Lecture # 20

Biot Savart Law
Motion is parallel to current...

The direction of the force on the electron is:
(a) up  (b) down  (c) out of the screen  (d) into screen.
Ampere's Law

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{total}} \]

In the above figure we have, \( \oint \mathbf{B} \cdot d\mathbf{s} \), around the red boundary, equal to

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \begin{cases} I_1 + I_2 + I_3 + I_4 \quad (a) \\ -I_1 + I_2 \quad (b) \\ I_1 + I_2 + 2I_3 \quad (c) \end{cases} \]
Magnetic field of a coaxial cable

Upward vertical current \( I \) on inner shell at radius \( r = a \)

Downward vertical current on outer shell at radius \( r = b \)

Uniform surface current density on shells

Find Magnetic Field at \( r_1, r_2, r_3 \)

\( a < r_1 < b; \quad r_2 < a, \quad r_3 > b \)

(a) 0,  (b) \( \frac{\mu_0 I}{2\pi r} \),  (c) \( \frac{\mu I}{2\pi a} \)

For, \( r = r_1 \)

\( r = r_2 \)

\( r = r_3 \)

How would the above magnetic field change if the inner vertical current \( I \) was uniformly distributed throughout inner wire?
**Solenoid**

Wire has $N$ turns with current $I$, so $n = \frac{N}{L} \equiv \# \text{ turns} / \text{length}$.

What is $\mathbf{B}$ field in wire?

- Apply Gauss' law around path (a)
- Apply Gauss' law around path (b)

leads to $\mathbf{B} = \frac{n}{L} \mu_0 I$ inside toroid.

$\mathbf{B} = 0$ outside toroid.

Inner field independent of shape of cylindrical shell.
Biot-Savart Law

Most general way to calculate magnetic field from a steady current.

\[ d\mathbf{B} = \frac{\mu_0 I \, ds \times \mathbf{r}}{4\pi r^3} \]

What is the B-field from a wire of current I of length L?
\[
\begin{align*}
L &= s_2 + |s_1| \\
r \sin \theta &= R \sqrt{t} \\
\tan \theta &= \frac{R}{-s} \\
S &= -R \frac{1}{\tan \theta} \\
dB &= \frac{\mu_0 I ds}{4\pi r^3} r \sin \theta \\
ds &= -R d(\cot \theta) = +R \frac{d\theta}{\sin^2 \theta} \\
\therefore \ dB &= \frac{\mu_0 I R d\theta}{4\pi r^2 \sin \theta} \\
\therefore \ dB &= +\frac{\mu_0 I R \sin \theta}{4\pi R^2} d\theta \\
B &= +\frac{\mu_0 I}{4\pi R} \left[ \frac{\cos \theta}{\theta^2} \right]_0^\pi \\
B &= +\frac{\mu_0 I}{4\pi R} \left( \cos \theta_2 - \cos \theta_1 \right)
\end{align*}
\]
Magnetic Field on axis of loop

\[ ds = \rho d\theta \]

\[ d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{ds \times \hat{r}}{r^2} \]

\[ \hat{z} \cdot d\vec{B} = \frac{\mu_0 I}{4\pi} \rho d\theta \sin \phi \]

\[ \vec{B} = \frac{\mu_0 I A}{4\pi} \sin \phi \frac{2\pi}{r^2} \]

Now:

\[ \sin \phi = \frac{r}{(R^2 + L^2)^{1/2}} \]

\[ r^2 = R^2 + L^2 \]

\[ \vec{B} = \frac{\mu_0 \hat{z}}{2} \frac{I}{(R^2 + L^2)^{3/2}} \]

\[ \hat{B} = \hat{z} \]

(a) for \( L = 0 \) (center of loop):

\[ \hat{B} = \frac{\hat{z}}{2} \frac{\mu_0 I}{2} \]

(b) for \( L \gg R \)

\[ \hat{B} \approx \frac{\hat{z}}{2} \frac{\mu_0 I}{2} \frac{R^2}{L^3} \]