

Lecture # 17

Inductance

# Motional EMF and Stationary Loop EMF

combine together in one compact Law

Faraday's Law 
$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_m}{dt}$$

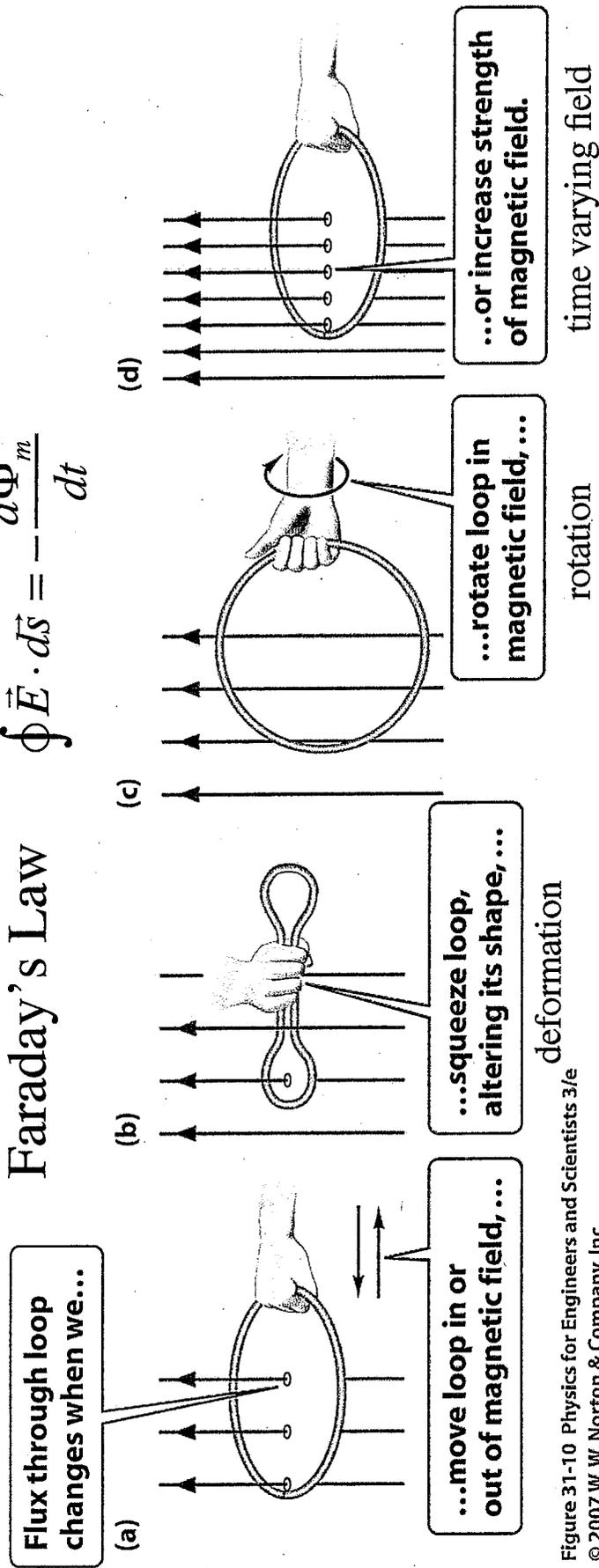


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Translation through non-uniform field

This Law is true whether the loop moves

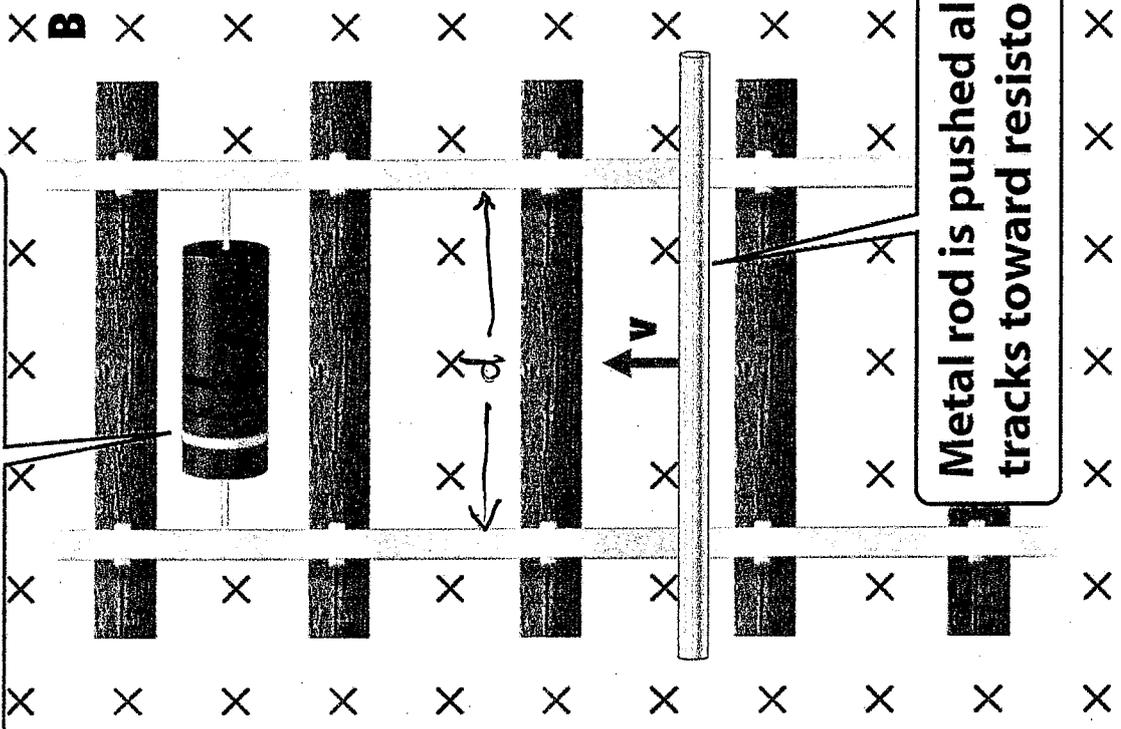
through a non-uniform magnetic field, deforms, rotates, and B-field changes in time.

