The Scaling of Forced, Collisionless Reconnection in a Simple 3D System

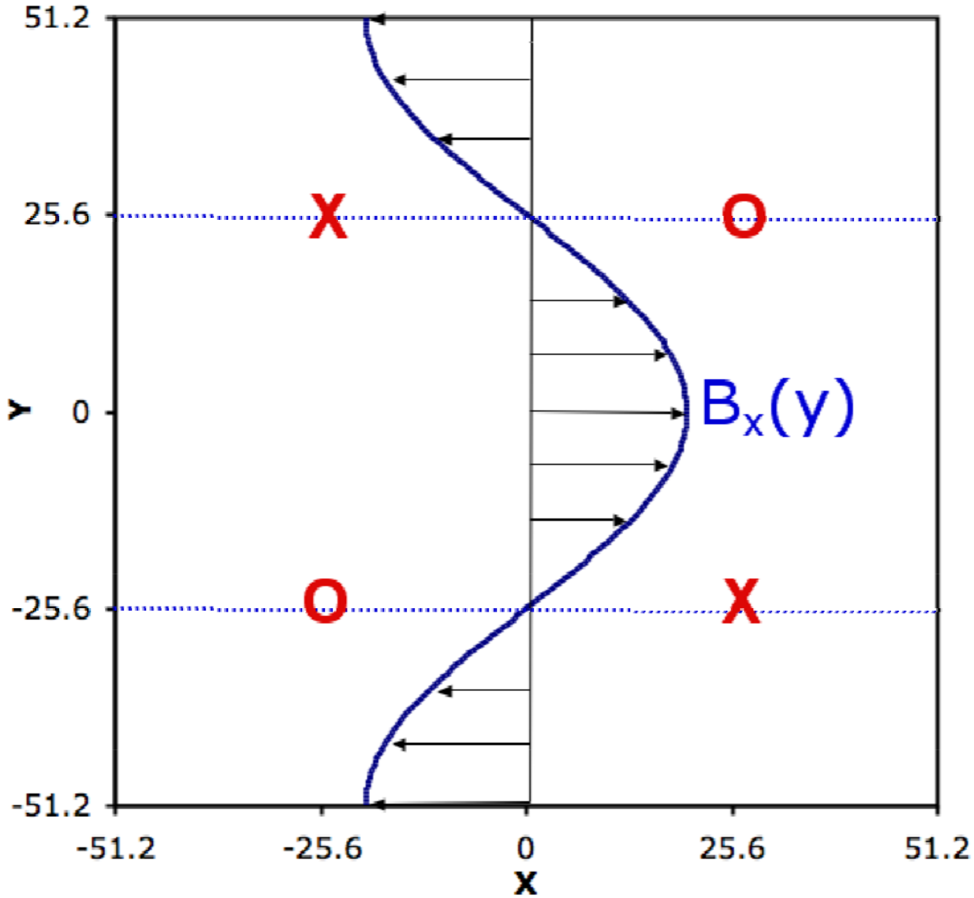
Brian Sullivan, University of New Hampshire
Barrett Rogers, Dartmouth College
22 Nov 2008

Objectives

- Observe scaling of the reconnection process in a simple 3D system
- Compare observed scaling to that in an analogous 2D system

1^{st} : 2D Scaling Study

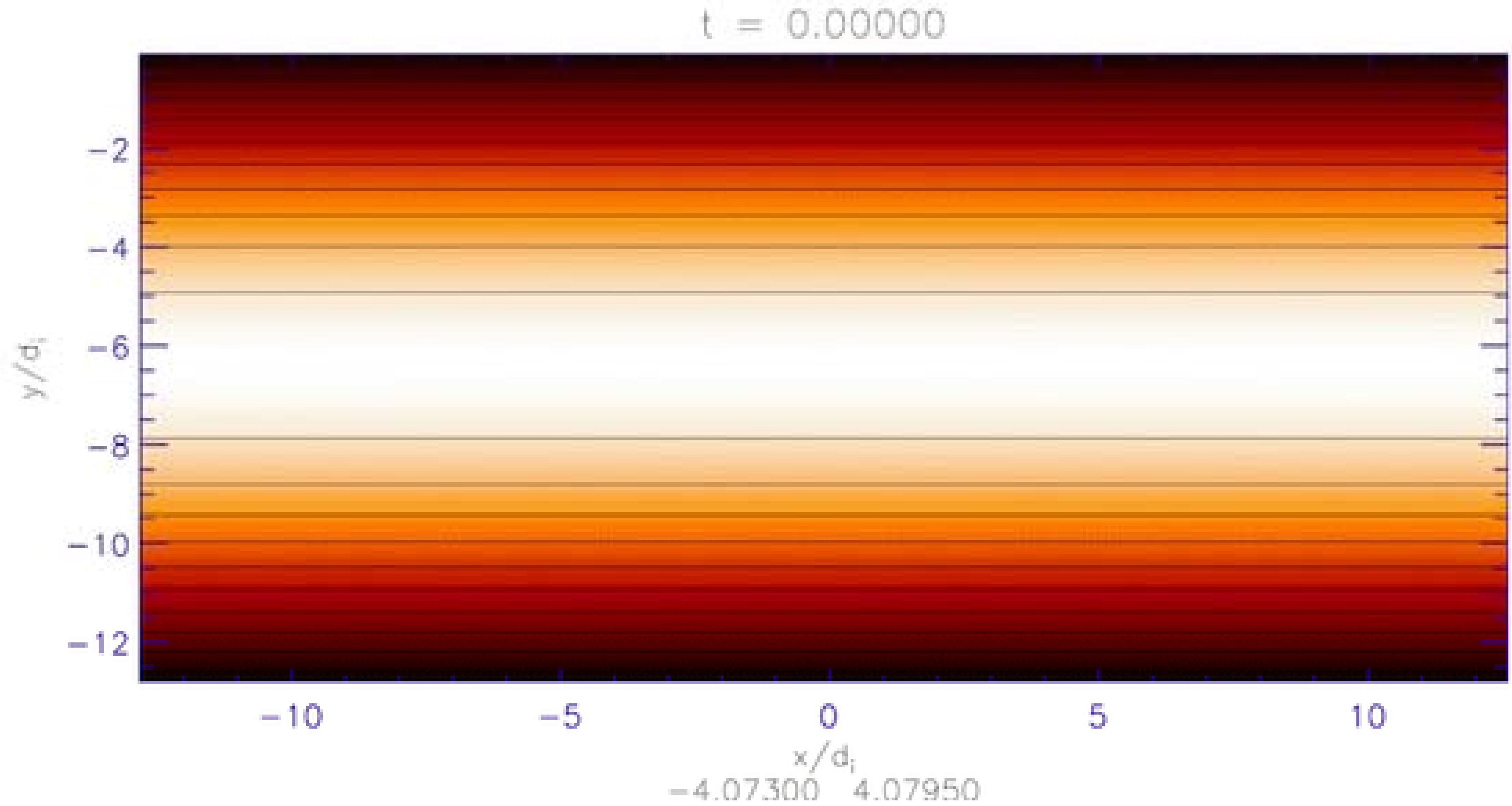
- 2D 2-fluid model with finite e^- inertia
- $\frac{m_e}{m_i} = \frac{1}{25}$
- $L_x = L_y = 102.4\ c/\omega_{pi}$
- Double Tearing mode equilibrium
- Periodic B.C.'s in both x and y



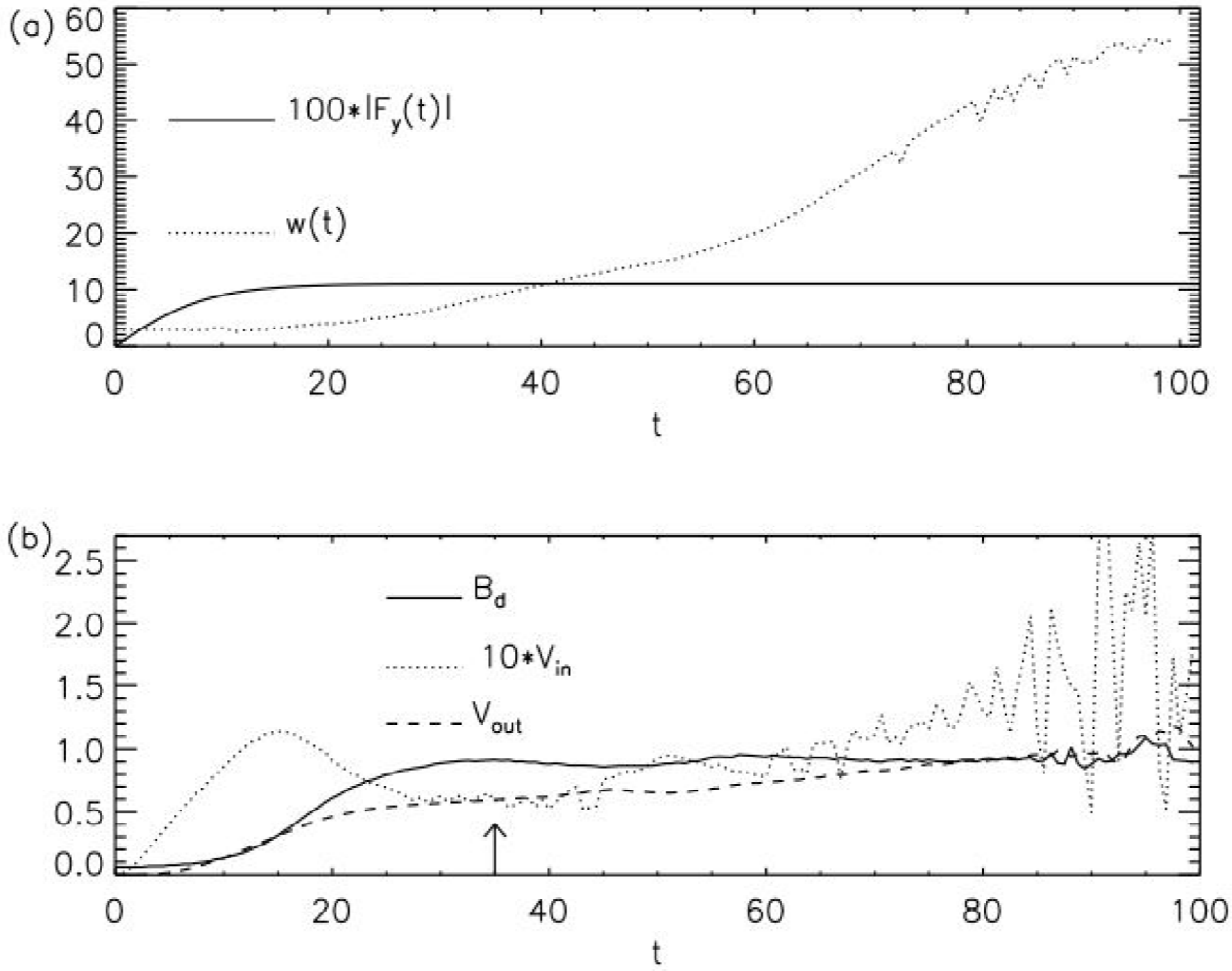
- Stable in absence of Forcing $(k_x\delta_J \geq 1)$

This system is stable, but if you force it...

