Lecture #12

Circuit Theory

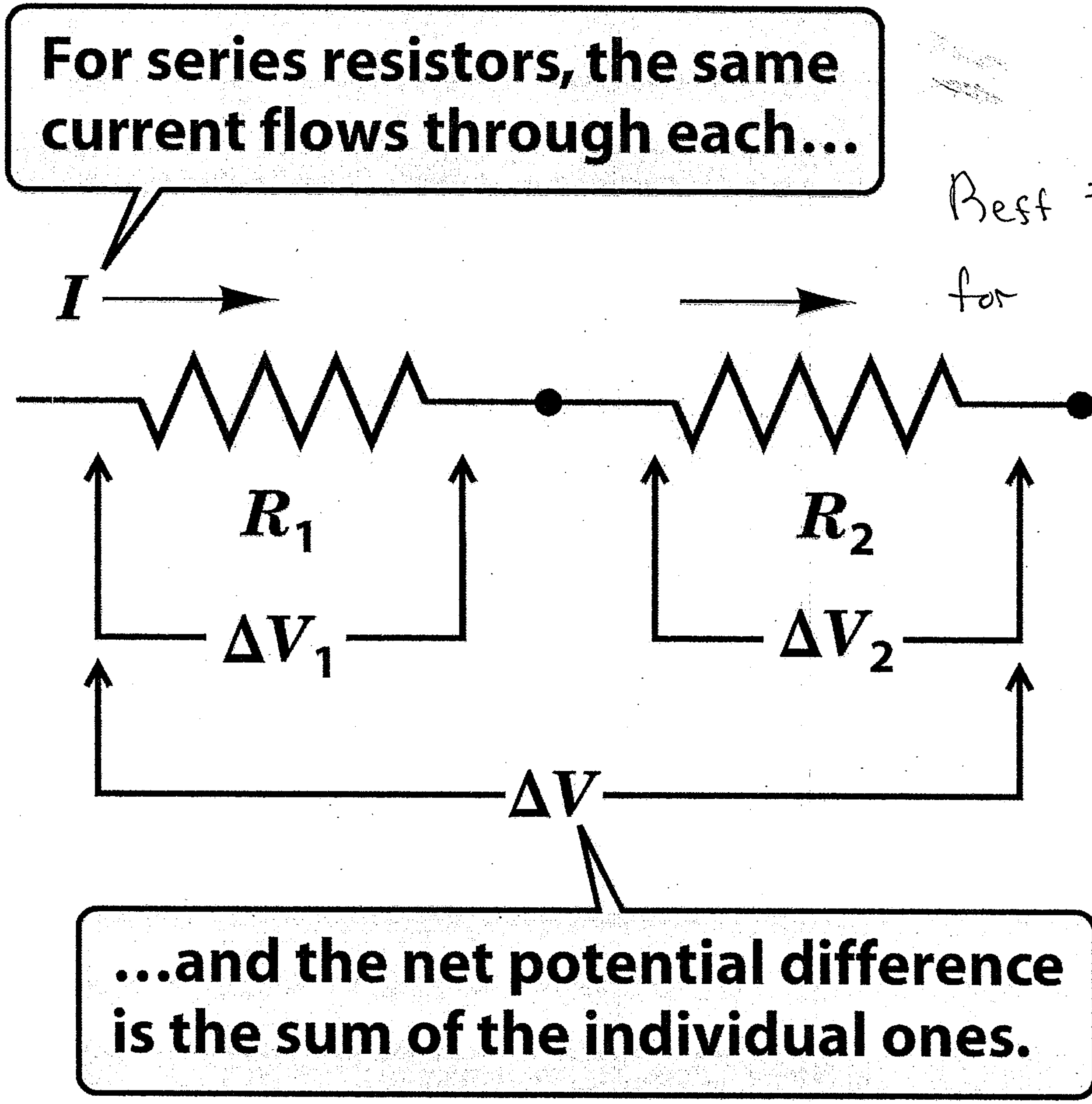



Figure 27-14 Physics for Engineers and Scientists 3/e


$R_{eff} = R_1 + R_2$

for resistor in series

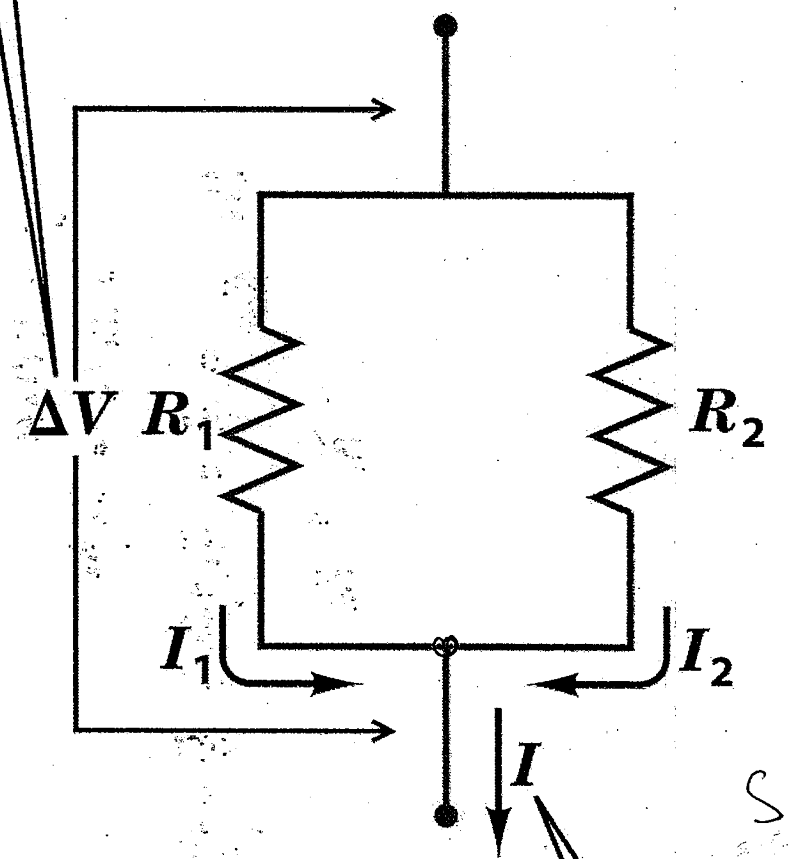
Follows because current the same in each resistor

