Lecture 11

D-C circuit theory
& Capacitors
If every resistor is 5 Ω, what is the effective resistance of this circuit?

\[ R_{\text{eff}} = \frac{\frac{V}{2} R + \frac{V}{6} R}{3} + \frac{\frac{V}{3} R}{3} \]

\[ R_{\text{eff}} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5}{6} R \]

\[ I_{\text{total}} \]

\[ \frac{I_0}{6} = \frac{5}{3} \]
What are circuit equations for diagram below

For this two-loop circuit...

\[ -\varepsilon_1 + I_1 R_1 + (I_1 - I_2) R_2 = 0 \]
\[ \varepsilon_2 + I_2 R_3 + (I_2 - I_1) R_2 = 0 \]
What are the circuit equations?

For a complicated circuit with several loops and branches...

(a)

\[ (1) \quad -E_1 + R_2 (I_1 - I_2) + I_1 R_1 = 0 \]

\[ (2) \quad I_0 = I_2 R_4 + R_3 (I_2 - I_3) + R_2 (I_2 - I_1) \]

\[ (3) \quad -E_2 + (I_3 - I_2) R_3 = 0 \]
Is true voltage across resistor measured in case (b)?

Is true voltage across resistor measured in case (a)?
Imperfect Ohmeter

\[
\frac{E}{R_x + R_{in}} = I
\]
To "balance" this bridge circuit, calibrated resistor $R_1$ is varied...

...until ammeter reads zero, ...

Does measurement change if battery has internal resistance?

(a) yes  (b) no

...which determines unknown resistance $R_x$.  

Figure 28-25  Physics for Engineers and Scientists 3/e  
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POWER DELIVERED BY A SOURCE OF emf

In time \( \Delta t \), battery delivers charge \( \Delta q \)

\[
\Delta E = P \Delta t = \Delta q \varepsilon \\
\frac{\Delta E}{\Delta t} = P = \frac{\Delta q}{\Delta t} \varepsilon \\

P = E I \\
\frac{\Delta q}{\Delta t} = \frac{dq}{dt} = I
\]
POWER DISSIPATED BY A RESISTOR
(JOULE HEAT)

$P = \Delta V I = I^2 R = \frac{(\Delta V)^2}{R}$

Energy Conservation

$E I = I^2 R_{\text{in}} + I^2 R$

Equation 28-23 derivatives  Physics for Engineers and Scientists 3/e
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(1) What is ratio of current through battery in circuit (a) to circuit (b)?

(A) 1  (B) 3  (C) 9

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(2) What is ratio of power delivered by battery in circuit (a) to circuit (b)?

(A) 1  (B) 3  (C) 9

---

True or False

Power dissipated in each resistor in circuit (b) is $E^2/R$?

(a) true  (b) false

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Figure 28-21 Physics for Engineers and Scientists 3/e
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If circuit is closed at time $t = 0_1$, will capacitor discharge instantaneously?

(a) yes  
(b) no
Khirchoff's Voltage Law still applies with capacitor in circuit.

a-b-c-d-a

Capacitor is initially uncharged.

When connected, charge flows from battery terminals to capacitor plates.

Figure 28-26 Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.
Kirkhoff's voltage rule:

\[ V_R = \mathcal{E} + \frac{q}{C} = 0 \]

But \[ I = \frac{dq}{dt} \]

\[ R \frac{dq}{dt} + \frac{q}{C} = \mathcal{E} \]

\[ \frac{dq}{dt} + \frac{q}{RC} = \frac{\mathcal{E}}{R} \; \Rightarrow \; \mathcal{Q}(t) \]

This is a first order differential equation:

The solution is a function of time plus an additional constant which can only be determined from initial conditions of how circuit is set up.