$\vec{J}_z(x,y)$



Time series Data



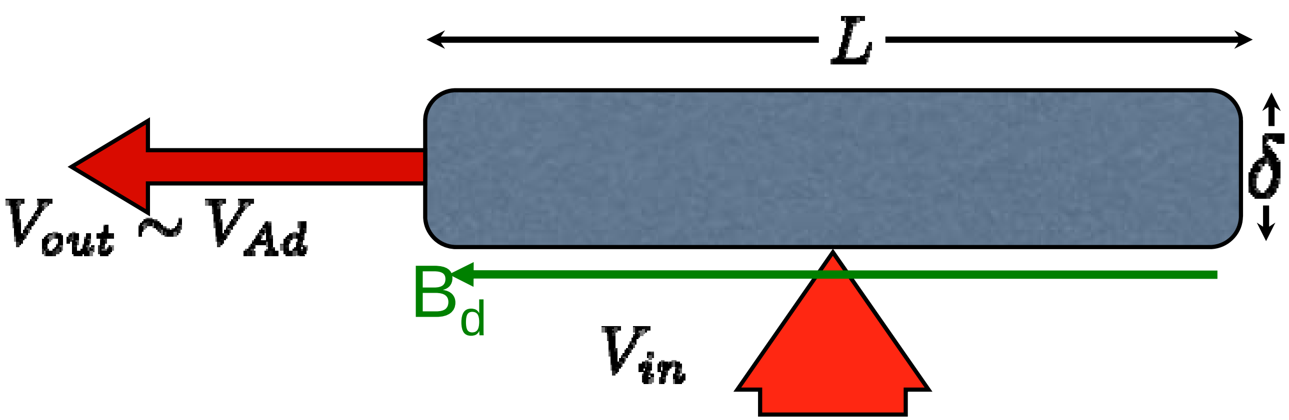
Expected R. Rate Scaling

$$R.Rate \sim E_z \sim V_{in} B_d$$

$$V_{in} \sim \left(\frac{\delta}{L}\right) V_{out}$$

$$V_{out} \sim V_{Ad} = \frac{B_d}{\sqrt{4\pi m_i n}}$$

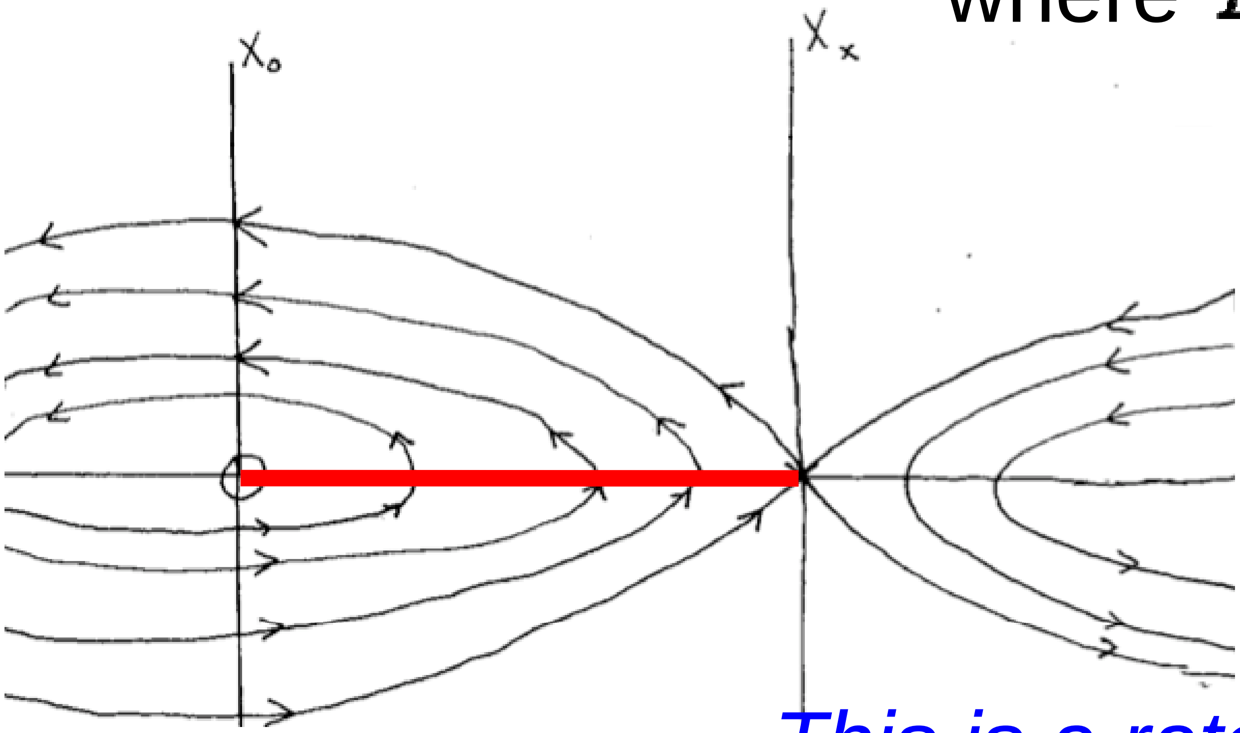
$$R.Rate \sim \frac{\delta}{L} \frac{B_d^2}{\sqrt{4\pi m_i n}}$$



Defining The Reconnection Rate

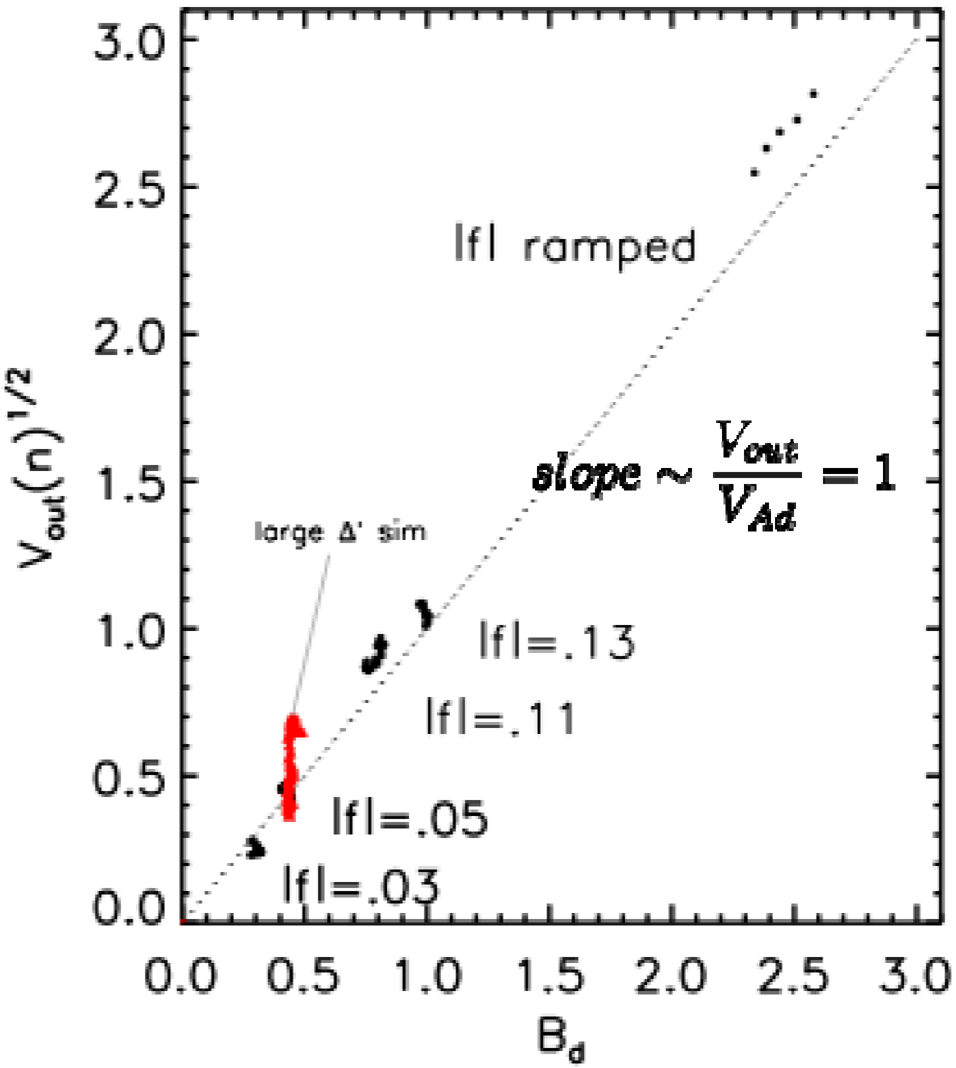
$$R_0 \equiv \frac{\partial}{\partial t}\left[\psi(x_x, -L_y/4) - \psi(x_o, -L_y/4)\right]$$
$$= \frac{\partial}{\partial t}\int_{x_o}^{x_x} B_y(x, -L_y/4)\,dx$$

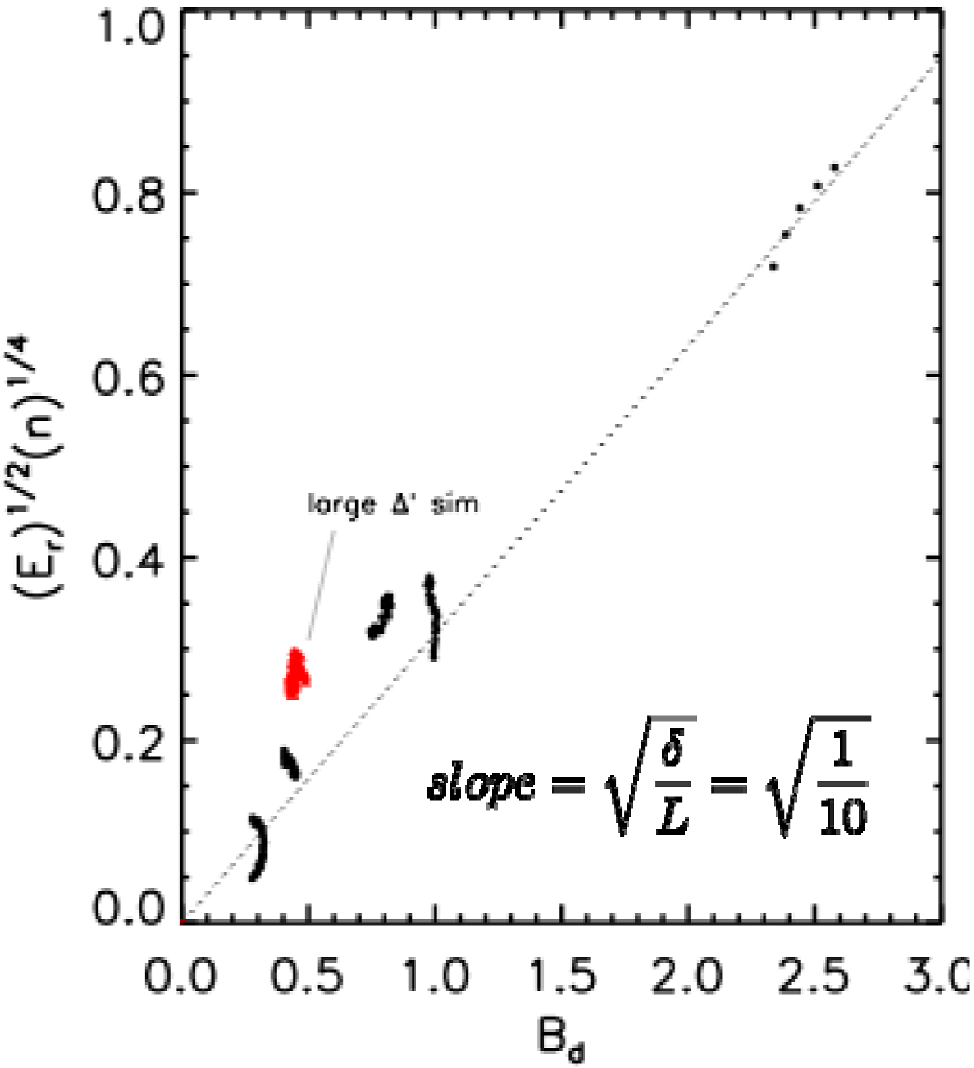
where $\vec{B} = \hat{z} \times \nabla\psi + B_z\hat{z}$
$= (-\partial_y\psi, \partial_x\psi, B_z)$



This is a rate per unit length!