Does the current flow is the clockwise or anti-clockwise direction?

(1) clockwise

(2) counter-clockwise

Does current flow in resistor? If so, which way?

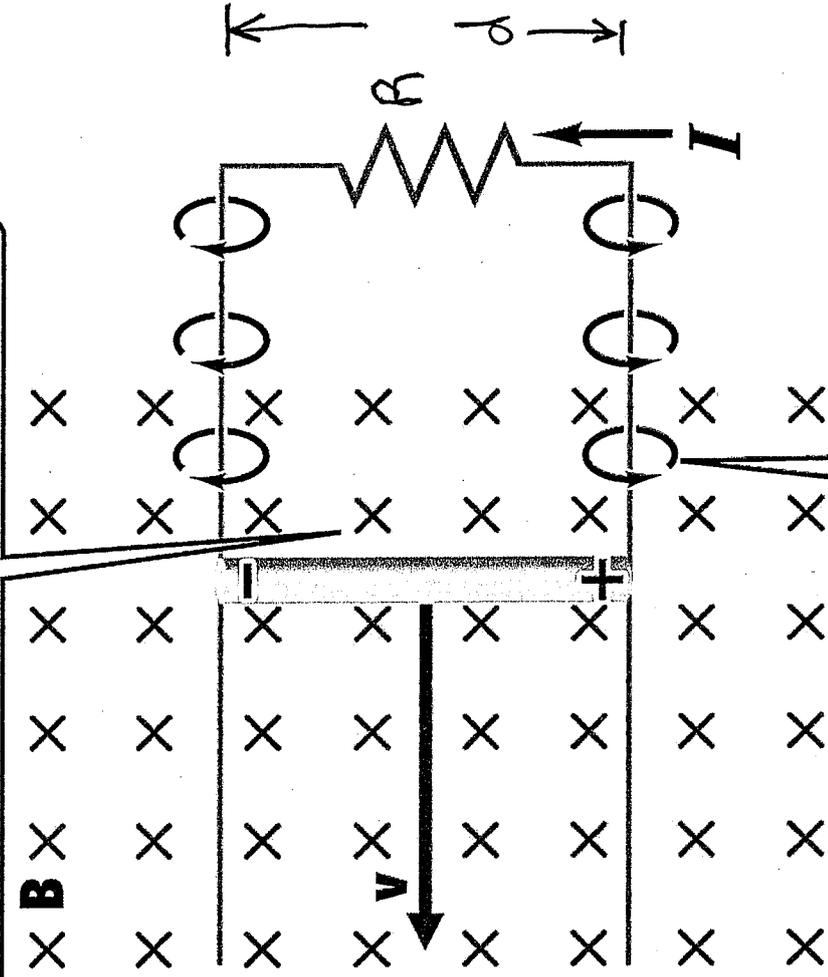


- a. Left to right
- b. Right to left
- c. Does not flow

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**When rod slides to left, area enclosed by circuit increases...**

Let us use energy arguments to find the force we need to move the rod at velocity  $v$ .



