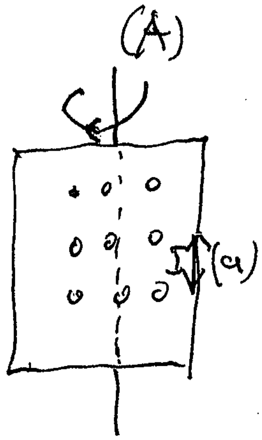
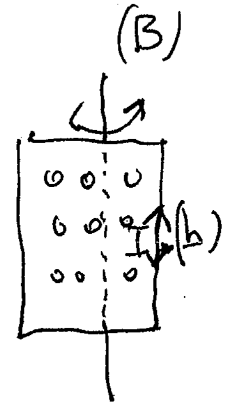
Lecture # 18

AC - currents





Loops (A) and (B) are in magnetic fields that point towards you, perpendicular to the plane of the loop. If you begin to turn the loop in the directions indicated, the current at points (a) and (b) move:

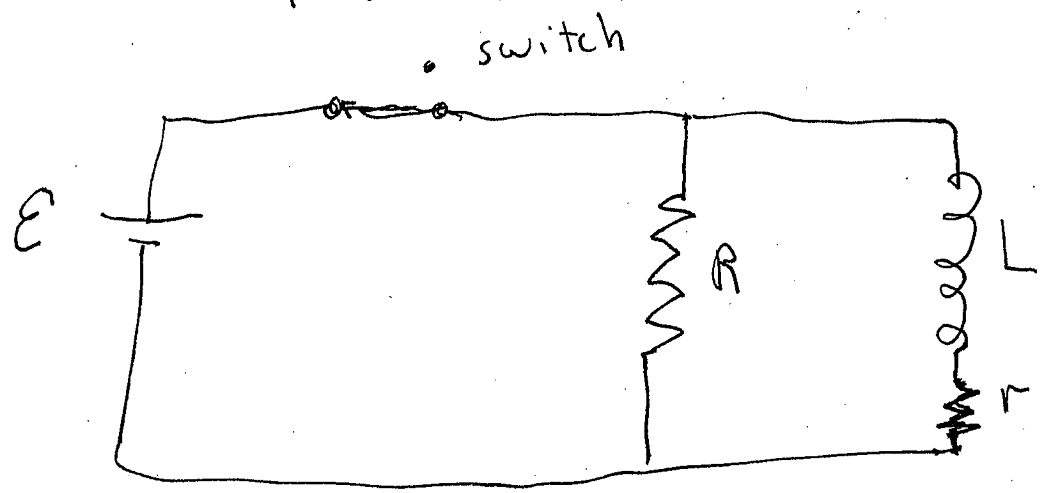
(1) (a) upward, (b) upward

(2) (a) upward, (b) downward

(3) (a) downward, (b) upward

(4) (a) downward, (b) downward

Consider the following circuit that has been established for a long time



What is the current in the inductor and resistor, respectively?

(1) $\frac{\mathcal{E}}{R}$ and $\frac{\mathcal{E}}{r}$

(b) $\frac{\mathcal{E}}{R}$ and $\frac{\mathcal{E}}{R+L}$

(c) $\frac{\mathcal{E}}{r}$ and $\frac{\mathcal{E}}{r+L}$

(d) $\frac{\mathcal{E}}{r}$ and $\frac{\mathcal{E}}{R}$

If the switch is suddenly opened will the current and voltage through the inductor instantaneously change?

(1) no & no

(2) no & yes

(3) yes & no

(4) yes & yes

What is the ratio of the current in the resistor, R, just before to just after the switch is opened?

(a) 1

(b) $\frac{r}{R}$

(c) $\frac{R}{r}$

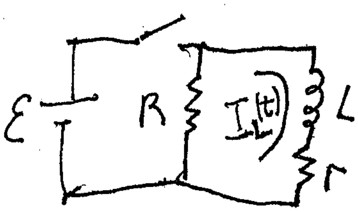
(d) indetermined because it depends on L

After the switch in opened
At what time, t_1, will the current in the resistor be equal to its initial current?

Total resistance of circuit is $R+r$

$$V = V_0 e^{-\frac{t_1(R+r)}{L}} = \mathcal{E} \qquad (1)$$

$$V_0 = I_L(t=0^+) R \qquad (2)$$ Voltage across resistor $t=0^+$



$$I_L(t=0^-) = \frac{\mathcal{E}}{r} \qquad (3)$$

$$I_L(t=0^-) = I_L(t=0^+)$$

$$\therefore \quad V_0 = \mathcal{E}\frac{R}{r} \qquad (4)$$

substituting (4) into (1) gives

$$\mathcal{E}\frac{R}{r} e^{-\frac{t_1(R+r)}{L}} = \mathcal{E}$$

$$e^{-t_1(R+r)/L} = \frac{r}{R}$$

$$\frac{t_1(R+r)}{L} = \ln\left(\frac{R}{r}\right)$$

$$t_1 = \frac{L}{R+r}\ln\left(\frac{R}{r}\right)$$

At $t=0^-$ no current flows in the circuit.

If at time t=0 the switch is suddenly ~~turned on~~ closed, (1) what is the initial current through the battery? (2) the final battery current?

