Towards Multi-Scale Models of Intense Laser-Plasma Interactions

B. A. Shadwick

Department of Physics and Astronomy

University of Nebraska – Lincoln

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Acknowledgements

Acknowledgements

Collaborators

- C. B. Schroeder, LBNL
- E. Esarey, LBNL

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Support

- US DoE
- UNL

Technological Applications of Laser-Plasma Interactions

- Accelerators

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  - Particle physics has insatiable appetite for higher energy colliders.

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  - Conventional metal cavities are limited to gradients of $10 - 100$ MeV$/$m.
  - Higher energy means (larger) longer machines and higher cost.
  - Plasma wave can easily support gradients greater than 10 GeV$/$m.

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  - Electron bunches can be produced by plasma instabilities.
  - Electron bunches can be produced direct manipulation of phase space (LILAC: Umstadter *et al.* PRL 96; CPI: Esarey *et al.* PRL 97.).

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  - “Exotic” schemes: laser undulators, Raman amplifiers, ...

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  - Plasma densities $10^{19} - 10^{20}$ cm^{-3}.

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  - 100% energy spread.
  - Bunch energy > 100 MeV.
  - Bunch charge ~ 10 nC.
  - Laser power $\lesssim 10$ TW.
  - Gas jet target.
  - Plasma densities $10^{19} - 10^{20}$ cm^{-3}.
  - Many experiment in this regime: Modena *et al.* (95); Nakajima *et al.* (95); Umstadter *et al.* (96); Ting *et al.* (97); Gahn *et al.* (99); Leemans *et al.* (01); Malka *et al.* (01).

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  - Laser power 10*s* TW.
  - Gas jet targets.
  - Plasma densities 10^{18}–10^{19} cm^{-3}.

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- 2006

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  - High quality electron beam production at 1 GeV. Leemans *et al.*, *Nature Physics*, September (2006).

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  - Bunch energy $\sim$ 1 GeV.
  - Bunch change $\sim$ 100 pC.
  - Laser power $\sim$ 40 TW.
  - Plasma channel $\sim$ 3 cm long, formed by capillary discharge.

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- 2006 —
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  - Narrow energy spread and small divergence.
  - Bunch energy $\sim$ 1 GeV.
  - Bunch change $\sim$ 100 pC.
  - Laser power $\sim$ 40 TW.
  - Plasma channel $\sim$ 3 cm long, formed by capillary discharge.
  - Plasma density 3×10^{18} cm^{-3}.

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- 2006 —— • Colliding pulse injection. Faure *et al.*, *Nature*, October (2006).

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    - Narrow energy spread, $\Delta E/E = 11\%$ and small divergence.

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  - Bunch energy 117 MeV.

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  - Bunch change $\sim$ 20 pC.

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  - Colliding pulse injection. Faure *et al.*, *Nature*, October (2006).
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  - Bunch change $\sim$ 20 pC.
  - Laser power $\sim$ 50 TW.

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  - Colliding pulse injection. Faure *et al.*, *Nature*, October (2006).
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  - Bunch energy 117 MeV.
  - Bunch change $\sim$ 20 pC.
  - Laser power $\sim$ 50 TW.
  - Plasma density 7.5×10^{18} cm^{-3}.

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  - High-quality beams in the “bubble regime.”

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  - High-quality beams in the “bubble regime.”
  - Narrow energy spread and small divergence.
  - Reproducible.
  - Many groups in US, Europe and Asia.

Introduction - Typical Example

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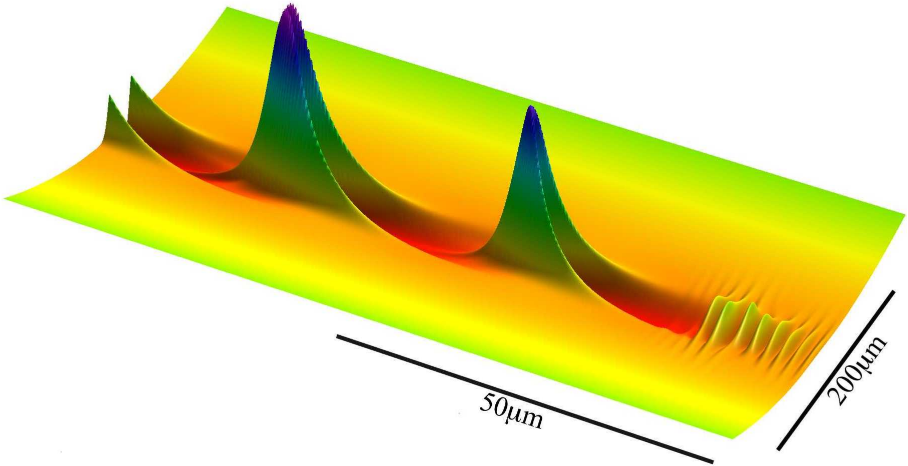

