Lecture # 17

Inductance
Motional EMF and Stationary Loop EMF combine together in one compact Law

Faraday’s Law \[ \int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \]

This Law is true whether the loop moves through a non-uniform magnetic field, deforms, rotates, and B-field changes in time.

Does the current flow is the clockwise or anti-clockwise direction?

(1) clockwise

(2) counter-clockwise
Does current flow in resistor? If so, which way?

Metal rod is pushed along tracks toward resistor.

a. Left to right
b. Right to left
c. Does not flow
When rod slides to left, area enclosed by circuit increases...

Let us use energy arguments to find the force we need to move the rod at velocity $v$.

...and to oppose increase of flux, magnetic field of induced current must be opposite to original magnetic field.

Figure 31-14 Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.
Generator's alternating emf oscillates sinusoidally between positive and negative values.
Dynamo is used to generate electrical currents from mechanical input power.

When loop of wire rotates in magnetic field...

...flux intercepted by surface within loop changes.

Sliding contacts connect loop to external circuit.

Mechanical power in

Induced electromagnetic torque is opposite applied torque (Lenz's law).
Sound wave moves diaphragm and coil back and forth,...

...changing flux through coil...

diaphragm

to amplifier

...and inducing an emf.
EMF of Rotating Loop

\[ \Theta = \omega t \]

\[ \Phi_m = B A \cos \Theta = B a b \cos \omega t \]

\[ \text{EMF} = -\frac{\Phi_m}{\partial t} = B a b \omega \sin \omega t \]

This EMF is used in a dynamo to generate electric currents.
We have a long solenoid, where current varies as \( I(t) = \frac{I_0 t}{T} \), and there are \( N \) coils per meter.

(a) What is magnetic field inside solenoid

\[ B = \mu_0 n I(t) = \mu_0 n I_0 \frac{t}{T} \]

\( B = 0 \) outside

(b) Is there electric field outside solenoid?

(a) yes

(b) no

(c) Find electric field inside and outside solenoid
$E$-field of a solenoid

$B = \frac{B_0 t}{T}$
$\mathbf{I} = \frac{I_0 t}{T}$, $B_0 = \mu I_0 n$

outside

$\mathbf{emf} = \oint \mathbf{E} \cdot d\mathbf{l} = -\dot{\Phi}_m = -\left(\frac{B_0 t \pi R^2}{T}\right)$

$L = 2\pi a R = -\frac{B_0 a R^2}{T}$

inside

$\mathbf{emf} = \oint \mathbf{E} \cdot d\mathbf{l} = -\dot{\Phi}_m$

$2\pi r E = -\dot{(B_0 t)} \frac{R^2}{T}$

$\mathbf{B} = -\frac{B_0}{2T} r$
Mutual induction

Time-dependent current in one coil produces a changing magnetic field...

\[ \Phi_{m_{21}} = M_{21} I_1 \]

\[ \Phi_{m_{12}} = M_{12} I_2 \]

\[ M_{12} = M_{21} \]

...and changing magnetic flux induces current in second coil.
Consider a coil in

Magnetic flux from $I(t)$ through coil (2) is (solenoid has $N_2$ turns)

$$\Phi_{m21} = B_0 \pi a^2 \approx \mu_0 N_2 I(t) \frac{\pi a^2}{d} \equiv M_{21} I(t)$$

$M_{21} = \text{mutual inductance} \equiv \frac{\mu_0 N_2 I(t)}{d}$ due to $M_{21}$

Theorem: Mutual inductance, $M_{21}$, from $\Phi$ and $I(2)$ current, though solenoid is given by $M_{21} = M_{21}$
In general mutual and self-inductance difficult to compute. But we can buy off the shelf, ready coils that have specified self-inductance and circuits, where components have a specified mutual inductance; and we need to understand:

\[ L_i I_i = B\text{-flux through coil one due to its own current} \]

\[ M_{12} I_2 = B\text{-flux through coil one due to current two} \]

\[ M_{21} I_1 (= M_{12} I_1) = B\text{-flux in coil two due to current in coil one} \]

\[ M_{12} I_2 = V_2 \equiv \text{voltage drop across coil one due to changing current in coil two} \]

\[ L_i I_i = V_i \equiv \text{voltage drop across coil one due to changing rate} \]
self inductance of solenoid of $N_1$ coils

$$N_1 \Phi_m = L I,$$

$$= N_1 B \pi a^2 = N_1 I_0 \mu_0 \pi a^2 \frac{N_1}{d}$$

$$\therefore L = \frac{N_1^2 \mu_0 \pi a^2}{d}$$

Emf across terminal

$$- \dot{\Phi}_m = - L \frac{dI_1}{dt}$$

In general, self inductance $\propto N_1^2$

mutual inductance $\propto N_1 N_2$
Because of inductance current cannot change instantaneously, but it takes a characteristic time, $\tau_{L/R} = L/R$.

Use Khirchoff's Law again

$$-\mathcal{E} + IR + L \frac{dI}{dt} = 0$$

$$L \frac{dI}{dt} + IR = \mathcal{E}$$

Solution

$$I(t) = \frac{\mathcal{E}}{R} + C \exp\left(-t \frac{R}{L}\right)$$

at $t = 0$, $I(t = 0) = 0$, thus

$$I(0) = \frac{\mathcal{E}}{R} + C = 0, \quad \therefore \quad C = -\frac{\mathcal{E}}{R}$$

$$I(t) = \frac{\mathcal{E}}{R} \left(1 - \exp(-t / \tau_{L/R})\right)$$

Circuit symbol for inductor is a coiled line.
...and it increases toward a final value of $\varepsilon/R$ (at $t = \infty$).

At characteristic time $\tau = L/R$, current is 63% of final value.

Current is initially zero...