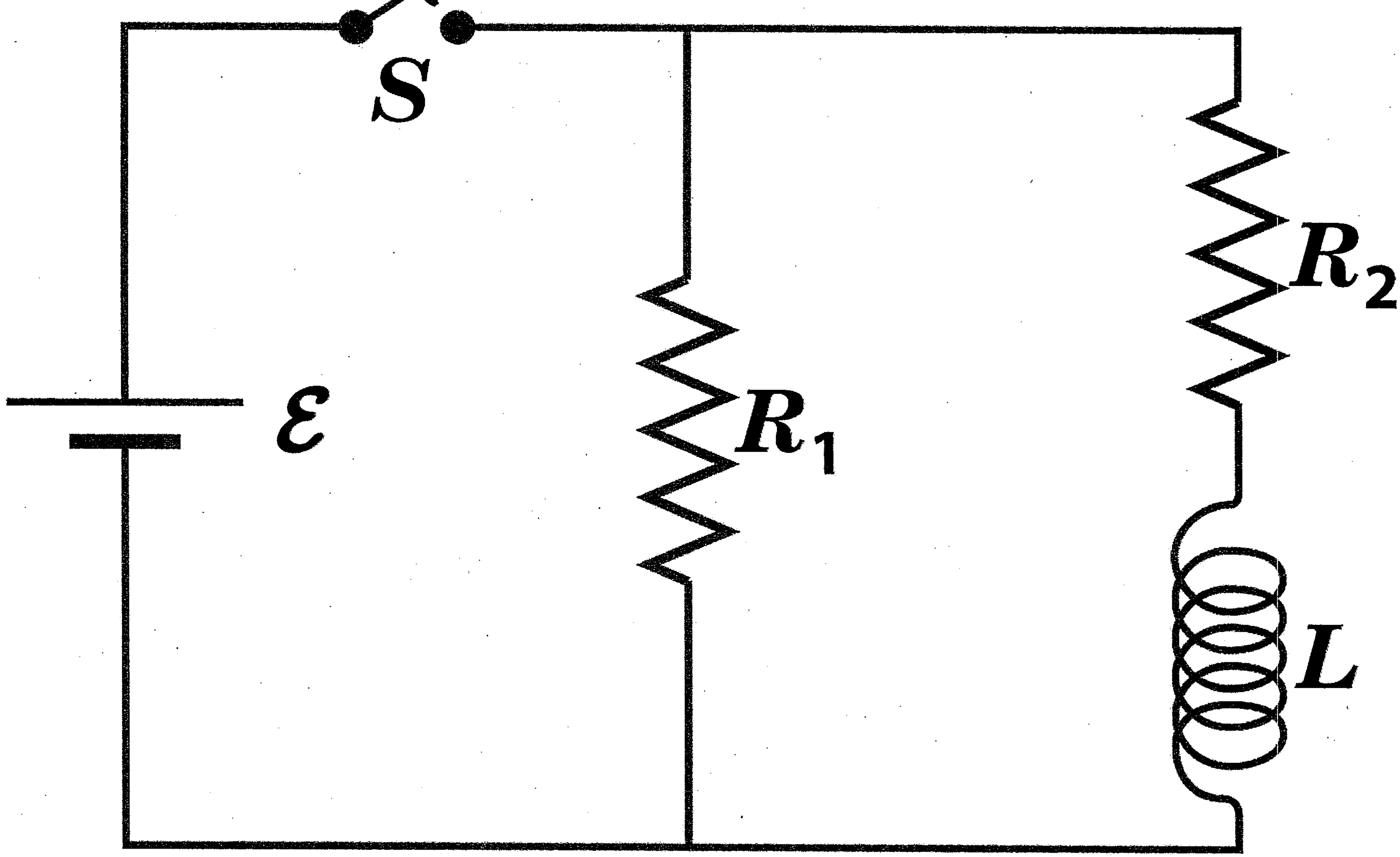


Figure 31-27 Physics for Engineers and Scientists 3/e

(a) $\mathcal{E}/R_1$ (b) $\mathcal{E}/R_2$ (c) $\mathcal{E}(R_2+R_1)/R_2R_1$

Capacitor Stores Electric Energy

Energy in an inductor

The text shows that the energy of an inductor is

$$U_M = \frac{1}{2} L I^2$$

Because voltage drop of an inductor the

Power supplied to an inductor is

$$\frac{dU_M}{dt} = P = \mathcal{E} I = L \frac{dI}{dt} I$$

$$= L \frac{d}{dt} I^2/2 \; ; \quad U_M = \int_{t_i}^{t_f} dt \, L \frac{d}{dt} I^2/2$$

Integrate

$$U_M(\text{final}) - U_M(\text{initial})$$

$$= L \frac{I_f^2}{2} - \underbrace{L \frac{I_i^2}{2}}_{=0} = L \frac{I_f^2}{2}$$

$f = \frac{1}{T}$ $\quad T = \frac{2\pi}{\omega}$; Root Mean Square Voltage (RMS Voltage)

$V = V_0 \cos \omega t$

$\langle V \rangle_T \equiv \left(\frac{1}{T} \int_0^T dt \, V^2 \right)^{1/2}$

$= \left(\int_0^T \frac{dt}{T} V_0^2 \cos^2 \omega t \right)^{1/2} = \frac{V_0}{\sqrt{2}}$