**For parallel resistors, the potential difference across each is the same…**

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$

for resistors in parallel

Follows from Voltage constant across both resistors




Basic Current "Law" (Khirchoff's current law)

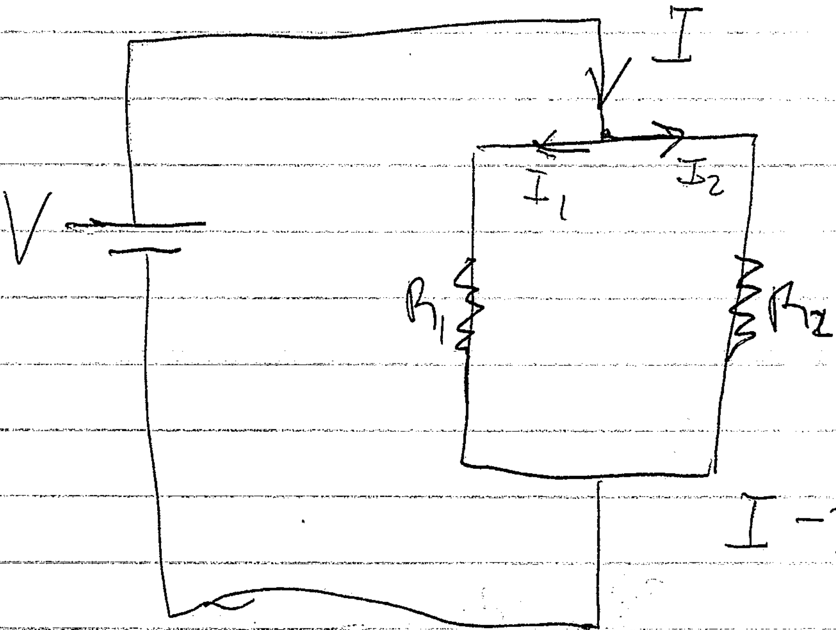
Sum of current into a junction vanishes

$$I_1 + I_2 - I_3 = 0$$

negative means current leaving junction

**…and the total current is the sum of the individual parallel currents.**

Figure 27-16 Physics for Engineers and Scientists 3/e

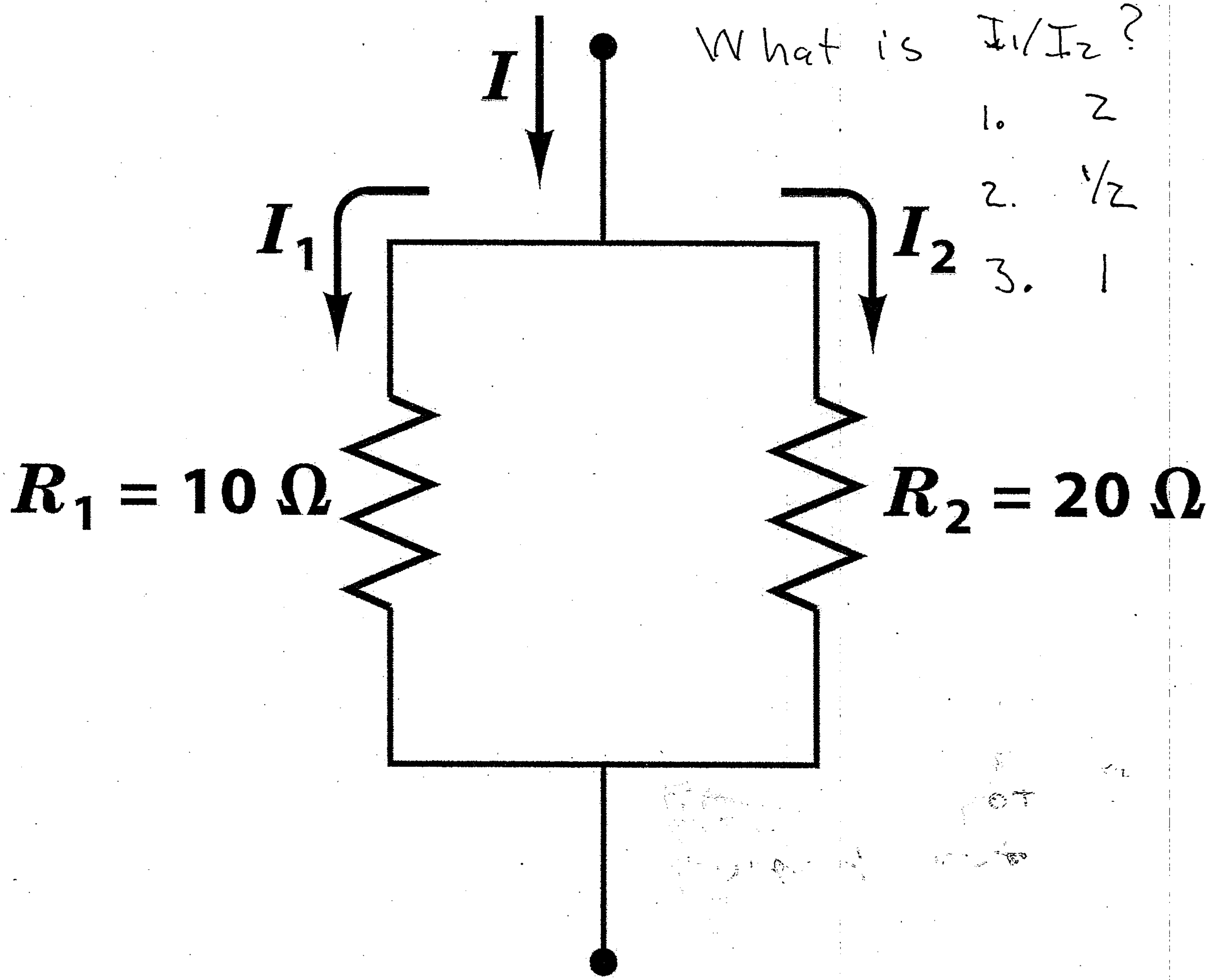



$$I - I_1 - I_2 = 0$$

$$I = I_1 + I_2$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$




What is $I_1/I_2$?

