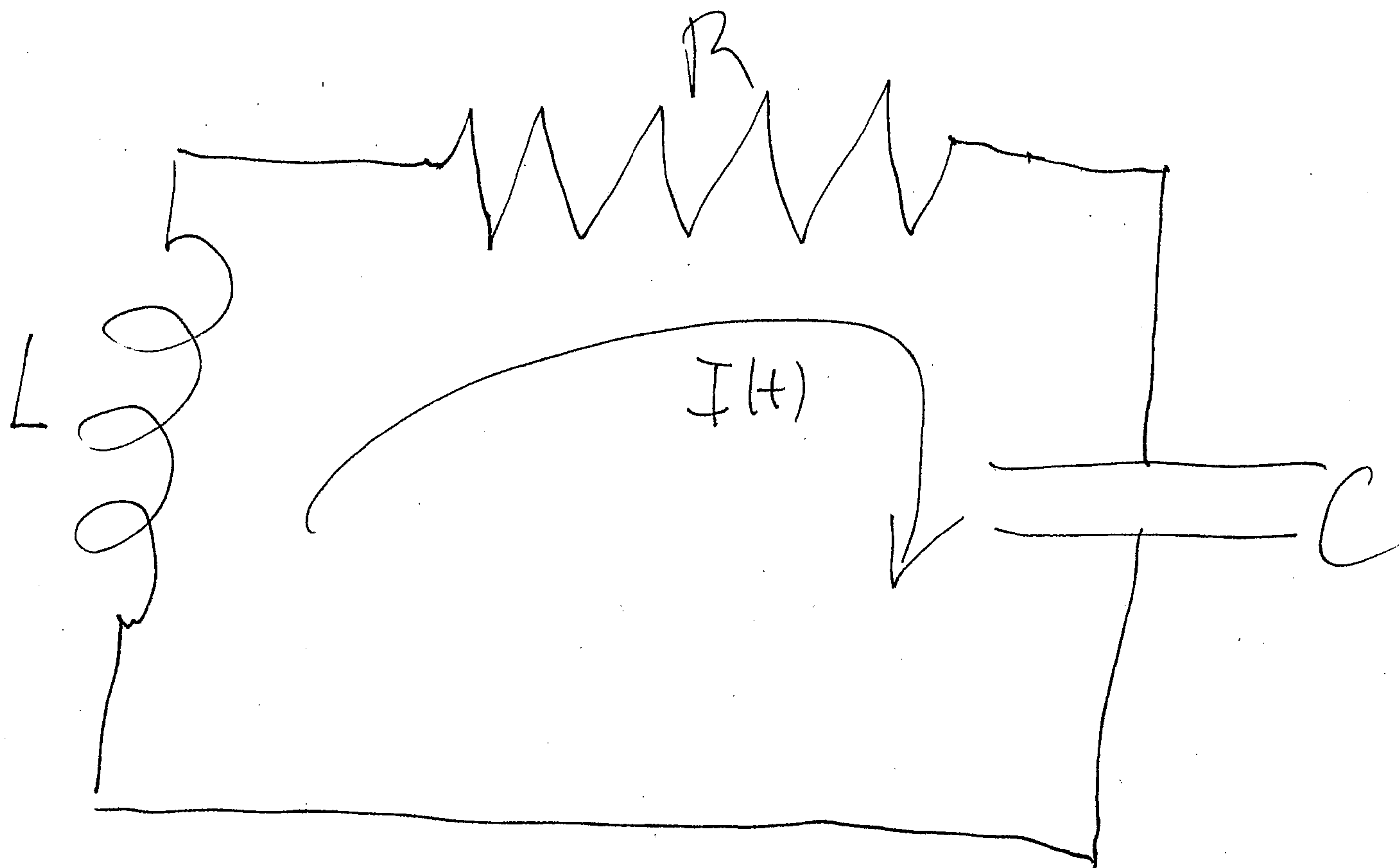
Khirchoff's Law

$$L\frac{dI}{dt} + \frac{q}{C} + IR = 0$$

$$I = dq/dt$$

$$L\frac{d^2q}{dt^2} +$$

L R C circuit



Khirchoff's Law again

$$L \frac{dI}{dt} + IR + \frac{q}{C} = 0$$

$$I = \frac{dq}{dt}$$

$$L \frac{d^2 q}{dt^2} + \frac{dq}{dt} R + \frac{q}{C} = 0$$

(6)