**...and to oppose increase of flux, magnetic field of induced current must be opposite to original magnetic field.**

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**Generator's alternating emf oscillates sinusoidally between positive and negative values.**

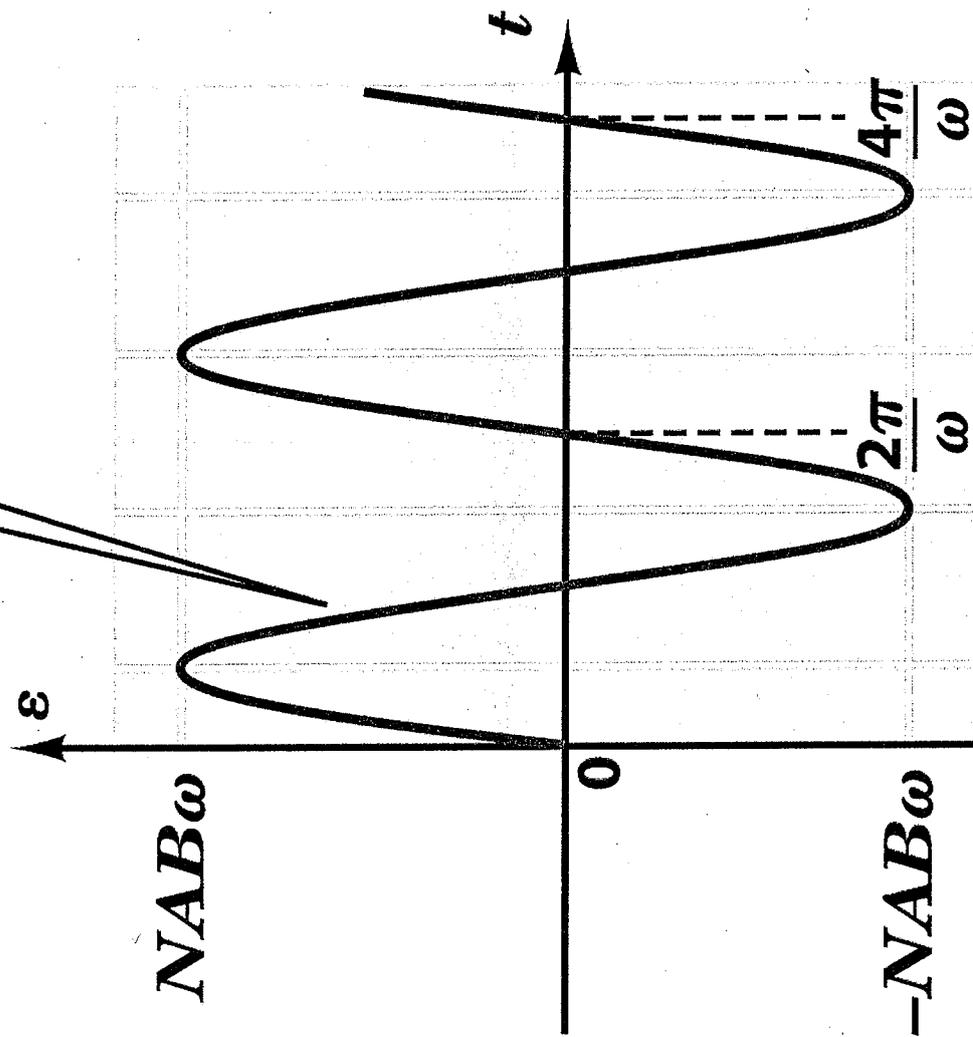


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Dynamo is used to generate electrical currents from mechanical input power