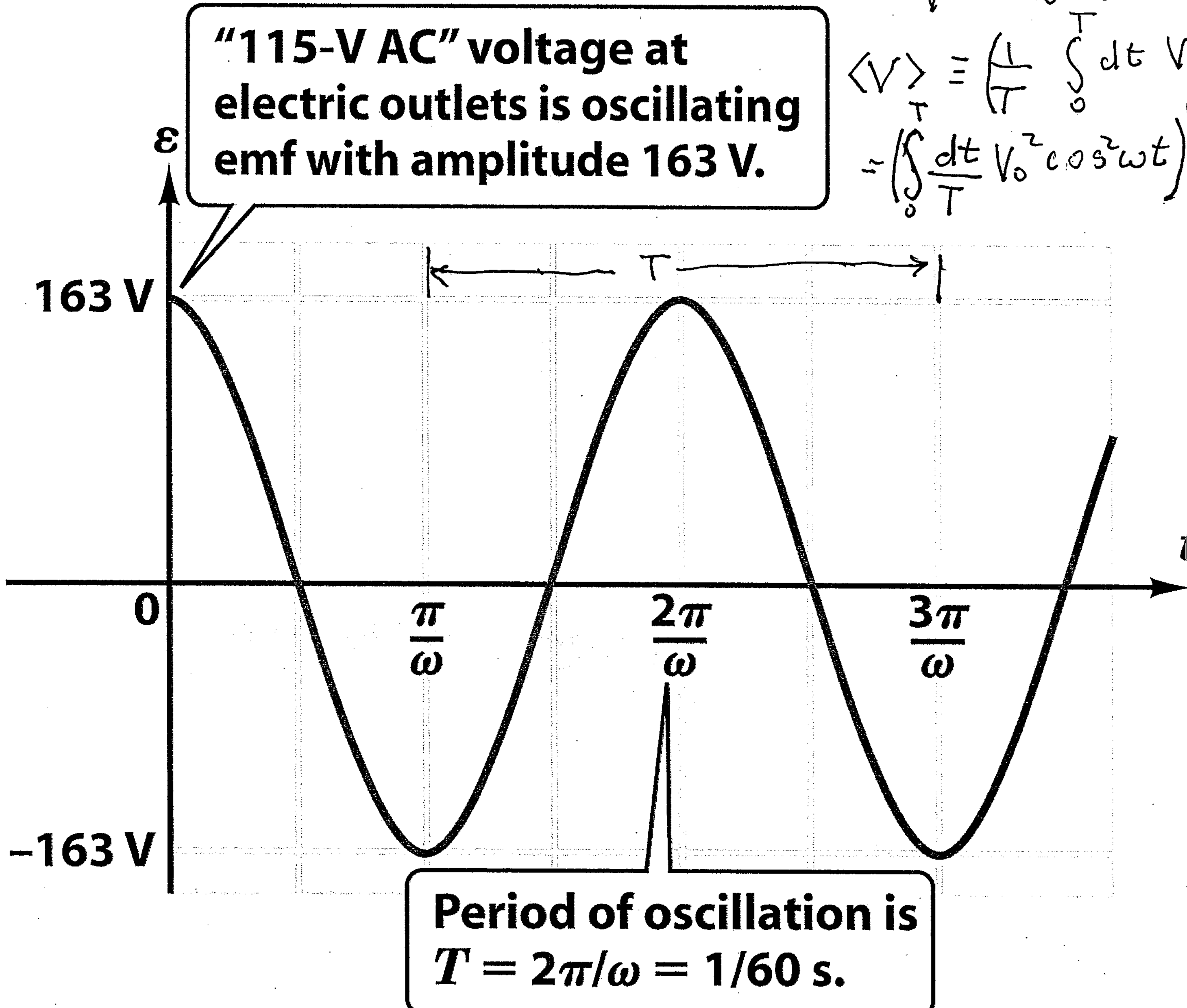


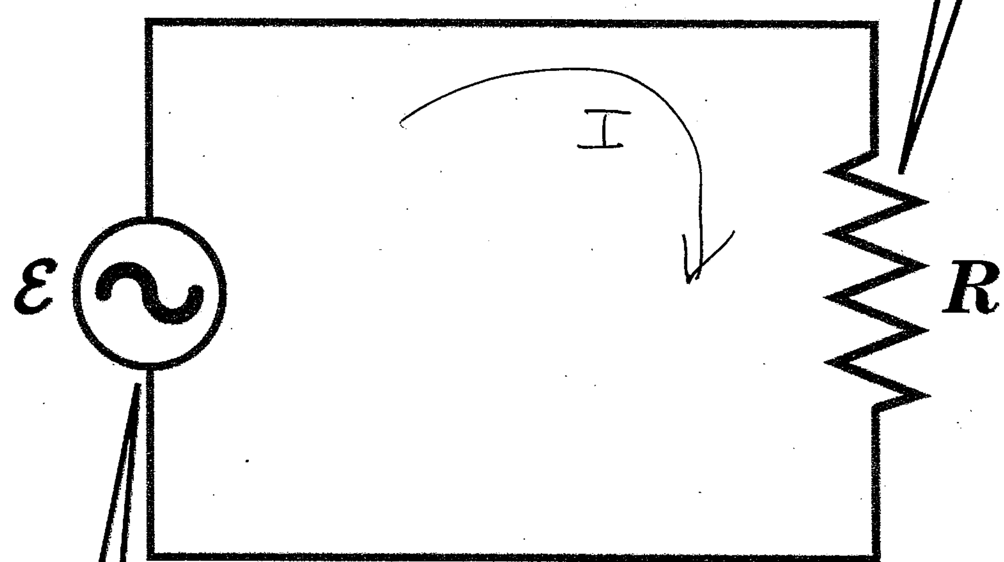
Figure 32-1 Physics for Engineers and Scientists 3/e

Khirchoff Voltage Rule still applicable

$-\mathcal{E}(t) + I(t)R = 0$

$\mathcal{E}(t) = \mathcal{E}_0 \cos\omega t$

$I(t) = \frac{\mathcal{E}_0}{R} \cos\omega t$



$\mathcal{E}(t) = \mathcal{E}_0 \cos\omega t$

