Non-hybrid computation of kinetic effects on linear MHD instabilities with AEGIS-K code

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Part I:

Revisiting linear gyrokinetics to recover MHD and missing FLR effects

Outline of key reasons:

1. Perpendicular MHD:

$$-\rho_m \omega^2 \boldsymbol{\xi}_\perp = \delta \mathbf{J} \times \mathbf{B} + \mathbf{J} \times \delta \mathbf{B} - \nabla \delta P$$

The conventionally-used equilibrium distribution function $F_{g0}(\mathbf{X}_\perp, \mu, \epsilon)$ does not yield the parallel current. Consequently, the Pfirsch-Schlüter current induced effect $(\mathbf{J} \times \delta \mathbf{B})$ is missing in the conventional gyrokinetic theory.

2. FLR:

The gyrophase-averaged gyrokinetic distribution function is insufficient. The high harmonic components with respect to the gyrophase Fourier decomposition are needed. Consequently, the conventional results about FLR effects based on the Bessel functions J_0 and J_1 are incomplete.

3. Parallel MHD:

$$\gamma P\mathbf{B}\cdot\nabla\left(\frac{1}{B^2}\mathbf{B}\cdot\nabla\nabla\cdot\boldsymbol{\xi}\right)+\rho_m\omega^2\nabla\cdot\boldsymbol{\xi}=\rho_m\omega^2\nabla\cdot\boldsymbol{\xi}_\perp.$$

The calculation of the gyrophase variation along particle orbit is incomplete in the conventional theory. The coupling between the gyrophase-averaged and gyrophase-dependent parts of the gyrokinetic distribution function has not been taken into account. Consequently, the source term of type $\rho_m\omega^2\nabla\cdot\boldsymbol{\xi}_\perp$ in the parallel equation of motion cannot be recovered in the conventional theory.

4. Drift kinetic equation:

We also note that even in the zero-order FLR expansion ($i.e.$, $k_\perp\rho_i\to 0$) our newly derived gyrokinetics equation is different from the conventional drift kinetic equation. Only guiding center theory with oscillating part α_1 of gyrophase included in the gyrophase definition can agree with gyrokinetic theory to lowest order.

The recovery of perpendicular MHD needs to include both
a) the next order equilibrium distribution function
b) the gyrophase-dependent part of the perturbed distribution

- Where MHD magnetic force balance occurs in the kinetic description:

$$\mathbf{v}\times\mathbf{B}\cdot\nabla_v\delta F = -\mathbf{v}\times\delta\mathbf{B}\cdot\nabla_v F \implies \nabla_v\cdot(\mathbf{v}\times\mathbf{B}\delta F) = -\nabla_v\cdot(\mathbf{v}\times\delta\mathbf{B}F)$$
$$\implies \mathbf{v}\times\mathbf{B}\delta F = -\mathbf{v}\times\delta\mathbf{B}F \implies \delta\mathbf{J}\times\mathbf{B} = -\mathbf{J}\times\delta\mathbf{B}.$$

- Without keeping the next order equilibrium distribution function, *viz.*,

$$\mathbf{v}\times\delta\mathbf{B}\cdot\nabla_v F(\mathbf{X}_\perp, E, \mu) = 0, \tag{1}$$

the $\mathbf{J}\times\delta\mathbf{B}$ effect in MHD cannot be retained.

- Note that

$$\frac{e}{m}\mathbf{v}\times\mathbf{B}\cdot\nabla_v\delta F = \frac{e}{m}\mathbf{v}\times\delta\mathbf{B}\cdot\nabla_v F_1 \implies$$
$$-\Omega\frac{\partial\delta F}{\partial\alpha} = -v_\perp(\mathbf{e}_1\cos\alpha + \mathbf{e}_2\sin\alpha)\times\delta\mathbf{B}\cdot\mathbf{e}_b\left(\frac{g}{B}\frac{\partial F_{g0}}{\partial\psi} + \frac{|v_\parallel|}{\Omega}\frac{\partial F_{g10}}{\partial\mu}\right).$$

Without keeping the gyrophase-dependent part of the perturbed distribution, the $\mathbf{J}\times\delta\mathbf{B}$ effect in MHD cannot be retained either.