1. 2
2. 1/2
3. 1

Figure 27-19 Physics for Engineers and Scientists 3/e

Simplifying a circuit diagram

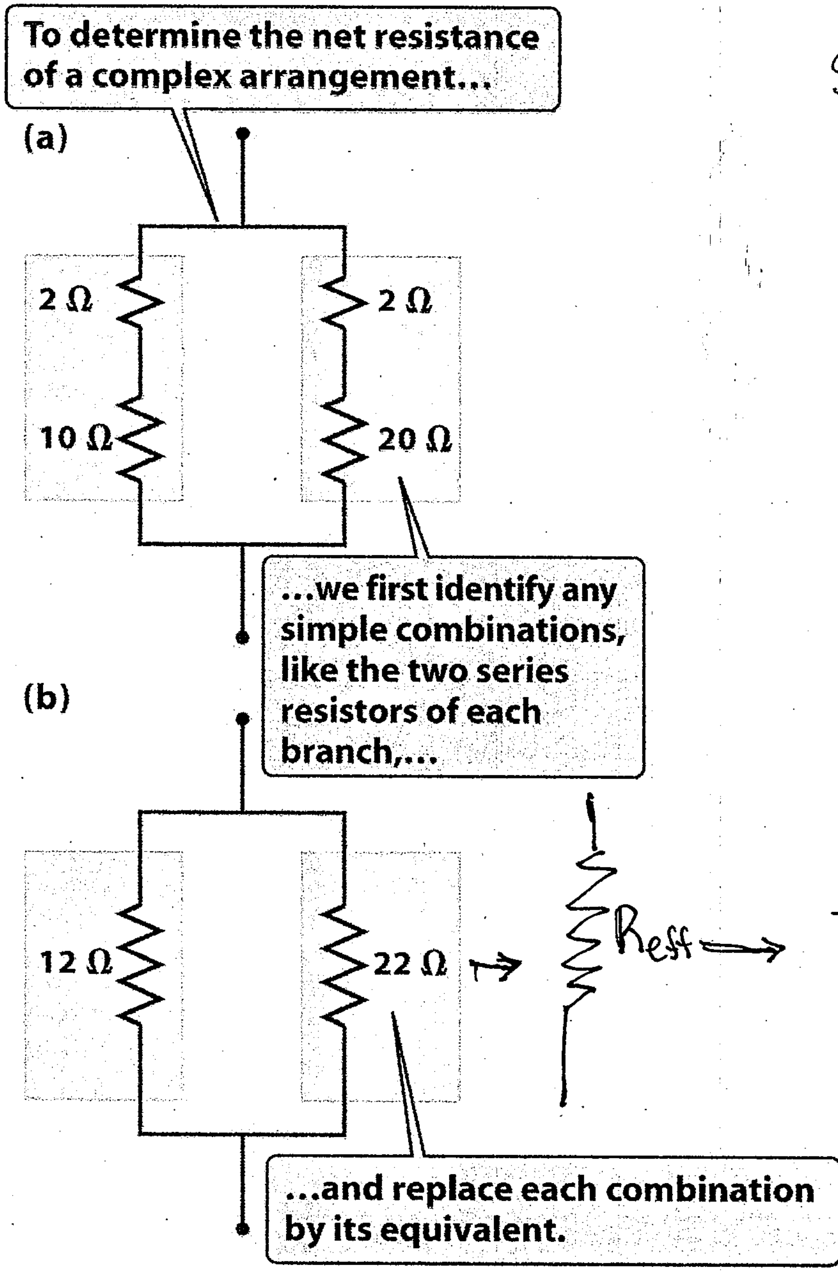



Figure 27-20 Physics for Engineers and Scientists 3/e