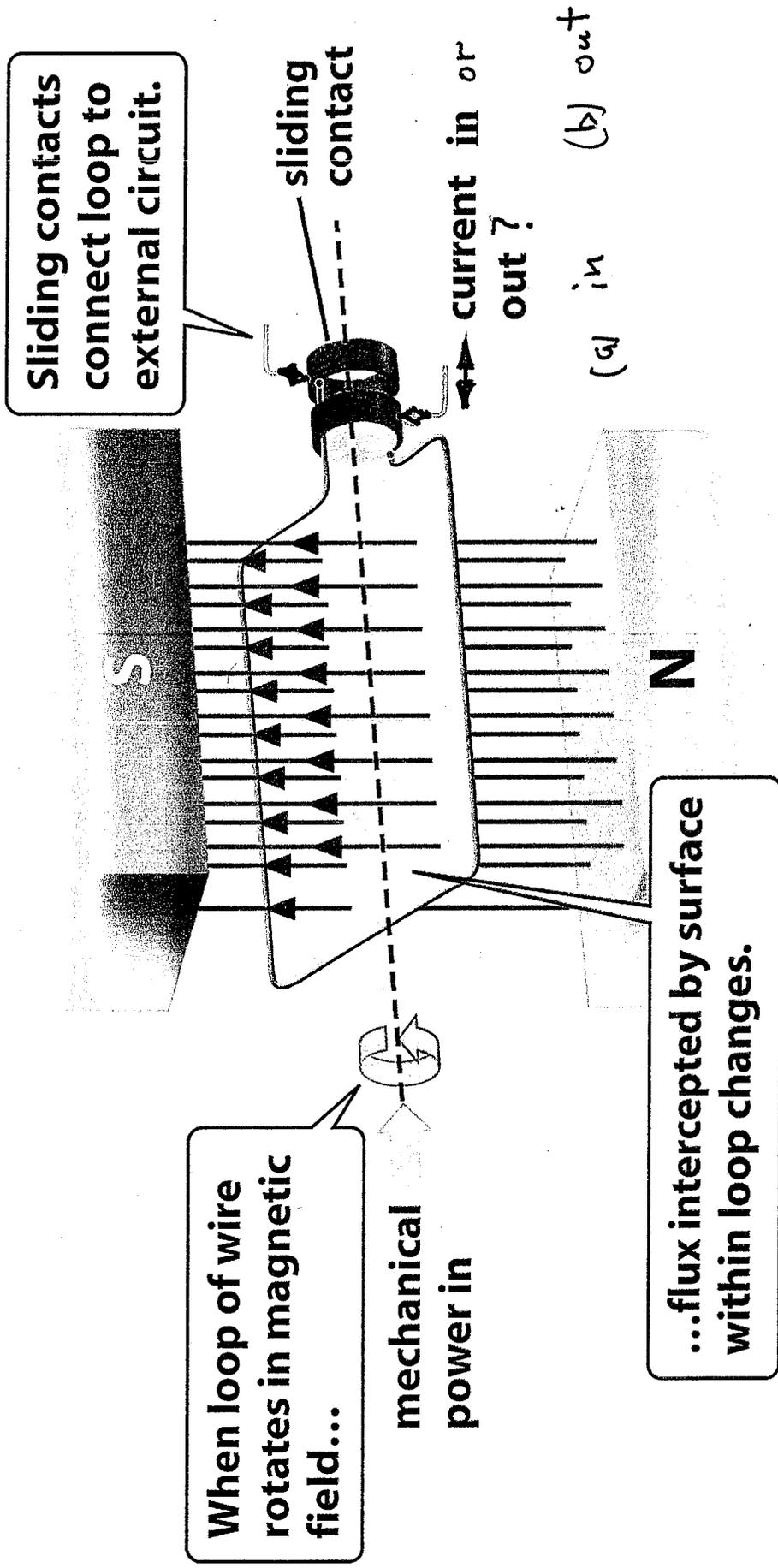


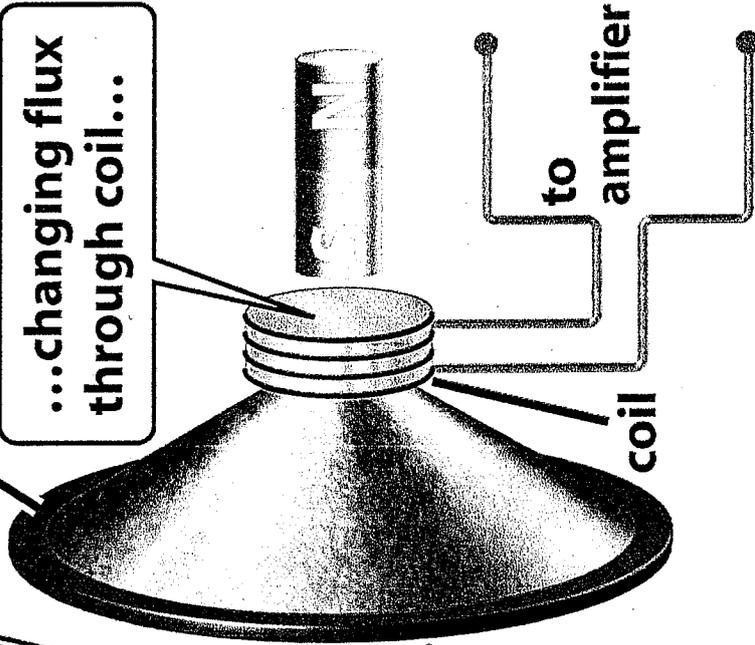
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Induced electromagnetic torque is opposite applied torque (Lenz's law).

Sound wave moves diaphragm and coil back and forth, ...

diaphragm

...changing flux through coil...



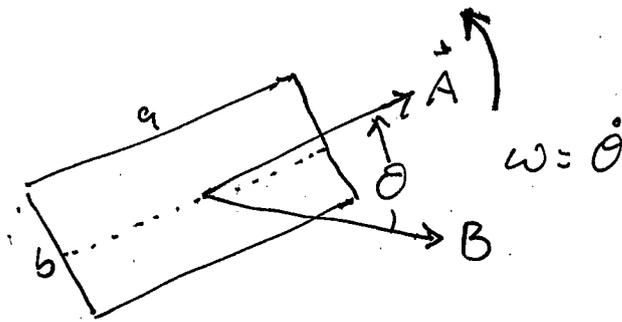
coil

to amplifier

...and inducing an emf.