Now the resistor brings in dissipation and electromagnetic energy slowly (if R is small enough) converts to heat energy, and oscillation dies away

$$L \frac{d^2q}{dt^2} + \frac{dq}{dt} R + \frac{q}{C} = 0$$

$\left(R < \sqrt{\frac{L}{C}}\right)$ One can show that decay rate is

$$e^{-\frac{R}{2L}t}$$

Decay rate of AC RLC circuit is

$$\tau = \frac{\quad}{2} \quad \text{if } R < \sqrt{\frac{L}{C}}$$

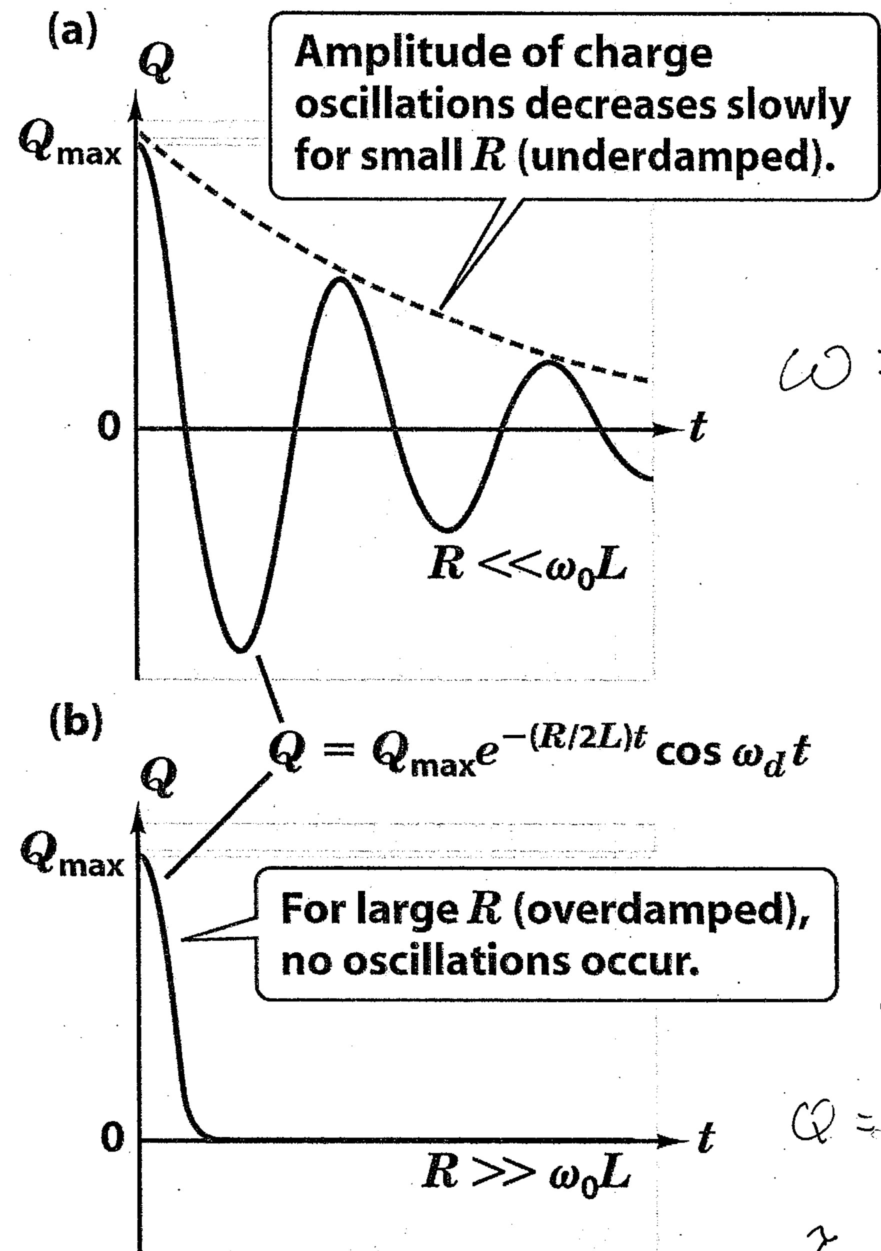
For $R > \sqrt{\frac{L}{C}}$, no oscillations but two different decay rates τ_C, τ_L. For $R \gg \sqrt{\frac{L}{C}}$ $\tau_C \to RC$, $\tau_L \to \frac{L}{R}$

