Lecture # 17

Inductance

Motional EMF and Stationary Loop EMF combine together in one compact Law

Faraday's Law $$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$$

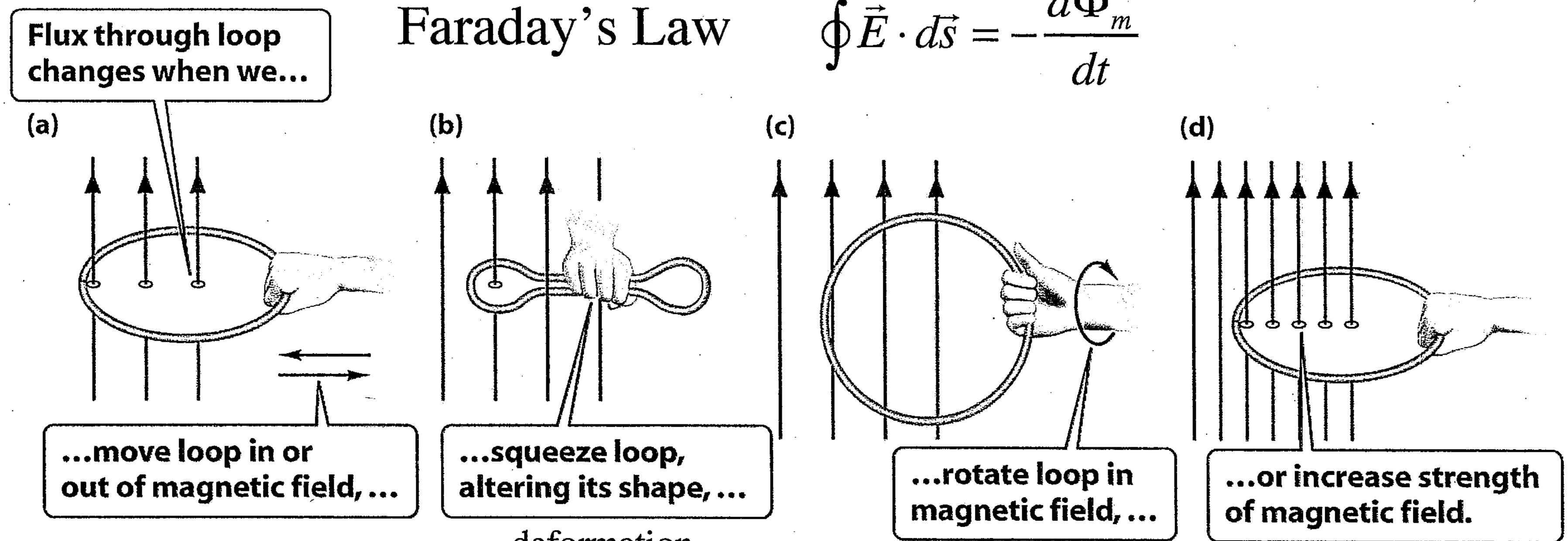

Figure 31-10 Physics for Engineers and Scientists 3/e

Translation through non-uniform field — deformation — rotation — time varying field

This Law is true whether the loop moves through a non-uniform magnetic field, deforms, rotates, and B-field changes in time.

Does the current flow is the clockwise or anti-clockwise direction?