Usual gyrophase average would eliminate the $\mathbf{J}\times\delta\mathbf{B}$ effect in MHD

Gyrophase-dependent part of gyrokinetic distribution function is essential to retain FLR effects

- Mathematical problem: The model equation is

$$\left(\partial/\partial t + \mathbf{v}_\parallel \cdot \nabla + \mathbf{v}_D \cdot \nabla\right) \delta f - \Omega \partial \delta f / \partial \alpha = (a_c \cos\alpha + a_s \sin\alpha) + b$$

and the corresponding current moment is

$$\delta \mathbf{j}_\perp = \int \frac{d\alpha dE d\mu B}{|v_\parallel|} v_\perp (\mathbf{e}_1 \cos\alpha + \mathbf{e}_2 \sin\alpha) \delta f. \tag{2}$$

- $\Omega \partial/\partial\alpha \gg (\partial/\partial t + \mathbf{v}_\parallel \cdot \nabla + \mathbf{v}_D \cdot \nabla)$ does not yield $\partial \delta f/\partial\alpha = 0$.

- Correct reasoning: The ordering $\Omega \partial/\partial\alpha \gg (\partial/\partial t + \mathbf{v}_\parallel \cdot \nabla + \mathbf{v}_D \cdot \nabla)$

  does not imply $\Omega \partial \delta f/\partial\alpha \gg a_c \cos\alpha + a_s \sin\alpha$, as required by $\partial \delta f/\partial\alpha = 0$,

  but only gives $-\Omega \partial \delta f/\partial\alpha = a_c \cos\alpha + a_s \sin\alpha$.

— Note that the contribution of the gyrophase-averaged distribution function to the current moment in Eq. (2) vanishes to lowest order.

Correction to the conventional gyrophase-averaged gyrokinetic equation to recover parallel MHD momentum equation

- The coupling of the gyrophase-dependent part of solution $(\delta\tilde{f})$ is essential to obtain an ordering consistent gyrophase-averaged gyrokinetic equation, since

$$\left\langle \dot{\alpha}_1 \frac{\partial \delta \tilde{f}}{\partial \alpha} \right\rangle \neq 0.$$

- Term $\left\langle \dot{\alpha}_1 \frac{\partial \delta \tilde{f}}{\partial \alpha} \right\rangle$ has contribution at lowest order, $i.e.$, to the drift equation. This leads to the following correction to the source terms:

$$\text{Conventional} \ : \ -i\omega\mu_0 B \frac{\partial F_{g0}}{\partial \varepsilon} \nabla_\perp \cdot \boldsymbol{\xi} - i\omega\mu_0 \frac{\partial F_{g0}}{\partial \varepsilon} \boldsymbol{\xi} \cdot \nabla B - i\omega \frac{\partial F_{g0}}{\partial \varepsilon} \left(\mu_0 B - \frac{v_\parallel^2}{2} \right) \boldsymbol{\kappa} \cdot \boldsymbol{\xi}.$$

$$\text{Ours} \ : \ -i\omega\mu_0 B \frac{\partial F_{g0}}{\partial \varepsilon} \nabla_\perp \cdot \boldsymbol{\xi} - i\omega \frac{\partial F_{g0}}{\partial \varepsilon} \left(\mu_0 B - v_\parallel^2 \right) \boldsymbol{\kappa} \cdot \boldsymbol{\xi}.$$

— Replacing the particle speed with thermal speed, our expression recovers the parallel MHD source term of type $\nabla_\perp \cdot \boldsymbol{\xi}$.

Key notes in rederiving the gyrokinetic formalism

1. Next order equilibrium distribution function is required:

$$F_0 = F(\mathbf{X}, E, \mu) - v_\parallel \frac{g}{\Omega}\frac{\partial F_{g0}}{\partial \psi_g} + \mathrm{sign}(v_\parallel) F_{g10}(\psi_g, \mu, \varepsilon)$$

— Only $F(\mathbf{X}, E, \mu)$ is used in conventional gyrokinetics.

2. Use the Fourier decomposition method to solve the gyrokinetic equation*

$$\{\delta H, \mathcal{R}\} = \sum_k \{\delta H_k, \mathcal{R}_k\} \exp\{ik\alpha\}.$$

— Only δH_0 is solved in conventional gyrokinetics.

3. $\dot{\alpha}_1 = \mathbf{v} \cdot \nabla_x \alpha + (1/\Omega)\mathbf{v} \times \mathbf{e}_b \cdot \nabla_x \Omega$ in gyrokinetic equation should contain the guiding center transform of the gyrofrequency

4. Some subtle points:

a) μ in $F(\mathbf{X}, E, \mu)$ needs to be carried to a sufficient order, not just $v_\perp^2/2B$;

b) It should be noted that, in the guiding center transform $\mathbf{X} = \mathbf{x} + \frac{1}{\Omega(\mathbf{x})}\mathbf{v} \times \mathbf{e}_b(\mathbf{x})$, Ω and $\mathbf{e}_b$ are given in terms of the particle coordinates.