$$\frac{1}{R_{eff}} = \frac{1}{12} + \frac{1}{22}$$

$$R_{eff} = \frac{132}{17} = 7.76\,\Omega$$

Find $R_{eff}$

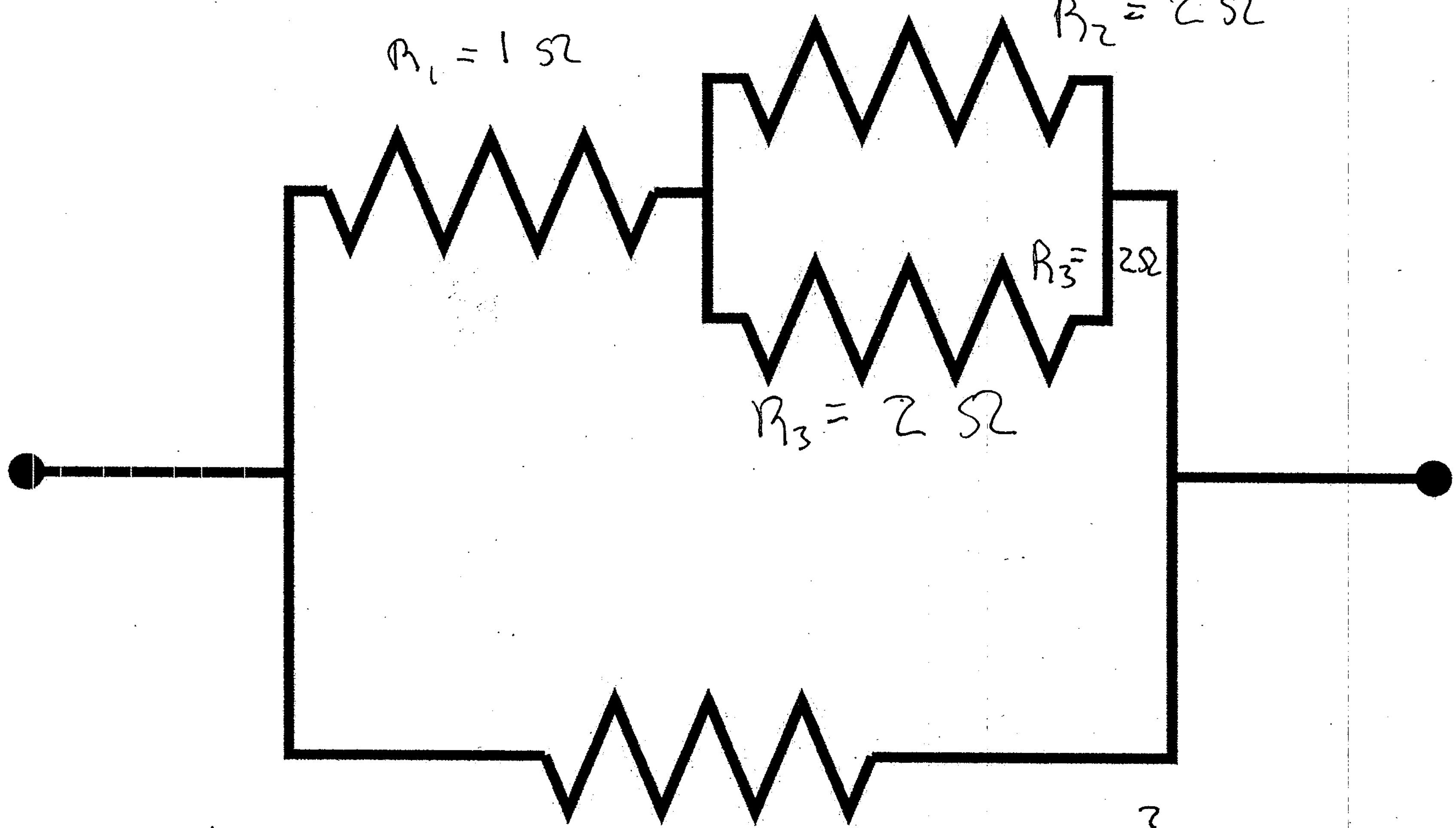
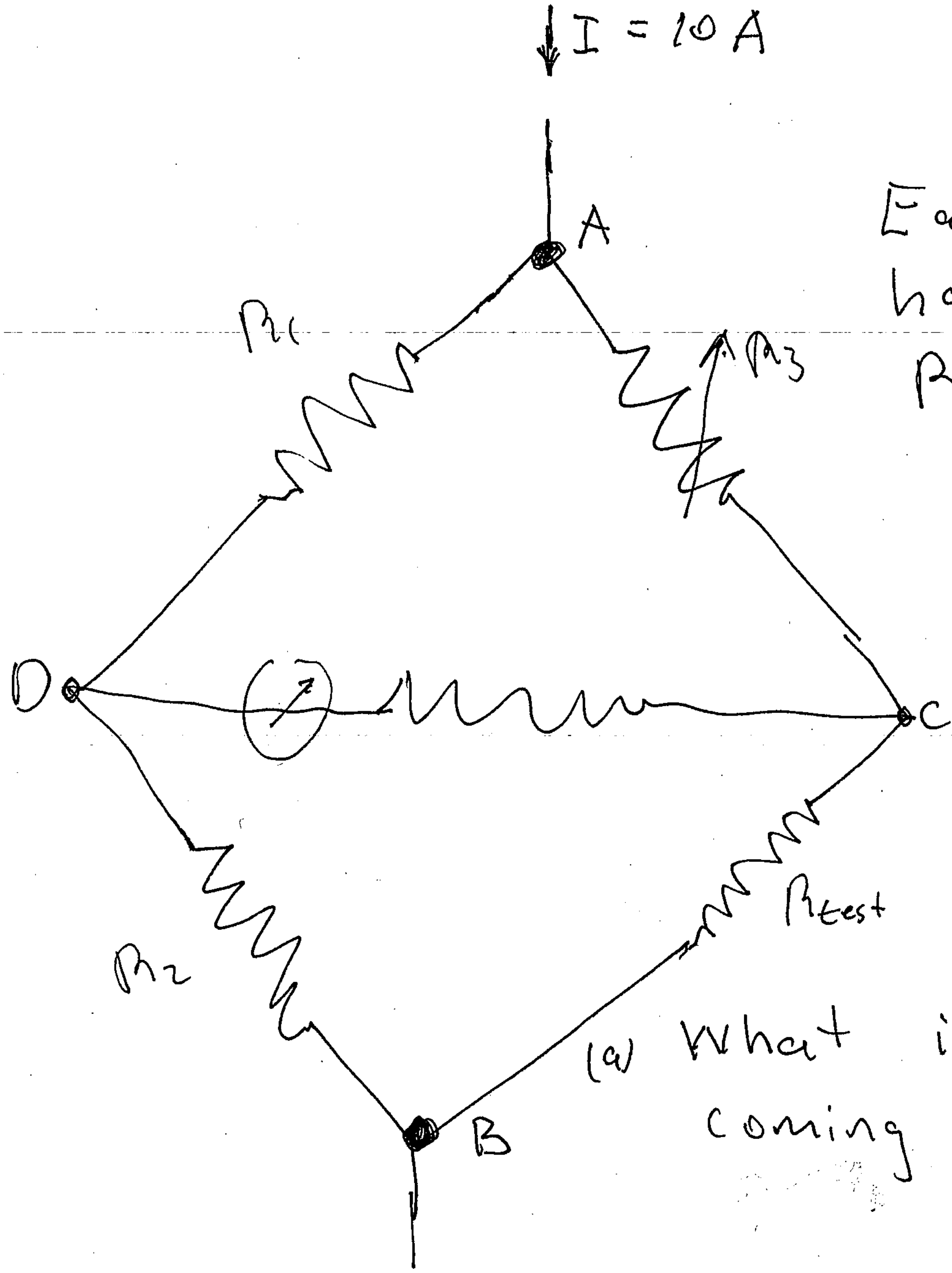