Forced 2D Data Agree with GEM Challenge Scaling

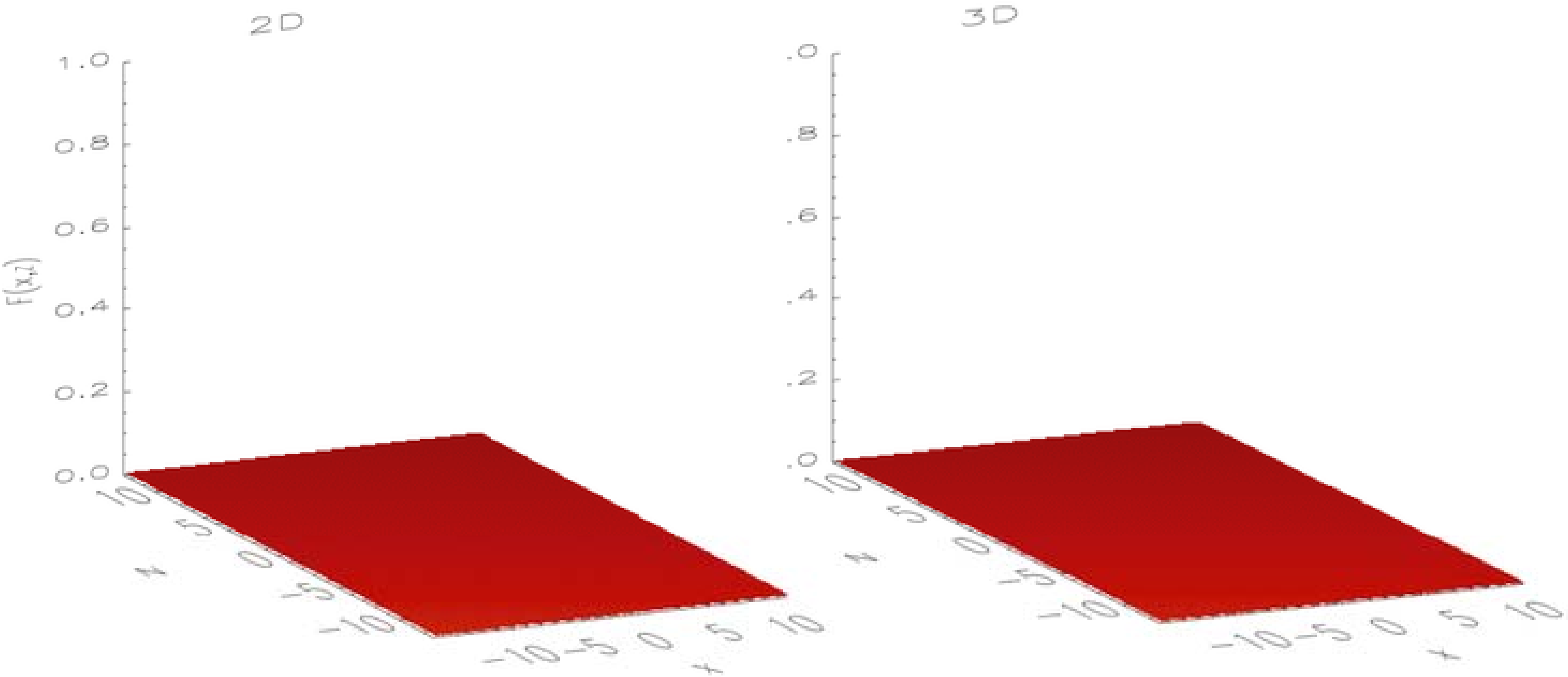




3D Study

- Same 1D initial equilibrium as in 2D study
- Smaller System size: $(25.6 d_i \times 25.6 d_i \times 25.6 d_i)$
  vs. $(102.4\ d_i \times 102.4\ d_i)$
- Forcing function scaled down with system
- Forcing localized in out-of-plane direction
- Comparisons made to a 2D analog

Out-Of-Plane Localization of Forcing



$$F_y(x,z)$$

Defining the 3D Reconnection Rate

In 3D we extend the 2D definition along z:

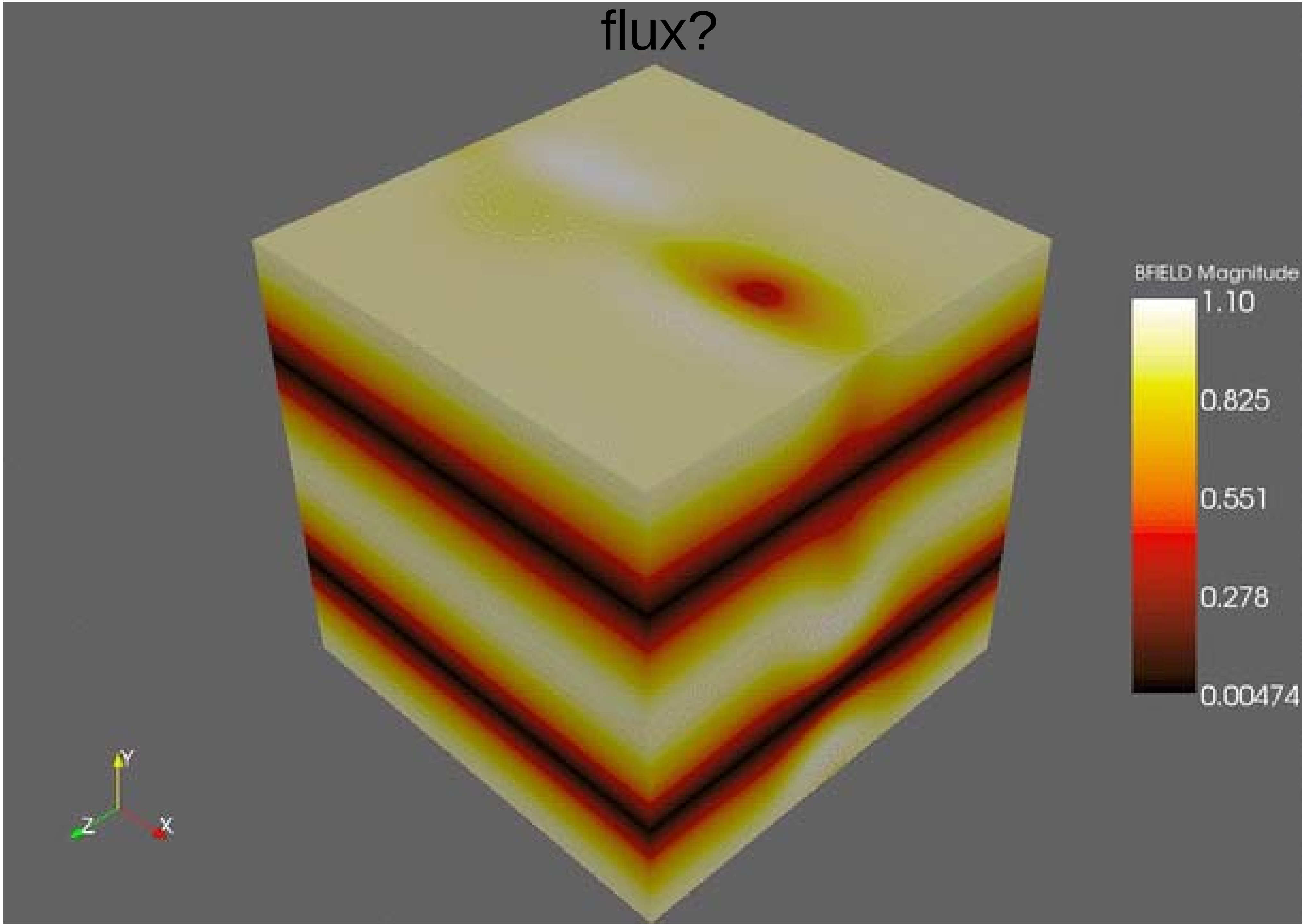
$$\Phi_{tot} = \int_A B_y(x,z)\,da$$

where A is the area of the magnetic island

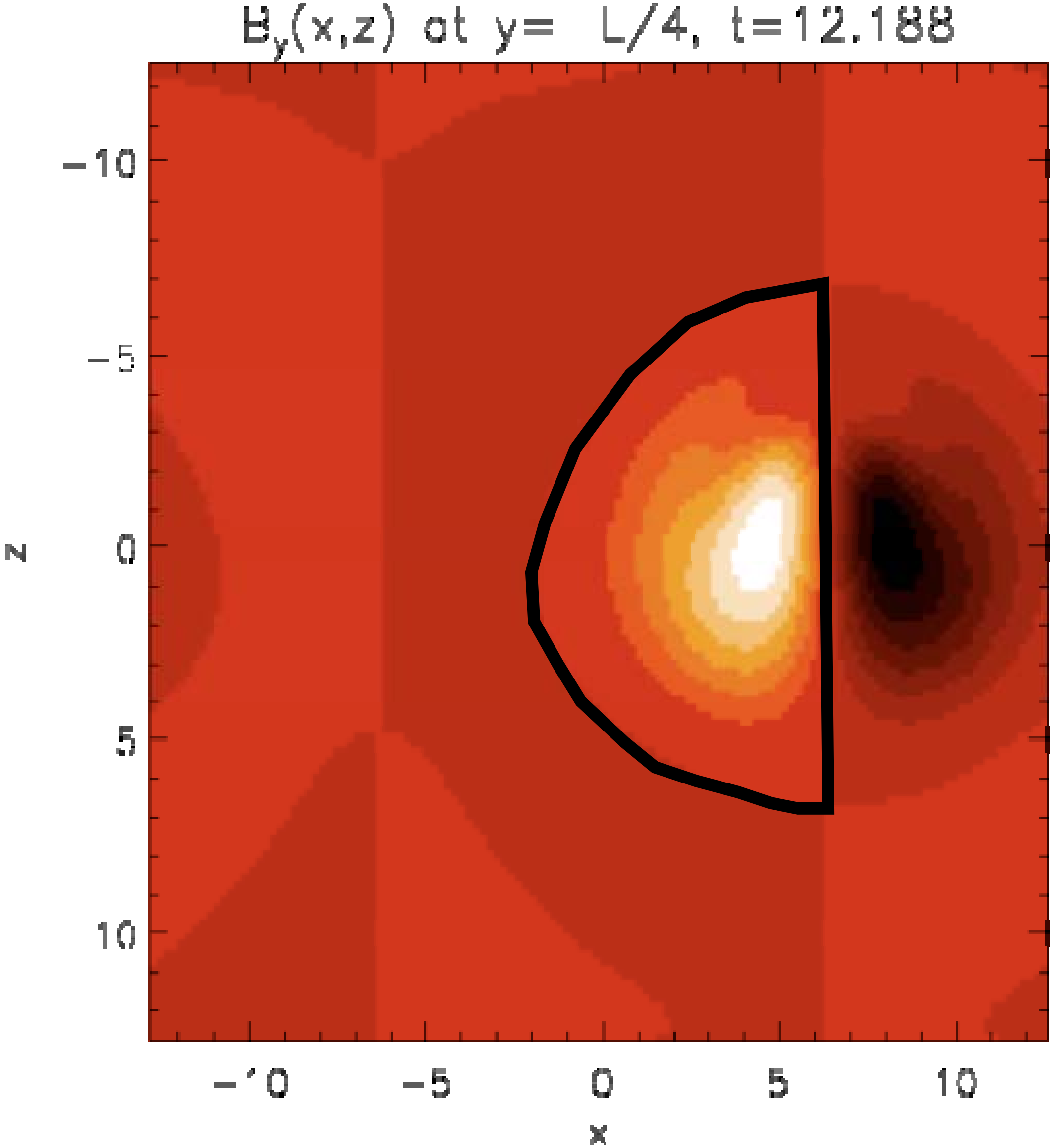
$$R_{tot} = \frac{\partial}{\partial t}\Phi_{tot} = \frac{\partial}{\partial t}\int_0^{Lz}(\Phi_{\mathrm{2D}})dz$$

