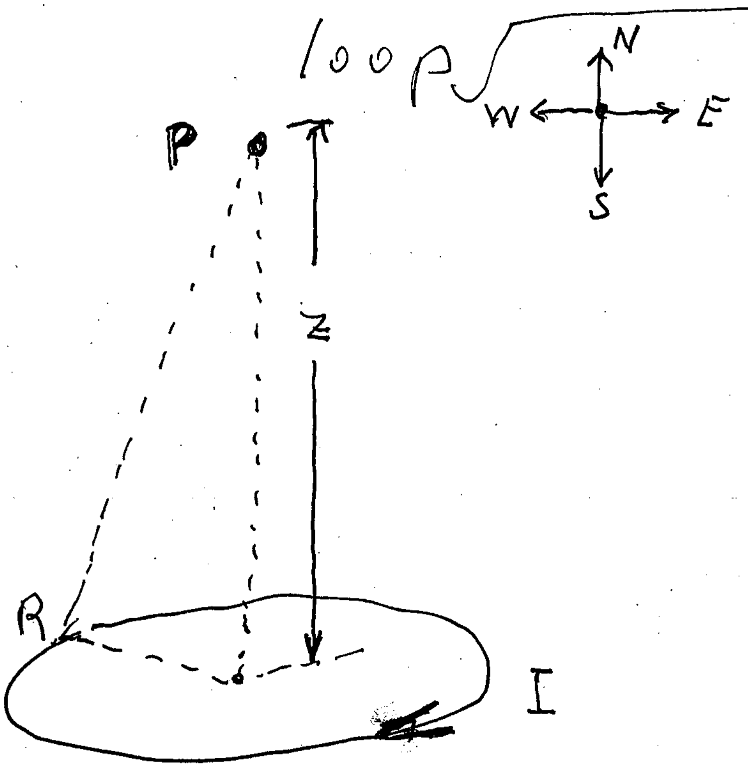
Lecture # 14

Magnetic Forces

Magnetic field on axis of loop



What is direction of magnetic field at point P

(a) East

(b) West

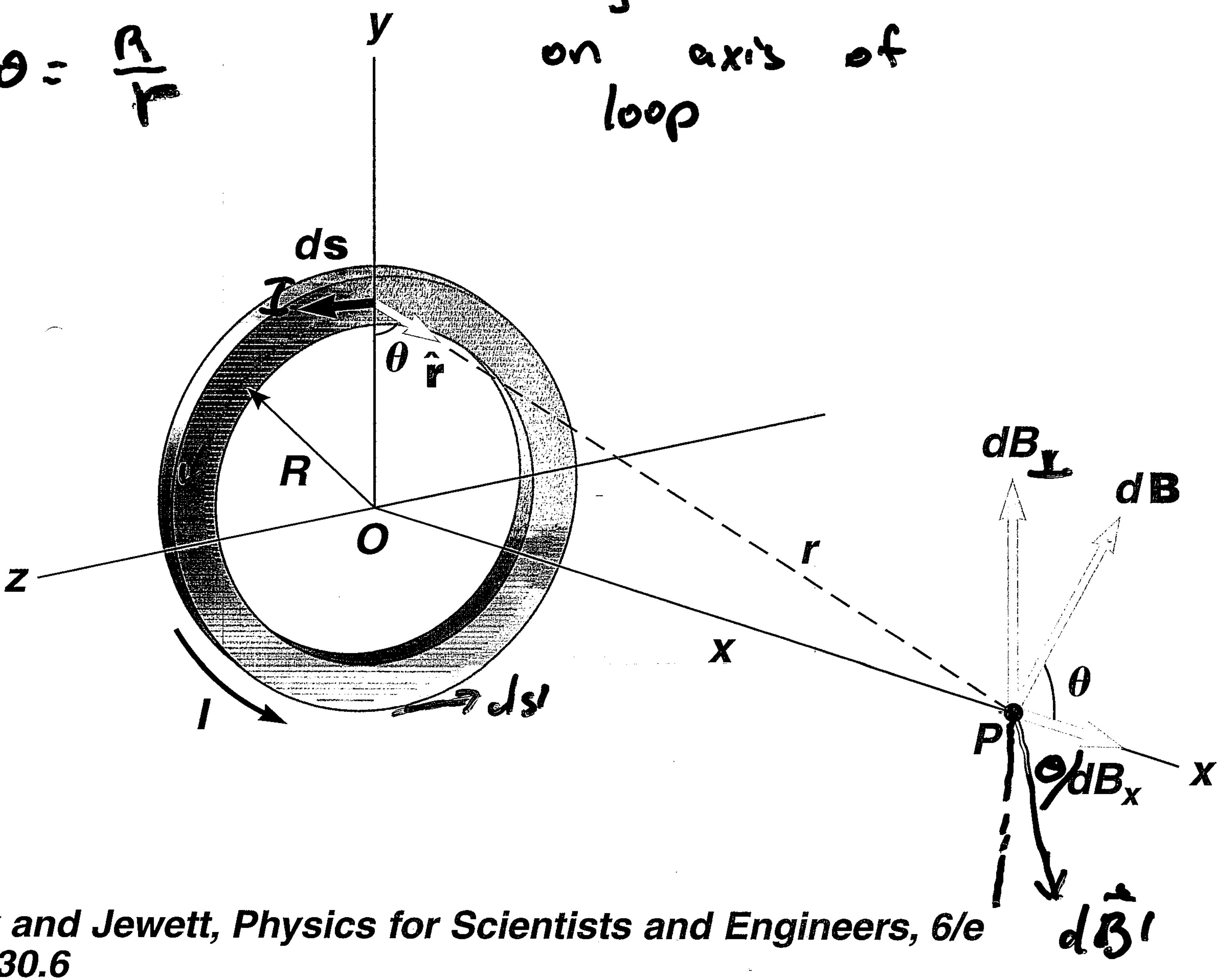
(c) North

(d) South

What is magnitude of B field at P?

Magnetic field on axis of loop

