Lecture 11

D-C circuit theory

& capacitors

If every resistor is $2\ \Omega$

what is effective resistance of this circuit?

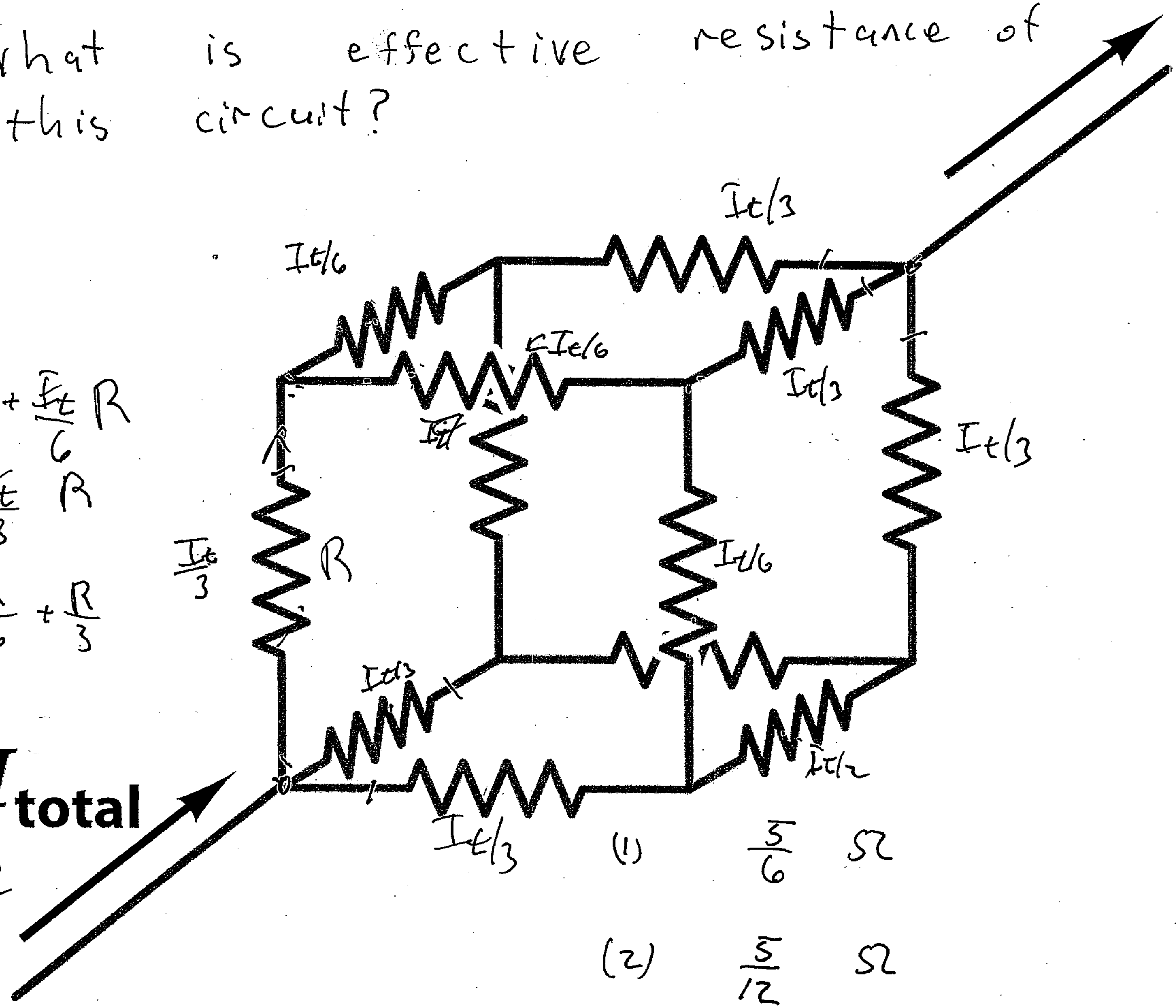


Figure 27-32 Physics for Engineers and Scientists 3/e

$I_t R_{eff}$

$= \frac{I_t}{3} R + \frac{I_t}{6} R + \frac{I_t}{3} R$

$R_{eff} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3}$

$= \frac{5}{6} R$

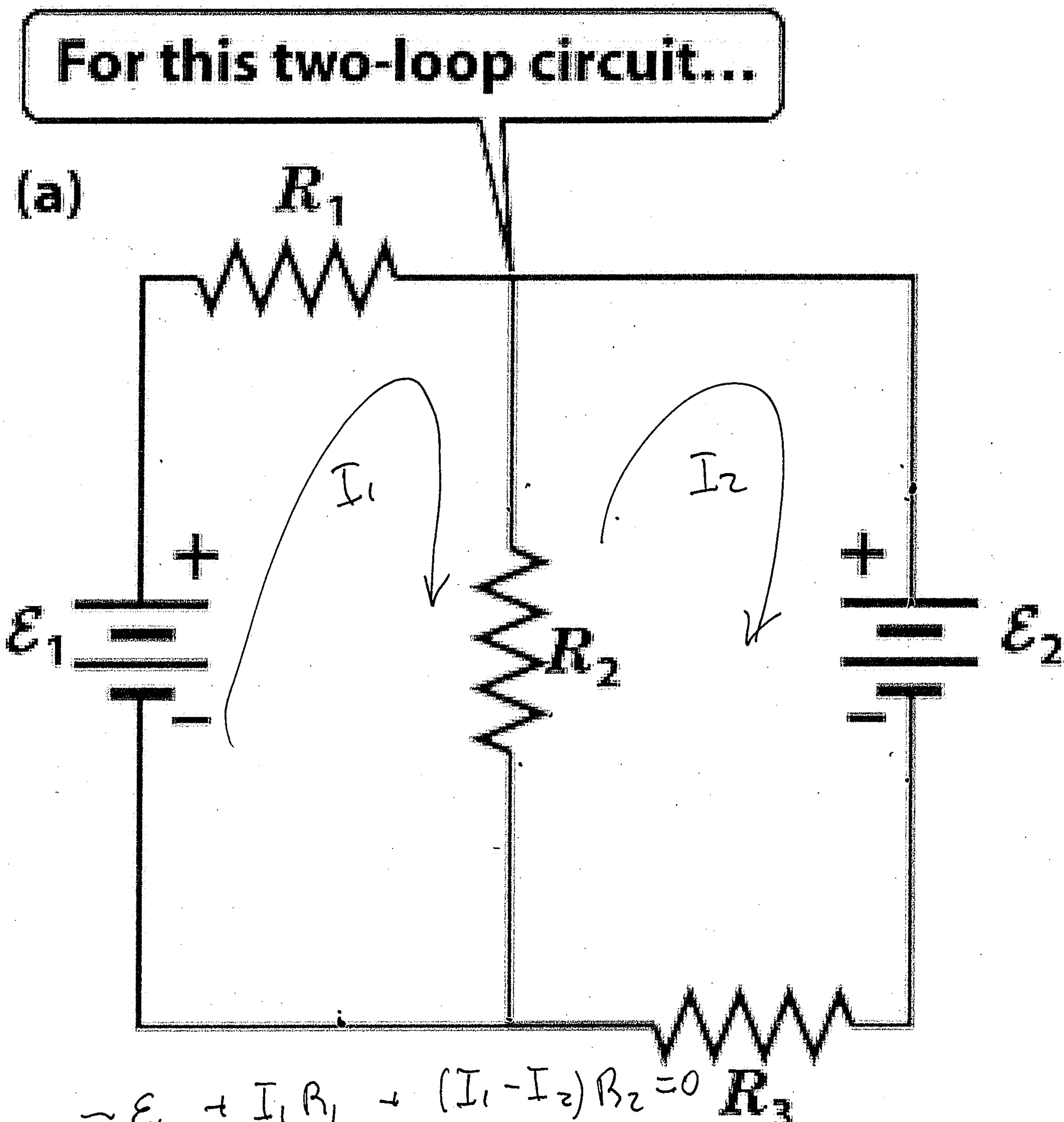
$\frac{10}{6} = \frac{5}{3}\Omega$

(1) $\frac{5}{6}\ \Omega$

(2) $\frac{5}{12}\ \Omega$

(3) $\frac{5}{3}\ \Omega$