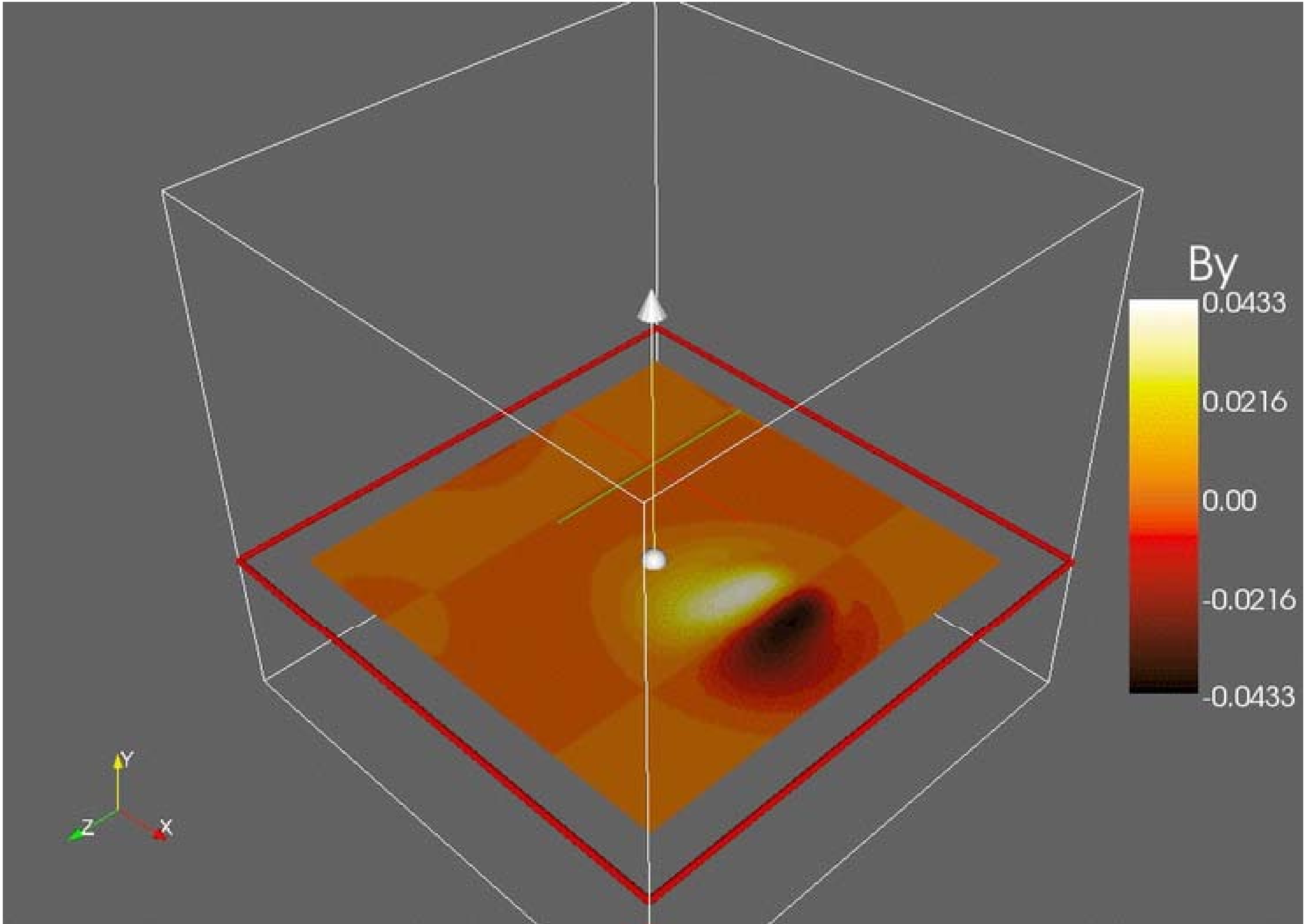
Careful:This is the total *rate, not per unit length.*

Where is the reconnected flux?

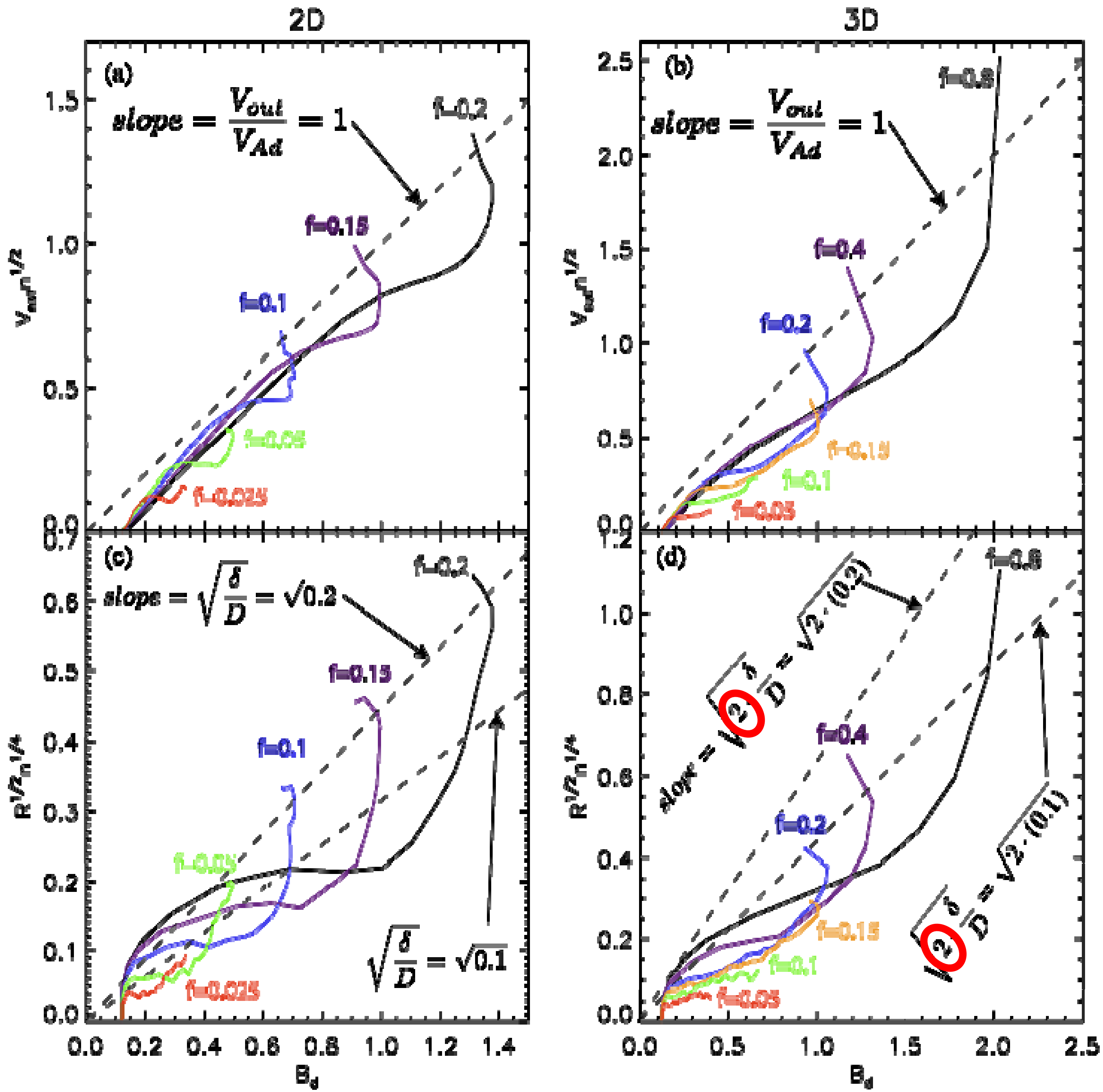


Shape of the Island





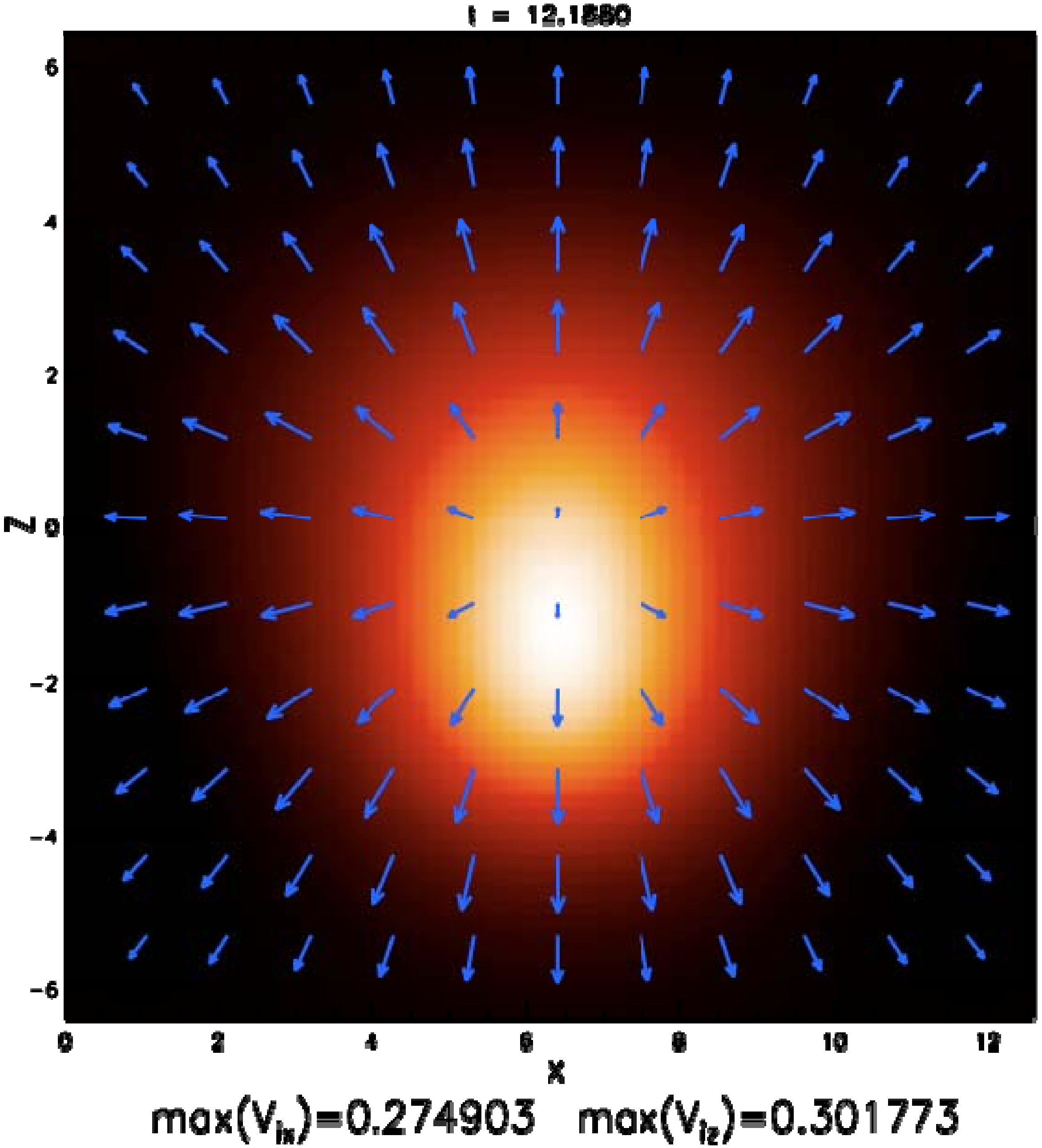
By
0.0433
0.0216
0.00
-0.0216
-0.0433
Y
Z
X



2D
3D
(a)
$slope = \frac{V_{out}}{V_{Ad}} = 1$
f=0.2
f=0.15
f=0.1
f=0.05
f=0.025
$V_{out} n^{1/2}$
(b)
$slope = \frac{V_{out}}{V_{Ad}} = 1$
f=0.8
f=0.4
f=0.2
f=0.15
f=0.1
f=0.05
(c)
$slope = \sqrt{\frac{\delta}{D}} = \sqrt{0.2}$
$\sqrt{\frac{\delta}{D}} = \sqrt{0.1}$
$R^{1/2} n^{1/4}$
(d)
$slope = \sqrt{2 \cdot \frac{\delta}{D}} = \sqrt{2 \cdot (0.2)}$
$\sqrt{2 \cdot \frac{\delta}{D}} = \sqrt{2 \cdot (0.1)}$
B_d

Why is inflow enhanced in 3D?

$$\frac{V_{in}}{V_{out}} \sim \frac{A_{out}}{A_{in}} = \frac{2\pi D\delta}{\pi D^2} = \frac{2\delta}{D}$$

