Lecture 26

L-R, L-C, RLC circuits
PhysiQuiz 31-6

In the situation shown here, solenoid #2 is placed inside the much larger solenoid #1. Solenoid #1 has \( N_1 \) turns, radius \( a \), and length \( d_1 \). Solenoid #2 has \( N_2 \) turns, radius \( b \), and length \( d_2 \). Determine the mutual inductance of the system:

\[ M = \frac{\mu_0 N_1 N_2 \pi a^2}{d_1} \]

\[ \frac{\mu_0 N_1 N_2 \pi b^2}{d_2} \]

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\[ \frac{\mu_0 N_1 N_2 \pi b^2}{d_2} \]

**Hint:** \( \phi_{ind} = -\frac{d\phi}{dt} = M \frac{di_1}{dt} \), where \( \phi \) is the magnetic flux at #2 due to \( i_1 \).
The circuit symbol for an inductor is a closed line.

When the battery is connected, the current in the circuit will gradually start to increase. The inductance current cannot change through the inductor instantaneously.

Because of inductance, the change in current is determined by the inductance current and the time.

\[ \frac{\Delta I}{\Delta t} = \frac{V}{L} \]

\[ I(t) = \frac{V}{L} t \]

The current at time \( t \) is given by:

\[ I(t) = \frac{V}{L} t \]

The inductance current is determined by the change in current over time, which is given by the equation:

\[ \frac{dI}{dt} = \frac{V}{L} \]

\[ I(t) = \int \frac{V}{L} dt \]

\[ I(t) = \frac{V}{L} t + C \]

The constant \( C \) is determined by the initial conditions of the circuit.

At \( t = 0 \), the current is:

\[ I(0) = \frac{V}{L} \cdot 0 + C = C \]

Thus, the current at any time \( t \) is:

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Initially zero...

Current is 
\[ \int \left( \frac{7}{\alpha^2} e^{-\alpha x} - 1 \right) \, \frac{dI}{3} = (\frac{7}{\alpha^2} - 1) \int \frac{dI}{3} \]

...and it increases...

At characteristic time \( t = L/R \),

Current is 63% of final value.

...of \( R/3 \) at \( t = \infty \).

\( 4L/R \) \( 3L/R \) \( 2L/R \) \( L/R \) \( 0 \)

\( R/3 \) \( 0.63R/3 \)
When battery is suddenly removed...

\[ I_a = \frac{\mathcal{E}}{R_a} \]

\[ I(t=0) = I_0 = \]

\[ I_R + L \frac{dT}{dt} = 0 \]

\[ I = I_0 e^{-\frac{t}{\tau_{LR}}} \]

\[ \tau_{LR} = \frac{L}{R} \]

...inductor will try to maintain current, but stored energy will be gradually dissipated in resistor.

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If at time $t=0$ the switch is suddenly turned on, (1) what is the initial current through the battery? (2) the final battery current?

\[ (a) \quad \frac{\mathcal{E}}{R_1} \quad (b) \quad \frac{\mathcal{E}}{R_2} \quad (c) \quad \frac{\mathcal{E}(R_2 + R_1)}{R_2 R_1} \]
After a current is established by discharge of capacitor...

inductor keeps current going for some time, ...

...resulting in reversed charge accumulation on capacitor plates.

\[
L \frac{d^2q}{dt^2} + \frac{q}{C} = 0
\]

\[
L \frac{dI}{dt} + \frac{q}{C} = 0
\]

Inductor stores \( B \)-field energy

\[
W_I = \frac{1}{2} LI^2
\]

\[
W_C = \frac{1}{2C} \approx E\text{-field}\text{ energy}
\]

L-C circuit
Mechanical analogy of LC circuit

\[ L \frac{d^2 q}{dt^2} + \frac{q}{C} = 0 \]

\[ \frac{1}{2} L I^2 = \frac{1}{2} L \left( \frac{dq}{dt} \right)^2 = \text{magnetic field energy} \]

\[ \frac{q^2}{2C} = \text{electric field energy} \]

---

Compare with motion of harmonic oscillator

\[ \frac{k}{m} \quad \text{Compare with motion of harmonic oscillator} \]

\[ T = \frac{1}{2} m v^2 = \frac{1}{2} m \left( \frac{dx}{dt} \right)^2 = KE \]

\[ U = \frac{1}{2} k x^2 = PE \]

\[ KE + PE = \text{constant} \]

\[ m \frac{d^2 x}{dt^2} + \frac{k}{m} x = 0 \]

\[ \omega_0^2 = \frac{k}{m} \]

\[ m \rightarrow L, \quad k \rightarrow \frac{1}{C}, \quad x \rightarrow \frac{q}{C} \]

The sum of magnetic field energy plus electric field energy remains constant in time

\[ \frac{1}{2} L \left( \frac{dq}{dt} \right)^2 + \frac{q^2(t)}{2C} = E \text{nergy} \]

\[ E = \text{total energy} \]

7b
When switch is turned on what is current into capacitor?

(a) 0

(b) $\frac{E}{R}$

(c) $\frac{1}{\sqrt{LC}}E$

Switch prevents interruption of current...

$I = \frac{E}{R}$

...when switching from steady-state $RL$ circuit...

...to oscillating $LC$ circuit.

Figure 32-21 Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.
In *RLC* circuit without any source of emf, current and charge again oscillate...

\[ P = RI^2 = R \left( \frac{dq}{dt} \right)^2 \]

\[ \dot{I}R + L \frac{dI}{dt} + \frac{q}{C} = 0 \]

\[ L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0 \]

...but now resistor causes damping.
Amplitude of charge oscillations decreases slowly for small $R$ (underdamped).

$$R << \omega_0 L = \sqrt{\frac{L}{C}}$$

For large $R$ (overdamped), no oscillations occur.

$$Q = Q_{\text{max}} e^{-\left(\frac{R}{2L}\right)t} \cos \omega_d t$$

$$\omega_d = \left[\frac{1}{LC} - \frac{B^2}{L^2}\right]^{1/2}$$

Critical damping

$$B = \sqrt{\frac{1}{LC}}$$

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