$\cos\theta = \frac{R}{r}$

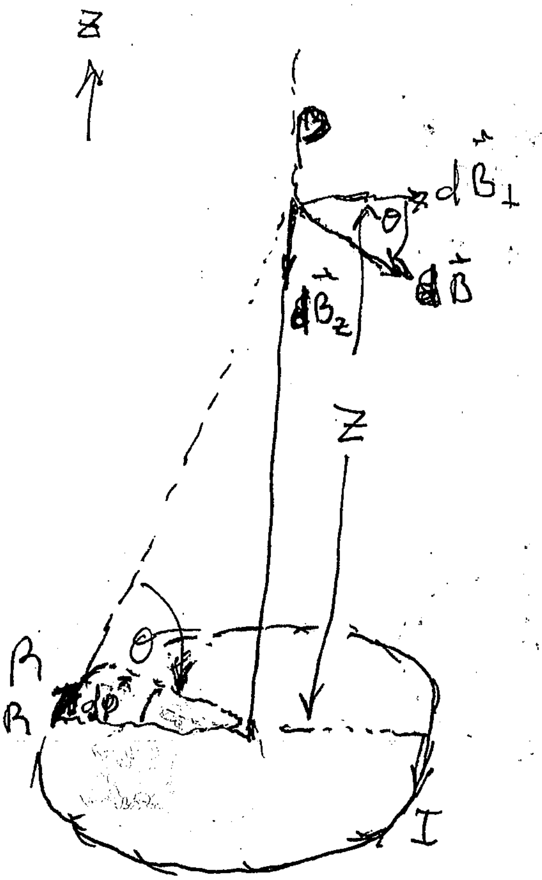


Serway and Jewett, Physics for Scientists and Engineers, 6/e
Figure 30.6

Figure 30.6

(Example 30.3) Geometry for calculating the magnetic field at a point *P* lying on the axis of a current loop. By symmetry, the total field **B** is along this axis.