Figure 27-29 Physics for Engineers and Scientists 3/e


$R_{eff} \stackrel{?}{=}$

(a) $1/2\ \Omega$
(b) $2\ \Omega$
(c) $1\ \Omega$




Each resistor has same resistance R

(a) What is current coming out of node B

(1) 10 A (2) 5 A (3) 20 A

(b) What is voltage drop between D & C?

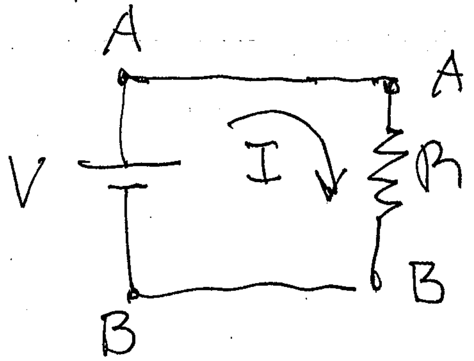
(a) IR (b) 0 (c) not determined

(c) What is effective resistance, Reff between terminals A and B?

(a) R/2 (b) R (c) 2R

# Khirchoff's Voltage Rule

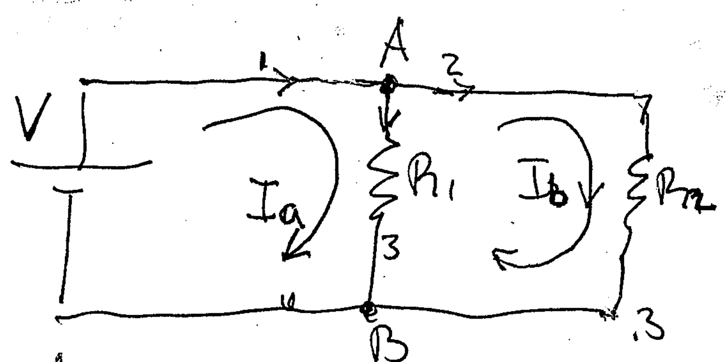
Total Voltage Drop around a circuit vanishes (battery give negative voltage drop, or voltage rise)




$\vec{AB} + \vec{BA}$

$IR - V = 0$

$IR = V$




Double E's method to shorten Kirchoff current rule

at A

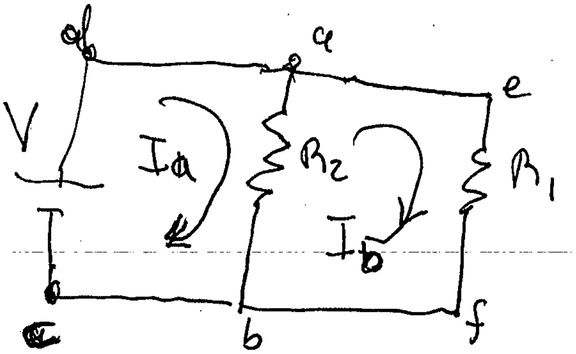
$I_a$ in leg 1

$-I_b$ in leg 2

$-I_a + I_b$ in leg 3

sums to zero

# Formal Procedure for simple parallel circuit




from a - b - c - d - a

$$(\alpha) \quad (I_a - I_b) R_2 - V = 0$$

from a - e - f - b - a

$$(\beta) \quad I_b R_1 + (I_b - I_a) R_2 = 0$$

$$R_{eff} = \frac{V}{I_a}$$

We need to solve for $I_a$ in terms of V. First eliminate $I_b$

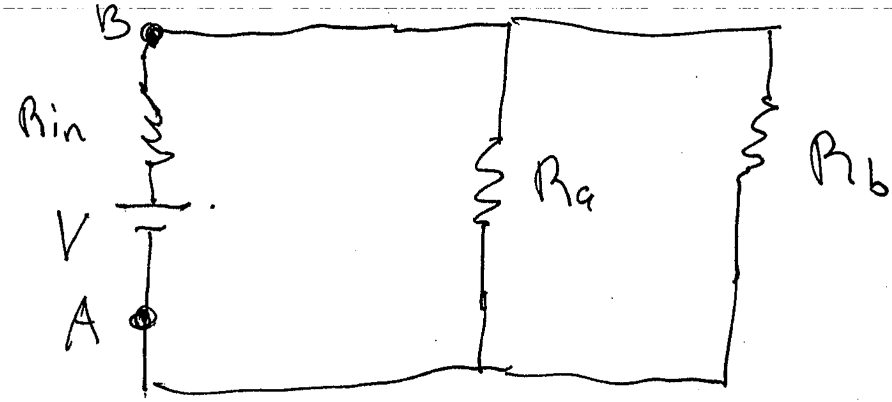
from $(\beta)$ $\quad I_b = I_a \dfrac{R_2}{R_1 + R_2}$