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* L. Chen and S. T. Tsai, *Plasma Phys.*, **25**, 349 (1983);
S. T. Tsai, J. W. Van Dam, and L. Chen, *Plasma Phys. Controlled Fusion*, **26**, 907 (1984).

Recovering MHD equilibrium

- MHD equilibrium equation $\mathbf{J}\times\mathbf{B} = \nabla P$ gives

$$\mathbf{J}_\perp = \frac{\mathbf{B}\times\nabla P}{B^2}, \qquad \mathbf{J}_\parallel = -\frac{gP'}{B} - g'B. \tag{3}$$

- Kinetic equilibrium

$$\left(v_\parallel \mathbf{e}_b\cdot\nabla_X + \mathbf{v}_D\cdot\nabla - \Omega\frac{\partial}{\partial\alpha}\right)F = 0.$$

— Usual gyrokinetics uses only equilibrium of the lowest order: $F_{g0}(\mathbf{X}_\perp,\varepsilon,\mu)$.

- Ref. [1] shows that the equilibrium MHD equation (3) can be recovered only with the next order neoclassical equilibrium F_{g1} taken into account. In Maxwellian example:

$$F_{g0}(\psi_g,\varepsilon) = n_0(\psi_g)\left(\frac{m_\rho}{2\pi T(\psi_g)}\right)^{3/2}\exp\left\{-\frac{m_\rho\varepsilon}{T(\psi_g)}\right\} \Longrightarrow \sum_{i,e} e\int d^3v\mathbf{v}F_{g0}(\psi_g,\varepsilon) = \frac{\mathbf{B}\times\nabla P}{B^2}.$$

$$F_{g1} = -v_\parallel\frac{g}{\Omega}\frac{\partial F_{g0}}{\partial\psi_g} + \mathrm{sign}(v_\parallel)F_{g10}(\psi_g,\mu,\varepsilon) \Longrightarrow \sum_{i,e} e\int d^3v\mathbf{v}F_{g1} = -\frac{gP'}{B} - g'B.$$

[1] R. D. Hazeltine, *Advances in Plasma Physics*, Ed. by A. Simon and W. B. Thompson (John Wiley and Son, Inc., New York, 1976), vol. **6**, p. 273.

Formulation of the basic set of equations

- Complete set of equations:

— Gyrokinetic equation

$$\left(-i\omega + \dot{\mathbf{X}} \cdot \nabla_X + \dot{\alpha}_1\right) \delta H - \Omega(\mathbf{X}) \frac{\partial \delta H}{\partial \alpha} = \mathcal{R};$$

— Ampere's law

$$\delta \mathbf{J} \cdot \mathbf{e}_{1,2} \;=\; \sum_{i,e} e \int d^3v \mathbf{v}_\perp \cdot \mathbf{e}_{1,2} \delta f(\mathbf{x});$$

— Quasineutrality condition

$$\sum_{i,e} e \int d^3v \delta f(\mathbf{x}) = 0.$$

- Reasons for directly using the two perpendicular components of Ampere's law:

a) The velocity moment equation contains both pressure and velocity moments. Calculating the velocity moment alone is equivalent to calculating the current density in Ampere's law.

b) Direct construction of the kinetic vorticity equation, by applying $\sum_{i,e} e \int d^3v$ on the gyrophase-averaged gyrokinetic equation needs a backward transform to particle coordinates, since $\mathbf{v}_\| \cdot \nabla_\mathbf{X}$ is in the guiding center coordinates.

c) The vorticity equation alone is insufficient for the completeness of the basic set of equations. At least one perpendicular component of Ampere's law is needed.