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## EMF of Rotating Loop



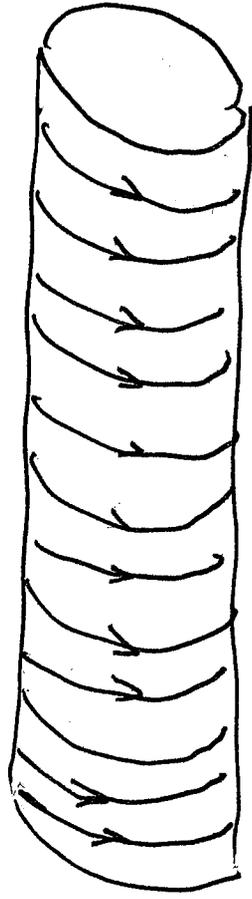
$$\theta = \omega t$$

$$\Phi_m = BA \cos \theta = Bab \cos \omega t$$

$$EMF = - \frac{\partial \Phi_m}{\partial t} = Bab \omega \sin \omega t$$

This EMF is used in  
a dynamo to generate  
electric currents

We have a long solenoid, where current varies as  $I(t) = I_0 t / \tau$ , and there are  $n$  coils per meter



(a) What is magnetic field inside solenoid

$$B = \mu_0 n I(t) = \mu_0 n I_0 t / \tau, \quad B \approx 0 \text{ outside}$$

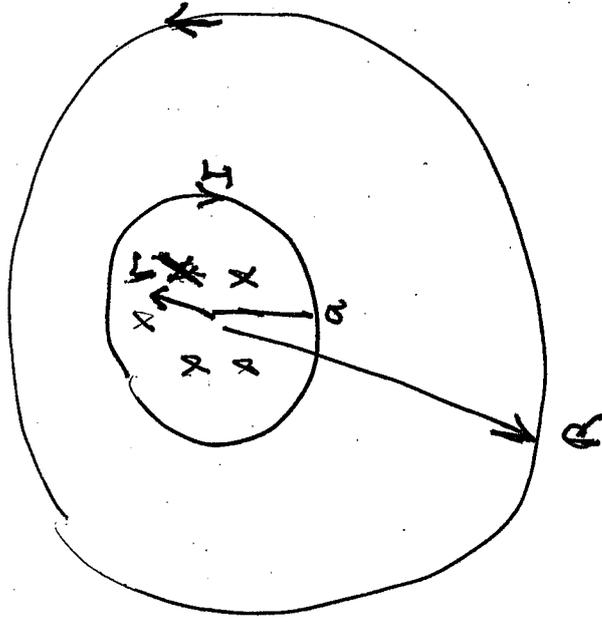
(b) Is there electric field outside solenoid?  
(a) yes (b) no

(c) Find electric field inside and outside solenoid

# E-field of a solenoid

$$B = B_0 t/T$$

$$I = I_0 t/T, \quad B_0 = \mu I_0 n$$



outside

$$\text{Emf} = \oint \vec{E} \cdot d\vec{l} = -\dot{\Phi}_m = -\left(\frac{B_0 t \pi R^2}{T}\right)$$

$$\vec{E} 2\pi R = -\frac{B_0 \pi R^2}{T}$$

$$\vec{E} = -B_0 \frac{R}{2T}$$

inside

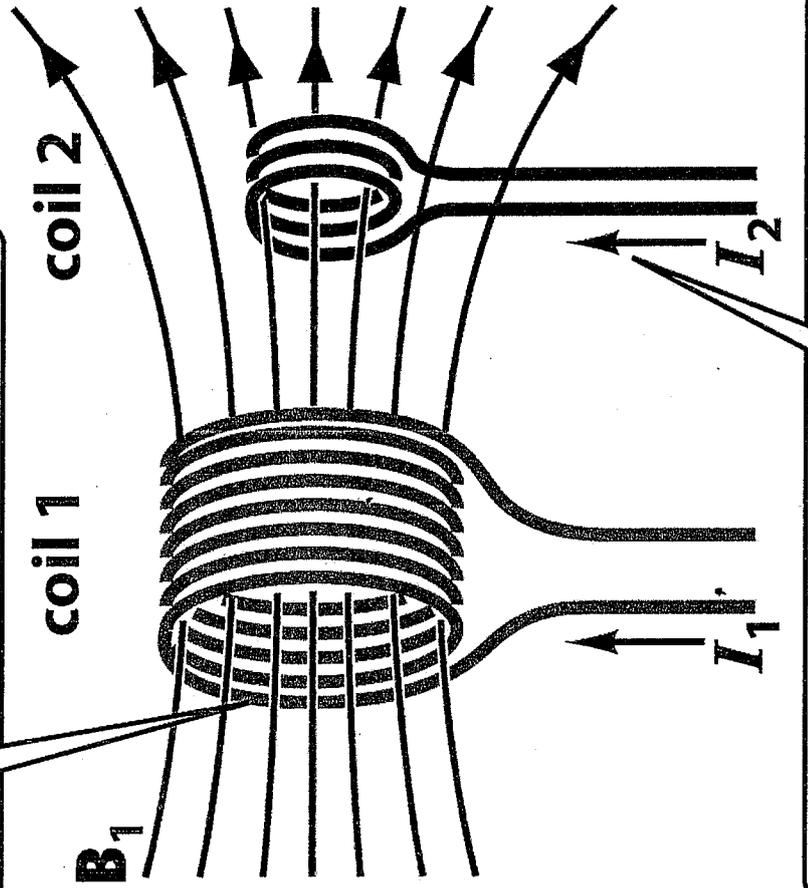
$$\text{Emf} = \oint \vec{E} \cdot d\vec{l} = -\dot{\Phi}_m$$

$$2\pi r E = -\left(\frac{B_0 t}{T}\right) \pi r^2$$

$$\vec{E} = -\frac{B_0 r}{2T}$$

# Mutual induction

Time-dependent current in one coil produces a changing magnetic field...