(2)



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, R\, d\phi}{Z^2 + R^2} \left[-\hat{z}\cos\theta + \hat{\perp}\sin\theta \right]$$

sum on $\hat{\perp}$ cancels to zero

sum on $\hat{z}$ adds $R\,d\phi \to R\,2\pi$

$$\vec{B} = -\frac{\mu_0 R}{2} \frac{I}{Z^2 + R^2} \cos\theta\ \hat{z}$$

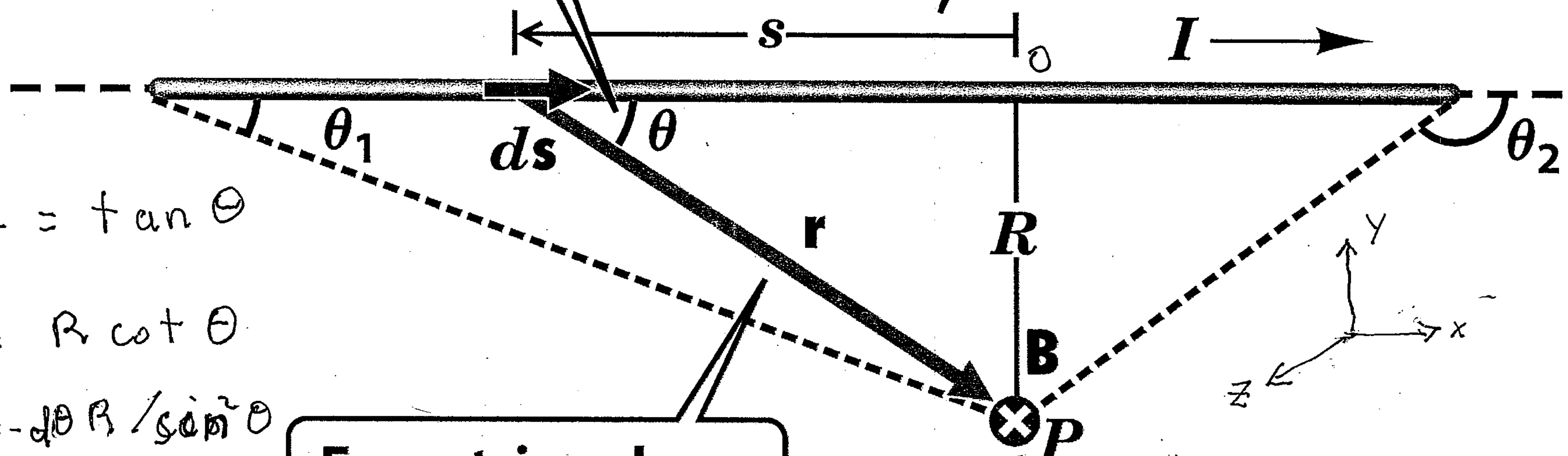
$$\cos\theta = \frac{R}{(Z^2 + R^2)^{3/2}}$$

$$\vec{B} = -\frac{I R^2 \mu_0\, \hat{z}}{2(Z^2 + R^2)^{3/2}} = \frac{\vec{m}\,\mu_0}{2\pi\,(Z^2+R^2)^{3/2}} \xrightarrow{(Z \gg R)} \frac{\vec{m}\,\mu_0}{2\pi Z^3}$$

$\vec{m} \parallel -\hat{z}$ $\qquad |\vec{m}| \equiv IA = I\pi R^2 \equiv$ magnetic moment

For field contributions from straight wire segment, both r and θ vary with position s.

For $\theta < \pi/2$, value of s is negative.



$\frac{R}{S} = \tan\theta$

$S = R\cot\theta$

$ds = -d\theta R/\sin^2\theta$

From triangle, $\tan\theta = R/(-s)$ and $\sin\theta = R/r$.

Each magnetic field contribution points in $d\mathbf{s} \times \mathbf{r}$ direction, here, into page.