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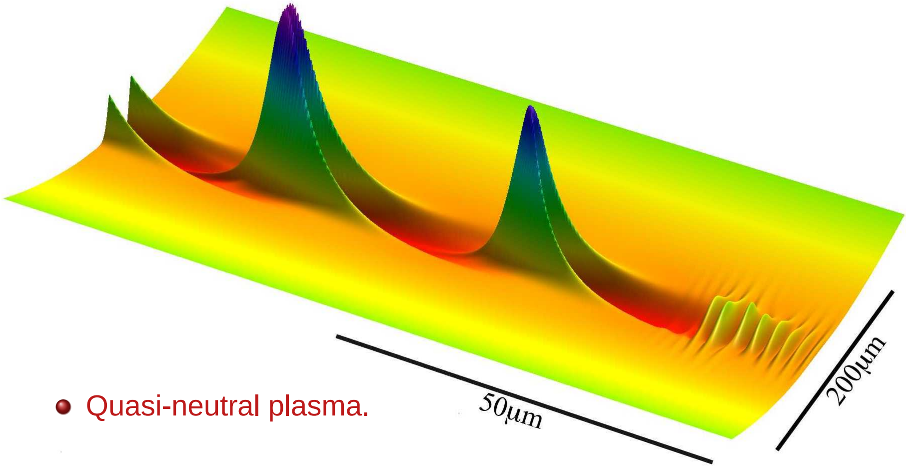

- Quasi-neutral plasma.

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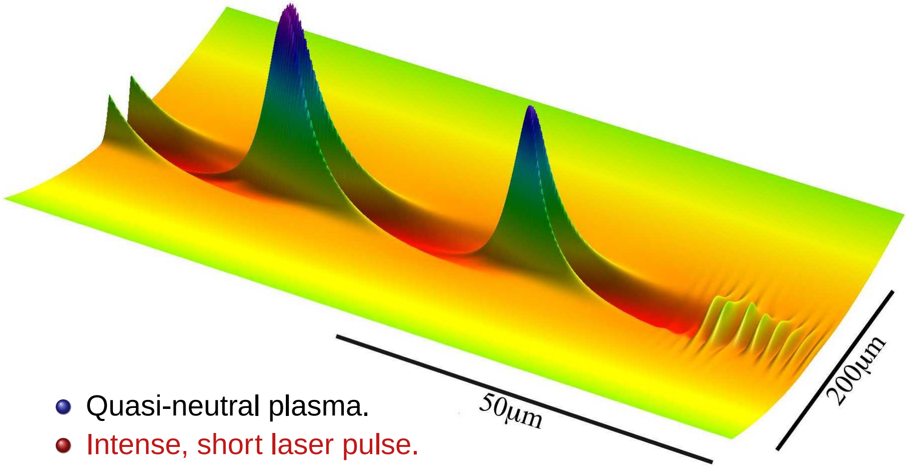

- Quasi-neutral plasma.
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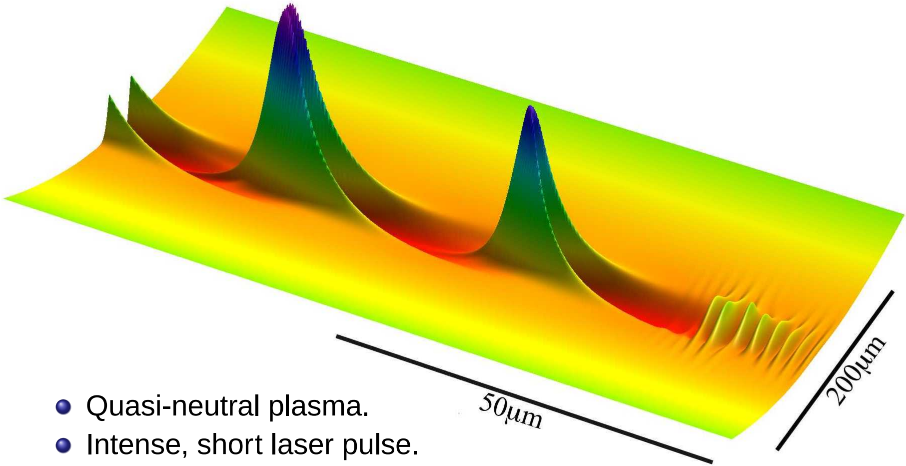