...and changing magnetic flux induces current in second coil.

Mutual inductance

$$\Phi_{I_2}$$

$\propto$

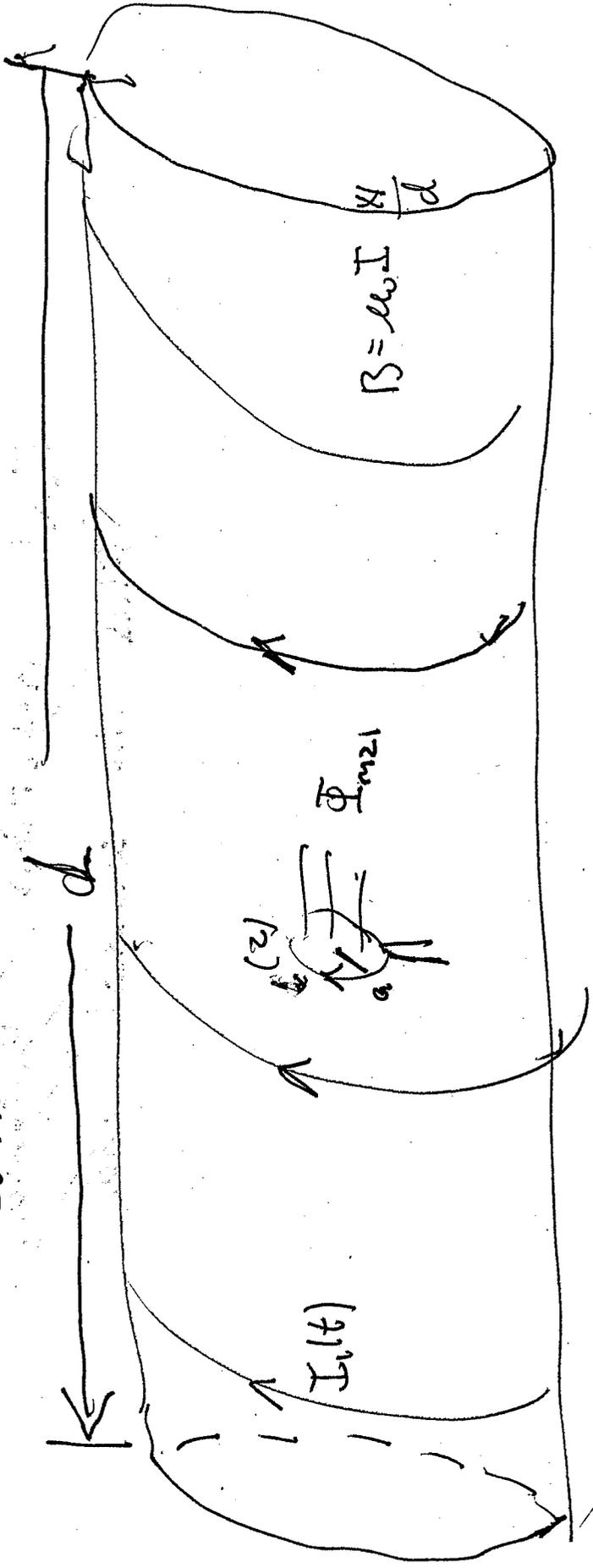
$$\Phi_{m21} = M_{21} I_1$$

$$\Phi_{m12} = M_{12} I_2$$

$$M_{12} = M_{21}$$

$$M_{12} = M_{21}$$

Consider a coil in



Magnetic flux from  $I_1(t)$  through coil (2) is (solenoid has  $N_1$  turns)

$$\Phi_{m21} = B_1 \pi a^2 = \mu_0 \frac{N_1 I_1(t)}{d} \pi a^2 \equiv M_{21} I_1(t)$$

$$M_{21} \equiv \text{mutual inductance} \equiv \mu_0 \frac{N_1 \pi a^2}{d} \equiv M_{21}$$

Theorem: Mutual inductance,  $M_{12}$ , from flux  $\Phi_{m12}$  (due to current, through solenoid) is given by  $M_{12} = M_{21}$

In general mutual and self-inductance difficult to compute: But we can buy off the shelf, ~~many~~ coils that have specified self-inductance

and circuits, where components have a specified mutual inductance; and we need to understand:

$L_1 I_1 \equiv$  B-flux through coil one due to its own current

$M_{12} I_2 \equiv$  B-flux through coil ~~two~~<sup>one</sup> due to current ~~in~~<sup>two</sup>

$M_{21} I_1 (= M_{12} I_1) \equiv$  B-flux in coil ~~one~~<sup>two</sup> due to current in coil ~~two~~<sup>one</sup>

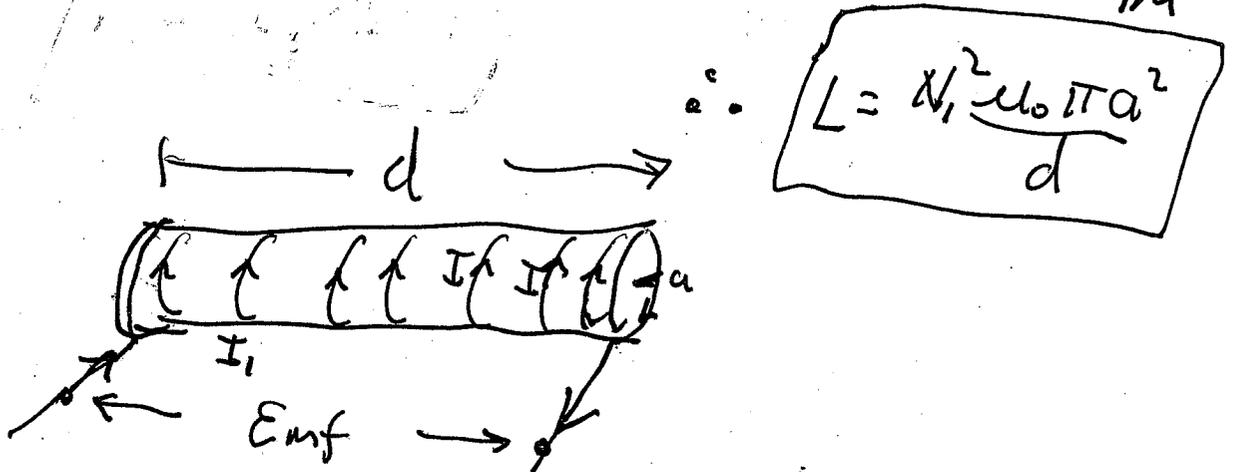
$M_{12} \dot{I}_2 = V_{12} \equiv$  voltage drop across coil one due to changing current in coil two

$L_1 \dot{I}_1 = V_1 \equiv$  voltage drop ~~across~~ across coil one due to change in ~~current~~<sup>current</sup>

self inductance of  
solenoid of  $N_1$  coils

$$N_1 \Phi_M \equiv L I_1$$

$$= N_1 B \pi a^2 = N_1 I_1 \mu_0 \pi a^2 N_1 / d$$



Emf across terminals

$$= - \dot{\Phi}_M = -L \frac{dI_1}{dt}$$

In general, self inductance  $\propto N_1^2$   
 " " , mutual inductance  $\propto N_1 N_2$

Because of inductance current cannot change instantaneously, but it takes a characteristic time,  $\tau_{L/R} = L/R$ .

Use Kirchoff's Law again

$$-\mathcal{E} + IR + L \frac{dI}{dt} = 0$$

$$L \frac{dI}{dt} + IR = \mathcal{E}$$

Solution

$$\exp\left(-t \frac{R}{L}\right)$$

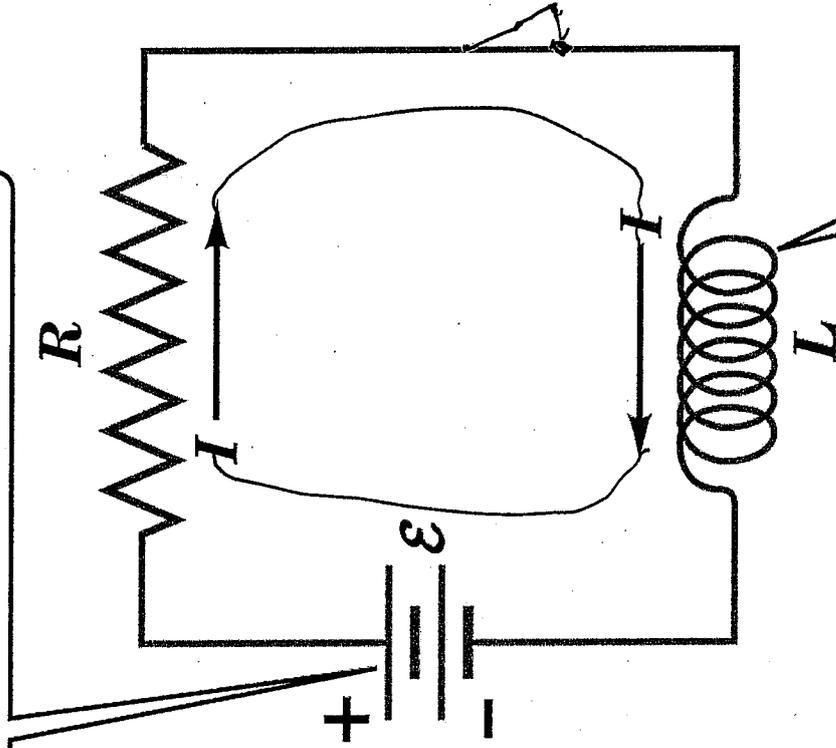
$$I(t) = \frac{\mathcal{E}}{R} + C \exp(-t / \tau_{L/R})$$