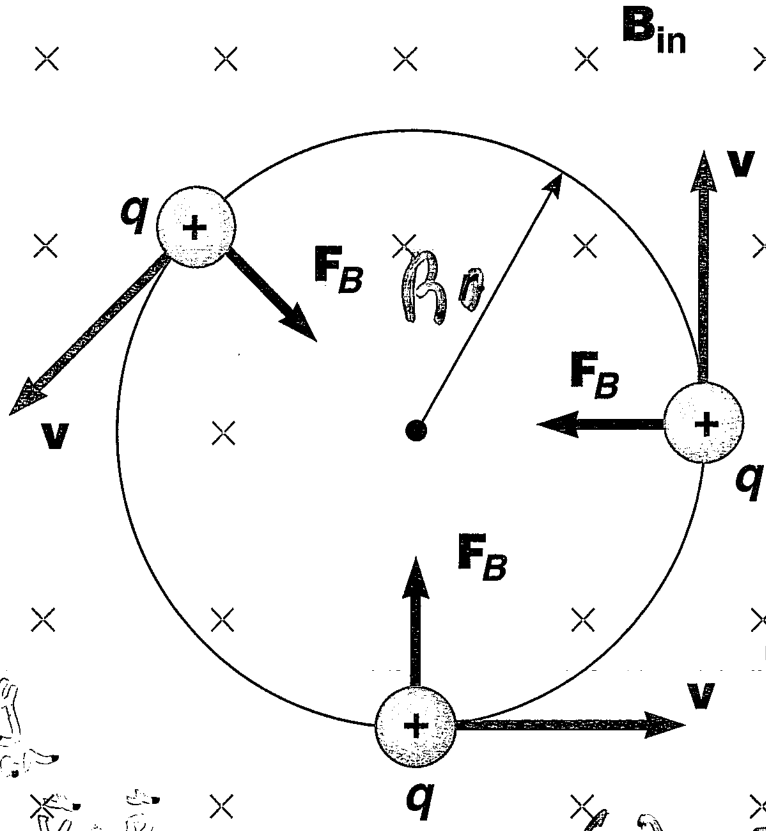
Figure 29-39 Physics for Engineers and Scientists 3/e

$\vec{F} = q\vec{v}\times\vec{B}$

$\vec{F}\cdot\vec{B} = 0$

$\vec{F}\cdot\vec{v} = 0$

∴ constant speed along B; circular motion ⊥ $\vec{v}\times\vec{B}$



$V_\parallel = V_{\parallel 0}$ constant

$\vec{F} = m\vec{a}$

$a = \frac{v_\perp^2}{R}$

∴ $m\frac{v_\perp^2}{R} = qv_\perp B$

$\omega_c \equiv \frac{v_\perp}{R} = \frac{qB}{m}$

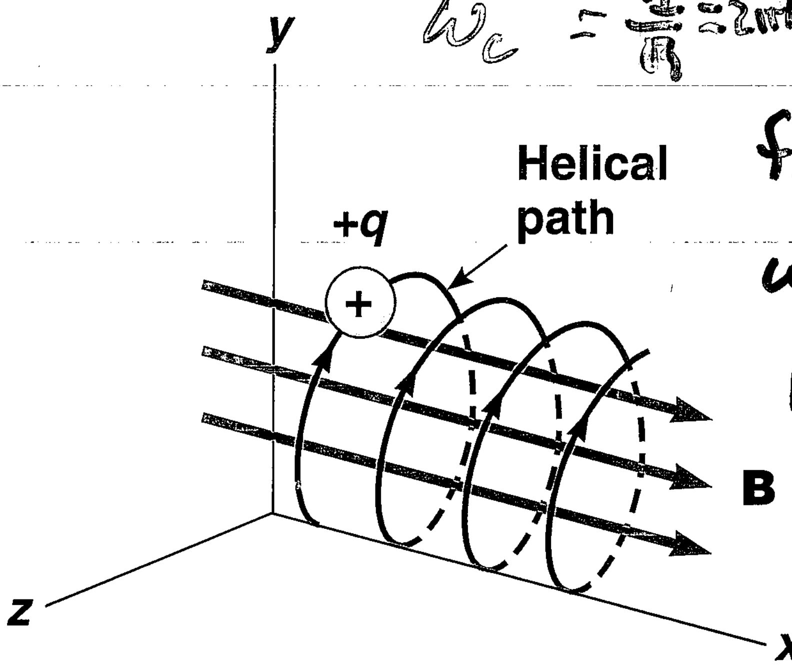
$\omega_c = \frac{v_\perp}{R} = 2\pi f_c = \frac{qB}{m}$