Substitute into $(\alpha)$

from $(\alpha)$ $\quad R_2 I_a - \dfrac{I_a R_2^2}{R_1 + R_2} = V$

$$\frac{R_1 R_2}{R_1 + R_2} I_a = V \quad ; \quad R_{eff} = \frac{V}{I_a} = \frac{R_1 R_2}{R_1 + R_2} \qquad (22)$$

# Batteries Have internal Resistance




What is current through battery?

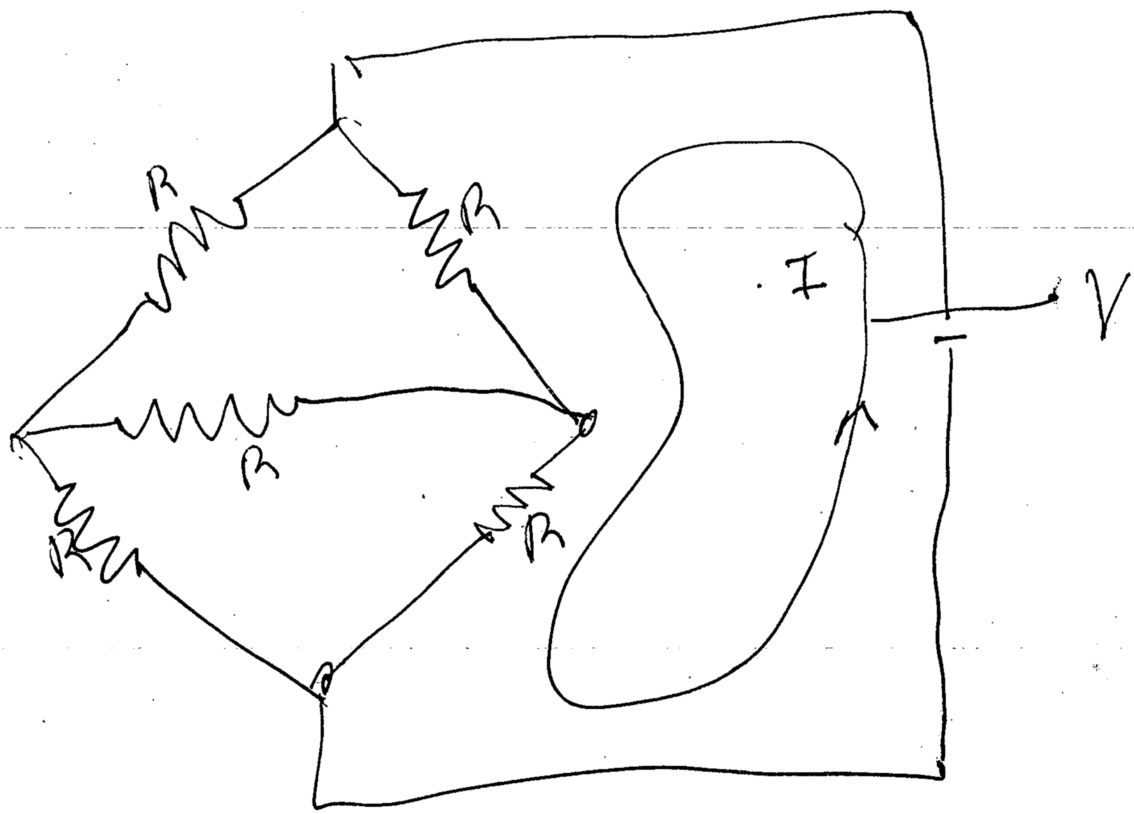
What is voltage at battery terminals A & B

$$R_{eff} = R_{in} + \frac{1}{\frac{1}{R_a} + 1/R_b}$$

$$= R_{in} + \frac{R_a R_b}{R_a + R_b}$$

$$I = \frac{V}{R_{eff}} = \frac{V}{R_{in} + R_a R_b / R_a + R_b} < \frac{V}{R_a R_b / R_a + R_b}$$

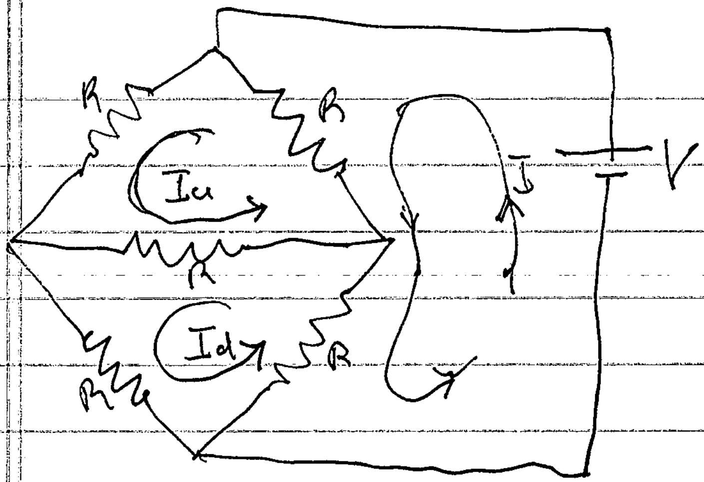
performance of battery lower than rated voltage without current