Complete set of equations:
1. Two components of the Ampere's law

- **Ampere's law** — compute the current density using the gyrophase dependent part of the distribution functions:

$$\mathbf{e}_1\cdot\nabla\times\delta\mathbf{B} = -\frac{gP'}{B^2}\mathbf{e}_1\cdot\delta\mathbf{B} - g'\mathbf{e}_1\cdot\delta\mathbf{B} + \frac{1}{B}\mathbf{e}_2\cdot\nabla\left(P'|\nabla\psi|\mathbf{e}_1\cdot\boldsymbol{\xi}\right) - \sum_{i,e} m_\rho\int d^3v\mu_0\mathbf{e}_2\cdot\nabla\delta G_0(\mathbf{x})$$
$$+\frac{\omega^2}{B}\rho_m\mathbf{e}_2\cdot\boldsymbol{\xi} - i\omega\frac{|\nabla\psi|}{B\Omega_i}P_i'\mathbf{e}_1\cdot\nabla(\mathbf{e}_1\cdot\boldsymbol{\xi}) + in_0m_{\rho i}\left[\omega-\omega_{*i}(1+\eta_i)\right]\frac{1}{B^2}\mathbf{e}_1\cdot\nabla\delta\varphi$$
$$+ \boxed{\text{additional FLR effects}}\,;$$

$$\mathbf{e}_2\cdot\nabla\times\delta\mathbf{B} = -\frac{gP'}{B^2}\mathbf{e}_2\cdot\delta\mathbf{B} - g'\mathbf{e}_2\cdot\delta\mathbf{B} - \frac{P'|\nabla\psi|}{B^2}\mathbf{e}_b\cdot\delta\mathbf{B} - \frac{1}{B}\mathbf{e}_1\cdot\nabla\left(P'|\nabla\psi|\mathbf{e}_1\cdot\boldsymbol{\xi}\right) + \sum_{i,e} m_\rho\int d^3v\mu_0\mathbf{e}_1\cdot\nabla\delta G_0(\mathbf{x})$$
$$-\frac{\omega^2}{B}\rho_m\mathbf{e}_1\cdot\boldsymbol{\xi} - i\omega\frac{|\nabla\psi|}{B\Omega_i}P_i'\mathbf{e}_2\cdot\nabla(\mathbf{e}_1\cdot\boldsymbol{\xi}) + in_0m_{\rho i}\left[\omega-\omega_{*i}(1+\eta_i)\right]\frac{1}{B^2}\mathbf{e}_2\cdot\nabla\delta\varphi$$
$$+ \boxed{\text{additional FLR effects}}\,.$$

— Magenta: missing MHD terms in the usual gyrokinetic formalism;

— Red: The coupling of parallel motion is given kinetically;

— Blue: The missing FLR terms in the conventional gyrokinetics, in which the only FLR effect in this order is $\omega^2 \to \omega\left[\omega-\omega_{*i}(1+\eta_i)\right]$.

Complete set of equations: 2. Quasineutrality condition 3. The gyrophase-averaged gyrokinetic equation

- **Quasineutrality condition**

$$\delta\varphi(\mathbf{x}) = -\frac{1}{\sum_{i,e} n_0 e^2/T} \sum_{i,e} e \int d^3v \delta G_0.$$

- **The gyrophase-averaged gyrokinetic equation**

$$\begin{aligned}\left(\mathbf{v}_\parallel \cdot \nabla - i\omega - i\omega_d\right) \delta G_0(\mathbf{X}) \;=\;& -i\omega\mu_0 B \frac{\partial F_{g0}}{\partial \varepsilon} \nabla_\perp \cdot \boldsymbol{\xi} - i\omega \frac{\partial F_{g0}}{\partial \varepsilon}\left(\mu_0 B - v_\parallel^2\right) \boldsymbol{\kappa} \cdot \boldsymbol{\xi} + i\omega_d \boldsymbol{\xi} \cdot \nabla F_{g0} + i\left(\omega - \omega_*^T\right) \frac{e}{m_\rho} \frac{\partial F_{g0}}{\partial \varepsilon} \delta\varphi \\ & - \frac{v_\perp^2}{\Omega} \mathbf{e}_1 \cdot \nabla_X F_{g0} \mathbf{e}_2 \cdot \nabla \left(\frac{1}{B} \mathbf{e}_b \cdot \delta\mathbf{B}\right) + \frac{v_\parallel v_\perp^2}{\Omega} (\mathbf{e}_1\mathbf{e}_1 + \mathbf{e}_2\mathbf{e}_2) : \nabla\nabla(\mathbf{e}_b \cdot \nabla\boldsymbol{\xi} \cdot \nabla F_{g0}) \\ & - \frac{v_\parallel v_\perp^2}{\Omega} \left[(\mathbf{e}_1 \cdot \nabla \mathbf{e}_b) \cdot \nabla(\mathbf{e}_1 \cdot \nabla\boldsymbol{\xi} \cdot \nabla F_{g0}) + (\mathbf{e}_2 \cdot \nabla \mathbf{e}_b) \cdot \nabla(\mathbf{e}_2 \cdot \nabla\boldsymbol{\xi} \cdot \nabla F_{g0})\right].\end{aligned}$$