$I(t) = \frac{\mathcal{E}_0}{R} \cos\omega t$

Figure 32-2 Physics for Engineers and Scientists 3/e

$0 = -\mathcal{E}(t) + IR$
$= -\mathcal{E}_0 \cos\omega t + IR$

$I = \frac{\mathcal{E}_0}{R} \cos \omega t$
$= \frac{\mathcal{E}_0}{R} \cos\left(\frac{2\pi}{T} t\right)$

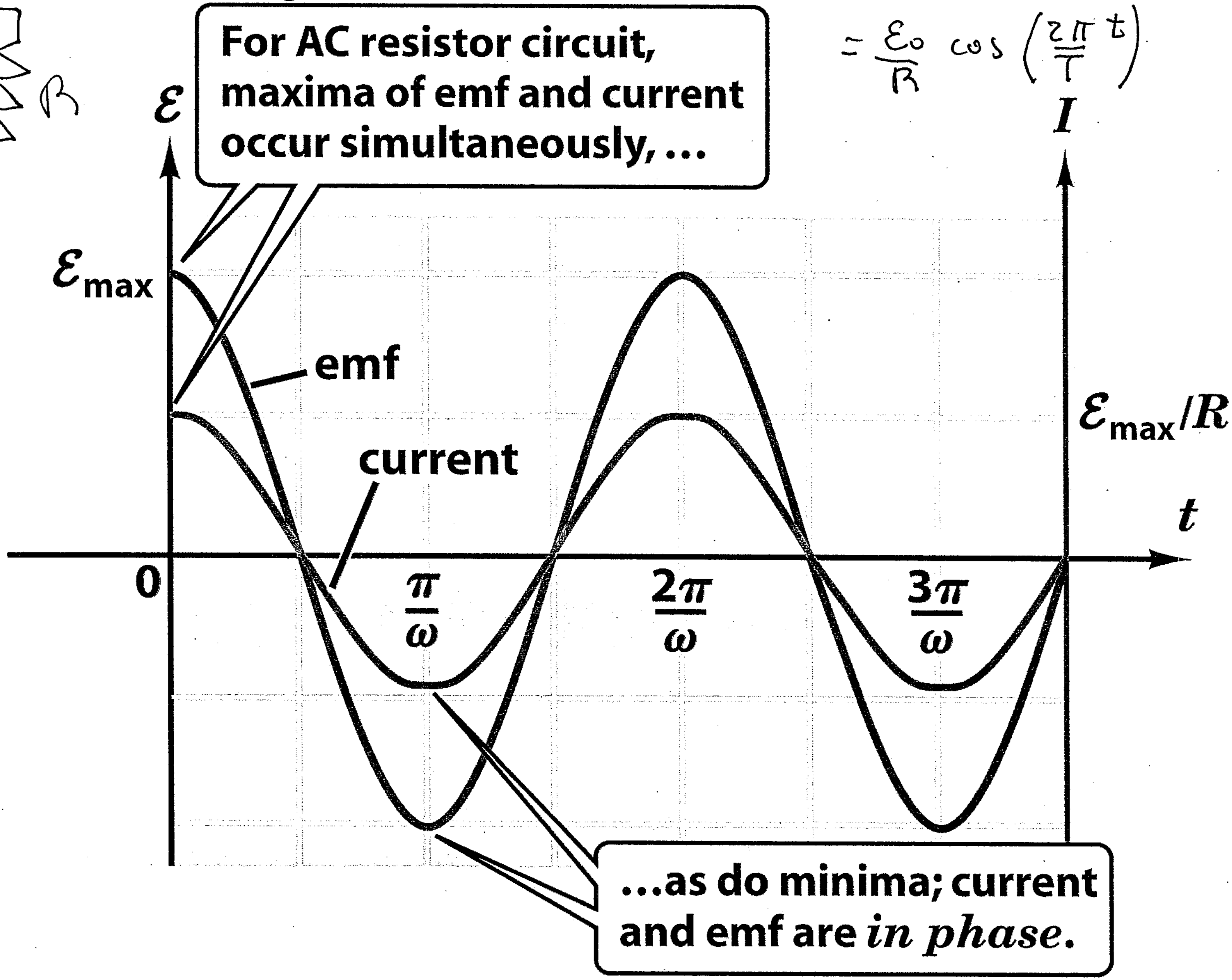
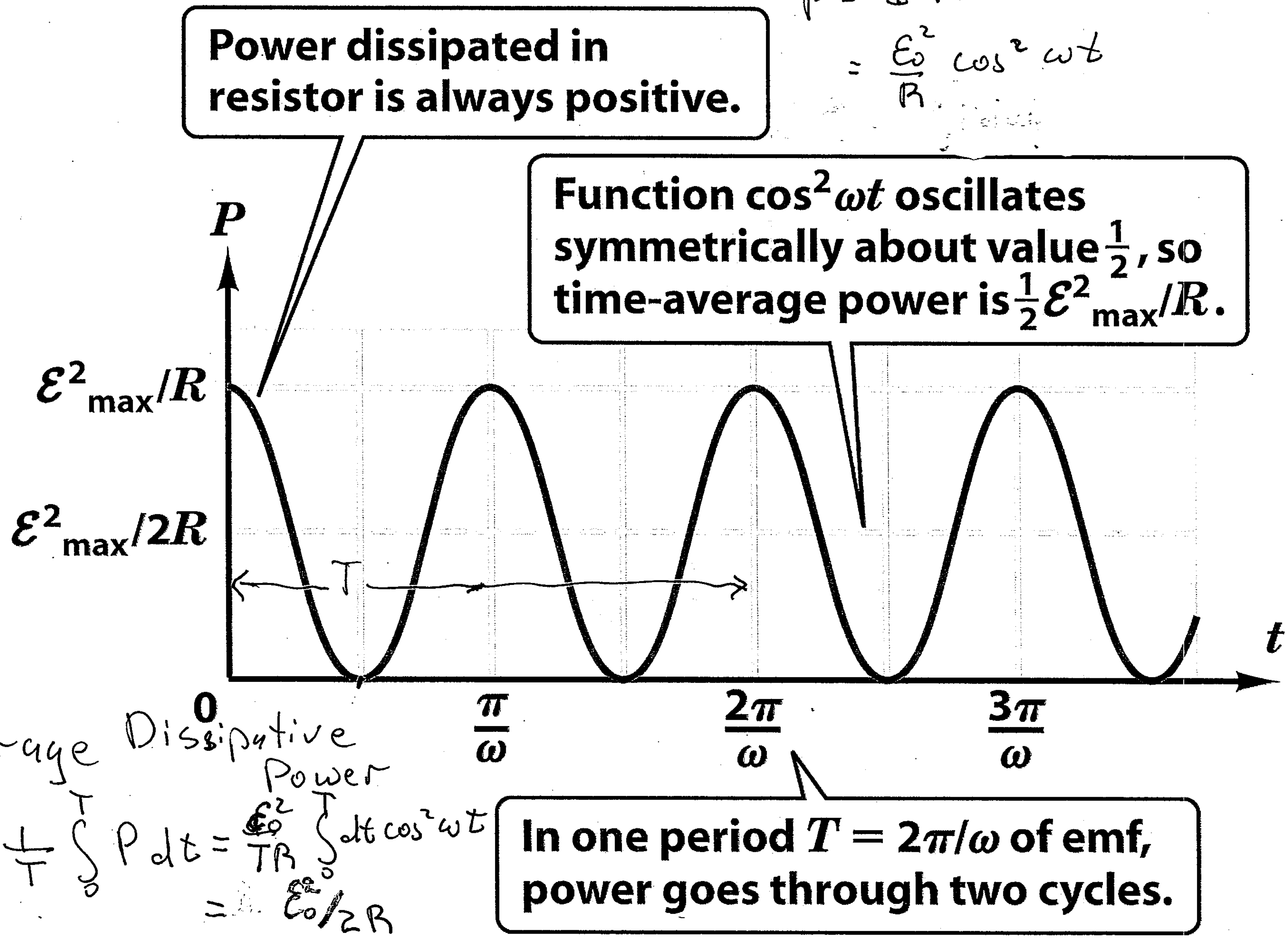