$f_c = qB/2\pi m$

$\omega_c = qB/m$

$R = \frac{v_\perp}{\omega_c}$

≡ Larmor radius



Serway and Jewett, Physics for Scientists and Engineers, 6/e
Figure 29.18 and Figure 29.19

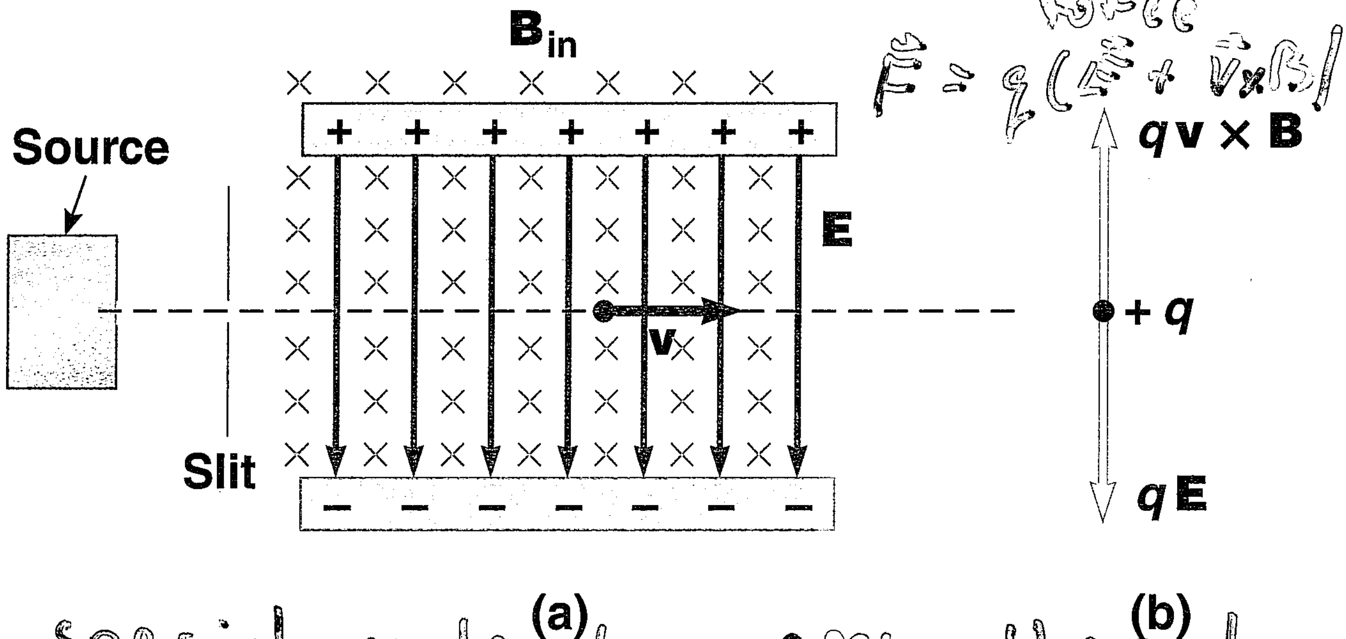
Figure 29.18

When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to **B**. The magnetic force $\mathbf{F}_B$ acting on the charge is always directed toward the center of the circle.