(1) clockwise (2) counter-clockwise

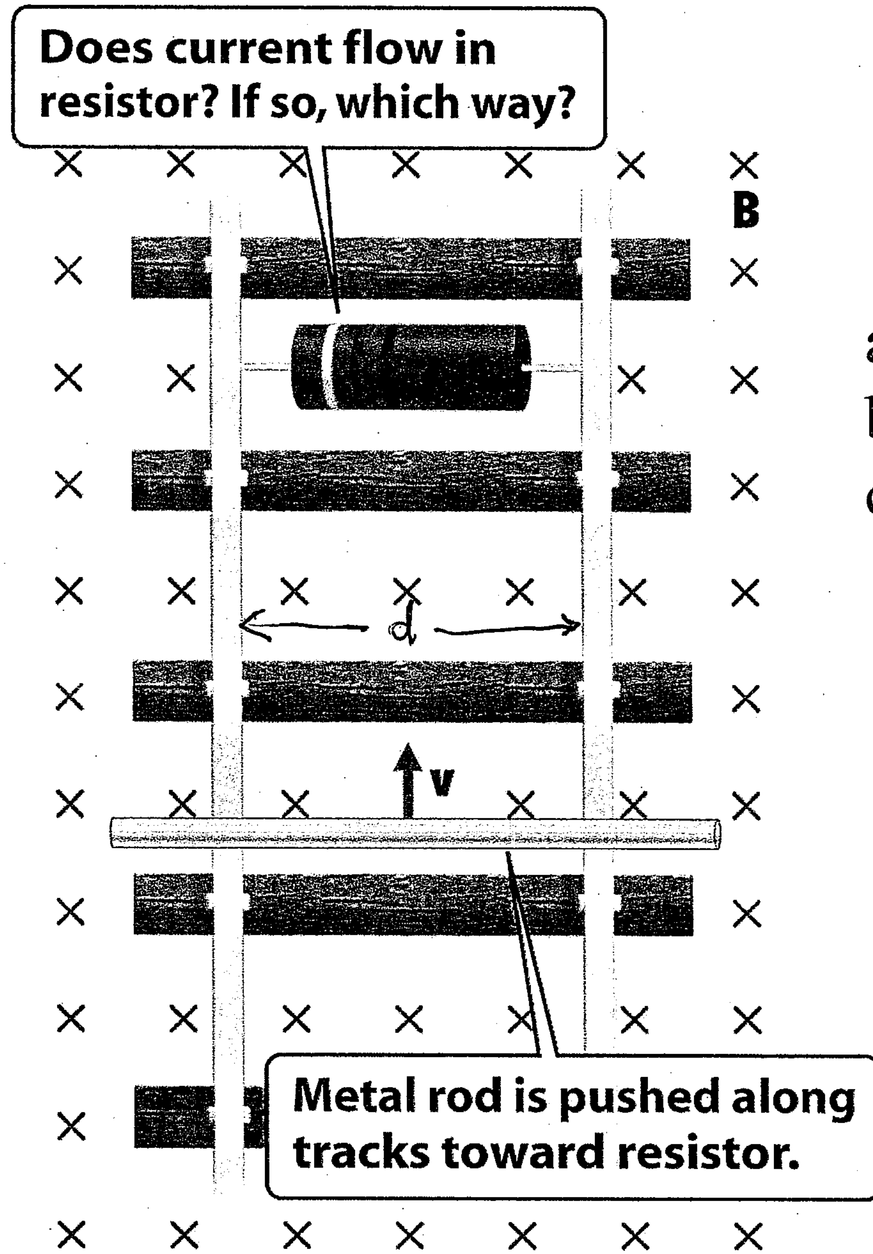

Figure 31-20 Physics for Engineers and Scientists 3/e

a. Left to right
b. Right to left
c. Does not flow

Let us use energy arguments to find the force we need to move the rod at velocity v.

