Lecture # 31
Preview Problems
Insight into Poynting’s Flux

A wire radius a, length l, carries a current $I$. Resistance is $R$.

Power, $P$, converted into heat:

$$P = I^2 R$$

Poynting Flux gives same result.

Voltage Drop:

$$V = IR$$

$$\mathbf{E} = \frac{IR}{l}$$

$$\mathbf{E} \cdot d\mathbf{r}$$

$$2\pi r \mathbf{B} = \mu_0 \mathbf{I}$$

at $r = a$

$$\mathbf{B}(a) = \frac{\mu_0 I}{2\pi a}$$

$$\mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0}$$

is into wire: Gives “wave” interpretation of dissipation

$$\oint_{S \cdot \mathbf{A}} \mathbf{E} \times \mathbf{B} \, d\mathbf{A} = \frac{\mathbf{E} \cdot \mathbf{B}}{\mu_0} 2\pi r dl = \frac{Ih}{\mu_0 l} \cdot \frac{\mu_0 I^2 r_{nad}}{2\pi}$$
Two protons are moving in the same uniform magnetic field. The orbital radius of the first proton is 10 cm, and the orbital radius of the second is 20 cm. Which proton has the larger speed? Which has the larger orbital frequency?

a. First, first
b. First, second
c. Second, first
d. Second, second
e. Second, both same

\[ l_{1}^{s^{+}} \quad 10 \text{ cm} = v_{L_{1}} \]

\[ l_{2}^{n^{0}} \quad 20 \text{ cm} = v_{L_{2}} \]

\[ ma = vBe \]

\[ \frac{mV}{B} = vBe \]

\[ R = \frac{mv}{eB} \]

\[ \omega = \frac{v}{R} = \frac{eB}{m} \]
What is the direction of the torque on coil?

The direction of the torque in the x-y plane as indicated. The direction shown in the figure below, and is shown in the x-z plane carries a current in the...
Transverse Hall voltage... and a corresponding balancing electric force... until charge builds up.

\[ \text{W is velocity of the}\]

\[ \text{Neither } W \text{ nor } E \]

\[ \text{Both } W \text{ and } E \]

\[ E \text{ only} \]

\[ W \text{ only} \]

Electrons with average vectors would reverse direction. Which current and magnetic field directions shown, consider instead positive charge carriers. Which is shown for negative charge carriers. For the case of negative charge carriers, the Hall effect geometry of the figure below shows... Checkup 30.5
Explain: Find the magnitude of $\mathbf{B}$ in the case described. What would be the direction of $\mathbf{B}$ if $g$ were negligible?

**Hint:** $F = q \mathbf{v} \times \mathbf{B}$.

<table>
<thead>
<tr>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

\[ \mathbf{E} \times \mathbf{B} = \mathbf{V} \]

through the E-field region without any deflection.

A charged particle with a positive charge $q$ is moving with velocity $\mathbf{v}$ along the negative $x$ direction. Find the direction of the magnetic field $\mathbf{B}$ such that $q$ may pass the positive $x$-direction as shown. $\mathbf{E}$ is pointing along the negative $y$ direction.
Hint: Use $A\vec{F} = \vec{I} \times \vec{B}$.

Dir of magnetic force

+! -!

1 2 3 4

Determine the direction of force on the current:
bar magnets. $\vec{B}$ is downward (−!) and the current $\vec{I}$ is into the page (−→).

The diagram shows an electric current passing between a gap between two
currents.

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The diagram shows a magnetic field passing along a current-carrying loop. The current \( \mathbf{l} \) is counterclockwise. The loop area is \( \mathbf{a} \times \mathbf{b} \). And \( \mathbf{B} \) is along \(-\mathbf{i}\). Find the direction of the torque \( \tau \) on the loop due to \( \mathbf{B} \):

\[
\mathbf{\tau} = \mathbf{l} \times \mathbf{B}
\]

\[
\hat{n} = \hat{\mathbf{k}}
\]

\[
\hat{\mathbf{z}} = \hat{\mathbf{\alpha}} \times \hat{\mathbf{B}}
\]

\[
\mathbf{\hat{k} \times (-\mathbf{i})} = -\mathbf{j}
\]

**Hint:** Use \( \mathbf{F} = \mathbf{l} \times \mathbf{B} \). For the direction of the torque, use the right-hand rule of rotation.

**Extra:** The following sketch is viewed from the bottom along \(+\mathbf{j}\):
Consider the setup shown of a Hall experiment. The cross section of the metal strip (the shaded area) is given by $t \times d$. $B$ is into the paper. Compare the potentials at $A$ and $C$:

\[
\begin{array}{cc}
A & B \\
V_A > V_C & V_A < V_C
\end{array}
\]

**Hint:** Use $F = qv \times B$. There should be excess charges at the upper and lower edges. This leads to a potential difference.

**Extra:** Show that the magnitude of Hall voltage $|V_A - V_C| = v_d B d$. The potential difference here is given by $V = \text{Work}/q = Fd/q$. 