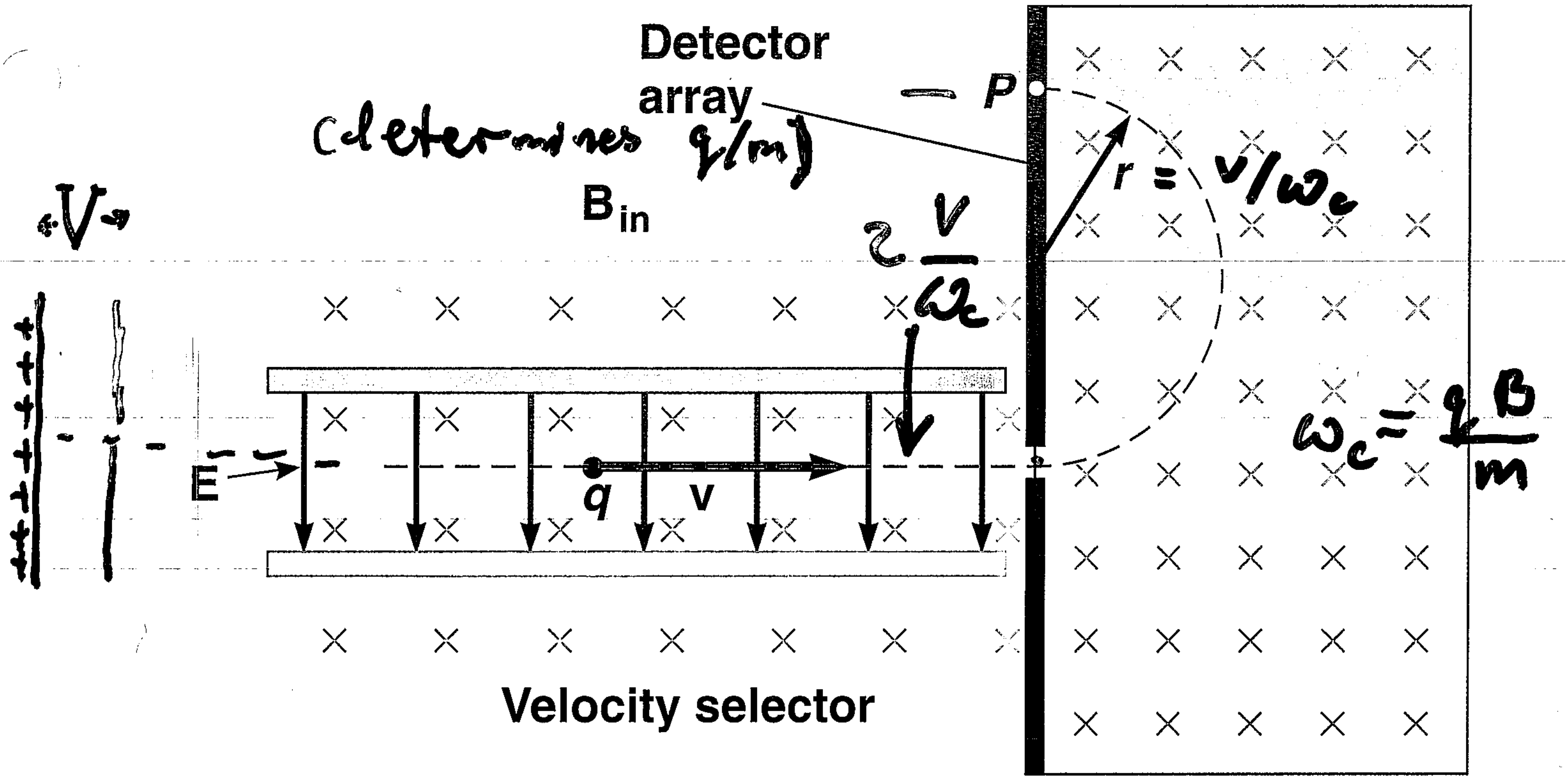
Figure 29.19

A charged particle having a velocity vector that has a component parallel to a uniform magnetic field moves in a helical path.