What are circuit equations for diagram below



$$-\mathcal{E}_1 + I_1 R_1 + (I_1 - I_2) R_2 = 0$$

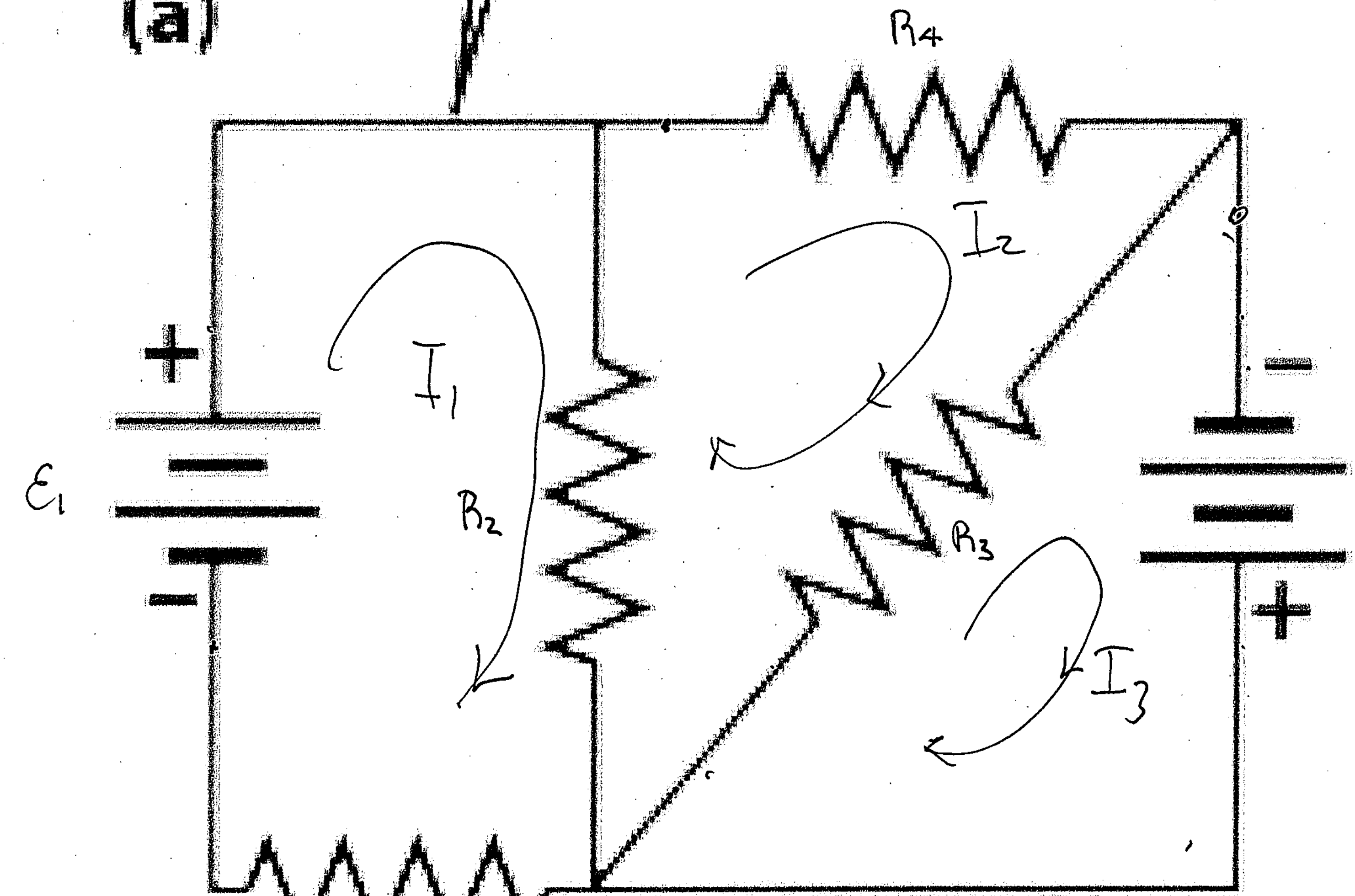
$$\mathcal{E}_2 + I_2 R_3 + (I_2 - I_1) R_2 = 0$$

What are the circuit equations?

For a complicated circuit with several loops and branches...

(1) $-\mathcal{E}_1 + R_2(I_1 - I_2) + I_1 R_1 = 0$

(a)



(1) $0 = I_2 R_4 + R_3(I_2 - I_3) + R_2(I_2 - I_1)$

$-\mathcal{E}_2 + (I_3 - I_2) R_3 = 0$

Measureing Voltages and Currents

Is true voltage across resistor measured in case (b) ?

Is true voltage across resistor measured in case (a)?

