Lecture 25
Mutual and Self Inductance
Do you understand the decrease in magnetic field. Induced current would oppose with direction such that an...

Induced electric field is tangent to circles around axis...

Decreasing with time, magnetic field is pulled apart when magnets...

\[ \mathbf{E} \cdot \mathbf{dA} = -\frac{\partial \mathbf{B}}{\partial t} \int \mathbf{dA} \]

Faraday Law
If a current flows in coil 1, the magnetic flux through coil 2, from coil 1, is proportional to

(a) $I_1$

(b) $N_1$

(c) $N_2$

(d) all of the above
The magnetic flux from coil 1 at coil 2 is proportional to

$$\Phi_{21} \propto N_1 N_2 I_1$$

$$\Phi_{21} = M_{21} I_1$$

$M_{21}$ = mutual induction coefficient

Now if $I_1$ changes in time there will be an EMF generated in coil 2.

$$E_{MF2} = -\frac{d\Phi_{21}}{dt} = -M_{21} \frac{dI_1}{dt}$$

What then is electric field in coil 2 if it is a circle with $N_2$ turns?

$$E_{MF2} = \oint \vec{E} \cdot d\vec{l} = N_2 (2\pi R_2) E_2 = -M_{21} \frac{dI_1}{dt}$$
Similarly mutual induction produces \( \Phi \) flux through coil 1 due to current in coil 2.

\[
\Phi_{12} = N_1 N_2 I_2 = M_{12} I_2
\]

It can be shown that

\[
M_{12} = M_{21}
\]

Emf in circuit "1" arises if \( I_2 \) changes

\[
E_{emf_1} = M_{12} \frac{dI_2}{dt} = M_{21} \frac{dI_2}{dt}
\]

Let us give a calculation that demonstrates \( M_{12} = M_{21} \)
Recall the magnetic field from a current loop of area \( A \) and current \( I \), on axis far from the loop, is given by

\[
B = \frac{\mu_0 I A}{2 d}
\]

\( \mu_0 I A \) = magnetic moment

If we had \( N \) loops, the field at \( P \) would be

\[
B = \frac{\mu_0 N I A}{2 d}
\]

\( N I A \) = magnetic moment

Now then consider two loops 1, 2, with area \( A_1 + A_2 \) and distance \( d \) apart, and calculate mutual flux through one loop with current \( I_1 \), \( N_1 \) coils through loop 2, with \( N_2 \) loops.
\[ B_{z1} = \frac{\mu_0 A_1 N_1}{2d}, \quad B_{z2} = \frac{\mu_0 A_2 N_2}{2d} \]

Flux through loop 1, from coil 2:

\[ \Phi_{12} = B_{z2} A_1 N_1 - \frac{\mu_0 A_2 N_2}{2d} A_1 N_1 \]

\[ = \left( \frac{\mu_0 A_2 A_1 N_1 N_2}{2d} \right) I_z = M_{12} I_2 \]

\[ M_{12} = \frac{\mu_0 A_2 A_1 N_1 N_2}{2d} \]

Interchange 1 \& 2 and you reach the conclusion that

\[ M_{21} = M_{12}, \quad M_{ij} = M_{ji} \]
Time-dependent current in one coil produces a changing magnetic field...

...and changing magnetic flux induces current in second coil.

Mutual induction
Find mutual inductance between solenoidal coil and internal ring.

What is the EMF induced in wire ring if \( I^1 \) is \( \frac{d\bar{I}}{dt} \)?

What is mutual induction between solenoid and inner solenoid? About the surface of wire ring...
A changing current in a coil will change its own magnetic field and magnetic flux, and will thereby induce an emf in the coil.

\[ E = -\frac{\partial \Phi}{\partial t} \]

Solenoid inductance of a self-induced self.
Self Inductance of a Solenoid

\[ B = \mu_0 \frac{I N}{l} \]

\[ \Phi_M = BAN = \mu_0 \frac{I N^2 A}{l} = LI \]

\[ L = \frac{\mu_0 N^2 A}{l} \]

In general self-inductance of a coil \( \propto N^2 \) (i.e. \# of turns in the coil)
Energy stored in an inductor

Delivered Power = $E_m F I$

\[ = L \frac{dI}{dt} I \]

\[ = \frac{L}{2} \frac{dI^2}{dt} \]

\[ \text{Stored energy} \]

\[ W_L = \int_{-\infty}^{t} \text{Power} \, dt = \int_{-\infty}^{t} dt \frac{L}{2} \frac{dI^2}{dt} = \frac{L}{2} I^2 \]

\[ W_L = \frac{L}{2} I^2 \]

In solenoid \[ L = \mu_0 \frac{A N^2}{D} \]

\[ W_L = \mu_0 \frac{(AD)N^2 I^2}{2D^2} = \frac{V}{2\mu_0} \left( \frac{\mu_0 N I}{D} \right)^2 = \frac{B^2V}{2\mu_0} \]

AD = V = Volume

\[ \text{Stored energy is B-field energy} \]
\[ W_L = W_B = \frac{1}{2} LI^2 \]

for solenoid

\[ W_L = \frac{B^2 V}{2 \mu_0} \]

\[ \frac{B^2}{2 \mu_0} = \text{magnetic energy density} \]

In general, the stored energy of inductors, can be expressed as

\[ W_B = \frac{1}{2} LI^2 = \frac{1}{2 \mu_0} \iint_S dV \left( \frac{B^2 l^2}{2 \mu_0} \right) \]

Recall electric field energy is

\[ W_E = \frac{q^2}{2 C} = \oint_S dV \frac{E_0 E^2(r)}{2} \]

\[ \frac{1}{2} E_0 E^2 = \text{electric field energy density} \]