Under damped

$R < \sqrt{L/C}$

Overdamped

$R > \sqrt{L/C}$



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$\omega = \left[\omega_0^2\left(1 - \frac{R^2C}{L}\right)\right]^{1/2}$

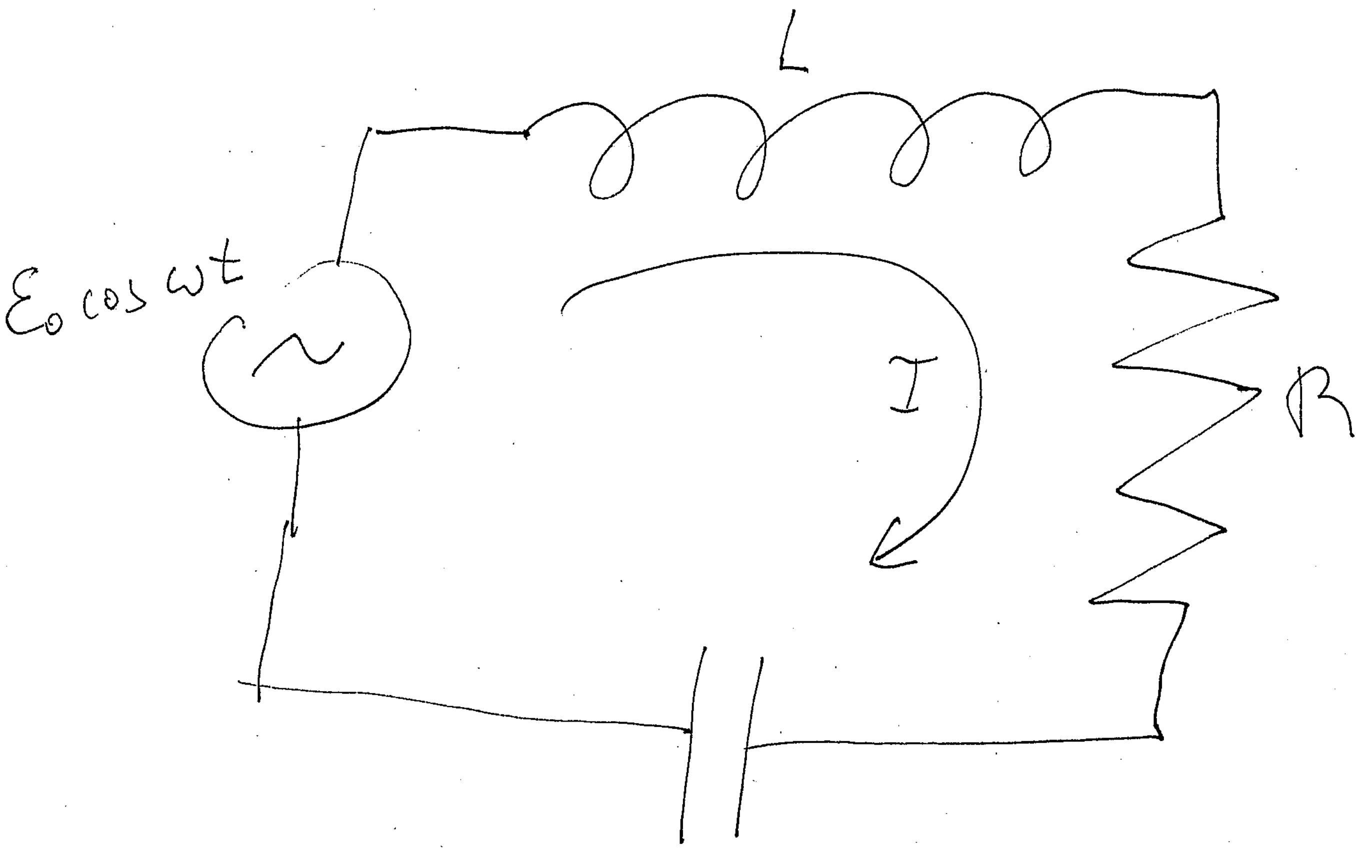
two characteristic damping time

$Q = Q_1 \exp(-t/\tau_1) + Q_2 e^{-t/\tau_2}$

$\tau_1 \rightarrow L/R$ when $\frac{R^2C}{L} >> 1$

$\tau_2 \rightarrow RC$

AC LRC circuit



C

Khirchoff's voltage Law

$$-\mathcal{E}_0 \cos\omega t + L\frac{dI}{dt} + IR + \frac{q}{C} = 0$$

Two different solutions

(a) Transient (homogeneous) solution: Eventually decays

(b) Particular Solution: With $\mathcal{E}_0$ finite remains as long as $\mathcal{E}_0$ present