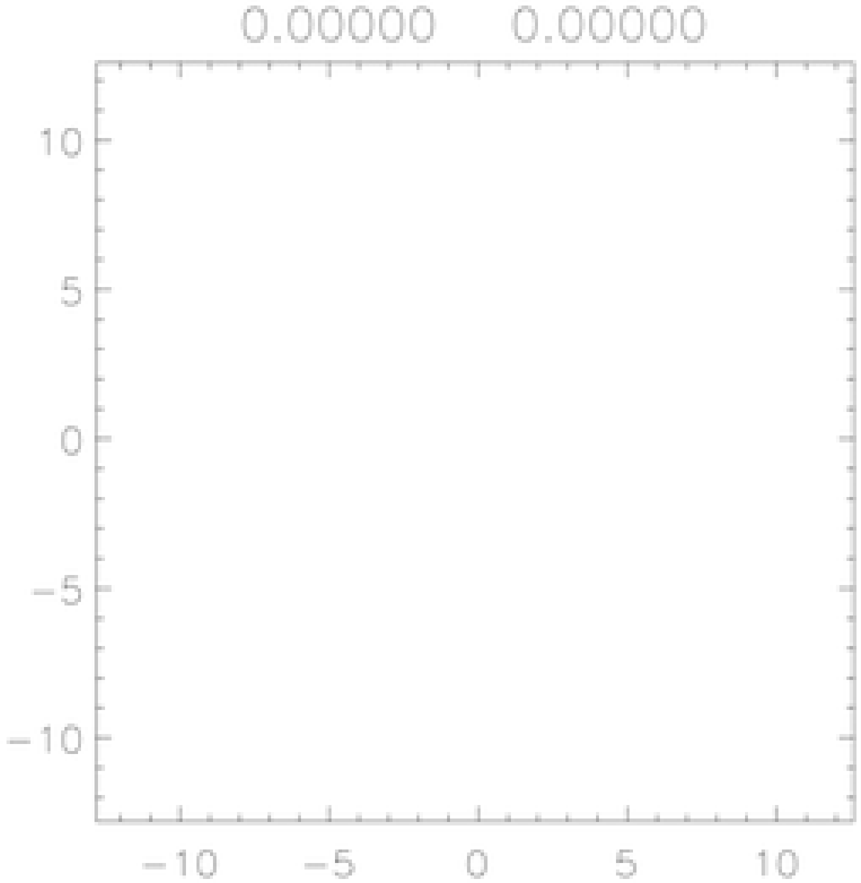


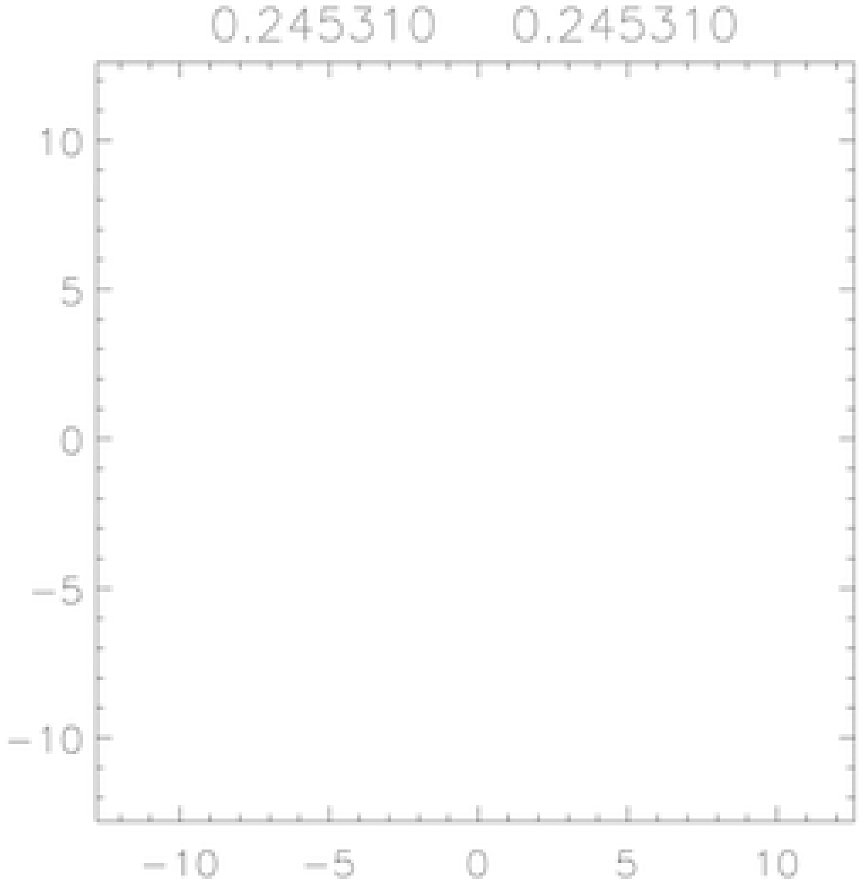
- Blue Arrows: $\vec{V}_i(x,z)$
- Greyscale :$J_z(x,z)$

Forced Reconnection in both 2D and 3D exhibit very similar scaling

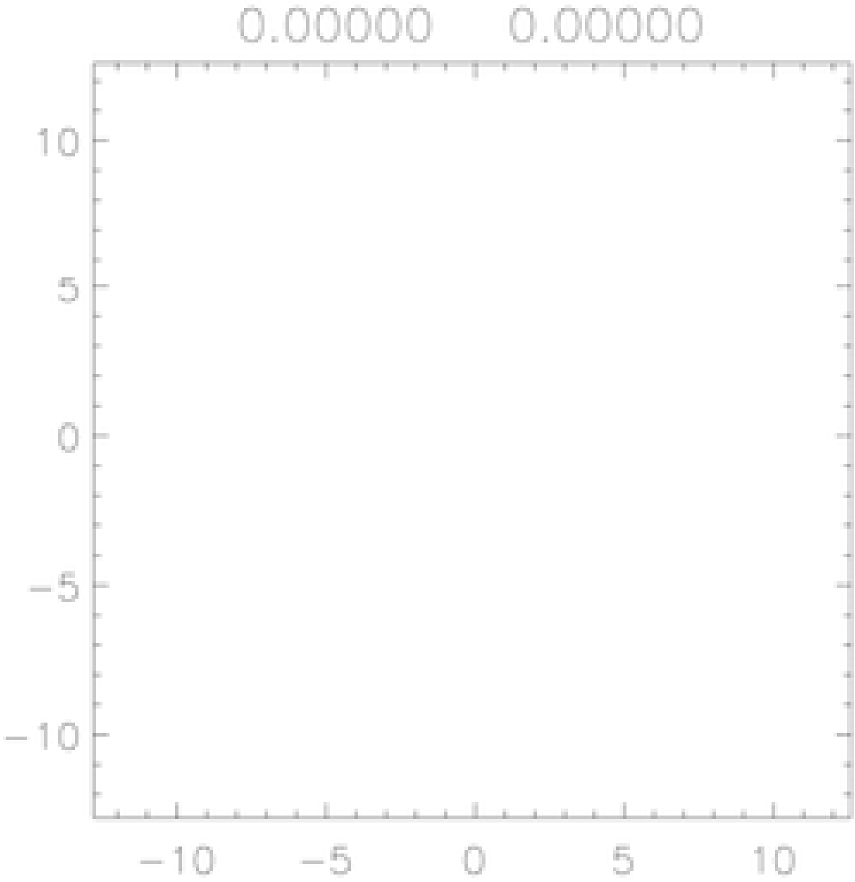
However, there are some qualitative differences in the dynamics...