Total electric-magnetic force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$



special velocity goes through undeflected $|\vec{v}| = E/B$



$r = v/\omega_c$

$2\frac{v}{\omega_c}$

$\omega_c = \frac{qB}{m}$

What potential V is needed to accelerate charge q of mass m, to move through velocity selector?

Serway and Jewett, Physics for Scientists and Engineers, 6/e
Figure 29.23 and Figure 29.24

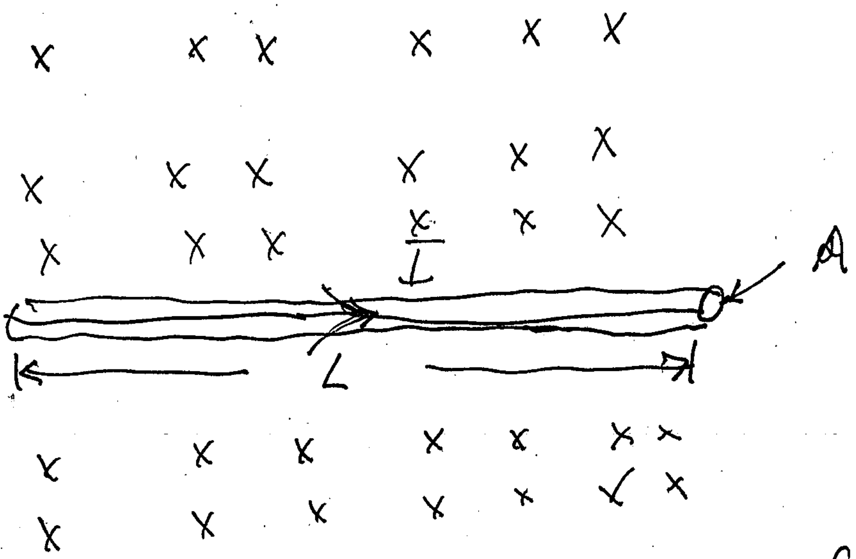
Figure 29.23

(a) A velocity selector. When a positively charged particle is moving with velocity **v** in the presence of a magnetic field directed into the page and an electric field directed downward, it experiences a downward electric force $q\mathbf{E}$ and an upward magnetic force $q\mathbf{v} \times \mathbf{B}$. (b) When these forces balance, the particle moves in a horizontal line through the fields.

Figure 29.24

A mass spectrometer. Positively charged particles are sent first through a velocity selector and then into a region where the magnetic field $\mathbf{B}_0$ causes the particles to move in a semicircular path and strike a detector array at *P*.

Force on a straight current carrying wire of length L



In which direction is force

(a) up
(b) down

Current in a wire causes, on wire, force in magnetic field!

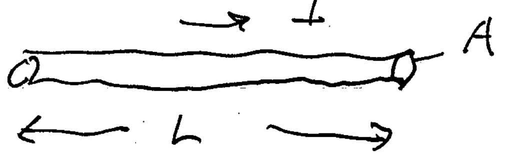
Derivation of Force

Force on a single charge carrier
(Take wire ⊥ $\vec{B}$), $\vec{v} \parallel$ to wire

$$\vec{f} = q\,\vec{v} \times \vec{B}$$

$$|\vec{f}| = qvB$$

Force from N charges



$$|\vec{F}| = N|\vec{f}| = NqvB$$

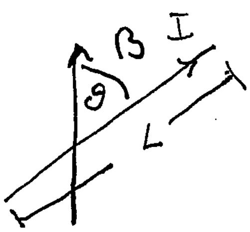
substitute: $N = nLA$ [$n \equiv$ density (particle/vol) of charge carriers]
into force equation:

current density

$$|\vec{F}| = (qnLA)\cancel{L}vB = \underset{\underset{I}{\shortparallel}}{(\boxed{qnv}A)}\,LB$$