$$\mathcal{E}_0 \cos \omega t = L \frac{dI}{dt} + IR + \frac{q}{C}$$

Forced oscillation

$$I = \underbrace{I_0}_{\text{amplitude}} \cos(\omega t + \underbrace{\phi}_{\text{phase}})$$

$$-\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

$$0 < \phi \leq \frac{\pi}{2}$$

capacitive - like circuit, current leads voltage

$$-\frac{\pi}{2} \leq \phi < 0$$

inductive - like circuit, current lags voltage

$$\mathcal{E}_0 \cos\omega t = L\frac{dI}{dt} + IR + \frac{q}{C} \quad (a)$$

If $\quad I(t) = I_0 \cos(\omega t + \phi) \quad (b)$

then $\quad q(t) = \dfrac{I_0}{\omega}\sin(\omega t + \phi) \quad (c)$

(note then $\dfrac{dq(t)}{dt} = I_0 \cos(\omega t + \phi) = I(t)$)

$$\frac{dI}{dt} = -\omega I_0 \sin(\omega t + \phi) \quad (d)$$

Substitute b, c, d into (a)

we obtain

$$\mathcal{E}_0 \cos\omega t = I_0\left[\left(\frac{1}{\omega C} - \omega L\right)\sin(\omega t + \phi) + R\cos(\omega t + \phi)\right]$$

$$\mathcal{E}_0 \cos\omega t = I_0\left[\left(\frac{1}{\omega C} - \omega L\right)\sin(\omega t + \phi) + R\cos(\omega t + \phi)\right] \quad (I)$$

Equation (I) has unique solution for I_0 and ϕ (please verify stated solution as exercise)

$$I_0 = \frac{\mathcal{E}_0}{X_{total}} \; ; \; Z = \left[\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2\right]^{1/2}$$

$$\phi = \tan^{-1}\left(\frac{-\frac{1}{\omega C} + \omega L}{R}\right)$$

If $\frac{1}{\omega C} > L$, $\phi > 0$ inductive (low frequency)

If $\frac{1}{\omega C} < L$, $\phi < 0$ capacitive (high frequency)

If $\frac{1}{\omega C} = L$, $\phi = 0$ purely resistive

resonance

For a given voltage, $\mathcal{E}_0$ and fixed resistance, the largest current, I_0, comes when the resonance condition is fulfilled $\omega = \omega_0 \equiv \frac{1}{\sqrt{LC}}$

$$I_0 = \frac{\mathcal{E}_0}{\left[R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2\right]^{1/2}}$$

Power delivered to resistor (load)

$$\boxed{P = \frac{I_0^2 R}{2} = \frac{\mathcal{E}_0^2}{2} \frac{R}{\left[R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2\right]}}$$

clearly largest when $1/\omega C - \omega L = 0$
Let us look near resonance

$$\omega = \omega_0 + \delta\omega$$

$$P = \frac{I_0^2 R}{2} = \frac{\mathcal{E}_0^2}{2} \frac{R}{\left[R^2 + \left(\frac{1}{(\omega_0+\delta\omega)C} - (\omega_0+\delta\omega)L\right)^2\right]}$$

$$\begin{pmatrix} \frac{1}{(\omega_0+\delta\omega)\,C} \\ - \\ (\omega_0+\delta\omega)L \end{pmatrix} \approx \begin{pmatrix} \frac{1}{\omega_0 C}\left(1 - \frac{\delta\omega}{\omega_0}\right) \\ - \\ \left(\omega_0 L\left(1 + \frac{\delta\omega}{\omega_0}\right)\right) \end{pmatrix}$$

$$\delta\omega = \omega - \omega_0 ; \quad \omega_0^2 = \frac{1}{LC}$$

$$\frac{1}{\omega_0 C} - \omega_0 L = \sqrt{\frac{L}{C}} - \sqrt{\frac{L}{C}} = 0$$