— This can be converted to a 2nd order equation (Zheng et al, PoP **1**, 2956 (1994)):

$$v_\parallel \mathbf{e}_b \cdot \nabla \frac{v_\parallel}{i(\omega - \omega_d)} \mathbf{e}_b \cdot \nabla \delta G_0^e - i(\omega - \omega_d)\delta G_0^e = \mathcal{R}_g^e + v_\parallel \mathbf{e}_b \cdot \nabla \frac{1}{i(\omega - \omega_d)} \mathcal{R}_g^o.$$

This shows the correspondence of the gyrophase-averaged gyrokinetic equation to the parallel MHD momentum equation.

— Note $\delta H(\mathbf{X}) = -\boldsymbol{\xi}(\mathbf{X}) \cdot \nabla F_{g0} + \delta G(\mathbf{X})$. One can show that $\delta G(\mathbf{X})$ corresponds to MHD $\gamma P \nabla \cdot \boldsymbol{\xi}$.

— The coupling of the gyrophase-dependent part of solution is essential to obtain the magenta-colored term ordering-consistently.

Part II:

AEGIS-K code and investigation of RWMs in ITER using our gyrokinetic theory

- **Basic set of equations for AEGIS-K**

— **the perpendicular momentum equation**

$$-\rho_m \omega^2 \boldsymbol{\xi}_\perp = \delta \mathbf{J} \times \mathbf{B} + \mathbf{J} \times \delta \mathbf{B} - \nabla \delta P - \nabla_\perp \int d^3 v (m_\rho \mu_0 B) \delta G_0(\mathbf{x}),$$

— **the quasineutrality condition**

$$\delta\varphi = -\frac{1}{1+Z\tau}\frac{T_e}{Ze_i}\frac{1}{n_0}\int d^3 v \delta G_{0i}.$$

— **the gyrophase-independent part of the gyrokinetic equation**

$$\begin{aligned} &\mathbf{v}_\parallel \cdot \nabla G_0(\mathbf{X}) - i\omega\delta G_0(\mathbf{X}) \\ &= i\omega \frac{m_\rho}{T_i}\mu_0 B F_{g0} \nabla_\perp \cdot \boldsymbol{\xi} + i\omega \frac{m_\rho}{T_i}\left(\mu_0 B - v_\parallel^2\right) F_{g0} \boldsymbol{\kappa} \cdot \boldsymbol{\xi} - i\omega \frac{Ze_i}{T_i} F_{g0} \delta\varphi, \end{aligned}$$

The source terms on the rhs of this equation are different from those of the conventional drift kinetic equation

AEGIS-K is a kinetic extension of the MHD AEGIS (Adaptive EiGenfunction Independent Solution shooting code) [J. Comp. Phys. 211, 748 (2006)]