J_{iz} and J_z in 3D

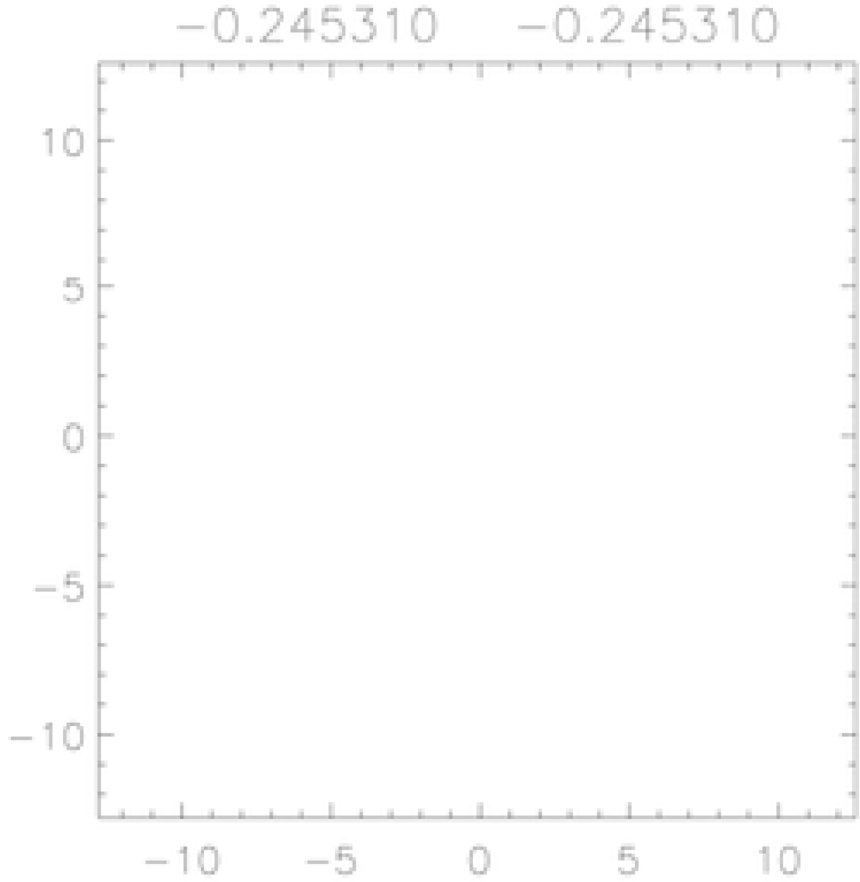




The electrons carry the Magnetic Island in the -*z* direction

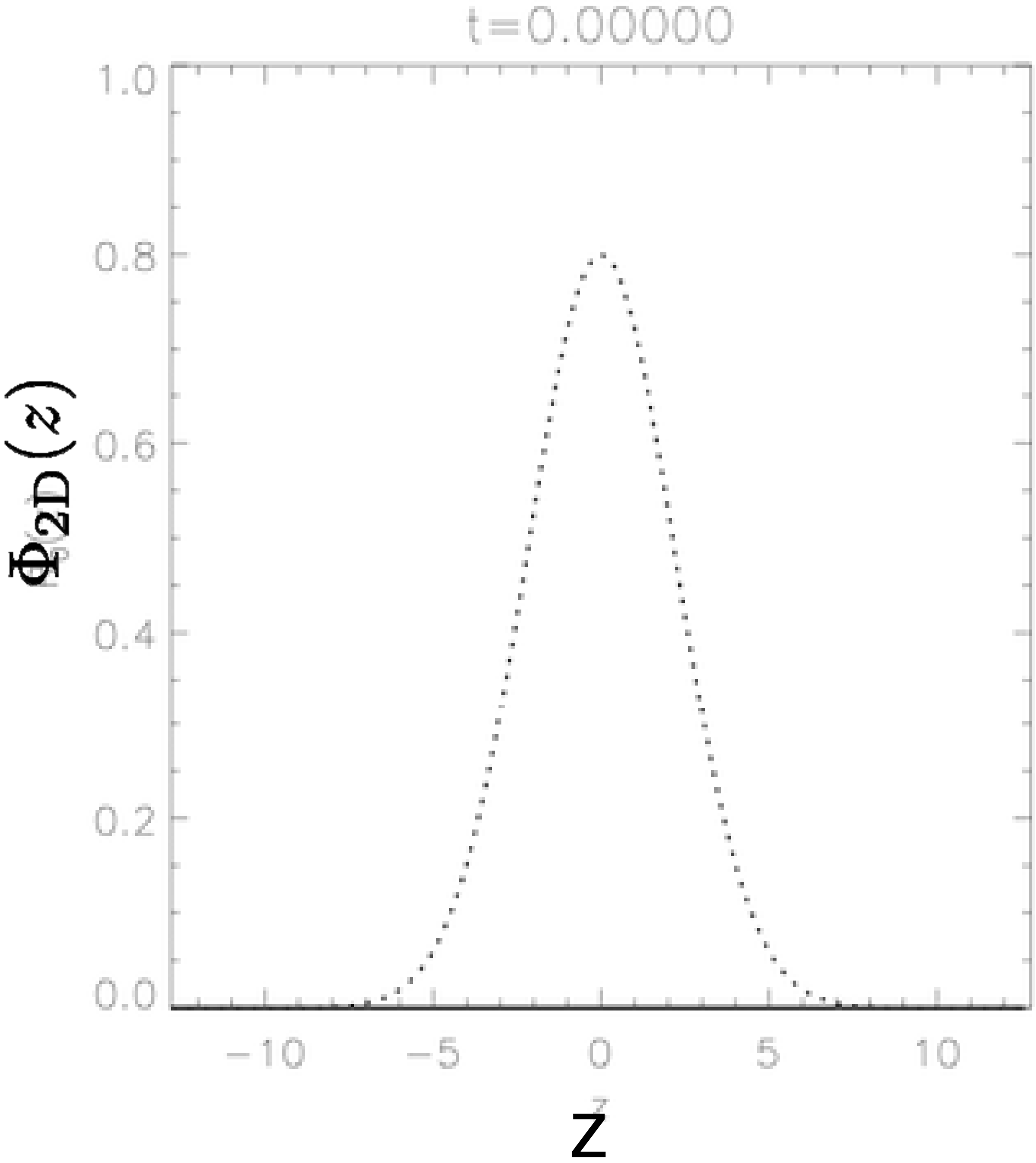


$B_y(x, z)$



$J_{ez}(x, z)$

Movement of Reconnected Flux along z

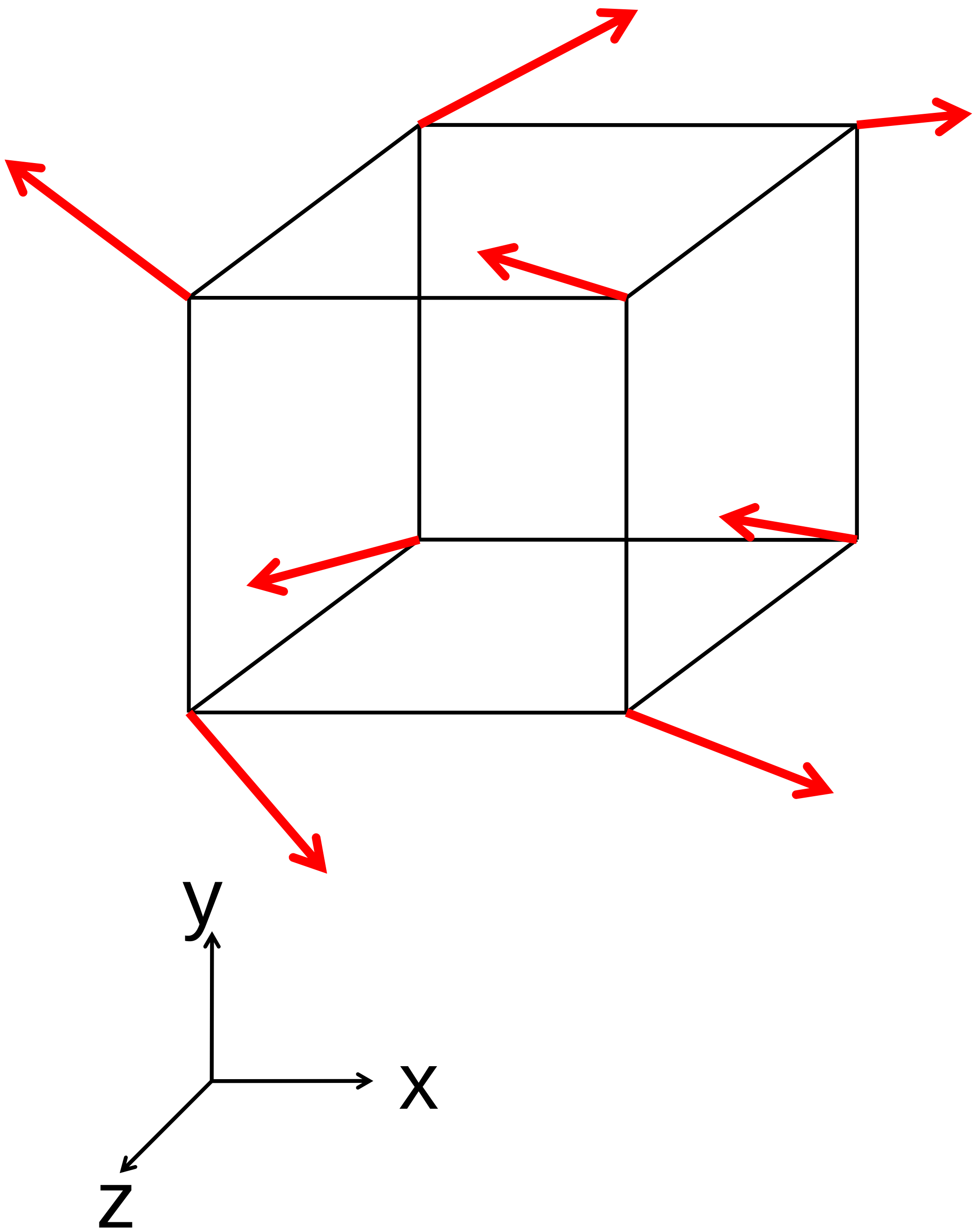


Dotted curve: Forcing $F(z)$

Solid curve: Flux $\Phi_{\mathbf{2D}}(z)$

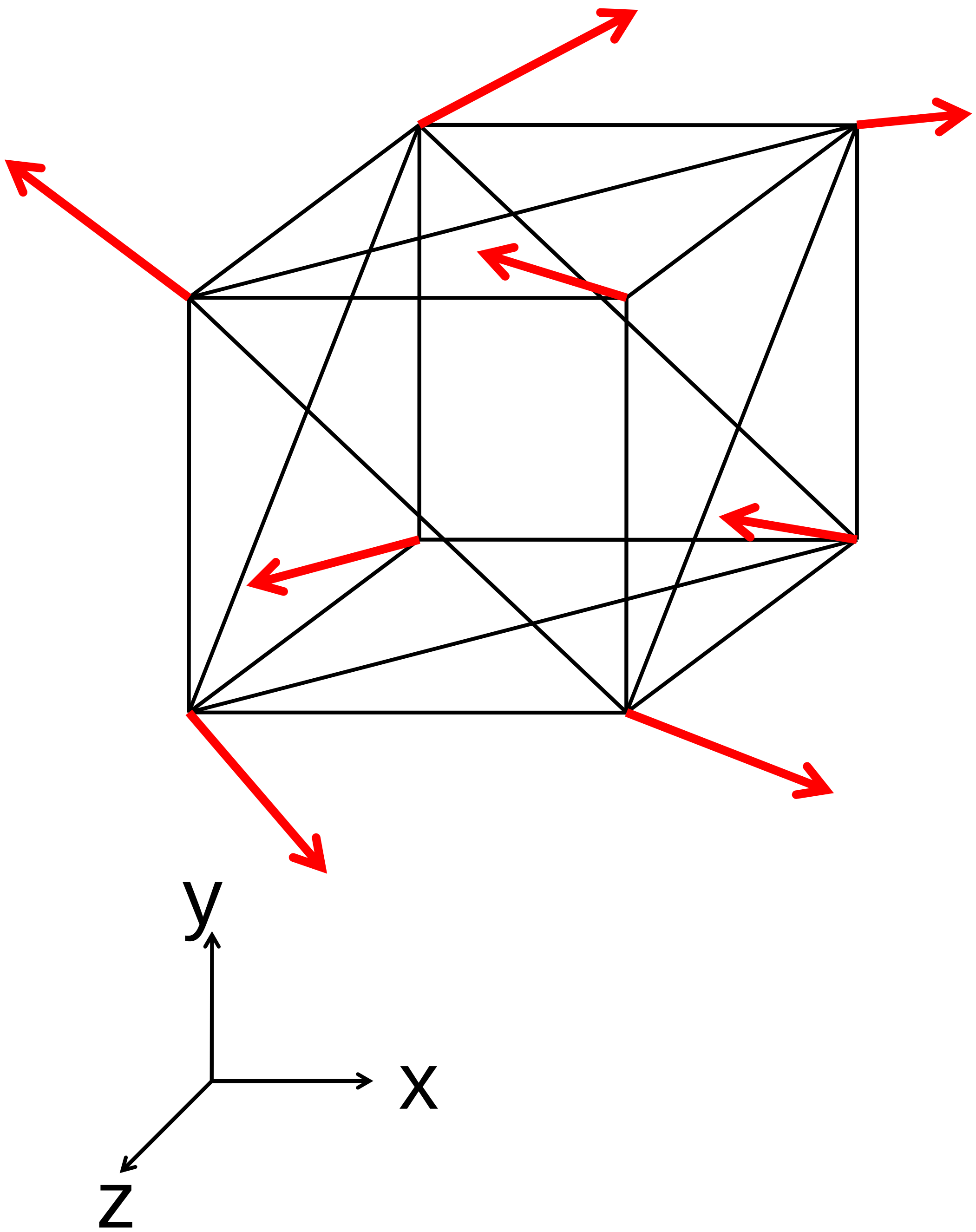
Where are the nulls of B?

- Nulls are located using an Algorithm by John Greene (*J. Comp. Phys.1992*)
- Algorithm implemented by John Dorelli
- Here’s how it works…

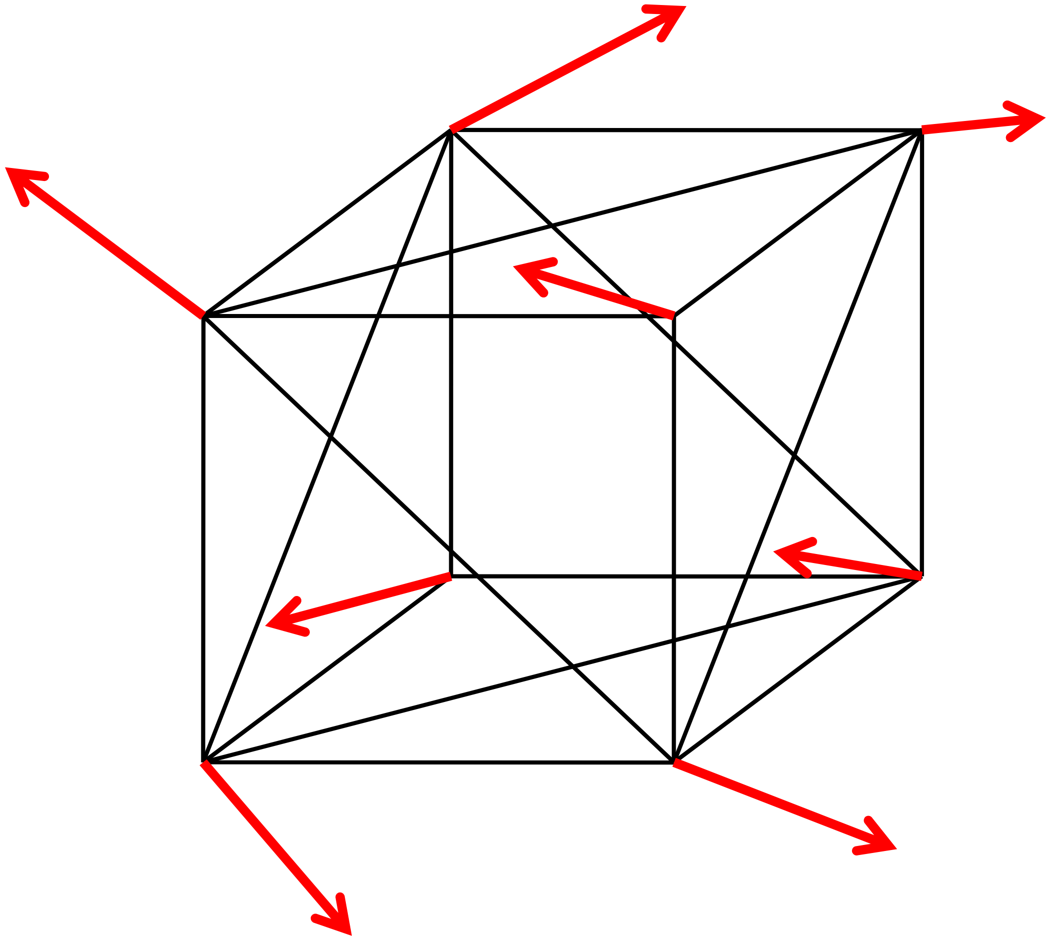


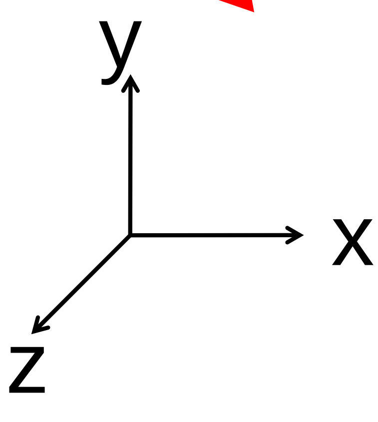
Magnetic field vectors at vertices of one grid cell

Does this cell contain a null of the field?