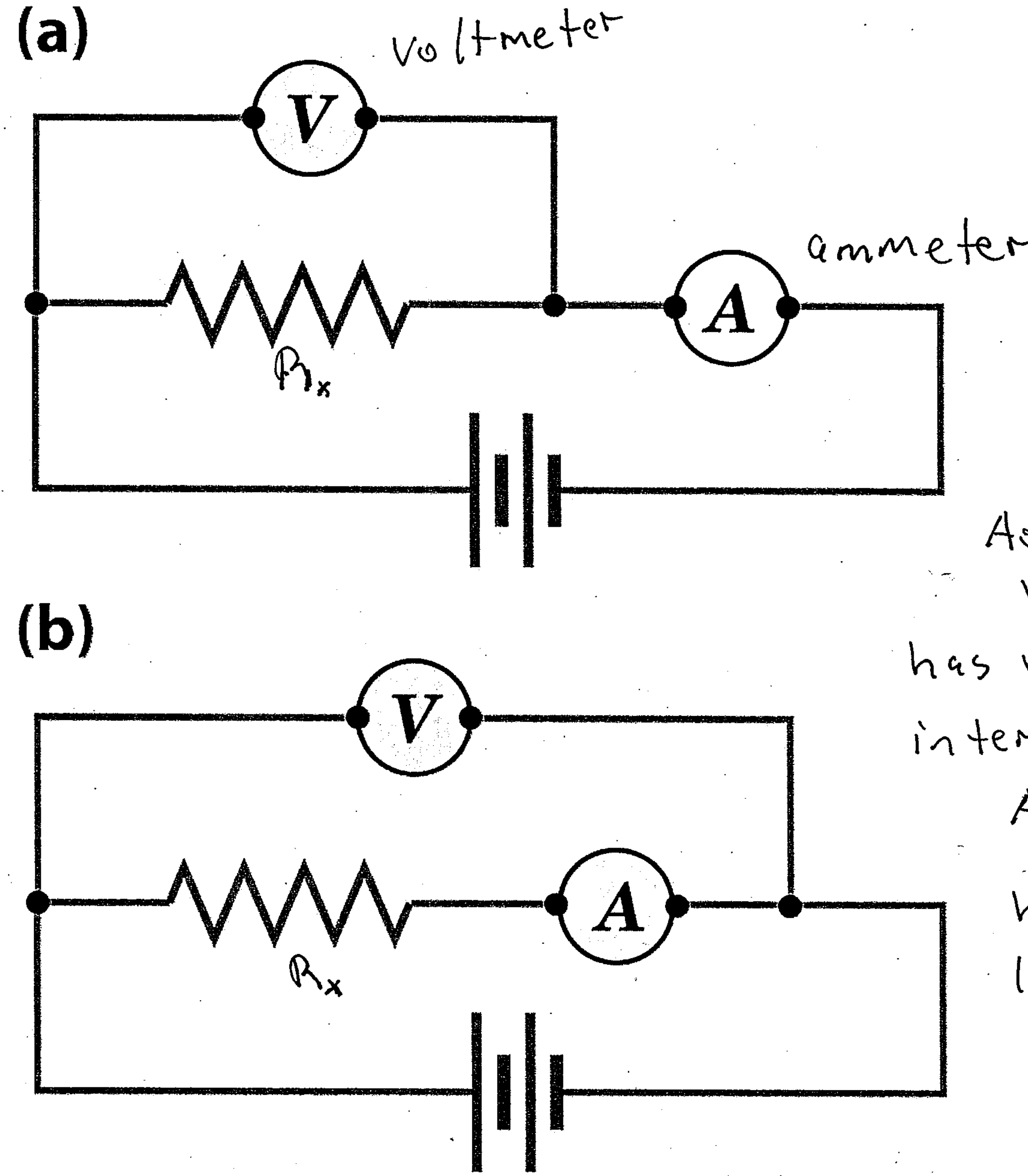
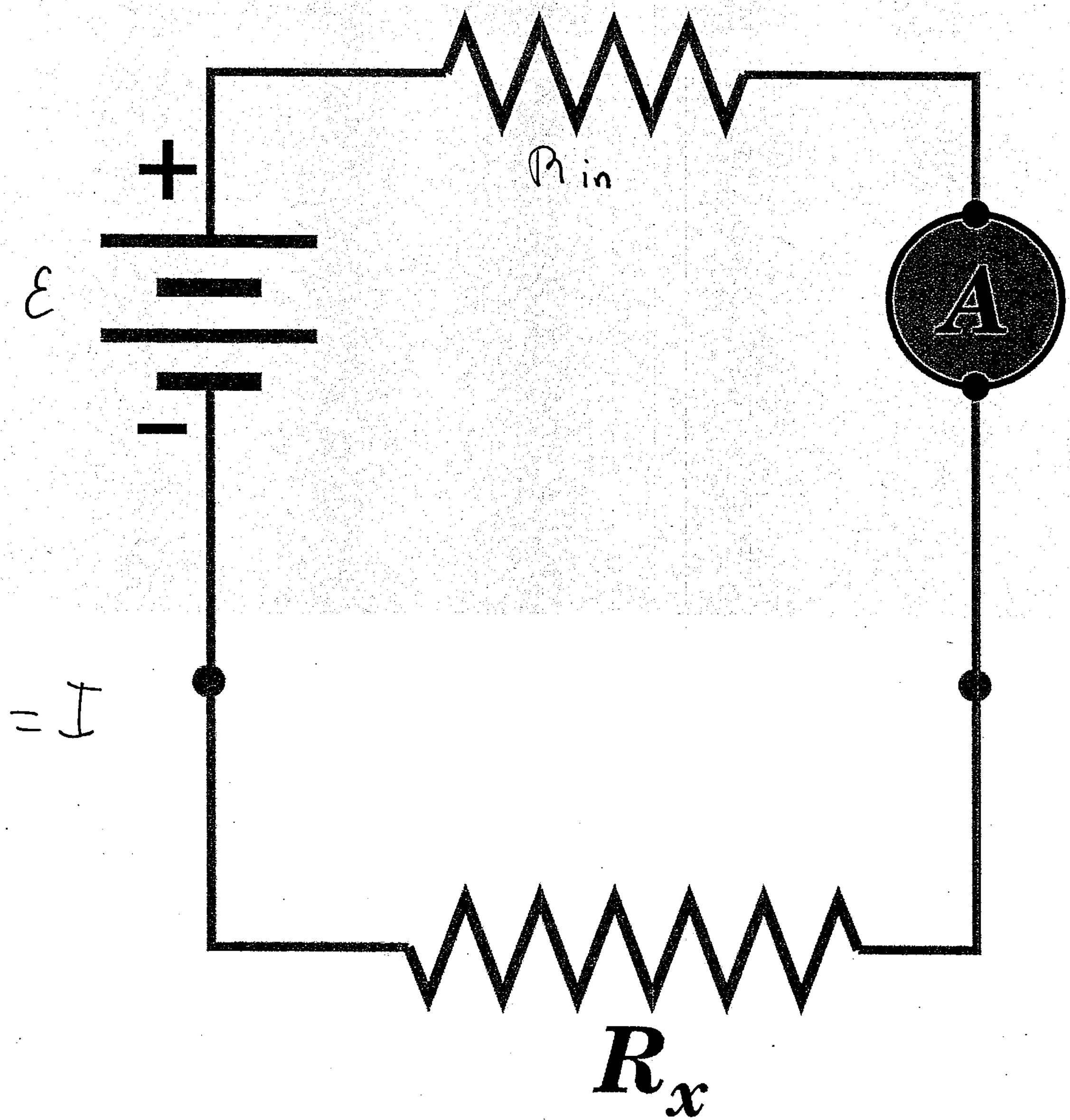


Figure 28-33 Physics for Engineers and Scientists 3/e

As good voltmeter has very high internal resistance

A good ammeter has a very low internal resistance

Imperfect Ohmeter



$$\frac{\mathcal{E}}{R_x + R_{in}} = I$$

Figure 28-32 Physics for Engineers and Scientists 3/e

Wheatstone Bridge

Does measurement change if battery has internal resistance?