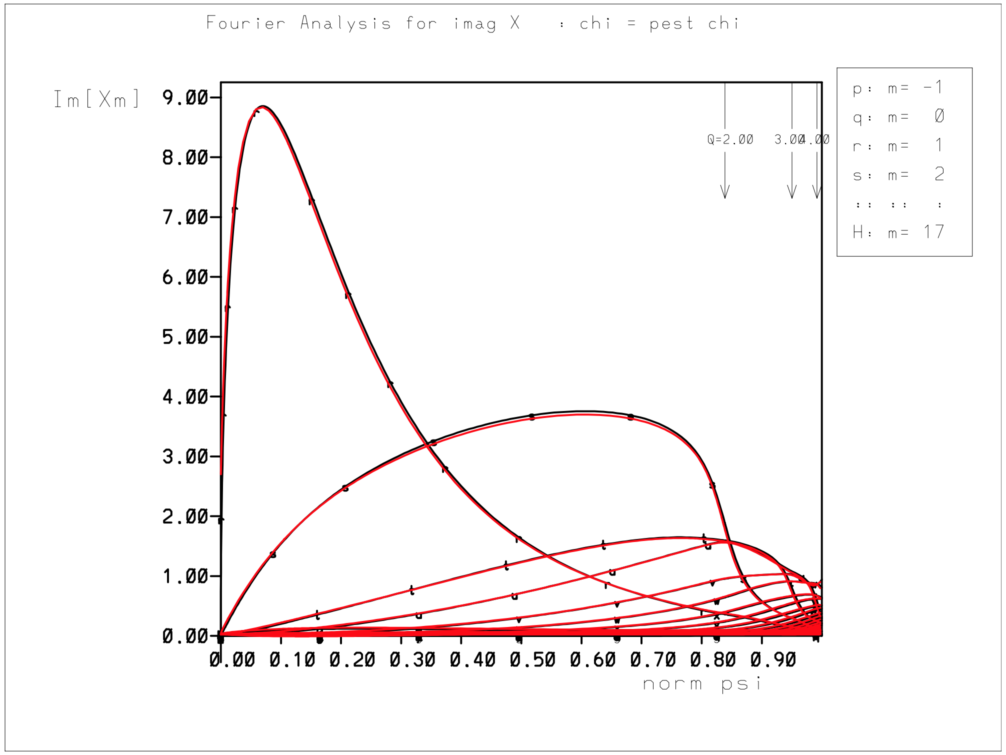

Figure 1: $n = 1$ eigenmode computed by AEGIS (red) and GATO (black)

- AEGIS numerical scheme:
  — Poloidal: Fourier decomposition
  — Radial: adaptive shooting based on the independent solution decomposition.

- AEGIS has been benchmarked with GATO in the ideal MHD limit.
  — Complete agreement is found for growth rate, stability beta limit, critical wall position, mode shape, *etc.*

ITER AT equilibrium

- **Typical parameters:** $I/aB = 1.14$, $q_0 = 2.48$, $q_a = 5.34$, $q_{\min} = 2.19$, $q_{95} = 4.19$, **elongation** $\kappa_a = 1.8$, **triangularity** $\delta_a = 0.41$, **and** $\beta_N^{no\ wall} = 2.94\%$.

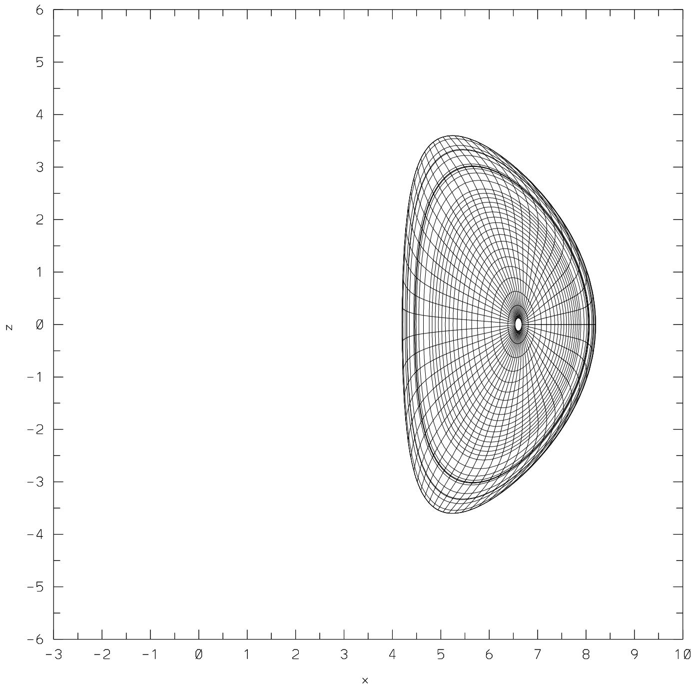

Figure 2: Cross section of the ITER AT configuration with poloidal Hamada and radial packed grids shown

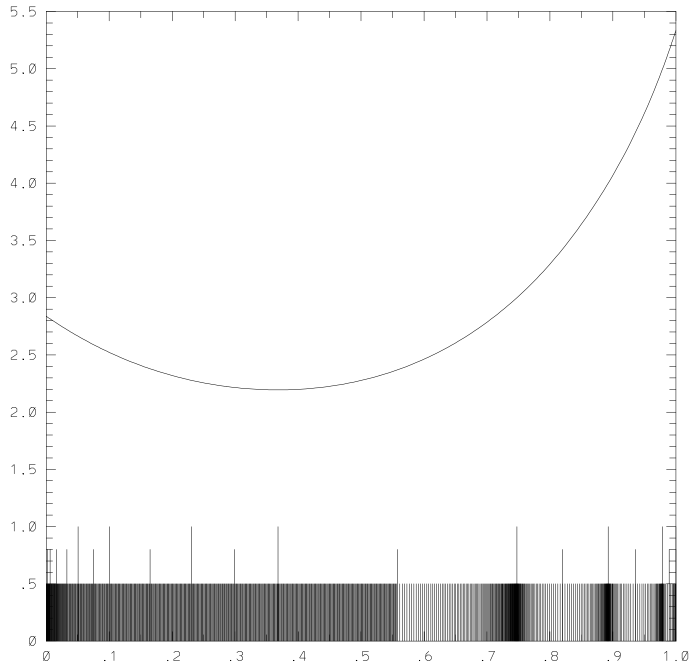

Figure 3: Safety factor profile, with the maximum step size and region separation for shooting shown.