Figure 32-4 Physics for Engineers and Scientists 3/e

$I = \frac{\mathcal{E}_0}{R} \cos \omega t$

$P = I^2 R$

$= \frac{\mathcal{E}_0^2}{R} \cos^2 \omega t$



Average Dissipative Power

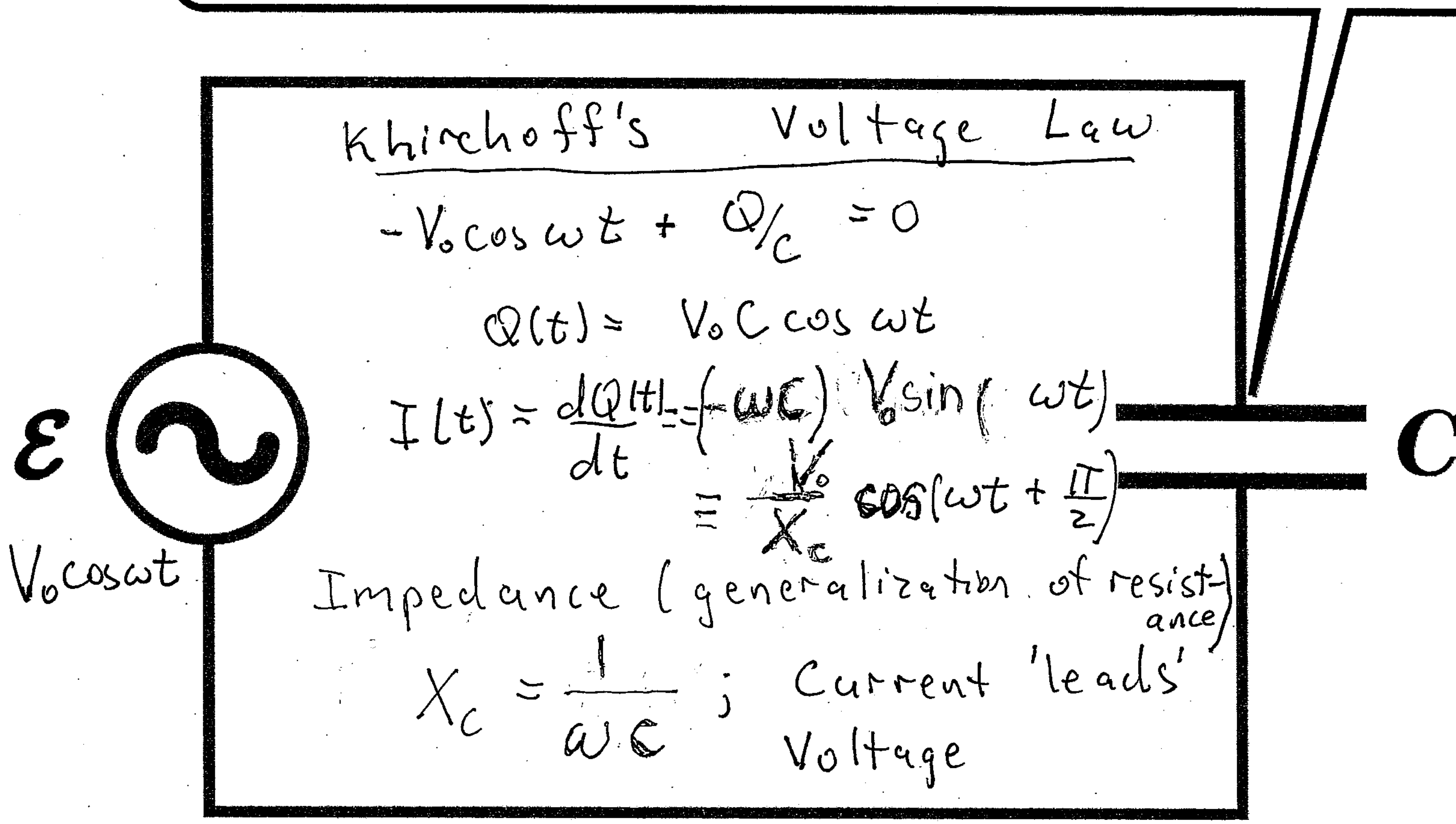
$\langle P \rangle = \frac{1}{T}\int_0^T P\,dt = \frac{\mathcal{E}_0^2}{TR}\int_0^T dt \cos^2 \omega t$