(a) yes (b) no

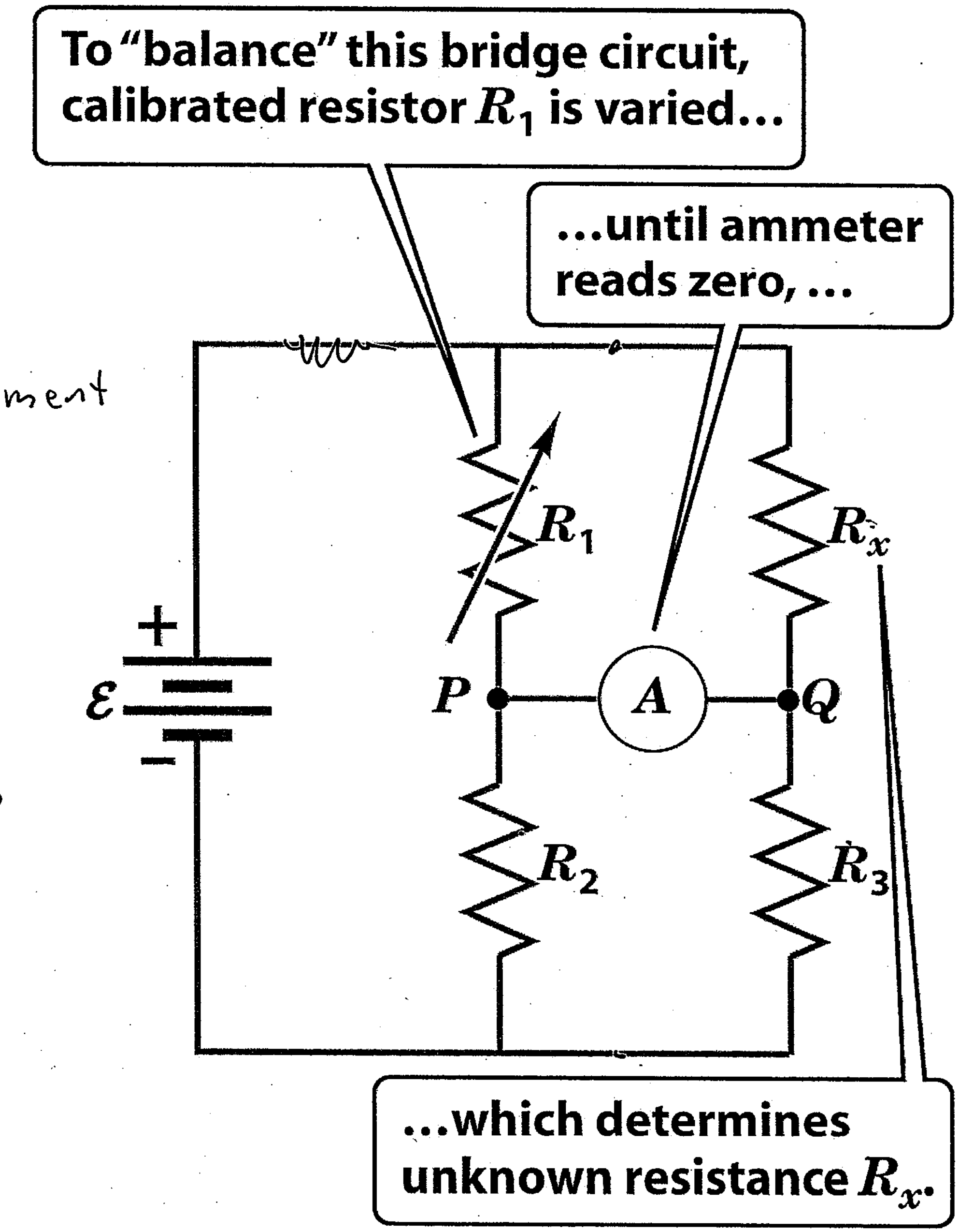
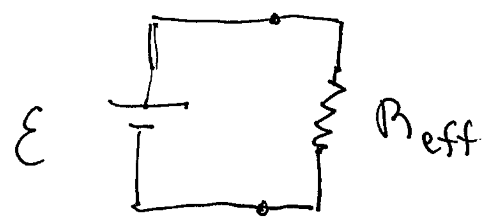


Figure 28-25 Physics for Engineers and Scientists 3/e

why?

POWER DELIVERED BY A SOURCE OF emf

In time Δt, battery delivers charge Δq



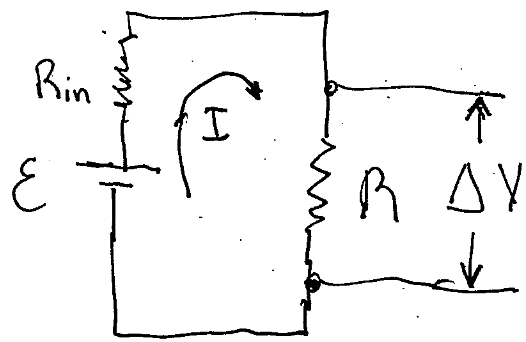
$$\Delta \mathcal{E} \equiv P \Delta t = \Delta q\, \mathcal{E}$$

$$\frac{\Delta \mathcal{E}}{\Delta t} = P = \frac{\Delta q}{\Delta t} \mathcal{E}$$

$$P = \mathcal{E} I$$

$$\frac{\Delta q}{\Delta t} = \frac{dq}{dt} =' I$$

POWER DISSIPATED BY A RESISTOR (JOULE HEAT)



$\Delta V = IR$; $I = \Delta V / R$

$$P = \Delta V I = I^2 R = \frac{(\Delta V)^2}{R}$$

Energy Conservation

$\mathcal{E} I = I^2 R_{in} + I^2 R$

Equation 28-23 derivatives Physics for Engineers and Scientists 3/e

Power Delivered by Battery
= Power Dissipated by Resistor R
+ Power Dissipated by Internal Resistor

(1) What is ratio of current through battery in circuit (a) to circuit (b)?

(A) 1 (B) 3 (C) 9

(2) What is ratio of power delivered by battery in circuit (a) to circuit (b)?

