Lecture # 15

DC - circuits
Resistors used in circuits are often carbon cylinders.

Resistor symbol is a zigzag line.

Arrow represents the movable contact of a variable resistor.
First stripes represent digits: yellow = 4 and violet = 7...

...and third stripe represents power of ten: orange = 3, so \( R = 47 \times 10^3 \, \Omega \).

Remaining stripe indicates tolerance: silver = ±10%.
Resistors in series

- Current the same in each resistor
- $V_{\text{total}} = V_1 + V_2$
- $R_{\text{eq}} = R_1 + R_2$

$I$

$R_1 = 10 \, \Omega \quad R_2 = 20 \, \Omega$

$V_2 = IR_2$

$V_{\text{total}} = V_1 + V_2$

$I_{\text{Res}} = IR_1 + IR_2$
For any number of resistors connected in series, the net resistance is the sum

\[ R = R_1 + R_2 + R_3 + \cdots \]
Resistors in Parallel

(1) Voltage drop the same across each resistor

(2) \( I = I_1 + I_2 \)

(3) \( \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \)

\( R_1 = 10 \Omega \)

\( V_1 = I_1 R_2 \)

\( V_1 = V_2 \)

\( I_{Reg} R_{eq} = V \)

\( I = \frac{V}{R_{eq}} \)

\( \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} \)

\( R_2 = 20 \Omega \)

\( V_2 = I_2 R_2 \)

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Figure 27-19 Physics for Engineers and Scientists 3/e
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For resistors connected in parallel, the inverse of the net resistance is the sum of the inverses of individual resistances,

\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \]
Composite resistance of circuit.

This circuit has two loops, but we can replace these two parallel resistors...

\[ R_1 = 2\Omega \quad R_2 = 4\Omega \]

...by a single effective resistance...

\[ R = \frac{4.0}{3}\Omega \]

...and obtain an equivalent single-loop circuit.

Current through battery = \( I_{\text{bat}} \) = \( \frac{\varepsilon}{R_{\text{eq}}} \)
\[
\frac{R_3R_1 + R_3R_2 + R_2R_1}{R_1} = \frac{R_3}{\frac{R_1R_2}{R_1} + R_2 + R_1}
\]

\[
\frac{R_3R_1 + R_3R_2 + R_2R_1}{R_1} = \frac{R_3}{\frac{R_1R_2}{R_1} + R_2 + R_1}
\]

\[
\frac{I_2}{E_1} + \frac{I_2}{E_2} = \frac{I}{E}
\]

\[
\frac{I_2}{E_1} + \frac{I_2}{E_2} = \frac{I}{E}
\]

\[
\text{Current} = I
\]

\[
\text{Current} = I
\]
What is equivalent resistance?

\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{R_1 R_2}{2}} \]

\[ R = \frac{R_2}{2} \]

1. \( 2 R_1 + R_2 \)
2. \( 2 R_2 + R_1 + \frac{R_1 R_2}{2} \)
3. \( 2 R_1 + R_2 / 2 \)
Current, I, Through R?

(a)

(b)

I = ?

(1) \( \frac{E}{R} \) in (a); \( \frac{E}{3R} \) in (b)

(2) \( \frac{E}{3R} \) in (a); \( \frac{E}{R} \) in (b)

(3) \( \frac{E}{R} \) in (a); \( \frac{E}{R} \) in (b)

Figure 28-21 Physics for Engineers and Scientists 3/e © 2007 W.W. Norton & Company, Inc.
Direction of current (flow of positive charge) is also direction of potential decrease across resistor.

Kirchhoff's Voltage Law: "Voltage Drop Around a circuit vanishes"
Then a starting point and direction are chosen for a path around loop.

Path around loop passes through this emf in backward direction.

What is correct equation?

a. \( E_1 + I R_1 + E_2 + I R_2 = 0 \)

b. \( -E_1 + I R_1 - E_2 + I R_2 = 0 \)

c. \( -E_1 + I R_1 + E_2 + I R_2 = 0 \)

Assumed current direction here is parallel to path chosen around loop.

What does negative I mean?

Figure 28-10b Physics for Engineers and Scientists 3/e
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When current $I_1$ encounters a junction...

- Kirchhoff's current law: Total current into a node is zero.
- Junction: It splits into two separate currents, $I_2$ and $I_3$.

Mathematically:

$$I_1 - I_2 - I_3 = 0$$

or

$$I_1 = I_2 + I_3$$
Write equations for circuit
3 unknowns $I_1, I_2, I_3$

For this two-loop circuit...

$E_1 - I_1 R_1 - I_2 R_2 - I_3 R_3 = 0$
$E_2 - I_2 R_2 - I_3 R_3 = 0$

...we first label and draw arrows for the three separate branch currents...

(b)

$E_1 - I_1 R_1 + I_2 R_2 = 0$
$E_2 - I_2 R_2 + I_3 R_3 = 0$

...and then choose a starting point and direction to go around each loop.
Write Equation for circuit

2 unknowns

$I_1$ and $I_3$

For this two-loop circuit...

\[ I_2 = I_1 - \delta I_3 \]

\[ -E_1 + I_1R_1 + R_2(I_1 - I_3) = 0 \]

...we first label and draw arrows for the three separate branch currents...

...and then choose a starting point and direction to go around each loop.

\[ E_2 + I_3R_3 + R_2(I_3 - I_1) = 0 \]
Write equation for circuit

For a complicated circuit with several loops and branches...

\[ -\varepsilon_1 + R_b (I_1 - I_2) + R_a I_1 = 0 \]
\[ I_2 R_c + R_d (I_2 - I_3) + R_b (I_3 - I_1) = 0 \]
\[ -\varepsilon_2 + R_d (I_3 - I_2) = 0 \]

Additional HW

Solve for \( I_1, I_2, I_3 \)!