Then consider next term

$$-\frac{1}{\omega_0 C}\frac{\delta\omega}{\omega_0} - \omega_0 L \frac{\delta\omega}{\omega_0} = 2L\,\delta\omega$$

$$P = \frac{\mathcal{E}_0^2}{2} \frac{R}{\left[R^2 + 4L^2\,\delta\omega^2\right]}$$

$$P = \frac{\mathcal{E}_0^2}{2} \frac{R}{\left[R^2 + 4L^2 \delta\omega^2\right]}$$

$$= \frac{\mathcal{E}_0^2}{2R} \frac{1}{\left[1 + \frac{4L^2\omega_0^2}{R^2} \frac{\delta\omega^2}{\omega_0^2}\right]}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$= \frac{\mathcal{E}_0^2}{2R_0} \frac{1}{\left[1 + \frac{4L}{CR^2} \frac{\delta\omega^2}{\omega_0^2}\right]}$$

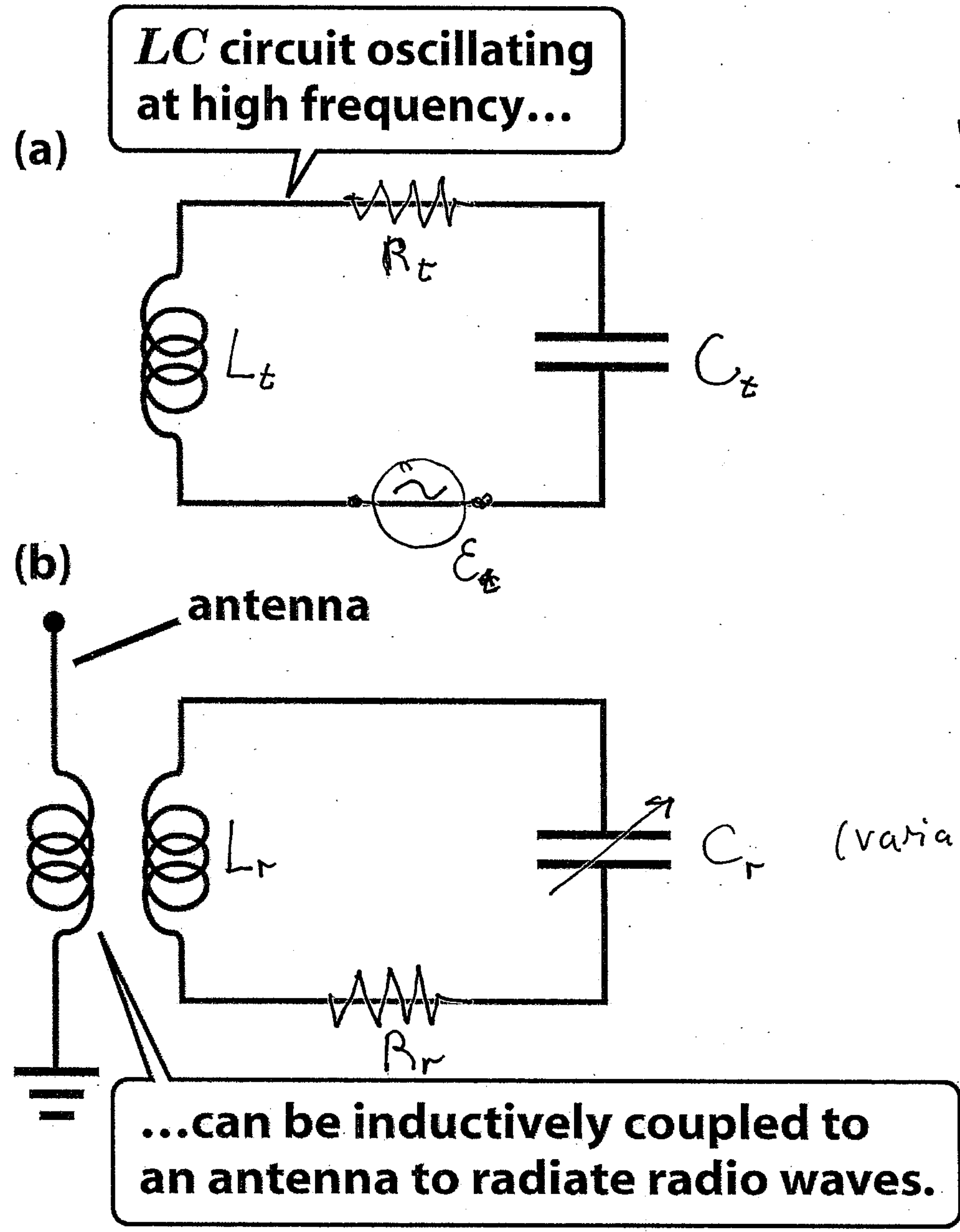
$$\boxed{P \equiv \frac{\mathcal{E}_0^2}{2R_0} \frac{1}{\left[1 + 4Q^2 \frac{(\omega-\omega_0)^2}{\omega_0^2}\right]}}$$

$$Q \equiv \frac{(L/C)^{1/2}}{R} \equiv \text{Quality Factor of Circuit}$$

When $\omega - \omega_0 = \frac{\omega_0}{2Q}$ power received by load is down by ½ (17

r - receiver
t - transmitter

Can Couple by mutual induction



(variable capacitor)