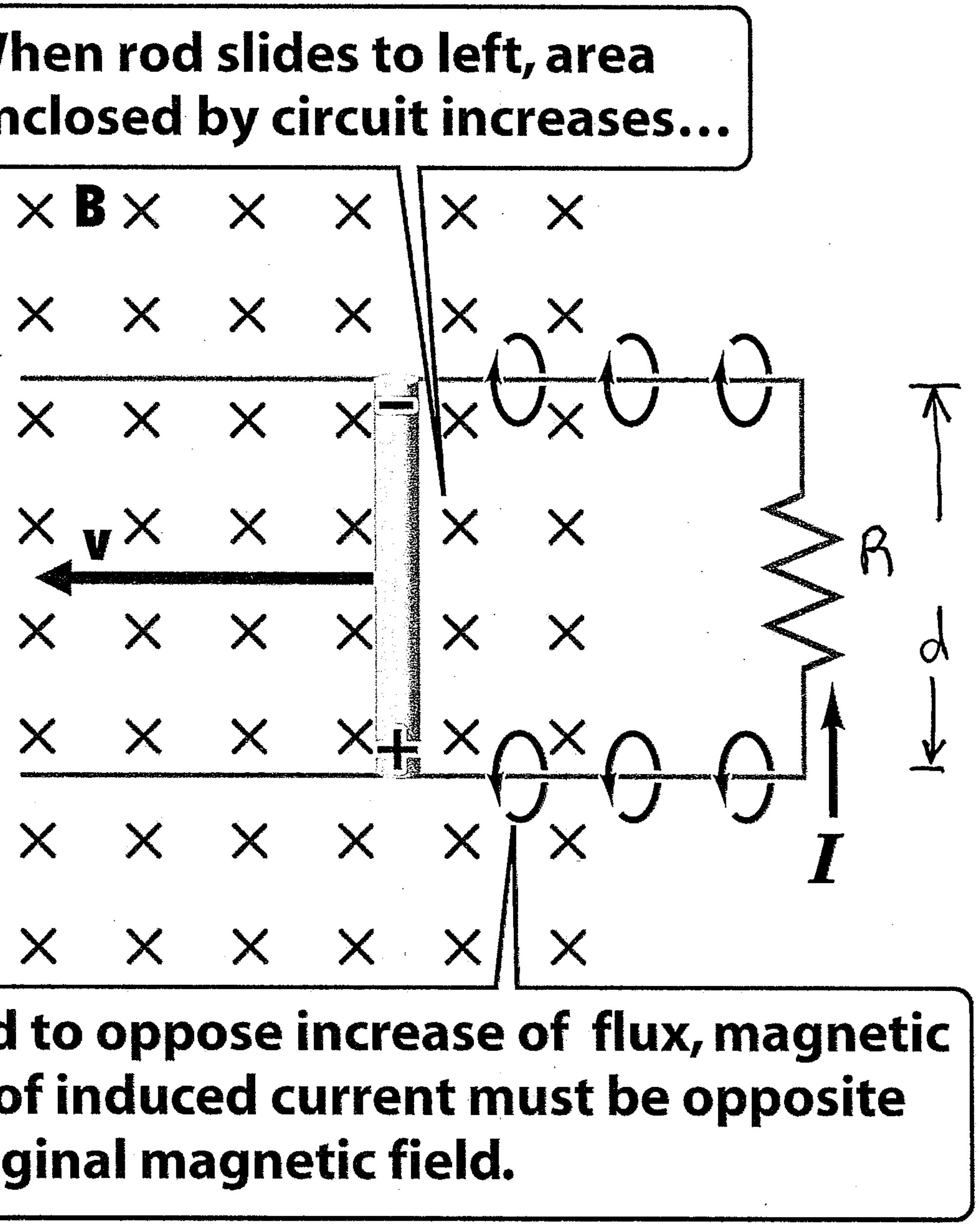

Figure 31-14 Physics for Engineers and Scientists 3/e

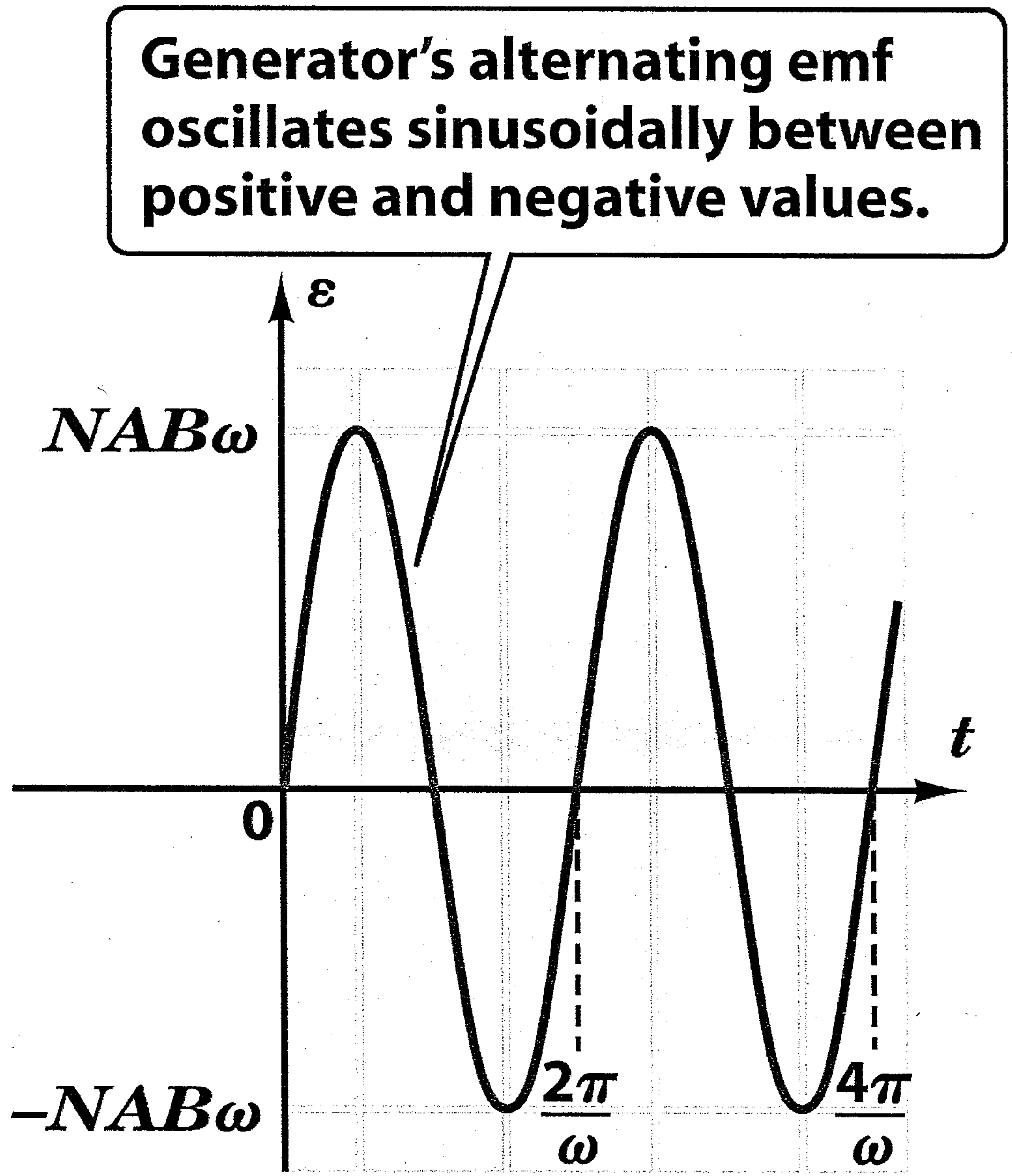

Figure 31-17 Physics for Engineers and Scientists 3/e

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Dynamo is used to generate electrical currents from mechanical input power