- Quasi-neutral plasma.
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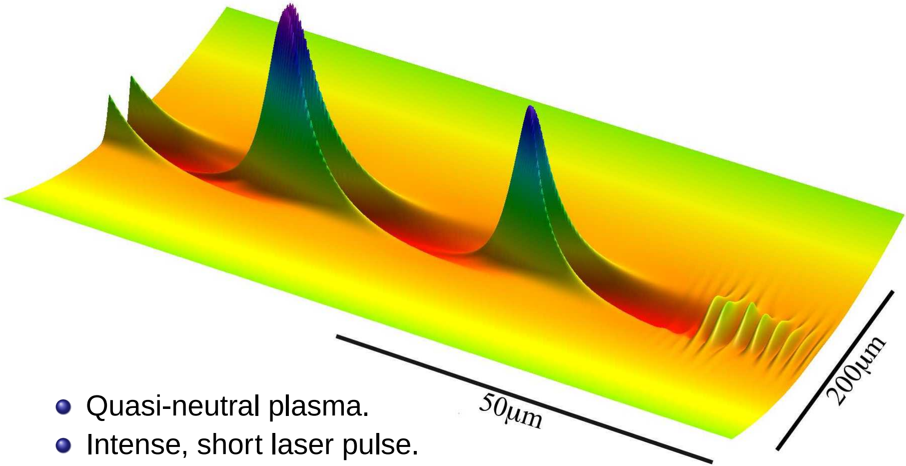

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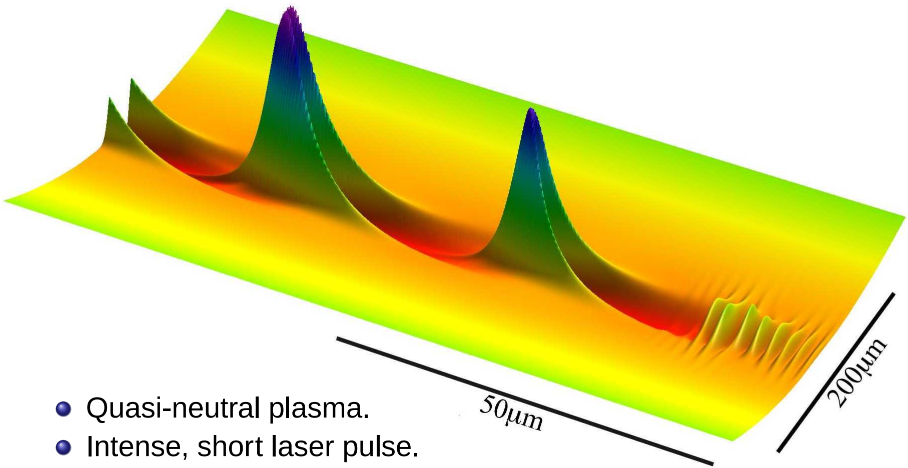

- Quasi-neutral plasma.
- Intense, short laser pulse.
- Transverse parabolic profile.
- Plasma acts like optical fiber guiding the laser pulse.
- Laser generates a large amplitude (nonlinear) plasma wave.

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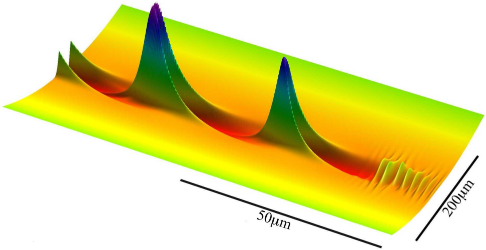
Typical parameters:



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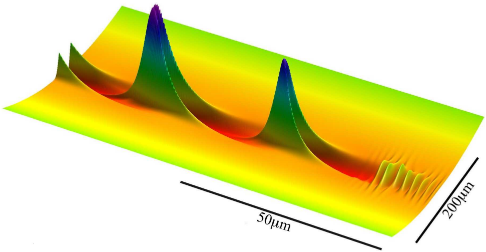

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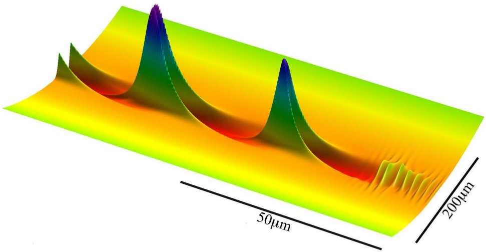

