Lecture # 18

A.C. - Currents
Loops (A) and (B) are in magnetic fields that point towards you, perpendicular to the plane of the loop. If you begin to turn the loop in the directions indicated, the current at points (a) and (b) move:

(1) (a) upward, (b) upward
(2) (a) upward, (b) downward
(3) (a) downward, (b) upward
(4) (a) downward, (b) downward
Consider the following circuit that has been established for a long time.

What is the current in the inductor and resistor, respectively?

\[
\begin{align*}
\text{(a)} & \quad \frac{E}{R} \quad \text{and} \quad \frac{E}{r} \\
\text{(b)} & \quad \frac{E}{R} \quad \text{and} \quad \frac{E}{R+L} \\
\text{(c)} & \quad \frac{E}{r} \quad \text{and} \quad \frac{E}{r+L} \\
\text{(d)} & \quad \frac{E}{r} \quad \text{and} \quad \frac{E}{R}
\end{align*}
\]

If the switch is suddenly opened, will the current through the inductor instantly change?

\[
\begin{align*}
\text{(1)} & \quad \text{no} \\
\text{(2)} & \quad \text{no} \\
\text{(3)} & \quad \text{no} \\
\text{(4)} & \quad \text{yes}
\end{align*}
\]
What is the ratio of the current in the resistor \( R \) just before to just after the switch is opened?

(a) \( \frac{r}{R} \)  
(b) \( \frac{R}{r} \)  
(c) \( \frac{R}{r} \)  
(d) in determined because it depends on \( L \)

After the switch is opened, at what time, \( t_s \), will the current in the resistor be equal to its initial current?
Total resistance of circuit is $R + r$

$$V = V_0 e^{-\frac{t}{L}} = E$$  \hspace{1cm} (1)

$$V_0 = I_L(t=0) R \hspace{1cm} (2) \text{ Voltage across resistor } \ z=0^+$$

$$I_L(t=0^-) = I_L(t=0^+) \hspace{1cm} (3)$$

$$\therefore V_0 = E \frac{R}{r} \hspace{1cm} (4)$$

Substituting (4) into (1) gives

$$E \frac{R}{r} e^{-\frac{t}{L}} = E$$

$$- t_i \frac{(R+r)}{L} \hspace{1cm} e^{-\frac{t}{(R+r)/L}} = \frac{r}{R}$$

$$t_i \frac{(R+r)}{L} = \ln \left( \frac{R}{r} \right)$$

$$t_i = \frac{L}{(R+r)} \ln \left( \frac{R}{r} \right)$$

\hspace{1cm} (4)
At \( t=0^- \) no current flows in the circuit closed.

If at time \( t=0 \) the switch is suddenly turned on, (1) what is the initial current through the battery? (2) the final battery current?

\[
(a) \frac{\mathcal{E}}{R_1} \quad (b) \frac{\mathcal{E}}{R_2} \quad (c) \frac{\mathcal{E}(R_2 + R_1)}{R_2 R_1}
\]
Energy in an inductor

The text shows that the energy of an inductor is

$$U_M = \frac{1}{2} LI^2$$

Because voltage drop of an inductor is the

Power supplied to an inductor is

$$\frac{dU_M}{dt} = P = EI = LI \frac{dI}{dt}$$

$$= L \frac{d}{dt} I^{\frac{3}{2}} \Rightarrow U_M = \int_{t_i}^{t_f} L \frac{dI^{\frac{3}{2}}}{dt}$$

Integrate

$$U_M (\text{final}) - U_M (\text{initial}) = L \int_0^I I^2 dt$$

$$= \frac{L I_0^2}{2} - \frac{L I_f^2}{2} = \frac{L I_f^2}{2}$$
Figure 32-1  Physics for Engineers and Scientists 3/e
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"115-V AC" voltage at electric outlets is oscillating emf with amplitude 163 V.

\[ f = \frac{1}{T} \quad T = \frac{2\pi}{\omega} \]  

\[ V = V_0 \cos \omega t \]  

\[ \langle V \rangle \equiv \left( \frac{1}{T} \int_0^T dt \, V^2 \right)^{1/2} \]  

\[ \approx \left( \frac{1}{T} \int_0^T dt \, V_0^2 \cos^2 \omega t \right)^{1/2} = \frac{V_0}{\sqrt{2}} \]

Period of oscillation is \[ T = \frac{2\pi}{\omega} = 1/60 \text{ s.} \]
Khurshoff Voltage Rule still applicable

Simplest AC circuit: oscillating emf supplies oscillating current through resistor.

\[-\mathcal{E}(t) + I(t)R = 0\]
\[\mathcal{E}(t) = \mathcal{E}_0 \cos \omega t\]
\[I(t) = \frac{\mathcal{E}_0}{R} \cos \omega t\]

Wave in circle is circuit symbol for source of oscillating emf.

Figure 32-2 Physics for Engineers and Scientists 3/e
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For AC resistor circuit, maxima of emf and current occur simultaneously, ...

...as do minima; current and emf are in phase.

\[ \varepsilon = -\varepsilon(t) + IR = -\varepsilon_0 \cos \omega t + IR \]

\[ I = \frac{\varepsilon_0}{R} \cos \omega t = \frac{\varepsilon_0}{R} \cos \left( \frac{2\pi}{\tau} t \right) \]
Power dissipated in resistor is always positive.

Function $\cos^2 \omega t$ oscillates symmetrically about value $\frac{1}{2}$, so time-average power is $\frac{1}{2} \frac{E_{\text{max}}^2}{R}$.

Average Dissipative Power

$$\langle P \rangle = \frac{1}{T} \int_0^T P \, dt = \frac{E_0^2}{R} \int_0^T \cos^2 \omega t \, dt = \frac{E_0^2}{2R}$$

In one period $T = \frac{2\pi}{\omega}$ of emf, power goes through two cycles.