(A) 1 (B) 3 (C) 9

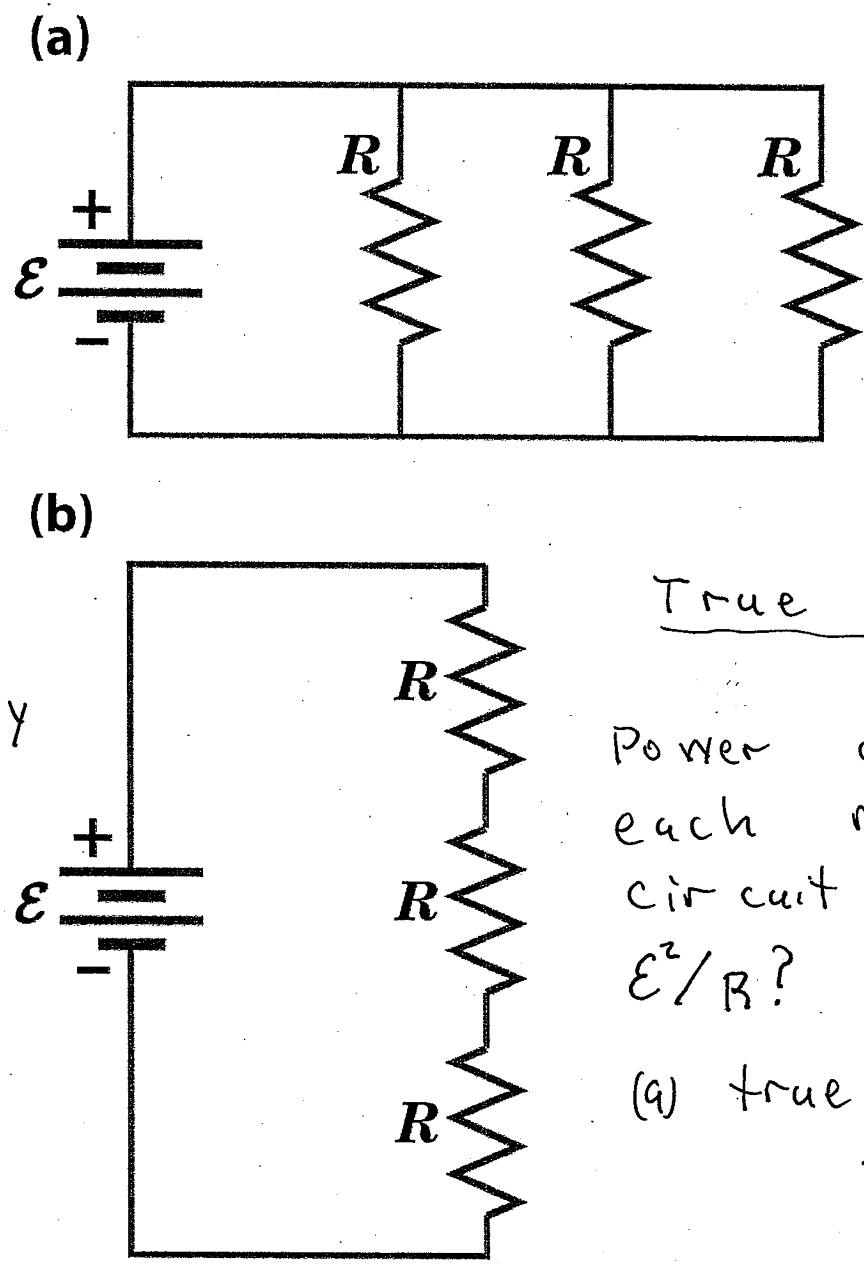
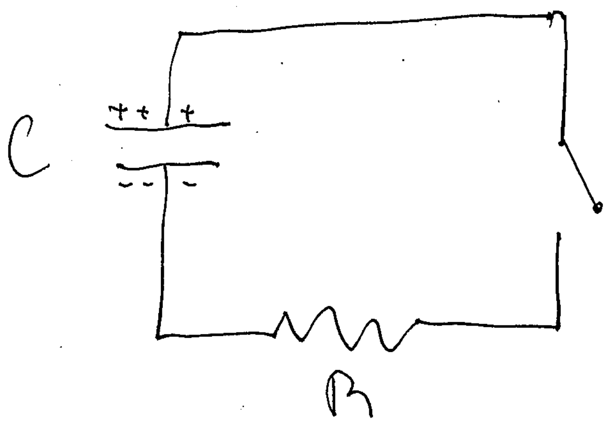


Figure 28-21 Physics for Engineers and Scientists 3/e

True or False

Power dissipated in each resistor in circuit (b) is $\mathcal{E}^2/R$?

(a) true (b) false



If circuit is closed at time $t=0$, will capacitor discharge instantaneously?

(a) yes

(b) no

Khirchoff's Voltage Law still applies with capacitor in circuit

a-b-c-d-a

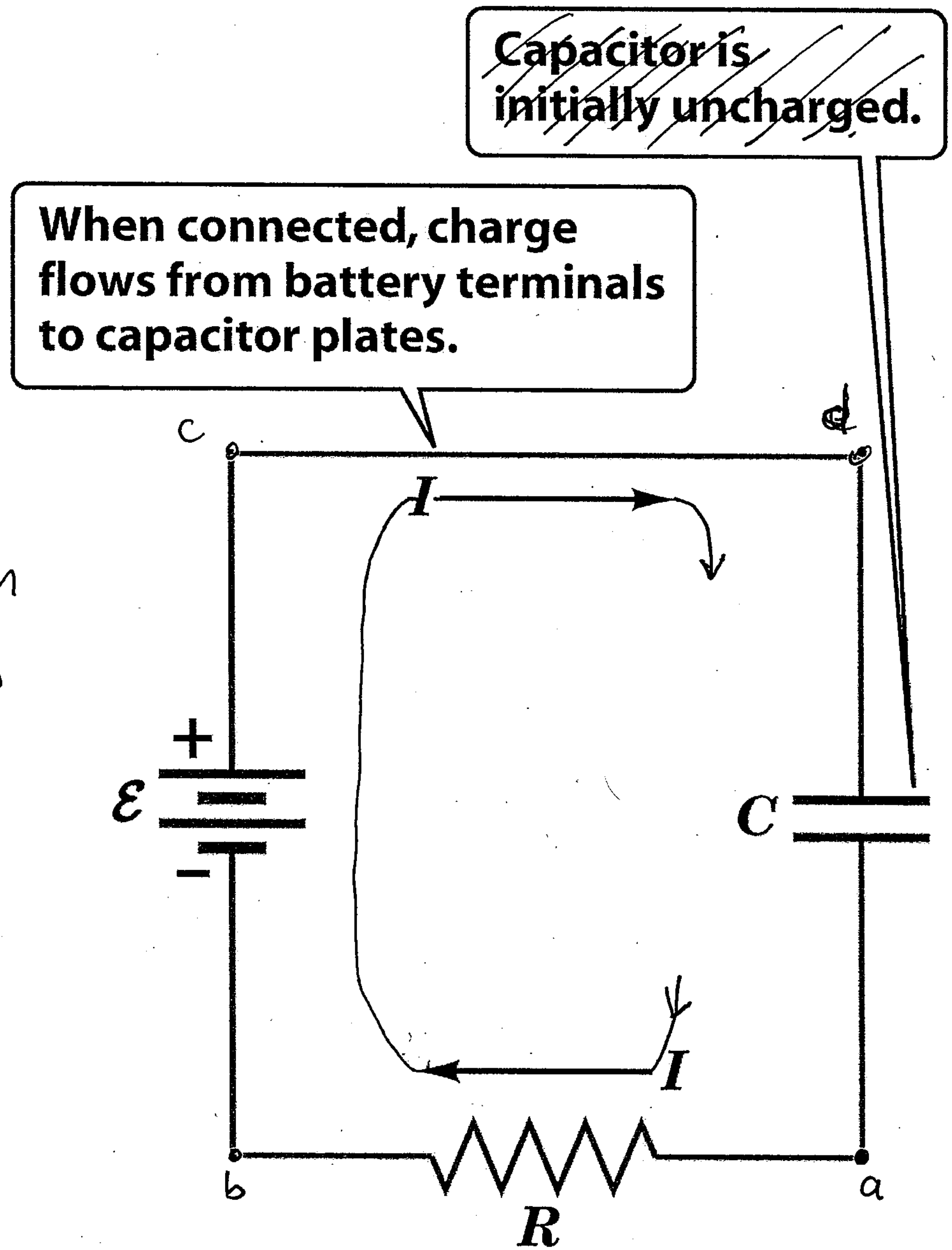
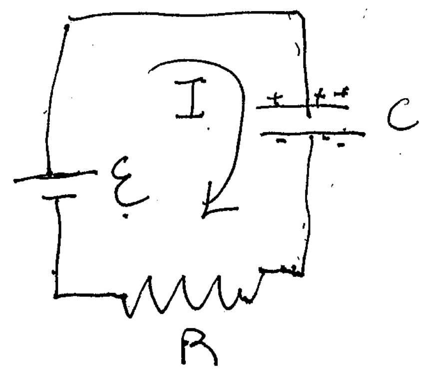