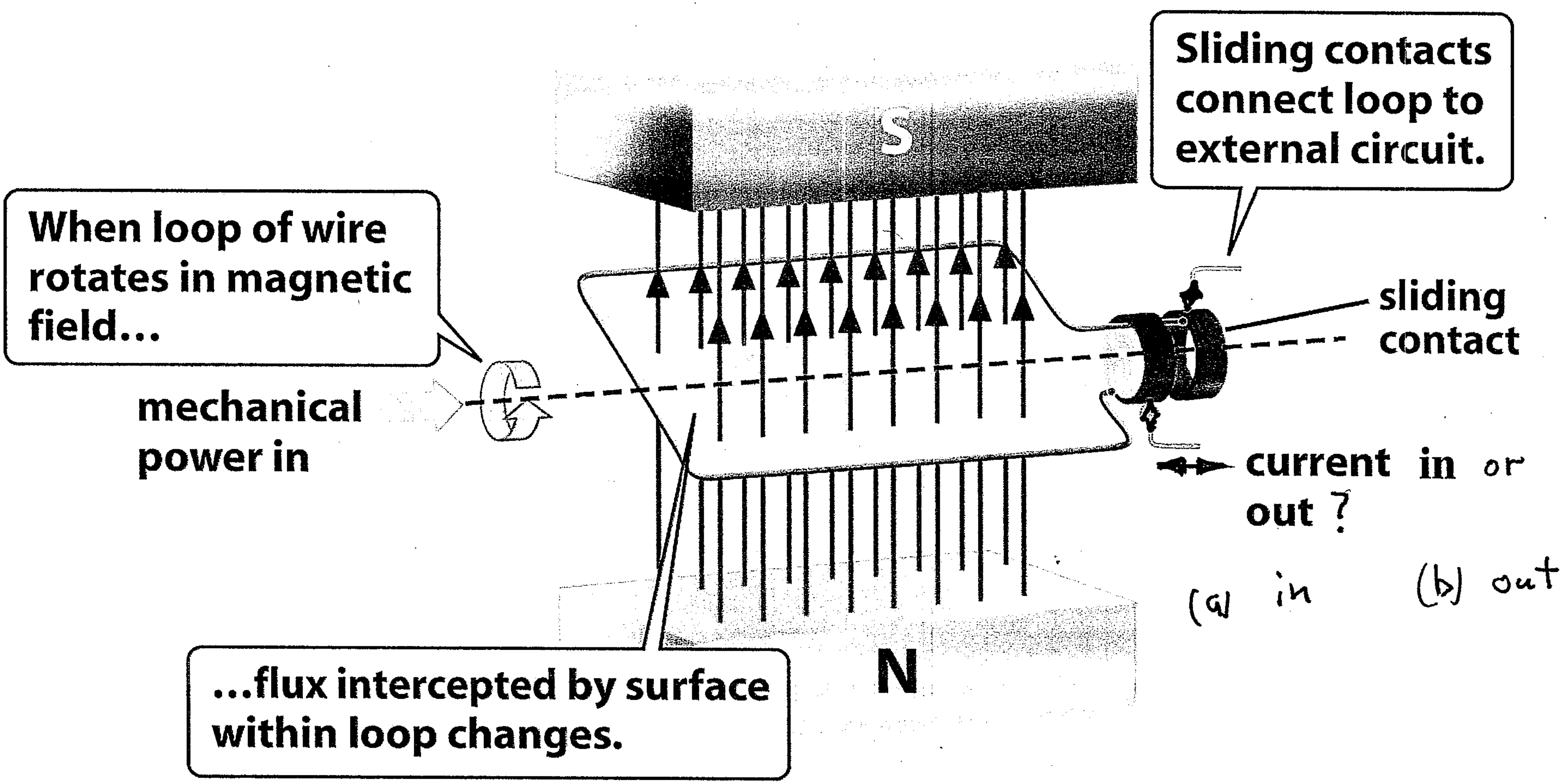


Figure 31-11 Physics for Engineers and Scientists 3/e

Induced electromagnetic torque is opposite applied torque (Lenz's law).

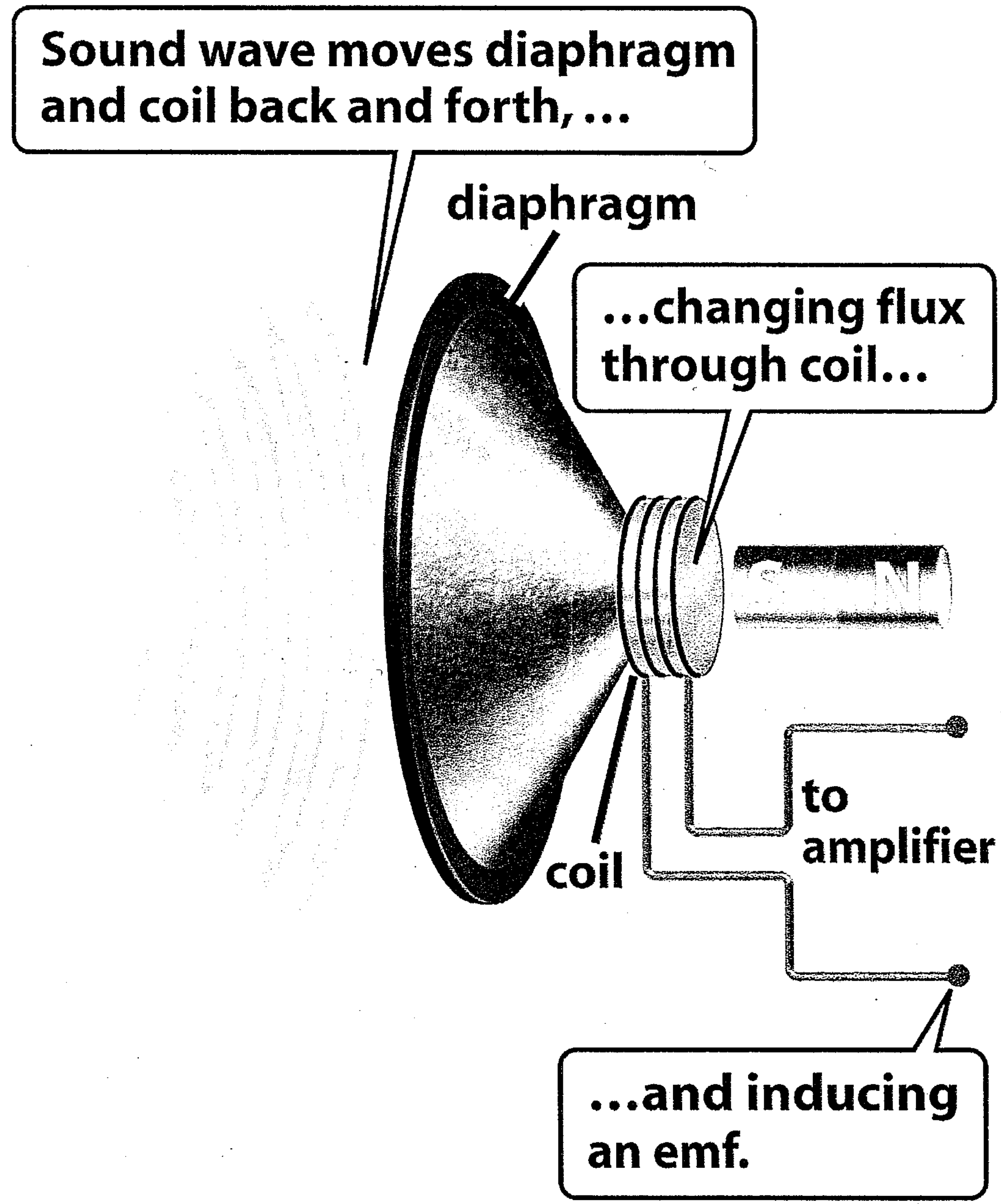
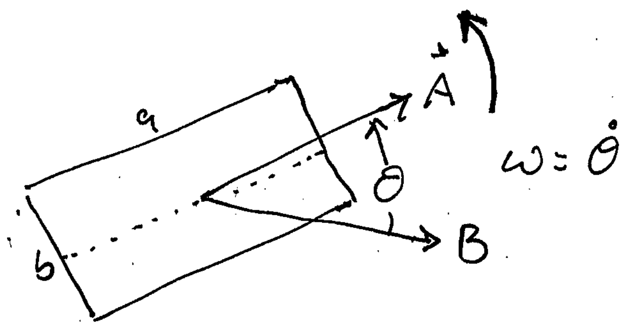