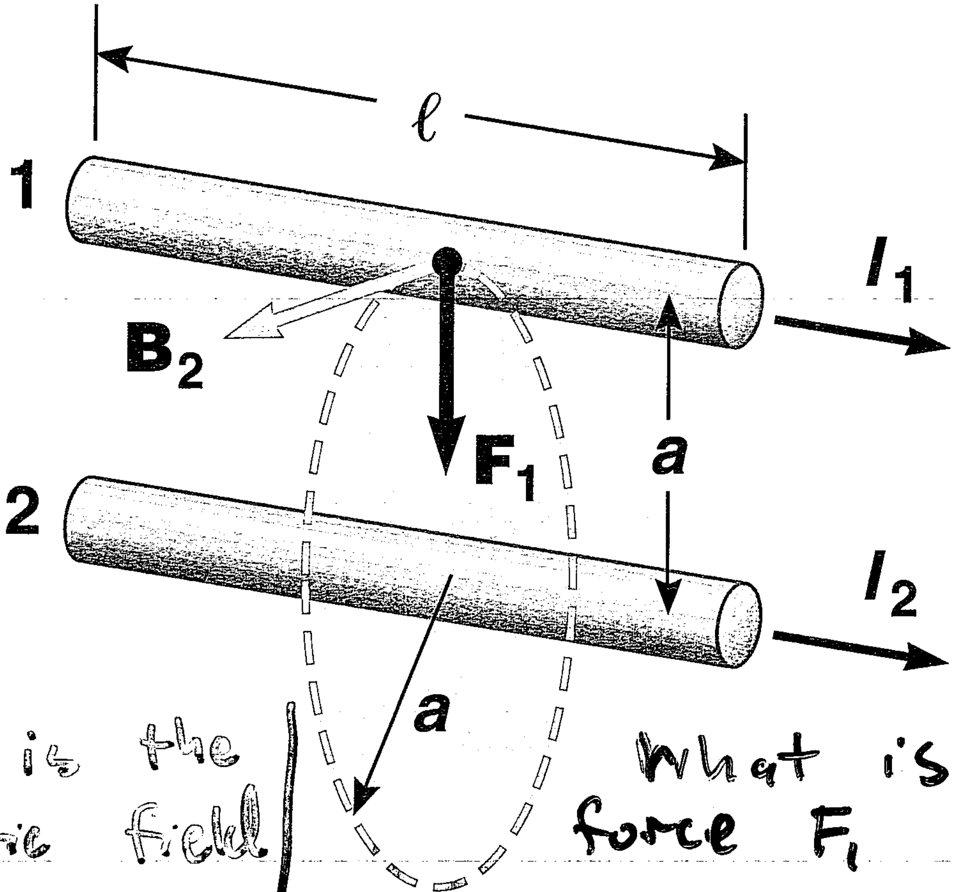
$$\boxed{|\vec{F}| = ILB}$$

More generally $\vec{F} = \vec{I} \times \vec{B}\,L$



Currents in the same direction attract

Currents on opposite direction repel.



What is the magnetic field B_2?

(a) $\mu_0 I_1 / a 2\pi$

/(b) $\mu_0 I_2 / a 2\pi$

(c) $\mu_0 I_1 \ell / a^2$, (d) $\mu_0 I_2 \ell / a^2$

What is the force F_1

(a) $\mu_0 I_1 I_2 / a 2\pi$

(b) $\mu_0 I_1 I_2 \ell / a^2 2\pi$

(c) $\mu_0 I_1 I_2 \ell / a 2\pi$

Serway and Jewett, Physics for Scientists and Engineers, 6/e
Figure 30.8

$|\vec{F}_2| = |\vec{F}_1|$?

(a) true (b) false

(a)

Figure 30.8

Two parallel wires that each carry a steady current exert a magnetic force on each other. The field $\mathbf{B}_2$ due to the current in wire 2 exerts a magnetic force of magnitude $F_1 = I_1 \ell B_2$ on wire 1. The force is attractive if the currents are parallel (as shown) and repulsive if the currents are antiparallel.