Figure 28-26 Physics for Engineers and Scientists 3/e

Khirkoff's voltage Rule:

$$IR - \mathcal{E} + \frac{Q}{C} = 0$$



But $I = \frac{dQ}{dt}$

$$R\frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E}$$

$$\frac{dQ}{dt} + \frac{Q}{RC} = \frac{\mathcal{E}}{R} \; ; \quad Q(t)$$

This is a first order differential equation:

The solution is a function of time plus an additional constant which can only be determined from initial conditions of how circuit is set up.

Let us first determine most general solution:

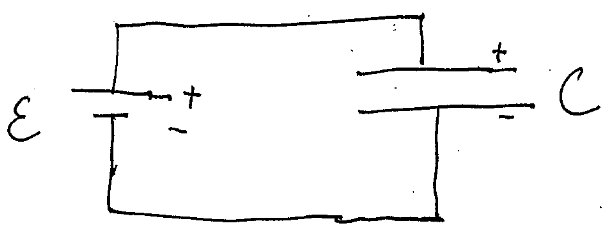
$$\frac{dQ}{dt} + \frac{Q}{RC} = \frac{\mathcal{E}}{R}$$

$$Q = Q_0 + Q_1(t) \; ;$$ Take Q_0 independent of t

$$\frac{dQ_1(t)}{dt} + \frac{Q_1(t)}{RC} + \frac{Q_0}{RC} = \frac{\mathcal{E}}{R}$$

equate

$$\boxed{Q_0 = \mathcal{E}\, C} \; ; \quad \frac{dQ_1(t)}{dt} + \frac{Q_1(t)}{RC} = 0$$



Q_0 is DC charge across capacitor

$$\frac{dQ_1(t)}{dt} + \frac{Q_1(t)}{RC} = 0$$

First order differential equation with constant coefficient

Solution will be of form

$$Q_1(t) = A \exp(-pt) \; ; \quad \frac{dQ_1(t)}{dt} = -p\left(A\, e^{-pt}\right) = -p\, Q_1(t)$$

$$\therefore \quad -p\, Q_1(t) + \frac{Q_1(t)}{RC} = 0$$

$$\left(-p + \frac{1}{RC}\right) Q_1(t) = 0$$

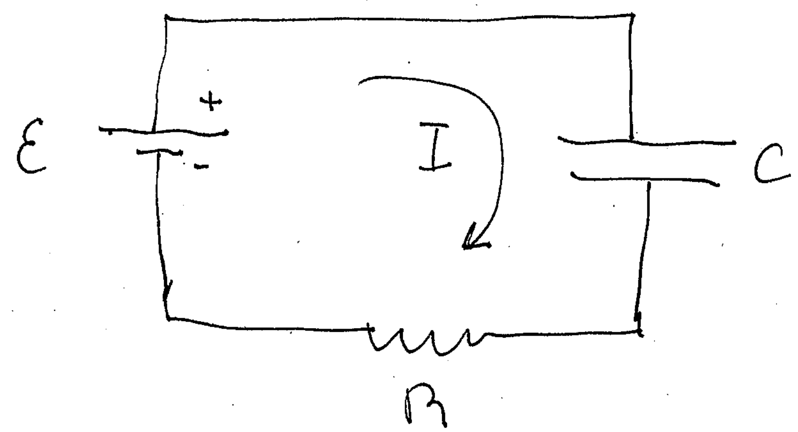
Nontrivial solution $\boxed{p = \frac{1}{RC}}$

Thus, most general solution of Equation

$$\frac{dQ(t)}{dt} + \frac{Q(t)}{RC} = \mathcal{E} \quad , \text{ is}$$

$$\boxed{Q(t) = \mathcal{E}C + A \exp(-t/RC)}$$

A is a constant that depends on initial conditions



Current, I, through resistor, through battery, and into (and out of) capacitor is:

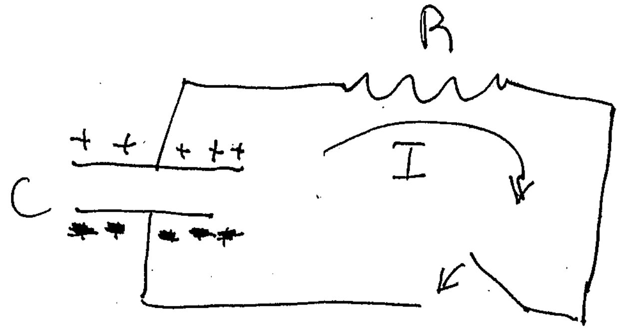
$$I(t) = \frac{dQ(t)}{dt} = \frac{d}{dt}\left[\mathcal{E}C + A \exp(-t/RC)\right]$$

$$\boxed{I(t) = -\frac{A}{RC}\exp\left(-\frac{t}{RC}\right) \quad (\text{independent of Voltage } \mathcal{E})}$$

If A is known, then
With charge and current known in time we can obtain voltage across each element of circuit at any time.