Figure 31-12 Physics for Engineers and Scientists 3/e

EMF of Rotating Loop



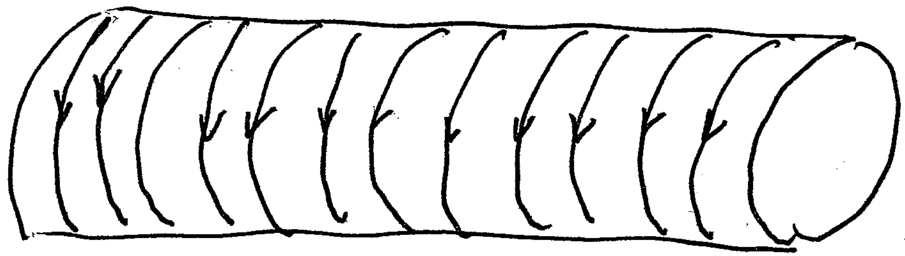
$$\theta = \omega t$$

$$\Phi_m = BA\cos\theta = Bab\cos\omega t$$

$$EMF = -\frac{\partial \Phi_m}{\partial t} = Bab\omega \sin\omega t$$

This EMF is used in a dynamo to generate electric currents

We have a long solenoid, where current varies as $I(t) = I_0 t/T$, and there are n coils per meter



(a) What is magnetic field inside solenoid

$$B = \mu_0 n I(t) = \mu_0 n I_0 t/T, \quad B \simeq 0 \text{ outside}$$

(b) Is their electric field outside solenoid?

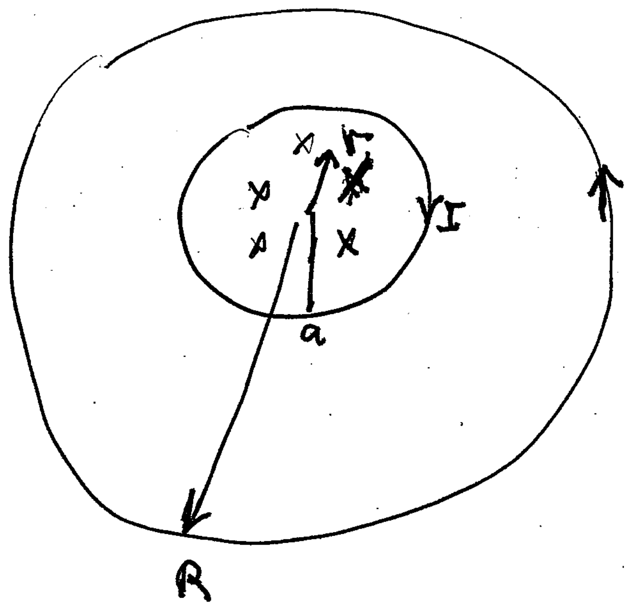
(a) yes (b) no

(c) Find electric field inside and outside solenoid

$\bar{E}$ - field of a solenoid

$B = B_0 t/T$

$I = I_0 t/T \quad , \quad B_0 = \mu I_0 n$



outside

$$\mathcal{E}mf = \oint \vec{E} \cdot d\vec{l} = -\dot{\Phi}_m = -\left(\dot{\frac{B_0 t}{T}} \pi R^2\right),$$