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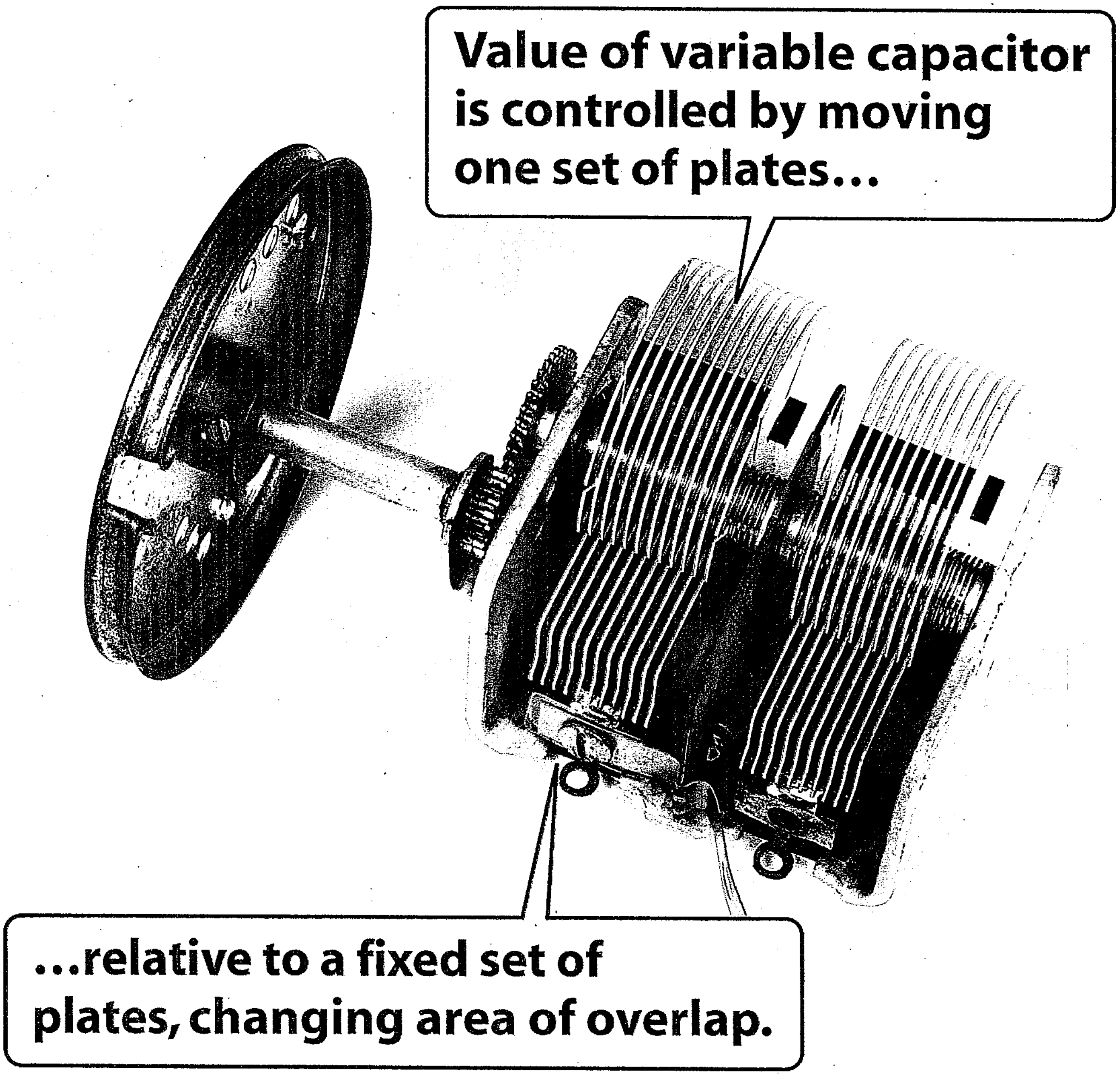

Figure 32-18 Physics for Engineers and Scientists 3/e
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