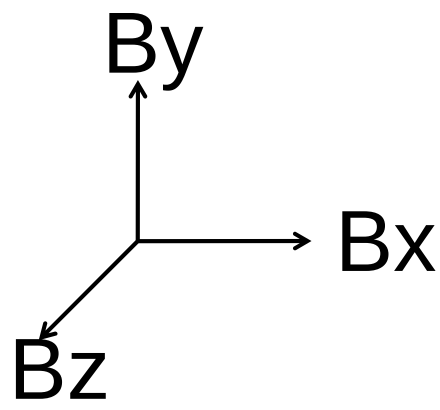


Divide cube faces into triangular simplexes

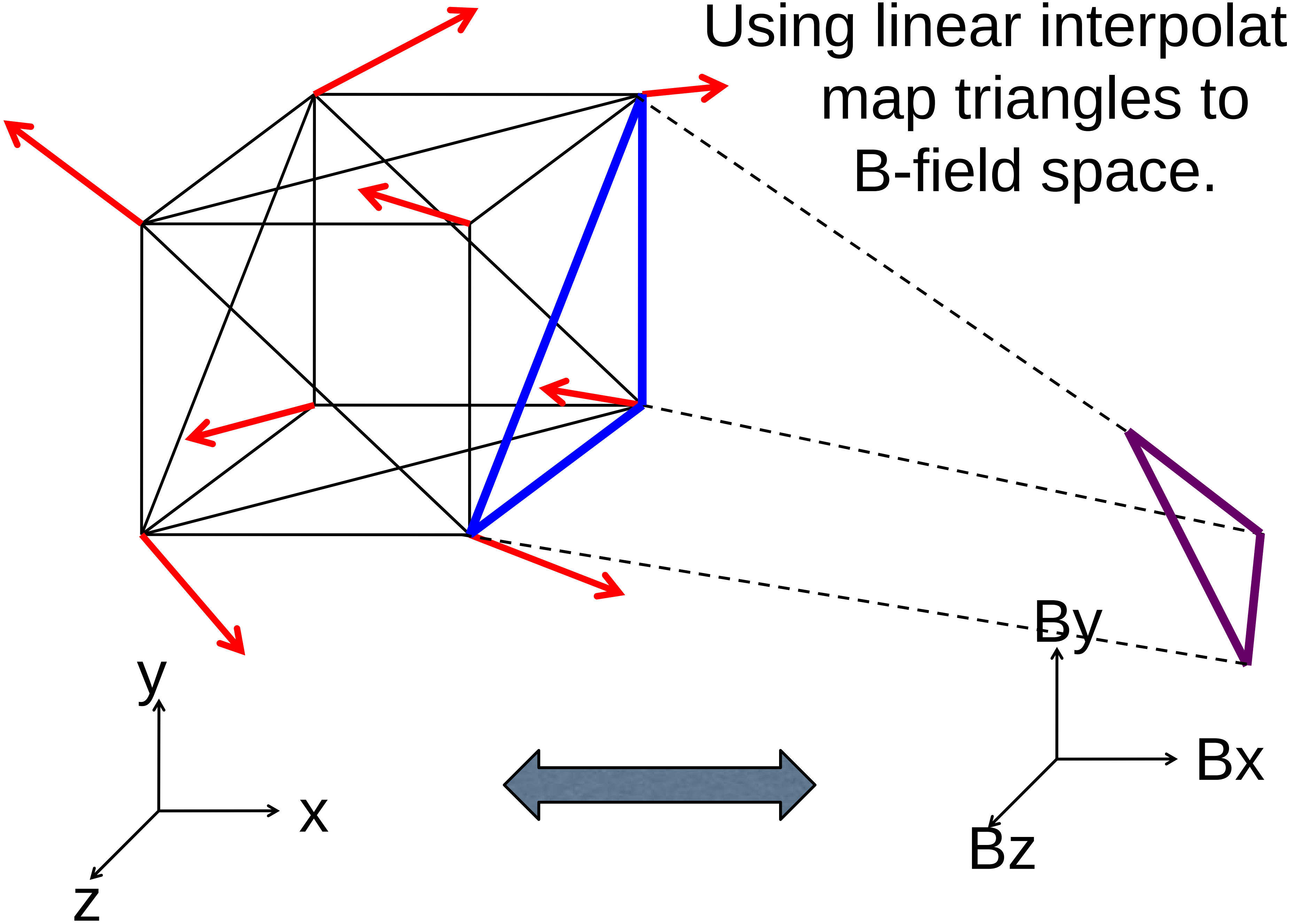


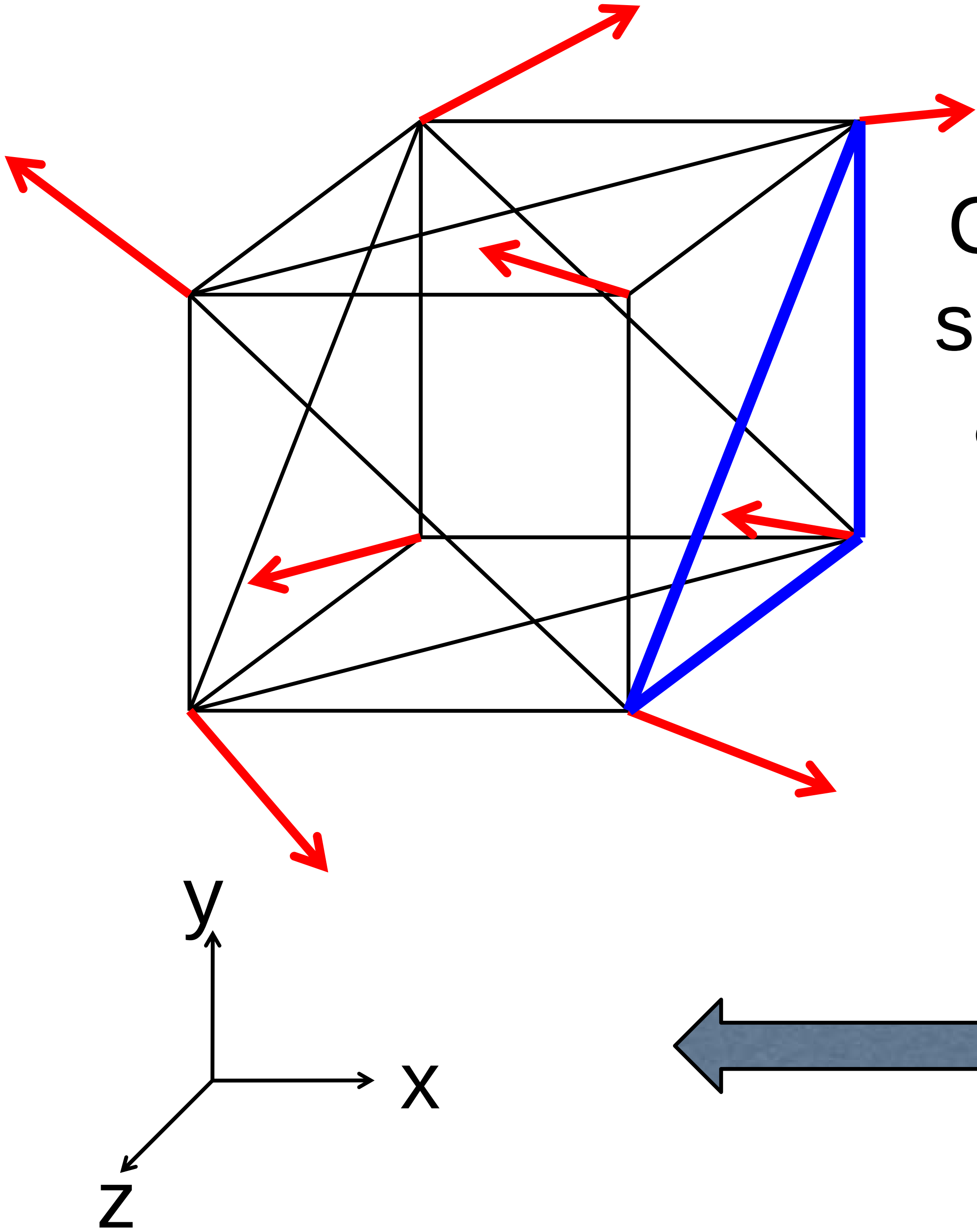


Suppose a mapping from configuration space to a B-field space:

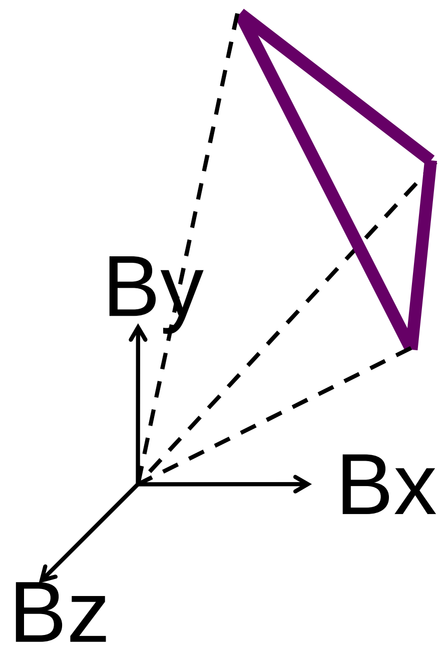


Using linear interpolation map triangles to B-field space.