Two important initial conditions

(a) Discharging a capacitor



Charge on capacitor at instant switch closed is Q_0

$\mathcal{E} = 0$

$$Q(t) = A \exp\left(-\frac{t}{RC}\right)$$

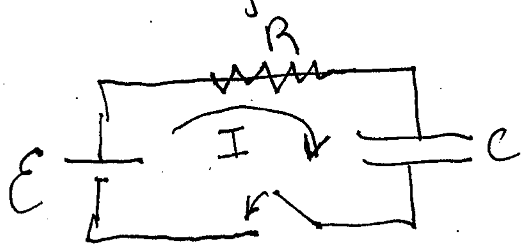
$$Q(t=0) = Q_0$$

$$Q(t=0) = Q_0 = A \exp\left(-\frac{t=0}{RC}\right)$$

$$= A \exp(0) = A$$

$$\boxed{Q(t) = Q_0 \exp(-t/RC)}$$

(b) Charging a capacitor; when zero charge across C, when switch closed



$$Q(t) = \mathcal{E}C + A \exp\left(-\frac{t}{RC}\right)$$

$$Q(t=0) = \mathcal{E}C + A = 0$$

$$A = -\mathcal{E}C$$

$$\boxed{Q(t) = \mathcal{E}C\left(1 - \exp(-t/RC)\right)}$$

Charging a capacitor

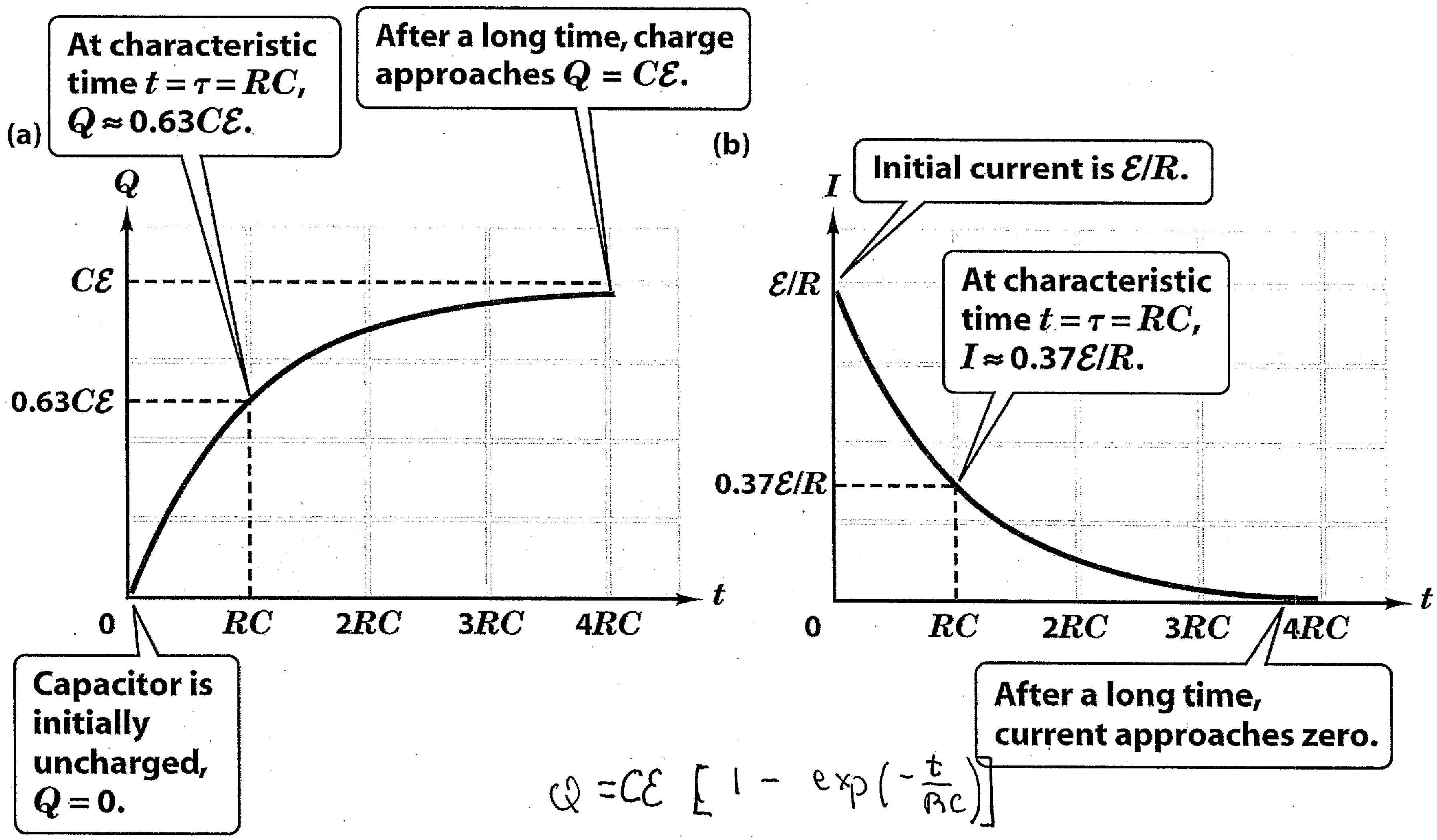


Figure 28-27 Physics for Engineers and Scientists 3/e

$$Q = C\mathcal{E}\left[1 - \exp\left(-\frac{t}{RC}\right)\right]$$

$$= C\mathcal{E}\left[1 - \exp\left(-\frac{t}{\tau_{RC}}\right)\right] \quad \tau = RC\text{-time}$$

$$I = \frac{\mathcal{E}}{R}\exp\left(-t/\tau\right)$$

When switch goes from 2 to 1,
capacitor charges as
$Q(t) = \mathcal{E}C[1 - \exp(t/\tau)]$