Typical unstable kinetic eigenmodes

- **Typical unstable eigenmodes with $\beta_N = 3.15\%$, wall position $b = 1.5$, and rotation frequency 0.0007.**

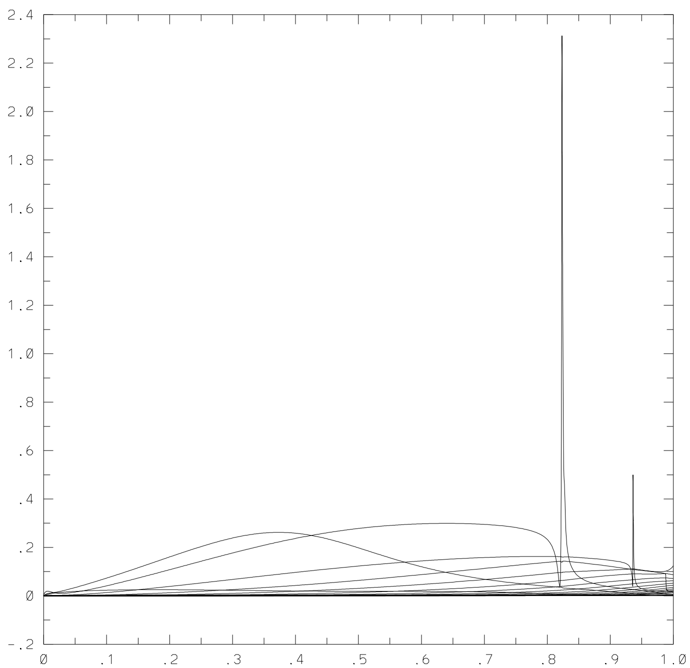

Figure 4: Real part of the unstable resistive wall mode in the presence of rotation and kinetic effects.

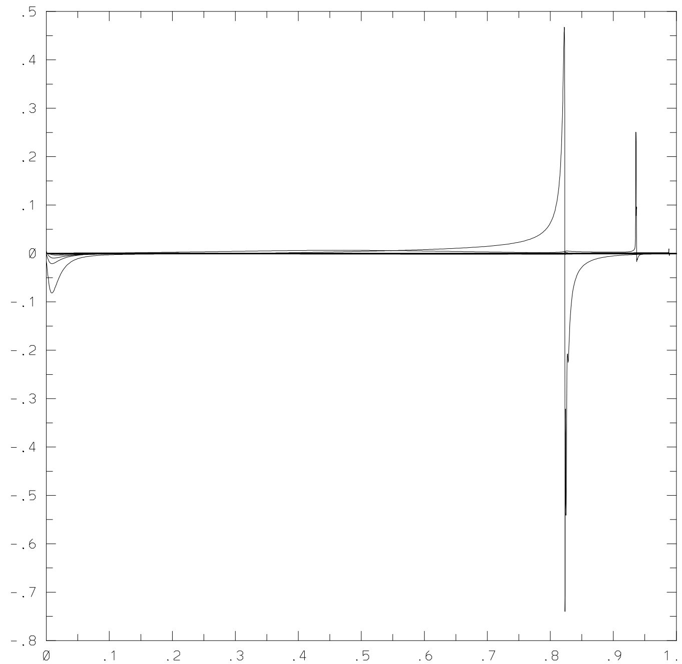

Figure 5: Imaginary part of the unstable resistive wall mode in the presence of rotation and kinetic effects.

