Lecture #10
Circuit Theory
For series resistors, the same current flows through each...

\[ R_1 \quad \Delta V_1 \]

\[ R_2 \quad \Delta V_2 \]

\[ \Delta V \]

...and the net potential difference is the sum of the individual ones.

\[ \text{Best} = R_1 + R_2 \] for resistor in series

Follows because current the same in each resistor
For parallel resistors, the potential difference across each is the same...

\[ \frac{1}{I_{\text{eff}}} = \frac{1}{R_1} + \frac{1}{R_2} \]

for resistors in parallel

Follows from Voltage constant across both resistors

\[ \Delta V = I_1 R_1 = I_2 R_2 \]

\[ I = I_1 + I_2 \]

Basic Current "Law" (Khirchhoff's current law)

Sum of current into junction vanishes
\[ I_1 + I_2 - I_3 = 0 \]

negative current leaving junction

...and the total current is the sum of the individual parallel currents.
\[ I - I_1 - I_2 = 0 \]
\[ I = I_1 + I_2 \]
\[ I = I_1 + I_2 \]
\[ V = \frac{V_0}{R_1} + \frac{V_i}{R_2} \]
\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \]
What is $I_1/I_2$?
1. 2
2. $1/2$
3. 1

$R_1 = 10 \, \Omega$

$R_2 = 20 \, \Omega$
To determine the net resistance of a complex arrangement...

(a)

...we first identify any simple combinations, like the two series resistors of each branch,...

(b)

...and replace each combination by its equivalent.

\[
\frac{1}{R_{\text{eff}}} = \frac{1}{12} + \frac{1}{22}
\]

\[
R_{\text{eff}} \approx \frac{\frac{132}{17}}{1} = 7.76 \Omega
\]

Figure 27-20 Physics for Engineers and Scientists 3/e
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I = 10 A

Each resistor has the same resistance R.

(a) What is current coming out of node B?

- 10 A
- 5 A
- 20 A

(b) What is voltage drop between D and C?

- 0 V
- IR
- not determined

(c) What is effective resistance, R_eff between terminals A and B?

- R/2
- R
- 2R
Kirchhoff's Voltage Rule

Total Voltage Drop around a circuit vanishes (battery give negative voltage drop, or voltage rise)

\[ \overrightarrow{AB} + \overrightarrow{BA} \]
\[ \overrightarrow{IR} - V = 0 \]
\[ \overrightarrow{IR} = V \]

Double E's method to shorten Kirchoff current rule

At A
- \( I_a \) in leg 1
- \( -I_b \) in leg 2
- \( -I_a + I_b \) in leg 3

Sums to zero
Formal Procedure for simple parallel circuit

\[
\begin{align*}
V & \quad I_a \quad R_2 \quad R_1 \\
\text{from } a \rightarrow b \rightarrow c \rightarrow d \rightarrow a & \\
(a) \quad (I_a - I_b)R_2 - V = 0 & \\
\text{from } a \rightarrow e \rightarrow f \rightarrow b \rightarrow a & \\
(b) \quad I_bR_1 + (I_b - I_a)R_2 = 0 & \\
R_{\text{eff}} &= \frac{V}{I_a} & \\
\text{We need to solve for } I_a \text{ in terms of } V. \text{ First eliminate } I_b & \\
\text{from (b)} \quad I_b &= I_a \frac{R_2}{R_1 + R_2} & \\
\text{Substitute into (a)} & \\
\text{from (a)} \quad R_2I_a - I_a \frac{R_2^2}{R_1 + R_2} &= V & \\
\frac{R_1R_2}{R_1 + R_2} I_a = V \quad ; \quad R_{\text{eff}} = \frac{V}{I_a} = \frac{R_1R_2}{R_1 + R_2} \quad (21)
\end{align*}
\]
Batteries have internal resistance.

What is current through battery?

What is voltage at battery terminals A and B?

\[ \text{Ieff} = \frac{1}{\text{Rin}} + \frac{1}{\frac{1}{\text{R_a}} + \frac{1}{\text{R_b}}} \]

\[ = \text{Rin} + \frac{\text{R_a R_b}}{\text{R_a} + \text{R_b}} \]

\[ I = \frac{V}{\text{Ieff}} = \frac{V}{\text{Rin} + \frac{\text{R_a R_b}}{\text{R_a} + \text{R_b}}} < \frac{V}{\text{R_a R_b} / \text{R_a} + \text{R_b}} \]

Performance of battery lower than rated voltage without current.
(1) Every loop should have at least one element with just one unique current.

(2) Every element should have at least one current passing through.
\[ V = -V + [(I_1 - I_u)\phi + I - I_d] \]

\[ 0 = I_u R + (I_u - I_d)R + (I_u - I)R \]

\[ 0 = I_d R + (I_d - I)R + (I_d - I_u)R \]

\[ V_R = 2I - I_u - I_d \]

\[ 0 = -I + 3I_u - I_d \]

\[ \Rightarrow \theta = I_u - I + 3I_u = 4I_u \]

\[ 0 = -4I + 8I_u \]

\[ I_u = \frac{1}{2} I \]

from middle \[ 0 = -I + \frac{3}{2}I - I_d \]

\[ -I_d = \frac{1}{2} I \]

\[ \text{Mett} = \frac{V}{I} = \frac{R}{I} \left[ 2I - \frac{1}{2}I - \frac{1}{2}I \right] = R \]

What is the fast way to get this answer?
More Complex Circuit

What is current in $R_{12}$?

" " $V_2$?

" " $V_1$?

What is voltage drop across $R_{12}$

$0 = -V_1 + I_a R_{ii} + (I_a - I_b) R_{12}$

$0 = V_2 + I_b R_{12} + (I_b - I_a) R_{12}$
\[ V_1 = I_a (R_{11} + R_{12}) - I_b R_{12} \]

\[ V_2 = -I_a R_{12} + I_b (R_{12} + R_{12}) \]

\[ V_1 = 1V, \quad R_{11} = 1\Omega, \quad R_{12} = 2\Omega, \quad R_{12} = 5\Omega \]

\[ 1 = 3I_a - 2I_b \]

\[ 2 = -2I_a + \frac{5}{2}I_b \]

\[ (-2 = -2I_a + \frac{5}{2}I_b)^{\frac{3}{2}} \Rightarrow -3 = -3I_a + \frac{15}{4}I_b \]

\[ 1 = 3I_a - 2I_b \]

\[ -2 = \frac{7}{4}I_b \]

\[ I_b = -\frac{8}{7}A \]

\[ 3I_a = 1 + 2I_b \]

\[ I_a = \frac{1}{3} \left[ 1 - \frac{16}{7} \right] = \frac{-1}{3} \frac{9}{7} = -\frac{3}{7}A = I_a \]

Current Through R_{12} (from \( \alpha \) to \( \beta \)), \( I_a - I_b = -\frac{3}{7} - (-\frac{8}{7}) = \frac{5}{7}A \)

\[ I_b (current \ through \ V_2) = -\frac{8}{7}A, \quad I_a = -\frac{3}{7}A \]

Voltage Drop \( \Delta V_{\alpha \beta} = \frac{5}{7}A \cdot 2 = \frac{10}{7}V \)
Each resistor has resistance $R$.

What is $R_{\text{eff}}$?

$I_{\text{total}}$ \[ P_{\text{eff}} = \begin{cases} (a) \frac{2RS}{5} \\ (b) \frac{3RS}{5} \\ (c) \frac{5RS}{6} \end{cases} \]