$$E \, 2\pi R = -\frac{B_0 \pi a^2}{T}$$

$$E = -B_0 a^2 / 2TR$$

inside

$$\mathcal{E}mf = \oint E \cdot dl = -\dot{\Phi}_m$$

$$2\pi r E = -\dot{\frac{(B_0 t)}{T}} \pi r^2$$

$$E = -\frac{B_0}{2T} r$$

Mutual induction

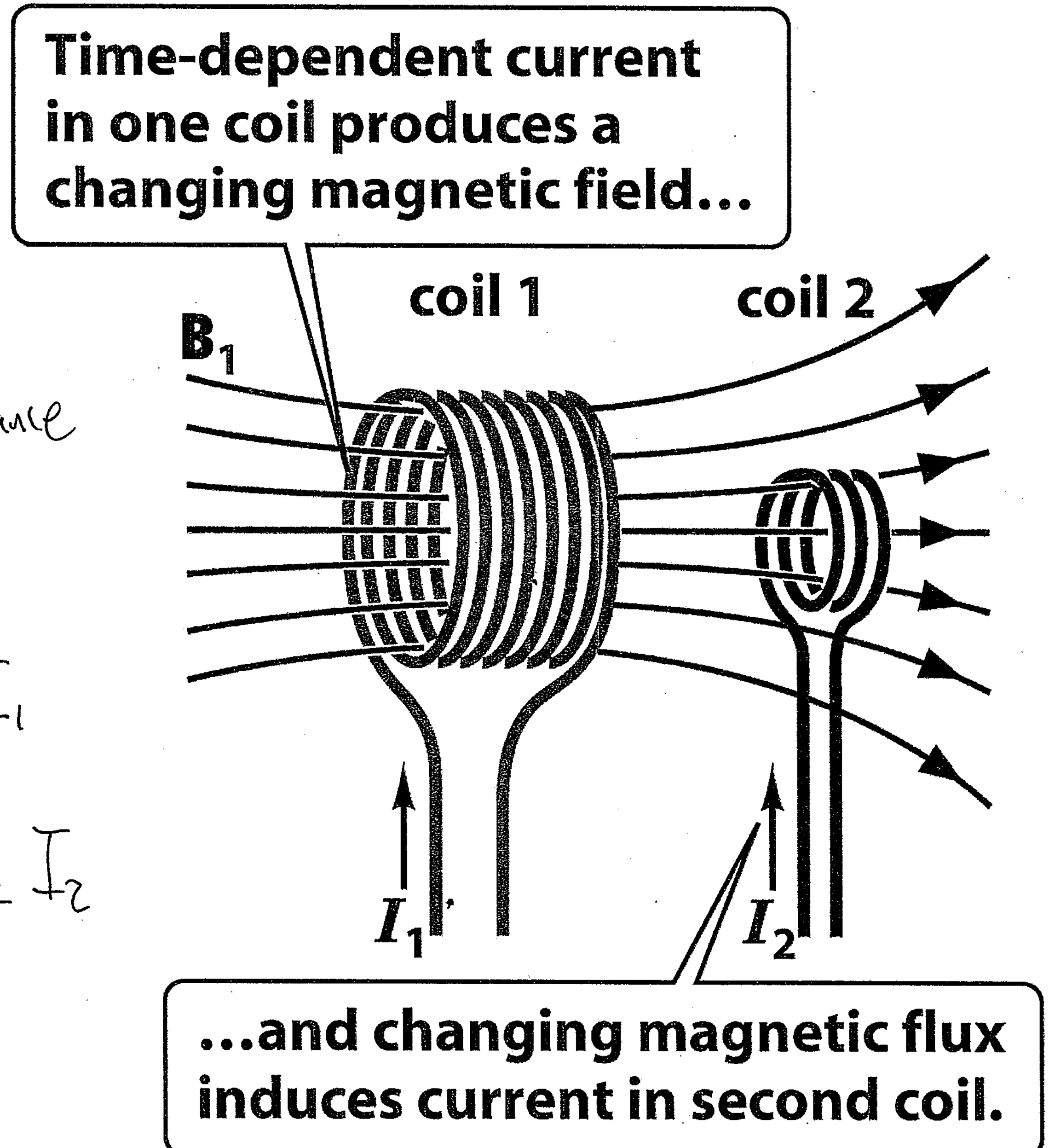
mutual inductance

Φ_{21}

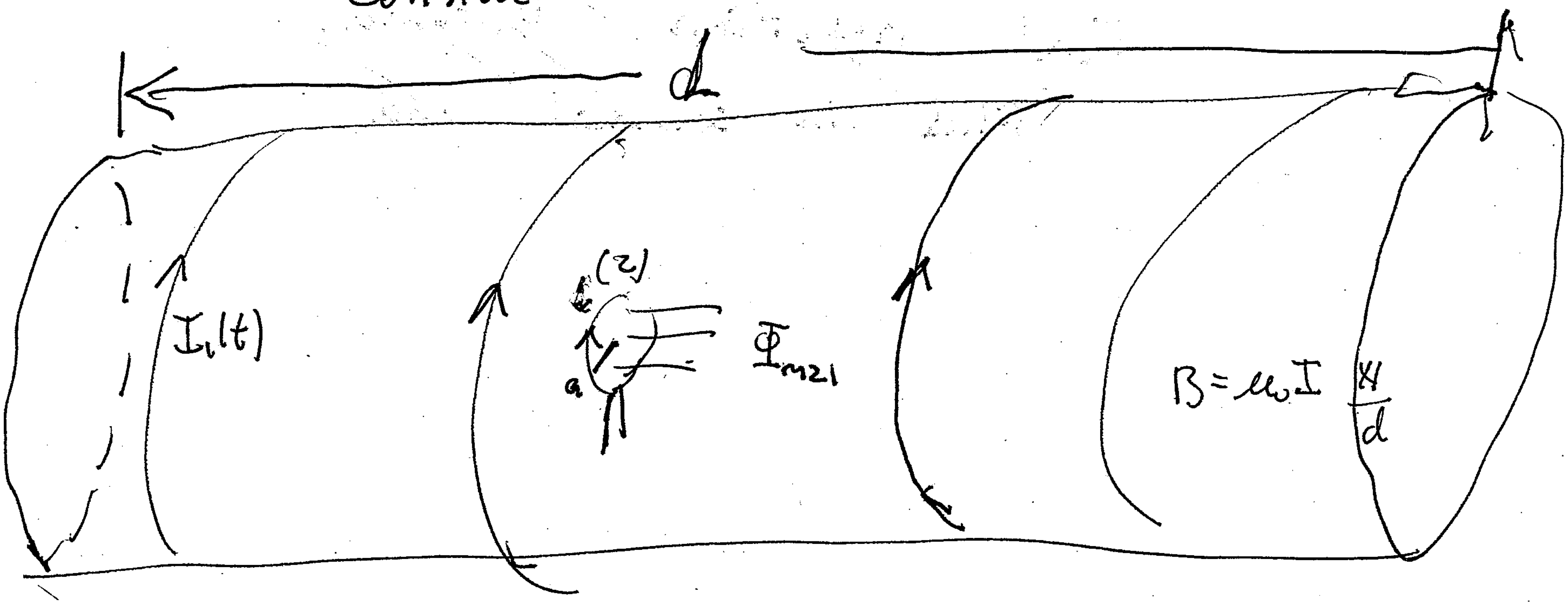
$\Phi_{m21} = M_{21} I_1$

$\Phi_{m12} = M_{12} I_2$

$M_{12} = M_{21}$



Consider a coil in



Magnetic flux from $I_1(t)$ through coil (2) is (solenoid has N_1 turns)

$$\Phi_{m21} = B_1 \pi a^2 = \frac{\mu_0 N}{d} I_1(t) \pi a^2 \equiv M_{21} I_1(t)$$

$$M_{21} \equiv \text{mutual inductance} \equiv \frac{\mu_0 N_1 \pi a^2}{d} \equiv M_{21}$$