(1) Every loop should have at least one element with just one unique current

(2) Every element should have at least one current passing them

α




$$0 = -V + \left[(I - I_u) + I - I_d\right]$$

$$0 = I_u R + (I_u - I_d)R + (I_u - I)R$$

$$0 = I_d R + (I_d - I)R + (I_d - I_u)R$$

$$V/R = 2I - I_u - I_d$$

$$0 = -I + 3I_u - I_d$$

$$(0 = -I - I_u + 3I_a)\,3$$

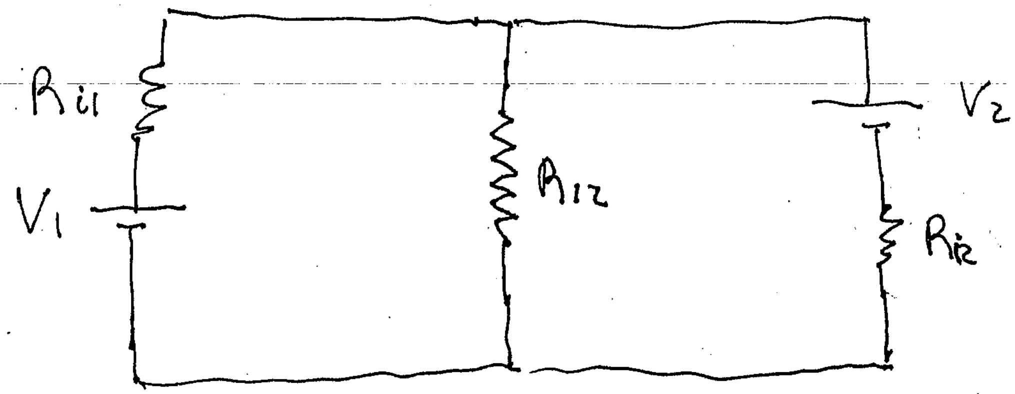
$$0 = -4I + 8I_a \;; \quad I_u = \tfrac{1}{2}I$$

from middle $0 = -I + \frac{3}{2}I - I_d \;\therefore\; I_d = \frac{1}{2}I$

$$R_{eff} = \frac{V}{I} = \frac{R}{I}\left[2I - \tfrac{1}{2}I - \tfrac{1}{2}I\right] = R$$

What is the fast way to get this answer?

# More Complex Circuit

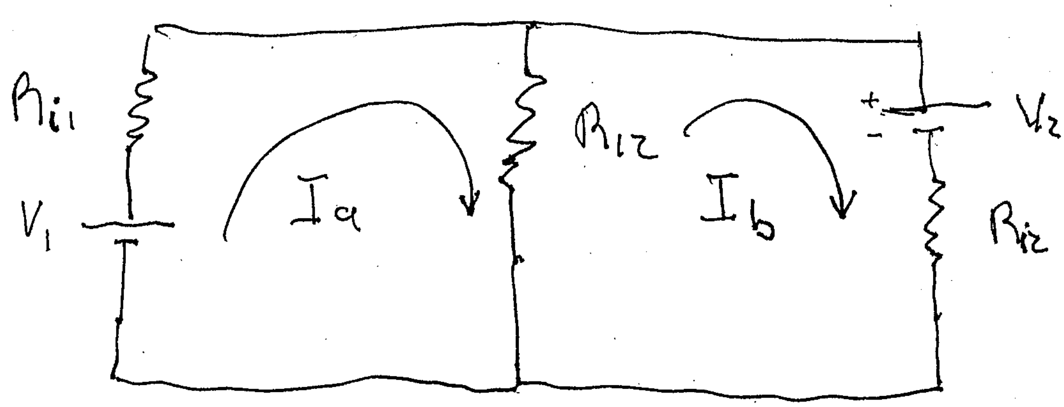



What is current in $R_{12}$ ?

" " $V_2$ ?

" " $V_1$ ?

What is voltage drop across $R_{12}$

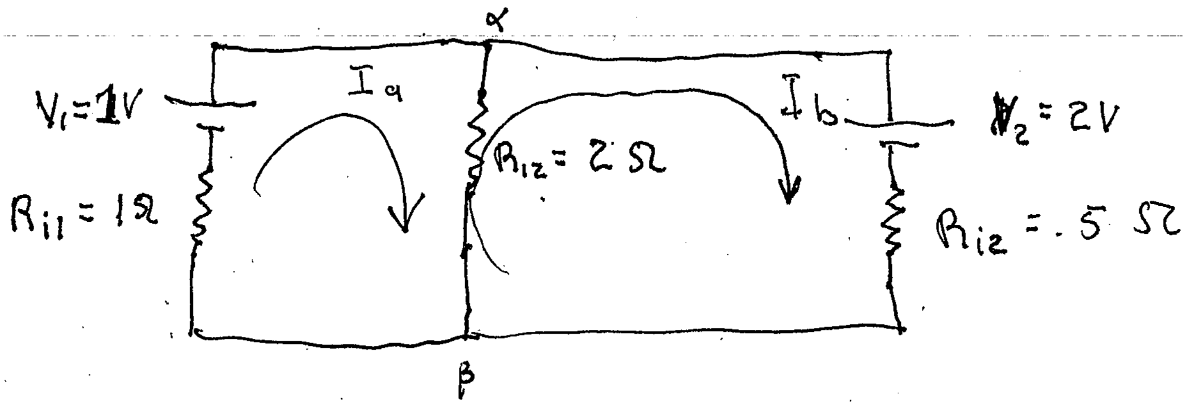



$$0 = -V_1 + I_a R_{i1} + (I_a - I_b) R_{12}$$

$$0 = V_2 + I_b R_{i2} + (I_b - I_a) R_{12}$$

$$V_1 = I_a(R_{i1} + R_{12}) - I_b R_{12}$$

$$-V_2 = -I_a R_{12} + I_b (R_{12} + R_{i2})$$




$$1 = 3I_a - 2I_b$$

$$-2 = -2I_a + 5/2\, I_b$$

$$\left(-2 = -2I_a + \tfrac{5}{2} I_b\right) 3/2 \Rightarrow -3 = -3I_a + \tfrac{15}{4} I_b$$