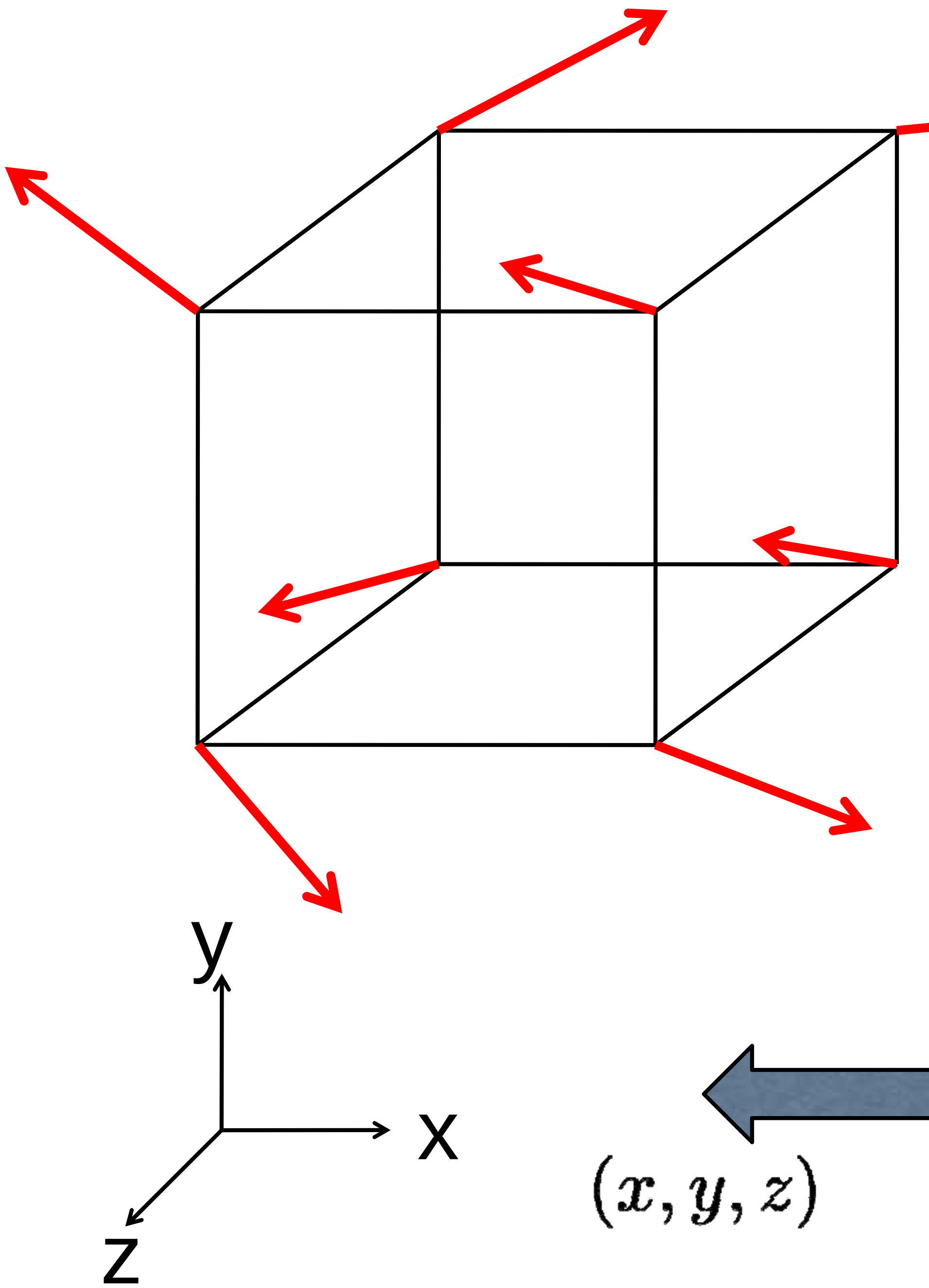


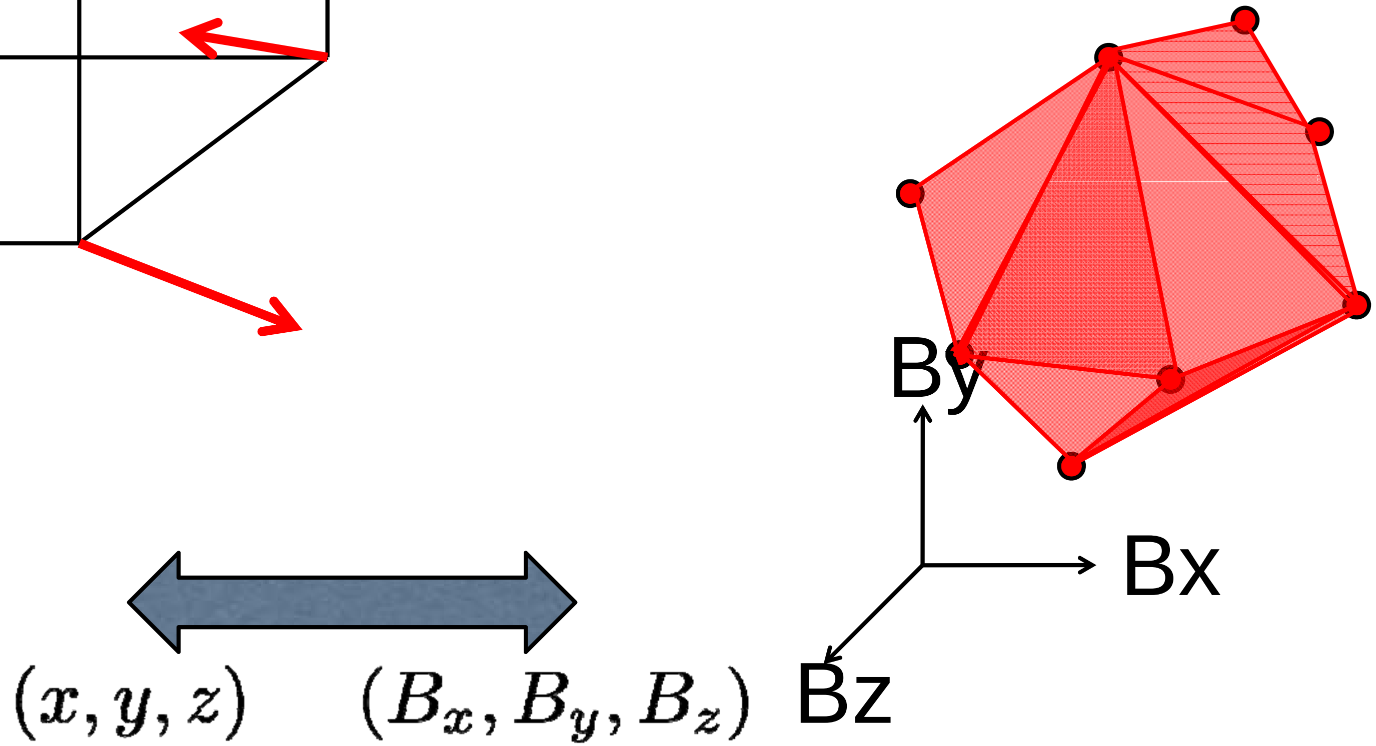


Calculate solid angle subtended by triangle at origin of B-space

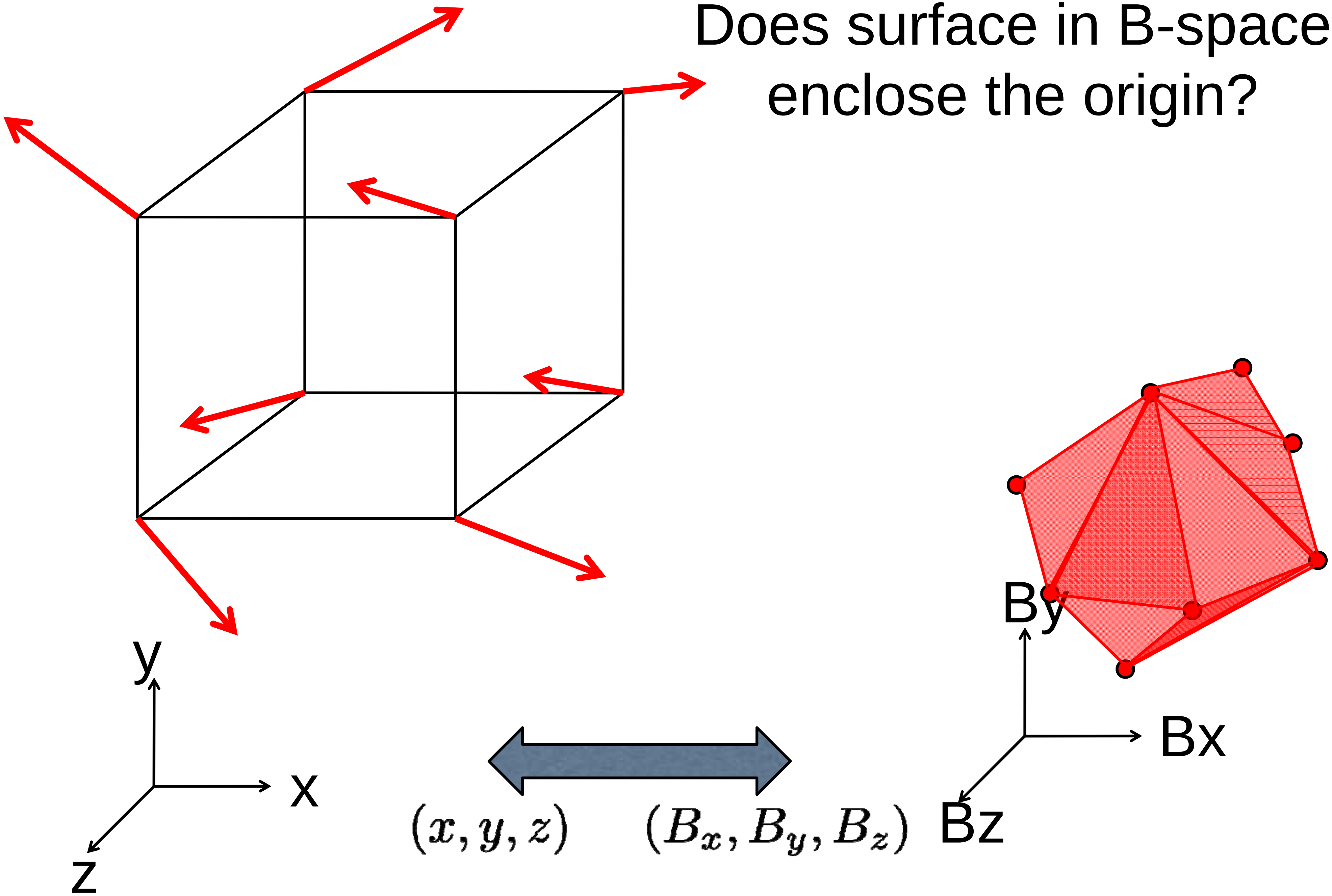


Repeat for all the triangles

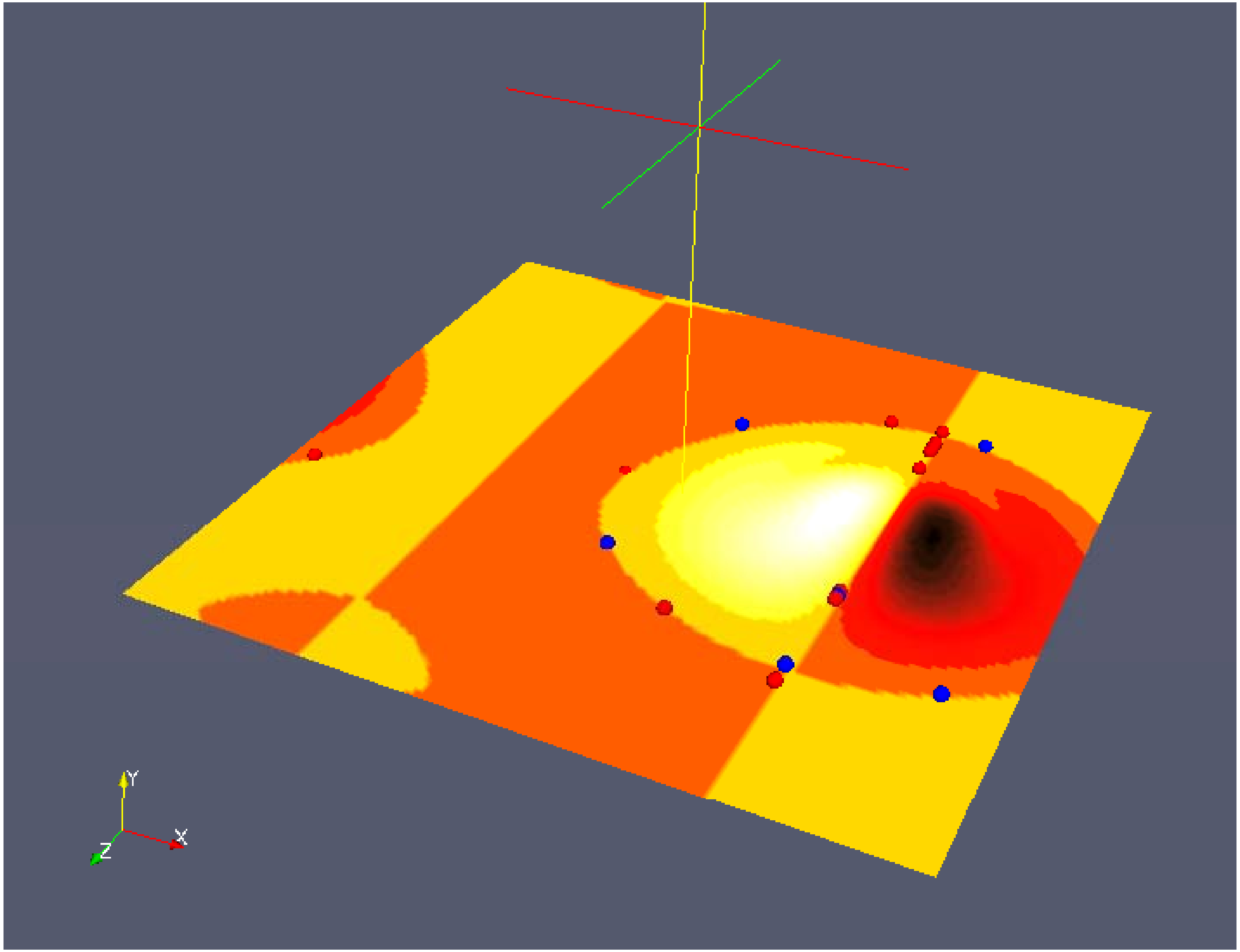




Does surface in B-space enclose the origin?



“String of pearls” pattern of nulls



Conclusions

- GEM-Challenge-like Scaling of R. Rate similar in 2D and 3D for the parameters in this study
- 3D system presents less back pressure because plasma can flow out in many directions direction
- Spread and drift of Reconnected flux along z makes comparison to 2D tricky
- Structure of 3D nulls is complex & has no direct 2D analog