Theorem: Mutual inductance, M_{12}, from flux (due to) coil's (2) current, ~~through~~ solenoid is given by $M_{12} = M_{21}$

In general mutual and self-inductance difficult to compute: But we can buy off the shelf, coils that have specified self-inductance

and circuits, where components have a specified mutual inductance; and we need to understand:

$L_1 I_1 \equiv$ B-flux through coil one due to its own current

$M_{12} I_2 \equiv$ B-flux through coil ~~two~~ one due to current two

$M_{21} I_1 \ (= M_{12} I_1) \equiv$ B-flux in coil ~~one~~ two due to current in coil ~~two~~ one

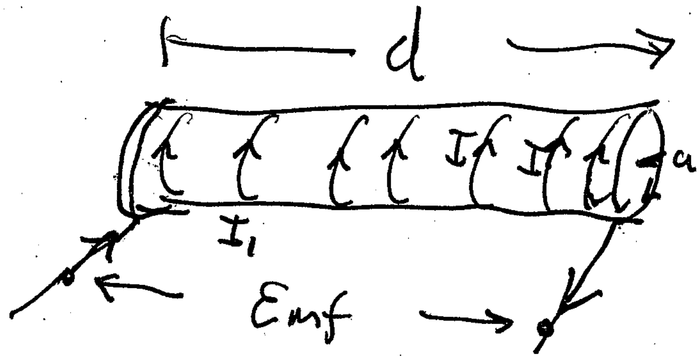
$M_{12} \dot{I}_2 = V_{12} \equiv$ voltage drop across coil one due to changing current in coil two

$L_1 \dot{I}_1 = V_1 \equiv$ voltage drop across coil one due to changing rate its own current self inductance of solenoid of N_1 coils

$$N_1 \Phi_M \equiv L I_1$$

$$= N_1 B \pi a^2 = N_1 I_1 \mu_0 \pi a^2 N_1/d$$

$$\therefore \boxed{L = N_1^2 \frac{\mu_0 \pi a^2}{d}}$$



Emf across terminal

$$= -\dot{\Phi}_M = -L \frac{dI_1}{dt}$$

In general, self inductance $\propto N_1^2$

" ", mutual inductance $\propto N_1 N_2$

Because of inductance current cannot change instantaneously, but it takes a characteistic time, $\tau_{L/R} = L/R$.

Use Khirchoff's Law again

$$-\mathcal{E} + IR + L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} + IR = \mathcal{E}$$

Solution

exp $\left(-t\frac{R}{L}\right)$

$$I(t) = \frac{\mathcal{E}}{R} + C\exp(-t/\tau_{L/R})$$

at $t = 0$, $I(t=0) = 0$, thus

$$I(0) = \frac{\mathcal{E}}{R} + C = 0, \quad \therefore\ C = -\frac{\mathcal{E}}{R}$$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - \exp(-t/\tau_{L/R})\right)$$