Let us first determine most general solution:
\[
\frac{dQ}{dt} + \frac{Q}{RC} = \frac{E}{R}
\]

\text{Take}

\[Q = Q_0 + Q_1(t)\]

\[Q_0\text{ independent of } t\]

\[
\frac{dQ_1(t)}{dt} + \frac{Q_1(t)}{RC} + \frac{Q_0}{RC} = \frac{E}{R}
\]

\text{equate}

\[Q_0 = EC\]

\[\frac{dQ_1(t)}{dt} + \frac{Q_1(t)}{RC} = 0\]

\text{\[Q_0\text{ is DC charge across capacitor}\]}

\text{First order differential equation with constant coefficient}

\text{Solution will be of form}

\[Q_1(t) = Ae^{-pt}\]

\[\frac{dQ_1(t)}{dt} = -p(Ae^{-pt})\]

\[\therefore -pQ_1(t) + \frac{Q_1(t)}{RC} = 0\]

\[(-p + \frac{1}{RC})Q_1(t) = 0\]

\text{Non-trivial solution}

\[p = \frac{1}{RC}\]
Thus, most general solution of Equation:

\[ \frac{dQ(t)}{dt} + \frac{Q(t)}{RC} = E \]

\[ Q(t) = EC + AE \exp\left(-\frac{t}{RC}\right) \]

is

\[ A \] is a constant that depends on initial conditions.

Current, \( I \), through resistor, through battery, and into (and out of) capacitor is:

\[ I(t) = \frac{dQ(t)}{dt} = \frac{d}{dt} \left[ EC + AE \exp\left(-\frac{t}{RC}\right) \right] \]

\[ I(t) = -\frac{A}{RC} \exp\left(-\frac{t}{RC}\right) \]

(\( \text{independent of voltage } E \))

If \( A \) is known, then with charge and current known in time we can obtain voltage across each element of circuit at any time.
Two important initial conditions

(a) Discharging a capacitor

\[ Q(t) = A \exp \left( -\frac{t}{RC} \right) \]
\[ Q(t=0) = Q_0 \]
\[ Q(\epsilon=0) = Q_0 = A \exp \left( -\frac{\epsilon}{RC} \right) \]
\[ = A \exp(0) = A \]
\[ Q(t) = Q_0 \exp \left( -\frac{t}{RC} \right) \]

(b) Charging a capacitor when zero charge across \( C \) when switch closed

\[ Q(t) = EC + A \exp \left( -\frac{t}{RC} \right) \]
\[ Q(t=0) = EC + A = 0 \]
\[ A = -EC \]
\[ Q(t) = EC \left( 1 - \exp \left( -\frac{t}{RC} \right) \right) \]
Charging a capacitor

(a) At characteristic time \( t = \tau = RC \), \( Q \approx 0.63CE \).

(b) After a long time, charge approaches \( Q = CE \).

Initial current is \( \frac{\varepsilon}{R} \).

At characteristic time \( t = \tau = RC \), \( I \approx 0.37\varepsilon/R \).

After a long time, current approaches zero.

\[
Q = CE \left[ 1 - \exp\left(-\frac{t}{RC}\right) \right] = CE \left[ 1 - \exp\left(-\frac{t}{\varepsilon_{AC}}\right) \right] \quad \varepsilon = AC - time
\]

\[
I = \frac{\varepsilon}{\tau} \exp\left(-\frac{t}{\tau}\right)
\]
When switch goes from 2 to 1,
capacitor charges as
\[ q(t) = EC \left[ 1 - \exp \left( \frac{t}{\tau} \right) \right] \]

\[ \tau = ? \]
(a) \( R_1 C \)
(b) \( R_2 C \)
(c) \( (R_1 + R_2) C \)

Capacitor is initially uncharged.

Figure 28-30a Physics for Engineers and Scientists 3/e
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When switch goes from (1) to (2)
Capacitor discharges as \( Q(t) = EC \exp(-t/\tau) \)

When disconnected from battery and connected to \( R_2 \)...

\[ \tau = ? \]

(a) \( R_1, C \)
(b) \( R_2, C \)
(c) \( (R_1 + R_2)/C \)

...charge flows through both \( R_1 \) and \( R_2 \) during discharge.

Figure 28-30c Physics for Engineers and Scientists 3/e © 2007 W.W. Norton & Company, Inc.