RWM stability diagram computed by AEGIS-K using our gyrokinetics theory

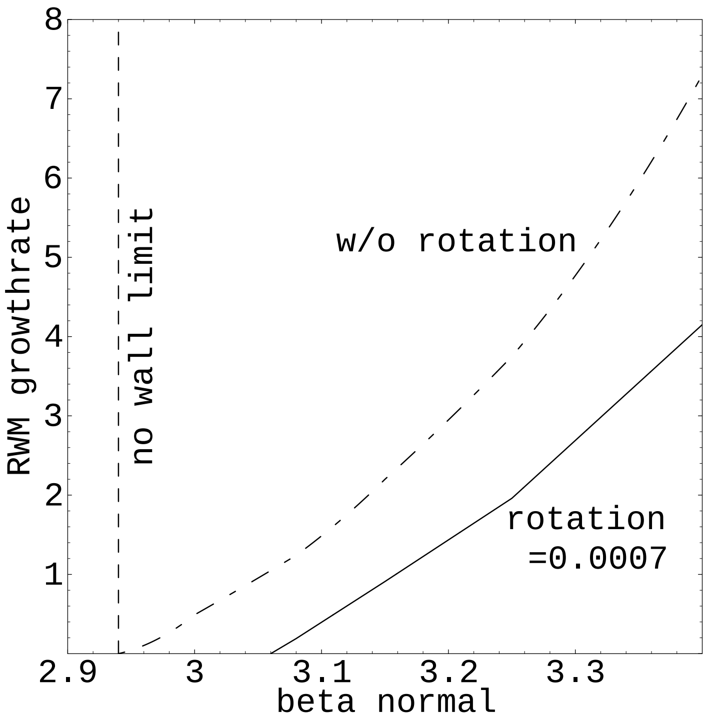

Figure 6: RWM growthrate versus β_N for ITER AT configuration computed by AEGIS-K. Dashed-dot curve represents the growth rate without rotation; Solid curve with normalized rotation frequency 0.0007.

- AEGIS-K features:
  — AEGIS-K is based on our newly derived gyrokinetics theory, which recovers full MHD and missing FLR effects;
  — AEGIS-K is a nonhybrid kinetic MHD code;
  — Adaptive shooting leads AEGIS-K to be able to treat the coupling between the shear Alfvén and kinetic resonances

- We find a modestly low rotation stabilization of RWMs in ITER AT is possible

Summary I: Gyrokinetics theory

We revisit the linear gyrokinetic theory with two major contributions

1. We recover full MHD from gyrokinetics

— In the perpendicular direction: $\mathbf{J}_0 \times \delta\mathbf{B}$ is recovered;

— In the parallel direction: If the particle speed is replaced by the thermal speed, the gyrophase-averaged gyrokinetic equation has similarity with the parallel MHD moment equation;

— Our gyrophase-averaged gyrokinetic equation of lowest order is different from conventional drift kinetic equation.

2. We recover missing FLR effects

— We find that the conventional results about FLR effects based on the Bessel functions J_0 and J_1 are incomplete.

— Especially, we point out that the conventional FLR modification in the 2nd order $\omega^2 \to \omega\left[\omega - \omega_{*i}(1+\eta_i)\right]$ applies only to the large aspect ratio configuration. The newly discovered FLR terms in this order has a structure similar to that of the Braginskii gyroviscous tensor.

Summary II: AEGIS-K code and numerical investigation of RWMs in ITER

- We have developed a nonhybrid kinetic MHD code AEGIS-K, using our newly developed gyrokinetic equation.

- We have applied our AEGIS-K code to study RWMs in an ITER AT configuration. Our preliminary results show that a low rotation stabilization of RWMs in ITER AT is possible for certain values of beta normal.

- The AEGIS-K code is still new. Further tests, especially to compare with existing codes like MARS, will be carried out in the future

References

- Linear gyrokinetic formalism was developed decades ago

— Electrostatic gyrokinetics:

P. H. Rutherford and E. A. Frieman, *Phys. Fluids*, **11**, 569 (1968).
J. B. Taylor and R. J. Hastie, *Plasma Phys.*, **10**, 479 (1968).

— Electromagnetic gyrokinetics:

T. M. Antonsen, Jr. and B. Lane, *Phys. Fluids*, **23**, 1205 (1980).
P. J. Catto, W. M. Tang, and D. E. Baldwin, *Plasma Phys.*, **23**, 639 (1981).

- Recent effort by H. Qin, W. M. Tang, and G. Rewoldt to derive so-called long wave-length gyrokinetics in order to recover MHD (about half dozen papers):

H. Qin, W. M. Tang, and G. Rewoldt, *Phys. Plasmas*, **5**, 1035 (1998).
H. Qin and W. M. Tang, *Phys. Plasmas*, **5**, 1052 (2004).

— However, the perpendicular MHD is actually not recovered completely in Qin, et al. formalism.

- Our newly derived gyrokinetic theory

L.-J. Zheng, M. T. Kotschenreuther, and J. W. Van Dam, Phys. Plasmas **14**, 072505 (2007).