Figure 32-5 Physics for Engineers and Scientists 3/e
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AC capacitor circuit: oscillating emf produces oscillating charge on capacitor, \( Q = C \mathcal{E} \).

\[
\mathcal{E} = V_0 \cos \omega t
\]

Khirchhoff's Voltage Law

\[
-V_0 \cos \omega t + \frac{\partial Q}{\partial t} = 0
\]

\[
Q(t) = V_0 C \cos \omega t
\]

\[
I(t) = \frac{dQ(t)}{dt} = (-\omega C) V_0 \sin(\omega t)
\]

Impedance (generalization of resistance)

\[
X_C = \frac{1}{\omega C}; \quad \text{Current 'leads' Voltage}
\]

\[
\frac{X_C}{V_0} \cos(\omega t + \frac{\pi}{2})
\]
Current in a capacitor leads voltage.

For AC capacitor circuit, current is zero when emf is maximum, ...

...and current peaks earlier in time than nearest voltage peak.

\[ \mathcal{E}(t) = \mathcal{E}_{\text{max}} \cos(\omega t) \]

\[ I(t) = -\omega C \mathcal{E}_{\text{max}} \sin(\omega t) \]

\[ = \frac{\mathcal{E}_{\text{max}} \cos(\omega t + \frac{\pi}{2})}{X_c} \]

\[ X_c = \frac{1}{\omega C} \]

Current amplitude is \( I_{\text{max}} = \omega C \mathcal{E}_{\text{max}} \).
A perfect capacitor conserves energy.

Half the time source is delivering energy, the other half time the source gets energy back.

Energy of capacitor

\[ \text{Energy} = \frac{1}{2} \frac{Q^2}{C} \]

\[ = \frac{1}{2} \left( \frac{Q_0 \cos \omega t}{C} \right)^2 \]

In one period \( T = \frac{2\pi}{\omega} \) of emf, power goes through two cycles.

\[ \text{Power} = \frac{d}{dt} \text{Energy} \]

\[ = \frac{Q_0^2 \omega \sin \omega t \cos \omega t}{C} = \frac{4}{2} \left( \frac{Q_0}{C} \right)^2 \frac{\omega C \sin(2\omega t)}{C} = \frac{1}{2} \omega C E_{\text{max}} \sin(2\omega t) \]
AC inductor circuit: oscillating emf produces oscillating current in inductor.

\[ -E + L \frac{dI}{dt} = 0 \]

\[ E = E_0 \cos \omega t = L \frac{dI}{dt} \]

\[ \frac{dI}{dt} = \frac{E_0}{L} \cos \omega t \]

\[ I(t) = \frac{E_0}{L} \int \cos \omega t \, dt = \frac{E_0}{\omega L} \sin \omega t = \frac{E_0}{\omega L} \cos (\omega t - \frac{\pi}{2}) \]

\[ I = I_0 \cos (\omega t - \frac{\pi}{2}) \]

\[ I_0 = \frac{E_0}{\omega L} = \frac{E_0}{X_L} \]

Current lags voltage

Inductive Impedance

\[ X_L = \omega L \]
Current - Voltage in Inductor

\[ E(t) = E_{\text{max}} \cos \omega t \]

...and voltage peaks earlier in time than nearest current peak.

Current amplitude is \( I_{\text{max}} = \frac{E_{\text{max}}}{\omega L} \).

For AC inductor circuit, current is zero when emf is maximum, ...

\[ I(t) = \frac{E_{\text{max}}}{\omega L} \cos \left( \omega t - \frac{\pi}{2} \right) \]
What is the current through each element?

\[ I_a = \frac{E_0}{R} \cos \omega t = \frac{E_0}{X_R} \cos \omega t \]

\[ I_c = \omega C E_0 \cos(\omega t + \frac{\pi}{2}) \]

\[ = \frac{E_0}{X_C} \cos(\omega t + \frac{\pi}{2}) \]

\[ I_L = \frac{E_0}{\omega L} \cos(\omega t - \frac{\pi}{2}) \]

\[ = \frac{E_0}{X_L} \cos(\omega t - \frac{\pi}{2}) \mathcal{E} \]

\[ X_R = R \]

\[ X_C = \frac{1}{\omega C} \]

\[ X_L = \omega L \]

When circuit elements are connected in parallel...

...the same instantaneous voltage is supplied to each.

Figure 32-14 Physics for Engineers and Scientists 3/e © 2007 W.W. Norton & Company, Inc.
If no resistance, such a circuit can have current for a long time, at a very special frequency

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

This behaves much like a mass on a spring, where inductor is akin to inertia, capacitor is akin to inverse of spring constant \(1/k\), and charge \(q\) akin to displacement \(x\).
\[ KE + PE = \frac{1}{2} kx^2 + \frac{1}{2} mx^2 \]

Energy oscillates between potential energy stored in the capacitor and kinetic energy of the stretched spring.

\[ x(t) = -A \sin(\omega t - \phi) \]

\[ \frac{dx}{dt} = -A \omega \cos(\omega t - \phi) \]

\[ \frac{d^2 x}{dt^2} = -A \omega^2 \sin(\omega t - \phi) \]

\[ C \frac{dv}{dt} + v = 0 \]

\[ L \frac{di}{dt} + i = 0 \]

\[ \frac{q}{C} = C \quad \text{or} \quad \frac{q}{L} = L \]

\[ \omega = \sqrt{\frac{1}{LC}} \]

\[ \phi = g \sin(\omega t - \phi) \]

\[ \phi = g \cos(\omega t - \phi) \]

\[ \phi = g \sin(\omega t - \phi) \]

\[ \phi = g \cos(\omega t - \phi) \]