$= \mathcal{E}_0^2/2R$

Figure 32-5 Physics for Engineers and Scientists 3/e

$(\mathcal{E}_0 = \mathcal{E}_{max})$

AC capacitor circuit: oscillating emf produces oscillating charge on capacitor, $Q = C\mathcal{E}$.



$\mathcal{E} = V_0 \cos\omega t$

Khirchoff's Voltage Law

$-V_0 \cos\omega t + Q/C = 0$

$Q(t) = V_0 C \cos\omega t$

$I(t) = \frac{dQ(t)}{dt} = -(\omega C) V_0 \sin(\omega t)$

$= \frac{V_0}{X_c} \cos(\omega t + \frac{\pi}{2})$

Impedance (generalization of resistance)

$X_c = \frac{1}{\omega C}$; Current 'leads' Voltage

Figure 32-7 Physics for Engineers and Scientists 3/e

Current in a capacitor "leads" voltage

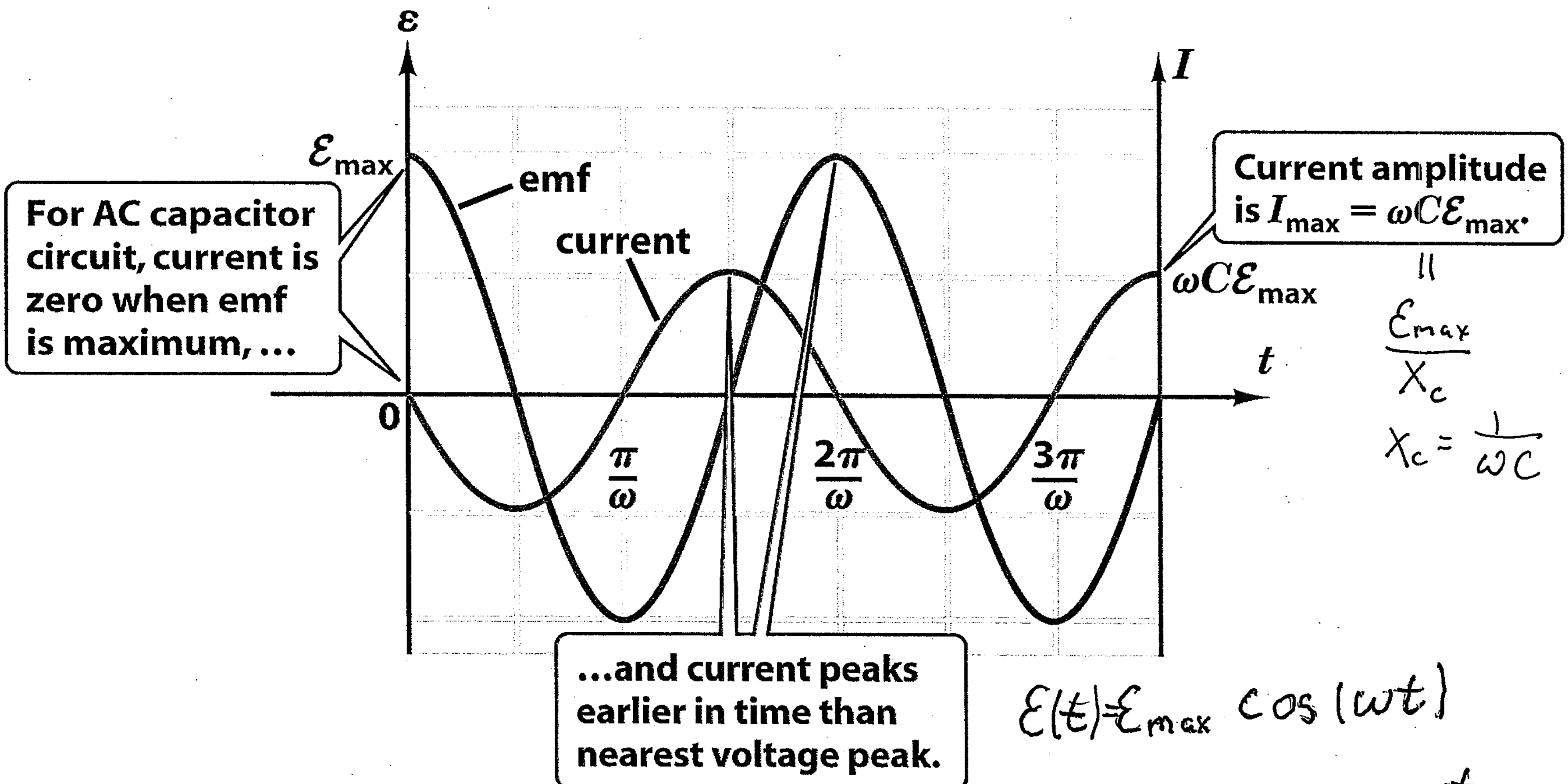


Figure 32-8 Physics for Engineers and Scientists 3/e

$$\omega C \mathcal{E}_{max} = \frac{\mathcal{E}_{max}}{X_c}$$

$$X_c = \frac{1}{\omega C}$$

$$\mathcal{E}(t) = \mathcal{E}_{max} \cos(\omega t)$$

$$I(t) = -\omega C \, \mathcal{E}_{max} \sin \omega t$$

$$= \frac{\mathcal{E}_{max}}{X_c} \cos\left(\omega t + \frac{\pi}{2}\right)$$

$$X_c = 1/\omega C$$

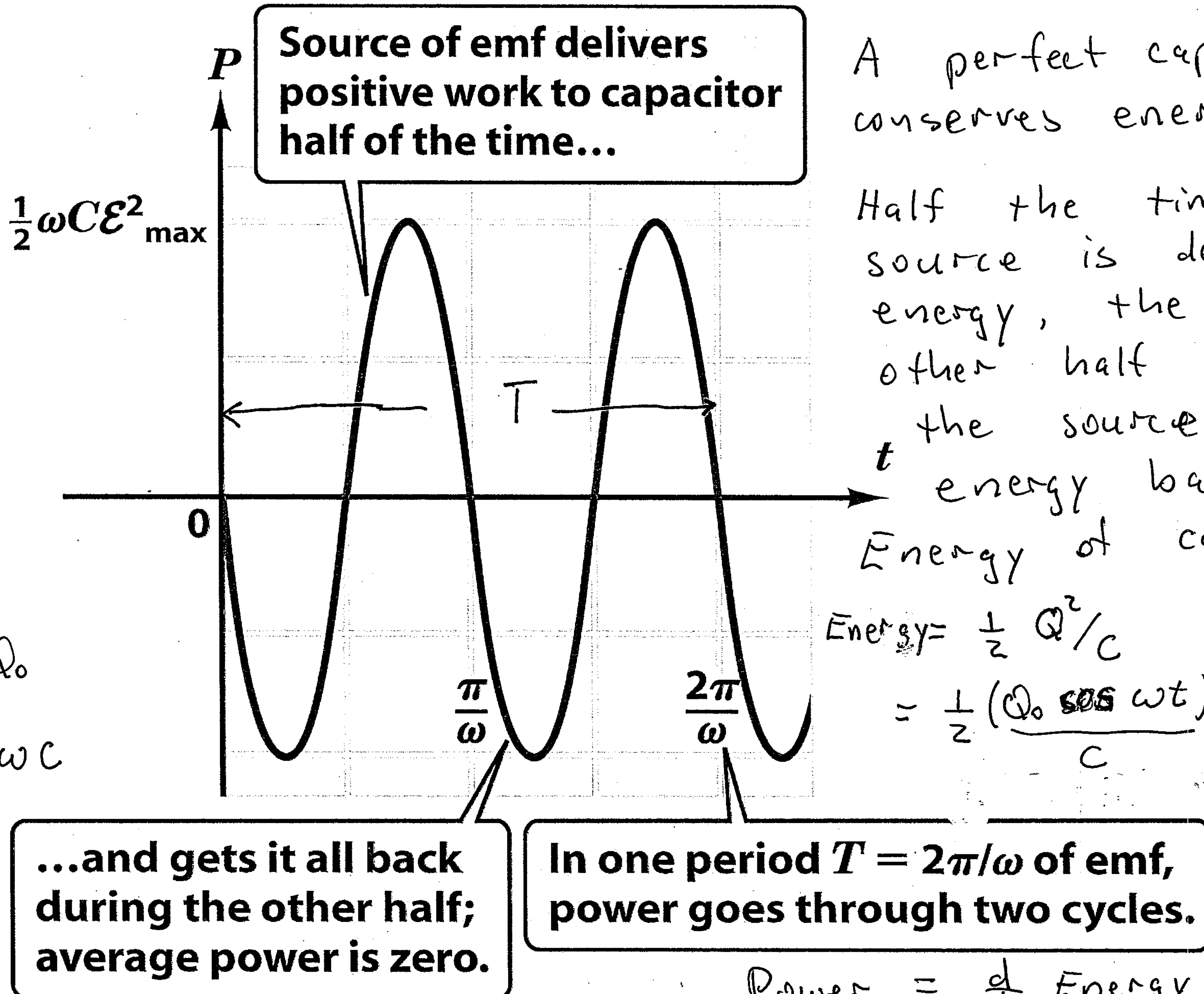


Figure 32-10 Physics for Engineers and Scientists 3/e

$\mathcal{E}_{max} = \frac{Q_0}{C}$

$I_0 = \omega Q_0$

$\mathcal{E}_{max} = I_0/\omega C$

A perfect capacitor conserves energy.

Half the time source is delivering energy, the other half time the source gets energy back.

Energy of capacitor

$$\text{Energy} = \frac{1}{2} Q^2/C$$

$$= \frac{1}{2}\frac{(Q_0 \cancel{\cos}\, \omega t)^2}{C}$$

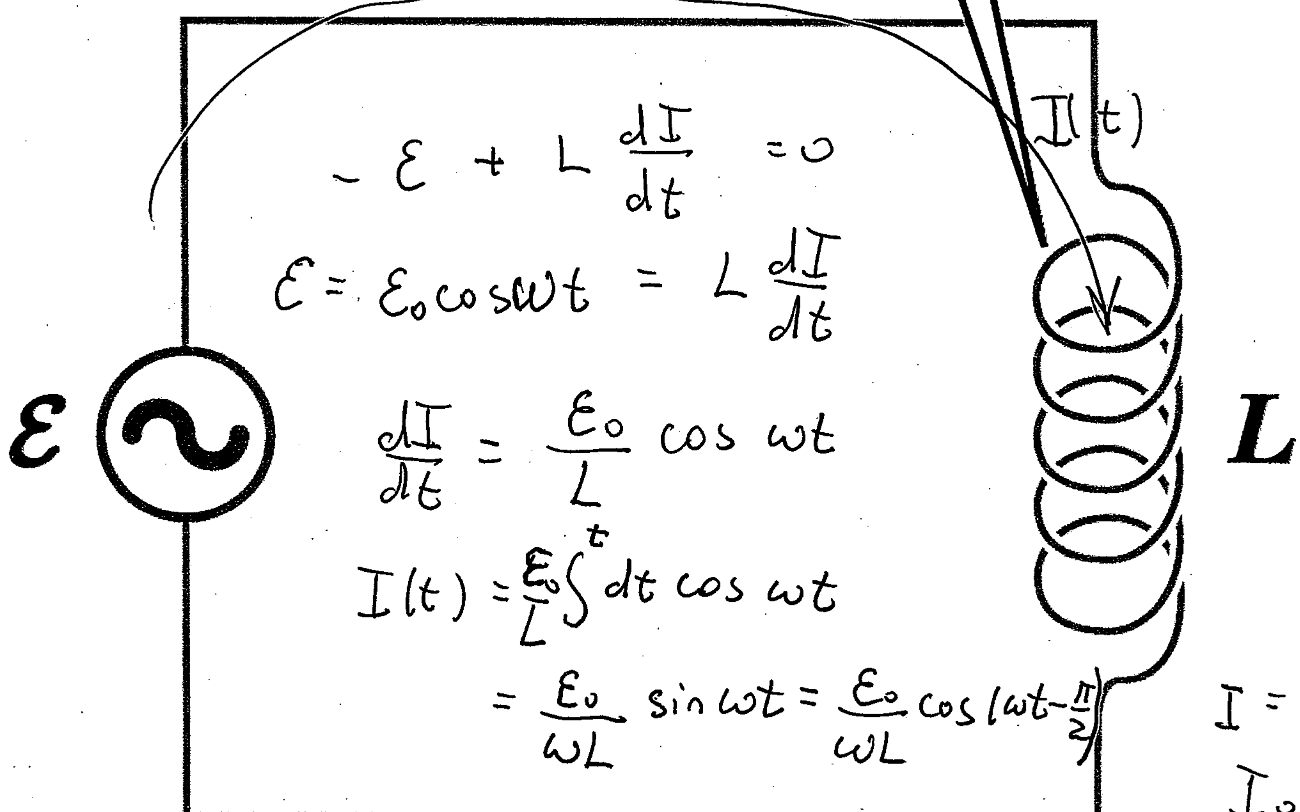
$$\text{Power} = \frac{d}{dt}\text{Energy}$$

$$= \frac{Q_0^2\omega}{C}\sin\omega t\cos\omega t = \frac{1}{2}\left(\frac{Q_0}{C}\right)^2\omega C\sin(2\omega t)$$

$$= \frac{1}{2}\omega C\,\mathcal{E}_{max}^2\sin(2\omega t)$$

Khirchoff's Law again

AC inductor circuit: oscillating emf produces oscillating current in inductor.