at  $t = 0$ ,  $I(t=0) = 0$ , thus

$$I(0) = \frac{\mathcal{E}}{R} + C = 0, \quad \therefore C = -\frac{\mathcal{E}}{R}$$

$$I(t) = \frac{\mathcal{E}}{R} (1 - \exp(-t / \tau_{L/R}))$$

When battery is connected, current in circuit will gradually start to increase.



Circuit symbol for inductor is a coiled line.

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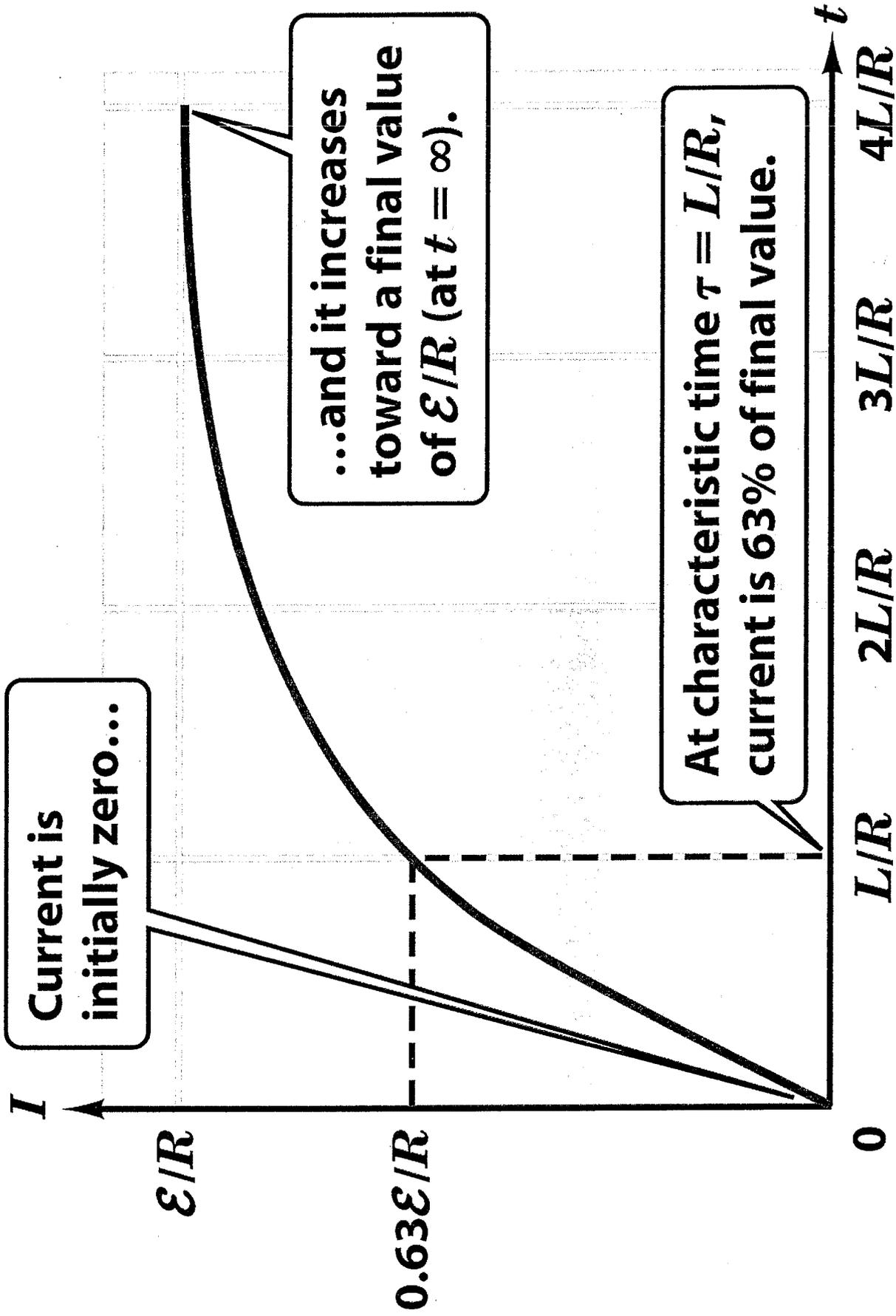


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