$\tau = ?$

(a) $R_1 C$

(b) $R_2 C$

(c) $(R_1 + R_2) C$

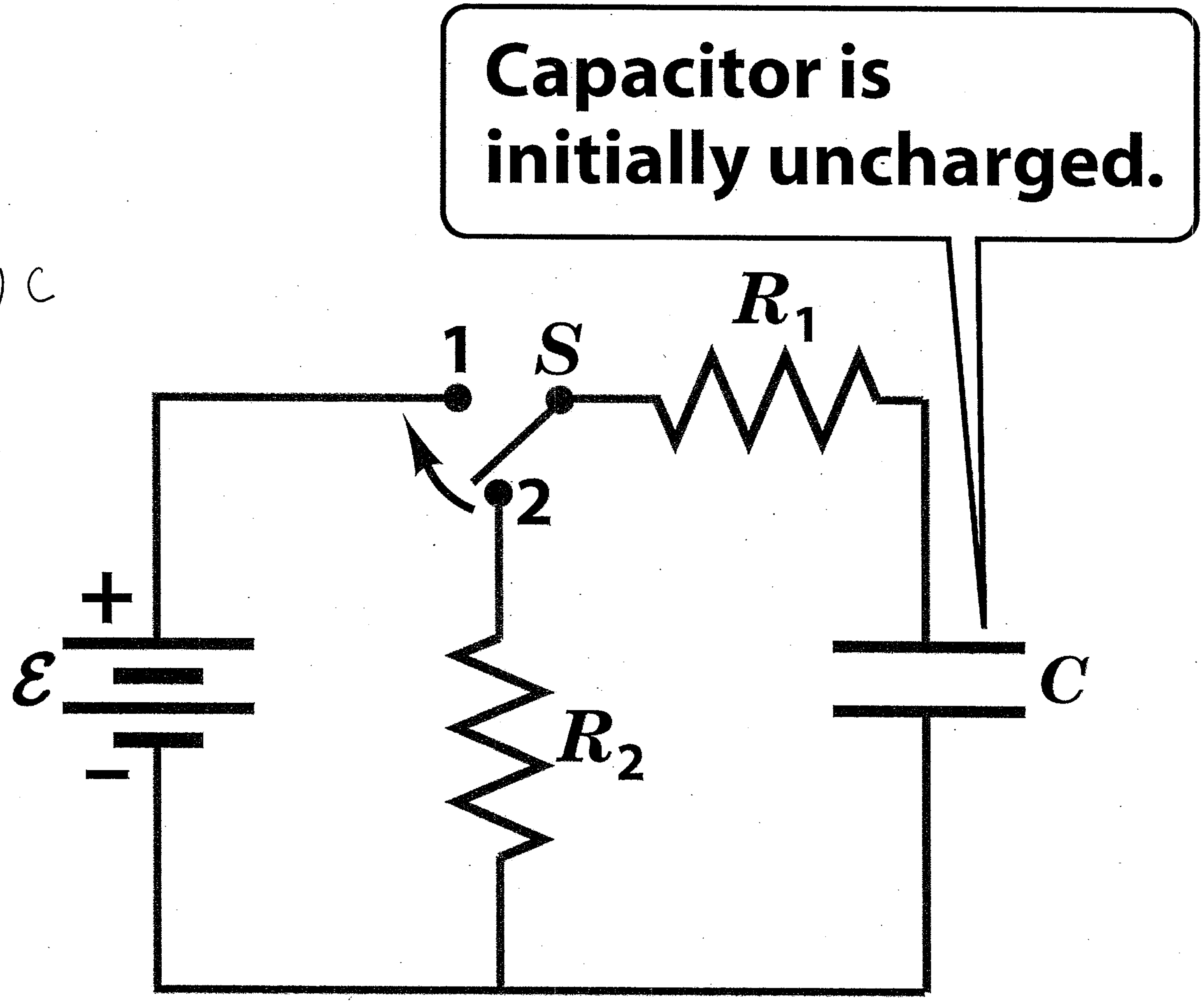
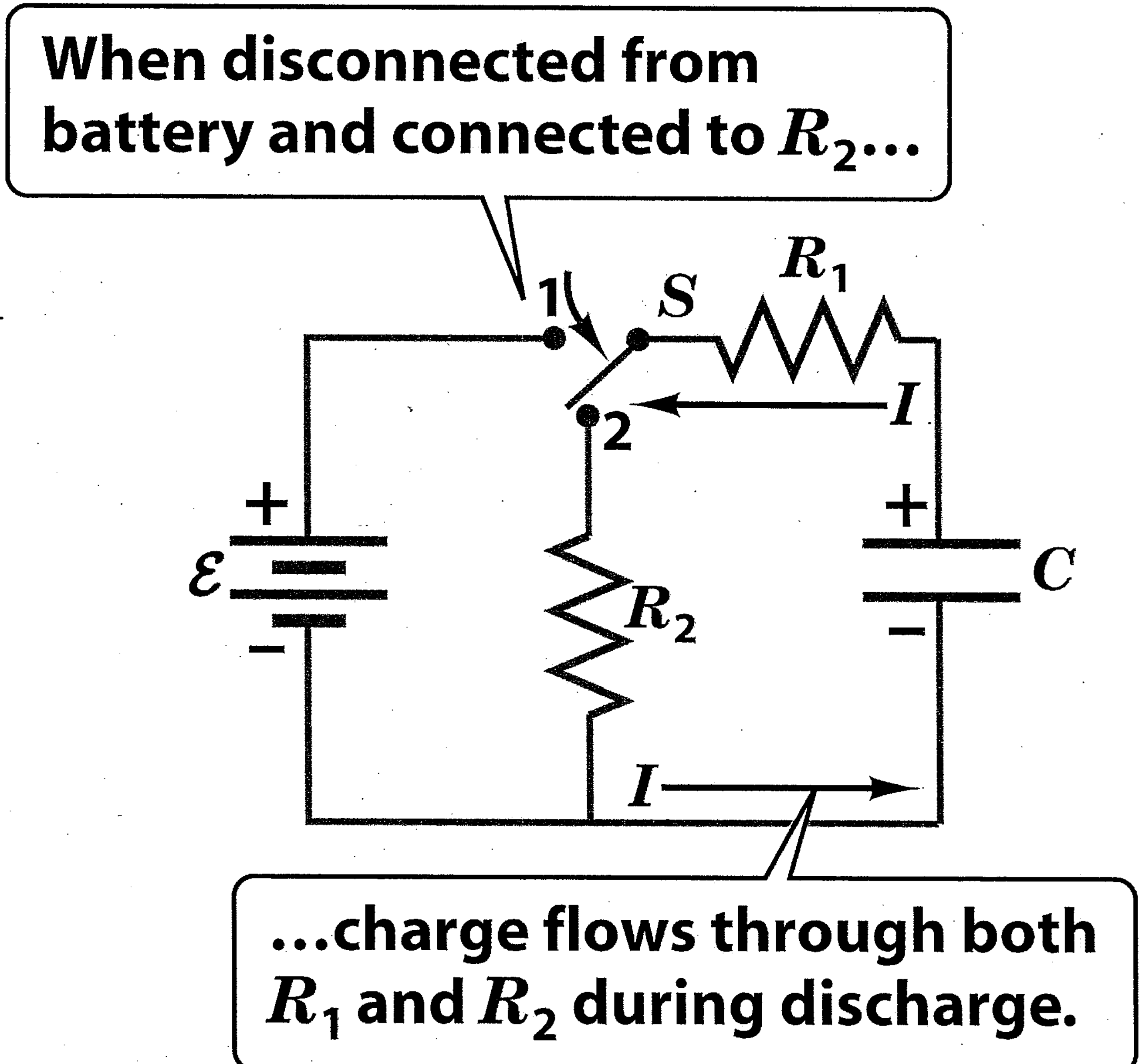


Figure 28-30a Physics for Engineers and Scientists 3/e

When switch goes from (1) to (2) capacitor discharges as $Q(t) = \mathcal{E} C \exp(-t/\tau)$



$\tau = ?$

(a) $R_1 C$

(b) $R_2 C$

(c) $(R_1 + R_2) C$

Figure 28-30c Physics for Engineers and Scientists 3/e