$$-\mathcal{E} + L\frac{dI}{dt} = 0$$

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t = L\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{\mathcal{E}_0}{L}\cos \omega t$$

$$I(t) = \frac{\mathcal{E}_0}{L}\int^t dt \cos \omega t$$

$$= \frac{\mathcal{E}_0}{\omega L}\sin \omega t = \frac{\mathcal{E}_0}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$I = I_0 \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$I_0 = \frac{\mathcal{E}_0}{\omega L} = \frac{\mathcal{E}_0}{X_L}$$

Figure 32-11 Physics for Engineers and Scientists 3/e

Current lags voltage

Inductive Impedance
$X_L = \omega L$

Current - Voltage in Inductor

$$\mathcal{E}(t) = \mathcal{E}_{max} \cos \omega t$$

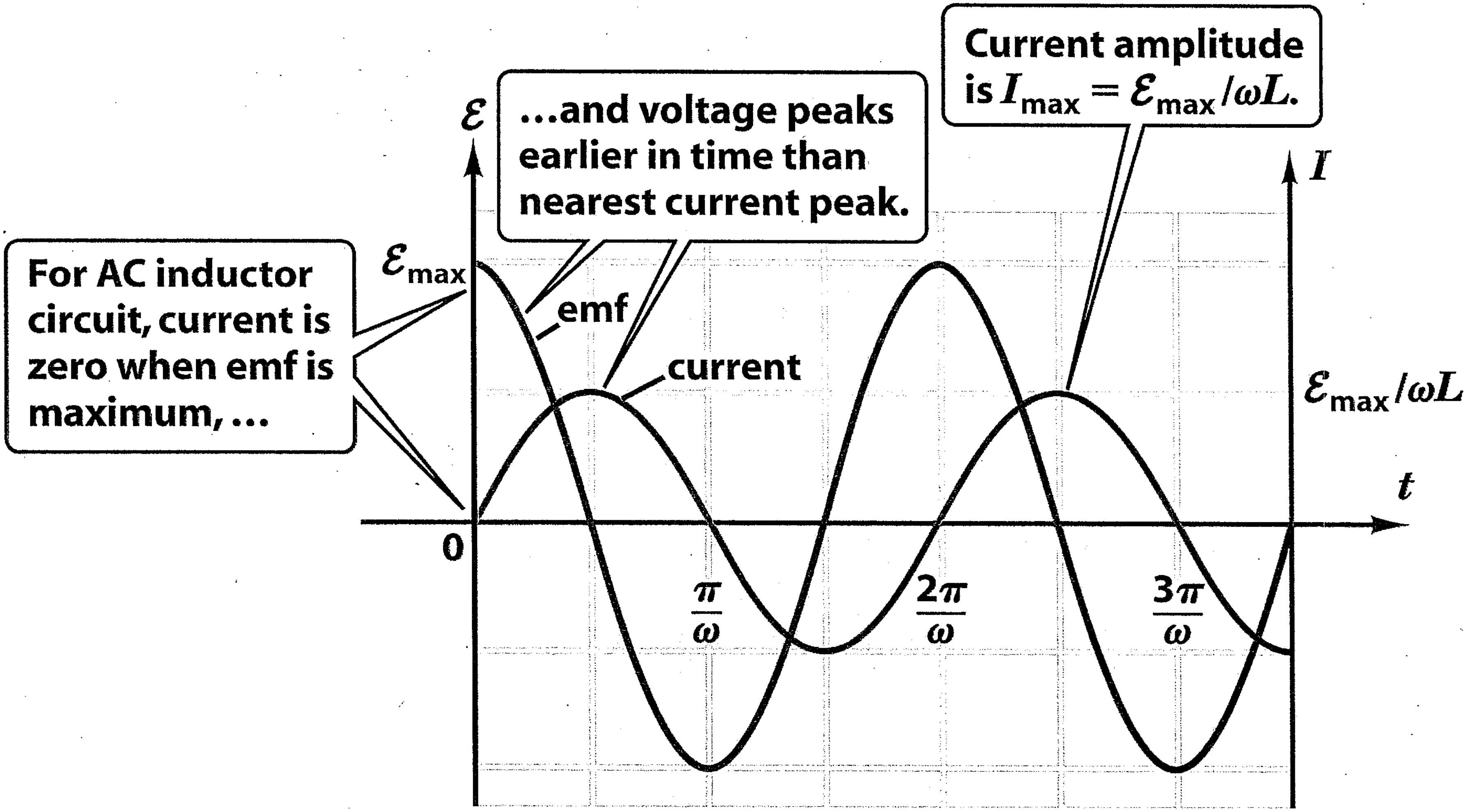


Figure 32-12 Physics for Engineers and Scientists 3/e

$$I(t) = \frac{\mathcal{E}_{max}}{\omega L} \cos\left(\omega t - \frac{\pi}{2}\right)$$

What is the current through each element?

$\mathcal{E} = \mathcal{E}_0 \cos \omega t$

$$I_R = \frac{\mathcal{E}_0}{R} \cos \omega t = \frac{\mathcal{E}_0}{X_R} \cos \omega t$$