$$1 = 3I_a - 2I_b$$

$$-2 = \tfrac{7}{4} I_b$$

$$I_b = -\tfrac{8}{7}\text{ A}$$

$$3I_a = 1 + 2I_b$$

$$I_a = \tfrac{1}{3}\left[1 - \tfrac{16}{7}\right] = \tfrac{-1}{3}\,\tfrac{9}{7} = -\tfrac{3}{7}\text{ A} = I_a$$

Current Through $R_{12}$ (from α to β), $I_a - I_b = \frac{-3}{7} - \left(-\frac{8}{7}\right) = \frac{5}{7}$ A

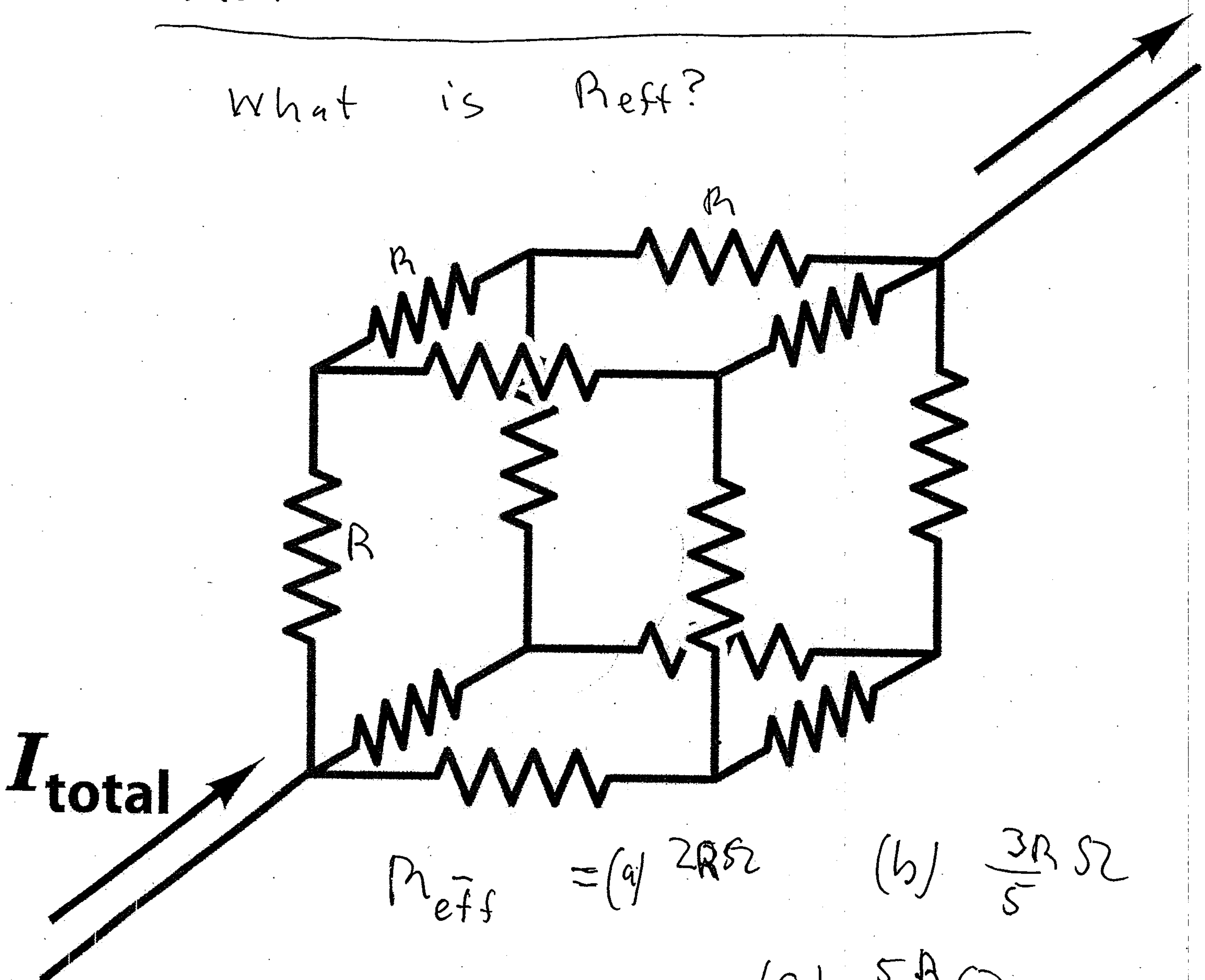
" 5/7 A

$I_b$ (Current Through $V_2$) $= -\frac{8}{7}$ A, $I_a = -\frac{3}{7}$ A

Voltage Drop $\Delta V_{\alpha\beta} = \frac{5}{7}\text{ A} \cdot 2 = \frac{10}{7}$ V

Each resistor has resistance R.

What is $R_{eff}$?




$R_{eff}$ = (a) $2R\,\Omega$  (b) $\frac{3R}{5}\,\Omega$

(c) $\frac{5}{6}R\,\Omega$