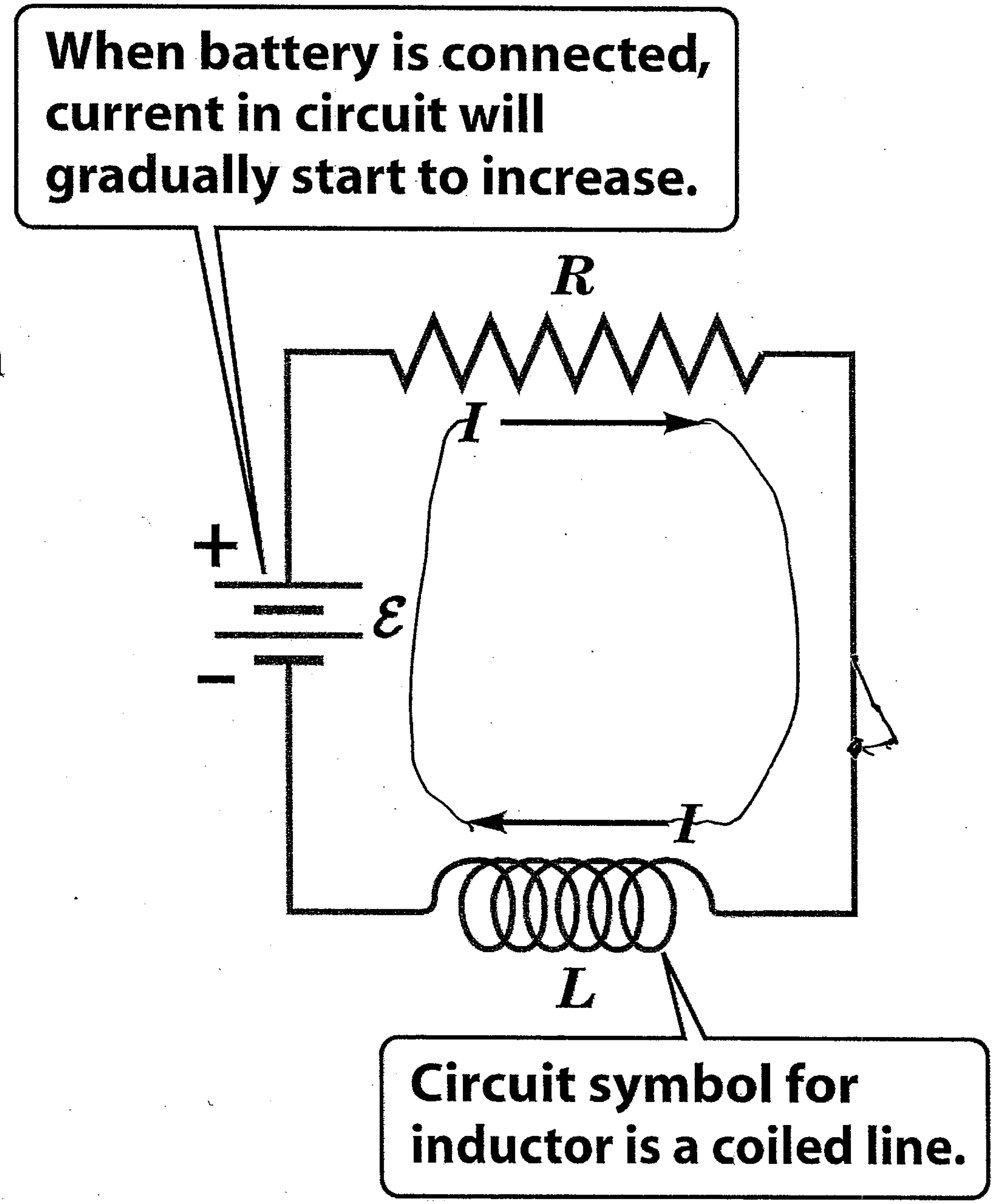


Figure 31-24 Physics for Engineers and Scientists 3/e

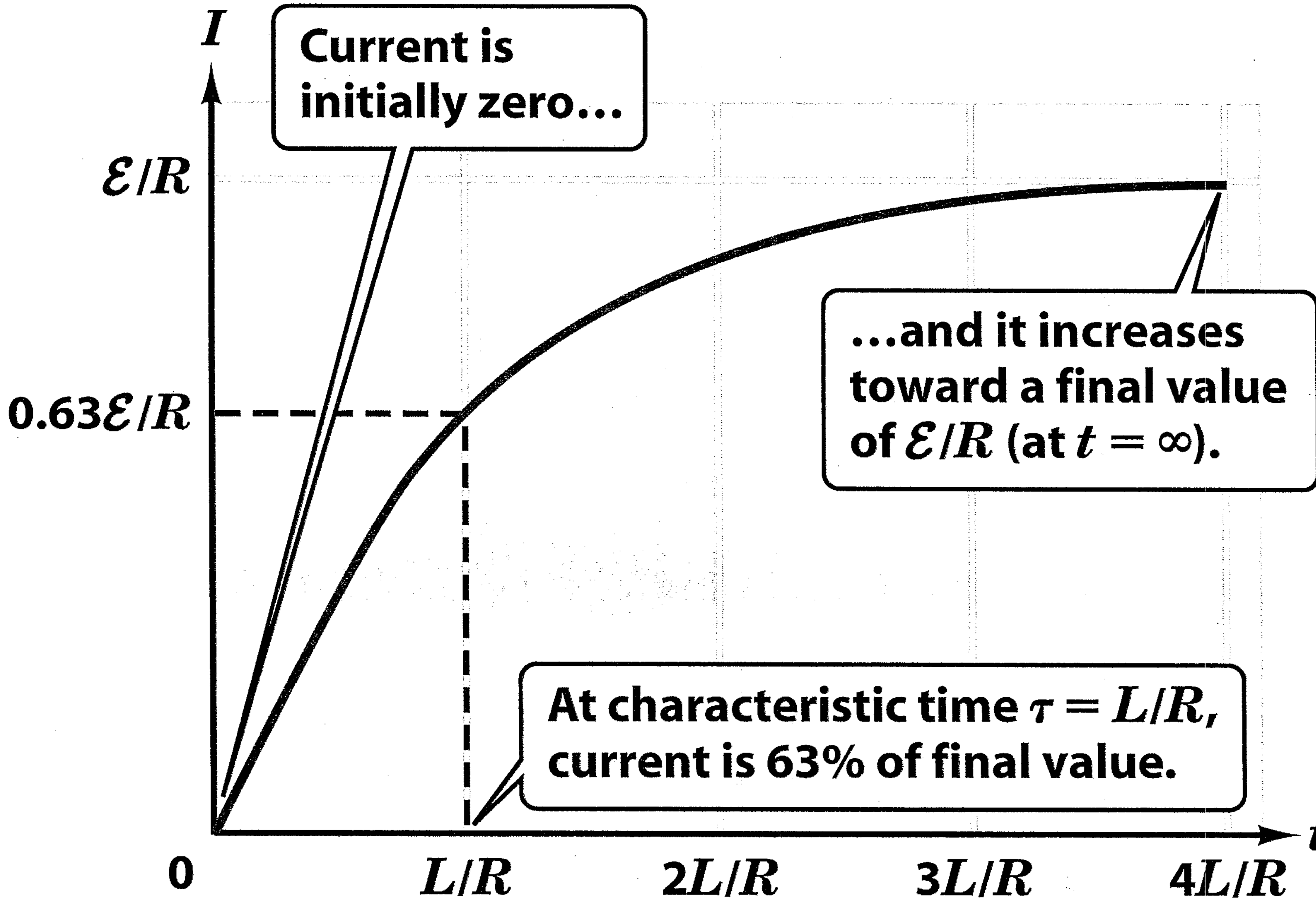


Figure 31-25 Physics for Engineers and Scientists 3/e