$$I_C = \omega C \mathcal{E}_0 \cos(\omega t + \tfrac{\pi}{2})$$

$$= \frac{\mathcal{E}_0}{X_C} \cos(\omega t + \tfrac{\pi}{2})$$

$$I_L = \frac{\mathcal{E}_0}{\omega L} \cos(\omega t - \tfrac{\pi}{2})$$

$$= \frac{\mathcal{E}_0}{X_L} \cos(\omega t - \tfrac{\pi}{2})$$

$$X_R = R$$

$$X_C = \frac{1}{\omega C}$$

$$X_L = \omega L$$

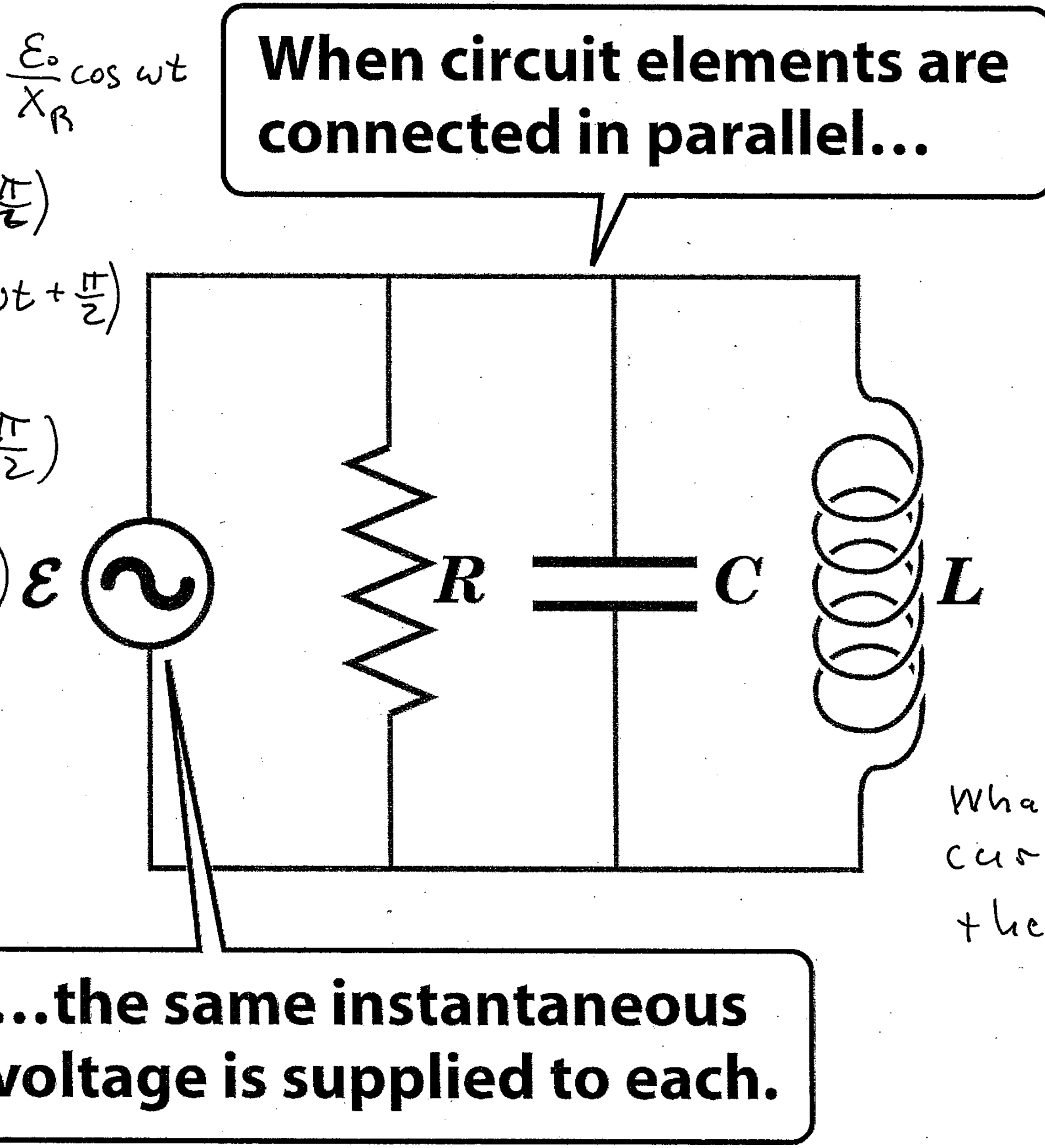
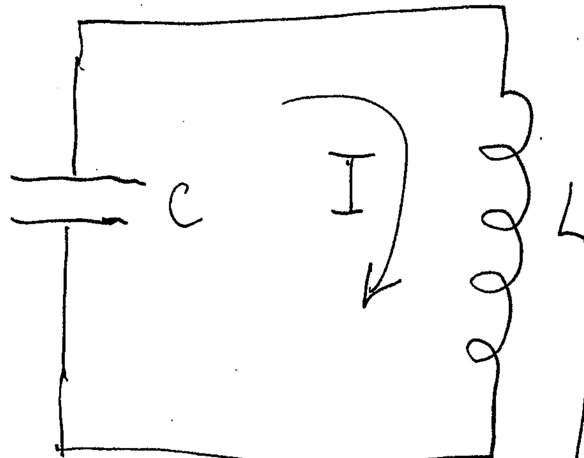


Figure 32-14 Physics for Engineers and Scientists 3/e

What is current through the battery?

LC circuit



$$\frac{q}{C} + L\frac{dI}{dt} = 0$$

$$I = \frac{dq}{dt}$$

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

If no resistance, such a circuit can have current for a long time, at a very special frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

This behaves much like a mass on a spring, where Inductor is akin to inertia, Capacitor is akin to inverse of spring constant $1/k$, and charge q akin to displacement x.

mass - spring

$$m \frac{d^2x}{dt^2} = -kx$$

$$m \frac{d^2x}{dt^2} + kx = 0$$

$$x = x_0 \sin(\omega t - \phi)$$

$$\frac{d^2x}{dt^2} = -\omega^2 x_0 \sin(\omega t - \phi)$$

$$= -\omega^2 x(t)$$

$\therefore$

$$(-\omega^2 m + k)\, x(t) = 0$$

$$\therefore\ -\omega^2 m + k = 0$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{k/m}$$

Energy oscillates between potential energy of fully stretched spring, to kinetic energy of unstretched spring

$$\boxed{KE = \qquad = m(\omega_0 x_0)^2 \cos^2(\omega t + \phi)}$$

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2 \sin^2(\omega t + \phi)$$

$$KE + PE = \frac{1}{2}kx_0^2 \text{ as } \omega_0^2 = \frac{k}{m}$$

LC - circuit

$$L \frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

$$q = q_0 \sin(\omega t - \phi)$$

$$\frac{d^2q}{dt} = -\omega^2 q_0 \sin(\omega t - \phi)$$

$$= -\omega^2 q(t)$$

$\therefore$

$$\left(-L\omega^2 + \frac{1}{C}\right) q(t) = 0$$

$$\omega^2 = \frac{1}{LC} \equiv \omega_0^2$$

$$\omega_0 = \frac{1}{(LC)^{1/2}}; \quad q = q_0 \sin(\omega t - \phi), \quad i = \frac{dq}{dt} = \omega_0 q_0 \cos(\omega t - \phi)$$

Energy oscillates between electric energy stored in the capacitor when there's no current in inductor $\boxed{U_E = \frac{q_0^2}{2C} \sin^2(\omega t - \phi)}$ to a magnetic energy

$$U_M = \frac{1}{2} L i^2 = \frac{q_0^2 \omega_0^2 L \cos^2(\omega t - \phi)}{2C}$$

in inductor $\boxed{U_M = q_0^2 \cos^2(\omega t - \phi)/2C}$

$$U_E + U_M = q_0^2/2C = i_0^2 L/2$$

(since $i_0 = \omega_0 q_0 = q_0/\sqrt{LC}$)