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The diagram here shows a current loop with a magnetic field passing through it. The angle between the $B$ field and the perpendicular projection of $B$ in the plane of the loop is $\alpha$. Determine the direction of the magnetic moment of the loop $\mu_{\text{loop}}$, and the angle between $\mu_{\text{loop}}$ and $B$:

<table>
<thead>
<tr>
<th>Direction of $\mu_{\text{loop}}$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle between $\mu_{\text{loop}}$ &amp; $B$</td>
<td>$\frac{\pi}{2} - \alpha$</td>
<td>$\frac{\pi}{2} - \alpha$</td>
<td>$\frac{\pi}{2} + \alpha$</td>
<td>$\frac{\pi}{2} + \alpha$</td>
</tr>
</tbody>
</table>

**Hint:** $\mu_{\text{loop}} = iA\mathbf{n}$, where $\mathbf{n}$ is the unit normal vector is defined by the right-hand rule.

**Extra:** Determine the magnetic torque vector.
Checkup 31.2

This figure shows several surfaces in a uniform magnetic field. Which has the largest magnetic flux?

A

B

C
The switch $S$ in the figure below has been open for a long time. At $t = 0$ it is closed. What is the current through the inductor immediately after closing the switch?

\[ I = \frac{E}{R_1} \]

\[ I = \frac{E}{R_2} \]

\[ I = \frac{E}{R_1 + R_2} \]

\[ E = \frac{E}{R_1} \cdot \frac{1}{R_1} \cdot R_2 \]

\[ E = \frac{E}{R_1} \cdot \frac{1}{R_2} \cdot R_2 \]

\[ E = \frac{E}{R_1 + R_2} \cdot \frac{1}{R_1 + R_2} \cdot R_2 \]
A metal rod AD is sliding to the right along the parallel metal railings shown. The resistance connecting the railings is R. Also, there is a magnetic field B directed into the paper. L = AD. Determine the direction of the induced current:

Direction of \( \mathbf{i}_{\text{ind}} \):
- Clockwise

Hint: Lenz's law states that \( B_{\text{ind}} \) tends to maintain the original flux.

Extra: Based on the equation \( \mathbf{e}_{\text{ind}} = -\frac{\mathbf{dB}}{\mathbf{dt}} \), \( \mathbf{B} \), \( \mathbf{i}_{\text{ind}} = \frac{\mathbf{e}_{\text{ind}}}{\mathbf{R}} \).

Show that \( \mathbf{i}_{\text{ind}} = \frac{\mathbf{B} \mathbf{L}}{\mathbf{R}} \).
Consider a metal bar OA, shown rotating in a plane perpendicular to a magnetic field $\mathbf{B}$, directed into the page. Which end of the bar has a higher potential?

<table>
<thead>
<tr>
<th>$V_O$ versus $V_A$</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_O &lt; V_A$</td>
<td>$V_O = V_A$</td>
<td>$V_O &gt; V_A$</td>
</tr>
</tbody>
</table>

**Hint:** The magnetic force due to $\mathbf{B}$ asserted on $q$, a positive charge on the rod, is given by $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. This force "pushes" $q$ from a low-potential point to a higher-potential point.

**Extra:** Show that $|V_O - V_A|$, the magnitude of the potential difference is $\omega BR^2/2$, where $R = OA$ and $\omega$ is the angular frequency of rotation.

**Hint:** When $q$ is pushed by a displacement $\Delta r$, the corresponding potential difference is given by $\Delta V = F\Delta r/q = qvB\Delta r/q = \omega Br\Delta r$. 

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In the situation shown here, solenoid #2 is placed inside the much larger solenoid #1. Solenoid #1 has \(N_1\) turns, radius \(a\), and length \(d_1\). Solenoid #2 has \(N_2\) turns, radius \(b\), and length \(d_2\). Determine the mutual inductance of the system:

\[
\nA \quad \mu_0 N_1 N_2 \pi a^2 / d_1 \\
B \quad \mu_0 N_1 N_2 \pi b^2 / d_1 \\
C \quad \mu_0 N_1 N_2 \pi a^2 / d_2 \\
D \quad \mu_0 N_1 N_2 \pi b^2 / d_2
\]

**Hint:** \(\varepsilon_{2,\text{ind}} = -N_2 \frac{d\phi}{dt} = M_{21} \frac{di_1}{dt}\), where \(\phi\) is the magnetic flux at #2 due to \(i_1\).

\[
\frac{\Phi_{b_2}}{I_1} = \frac{B_2 N_2 \pi b^2}{I_1} = \mu_0 \frac{E_0 N_1 \pi a^2}{E_1 \cdot d_1}
\]
For the resistor-inductor circuit shown here, the loop equation is \( \mathcal{E} - V_L - iR = 0 \), where \( V_L = L\frac{di}{dt} \). Find the current and voltage \( i \) and \( V_L \) at \( t = 0^+ \), that is immediately after the switch \( S \) was closed at \( t = 0 \):

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )</td>
<td>( \mathcal{E}/R )</td>
<td>0</td>
<td>( \mathcal{E}/R )</td>
<td>0</td>
</tr>
<tr>
<td>( V_L )</td>
<td>( \mathcal{E} )</td>
<td>( \mathcal{E} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Hint:** The inductor has an initial flux of zero. Lenz law implies that the inductor has magnetic inertia, meaning it has the tendency to maintain its no flux status. So at \( t = 0^+ \), we expect the flux through the inductor should still be approximately zero.

**Extra:** Find \( i \) at \( t = \infty \), where the current has reached a steady state in which \( \frac{di}{dt} \approx 0 \).