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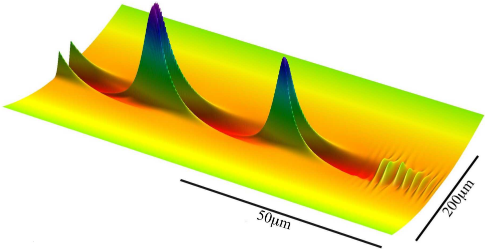

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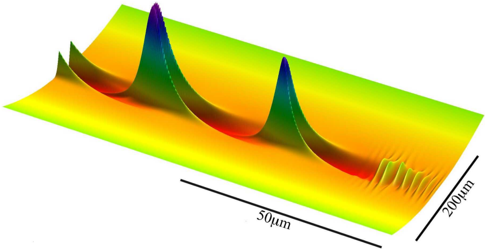

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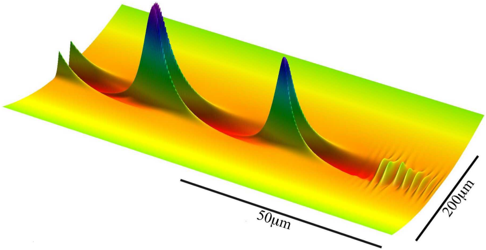

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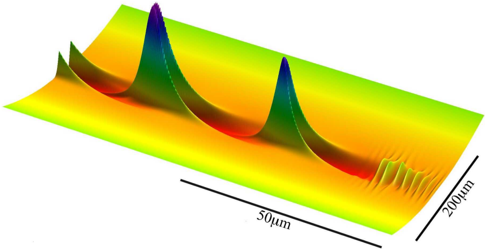

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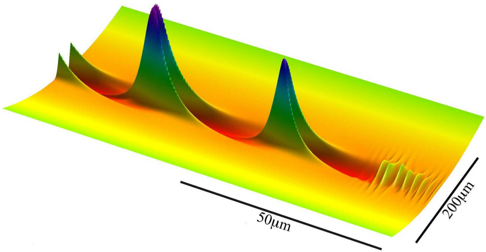

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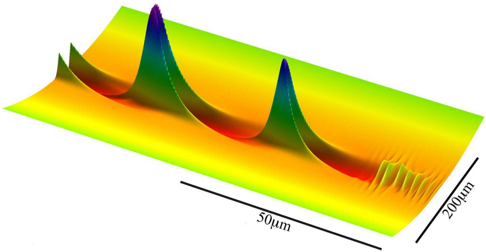

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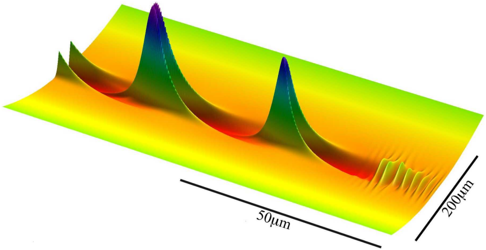

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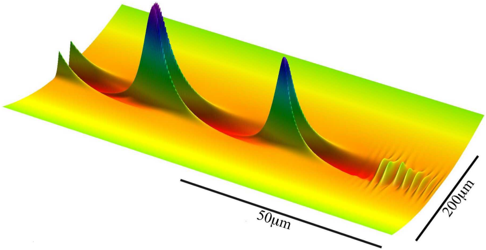

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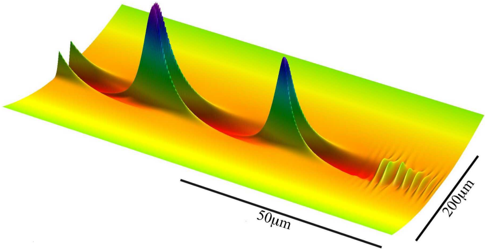

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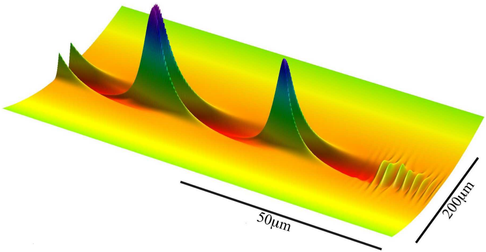

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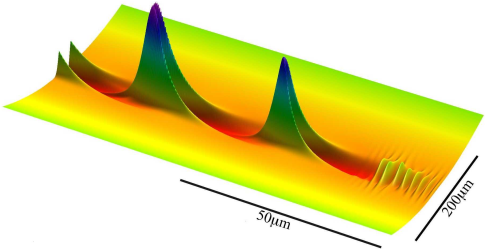

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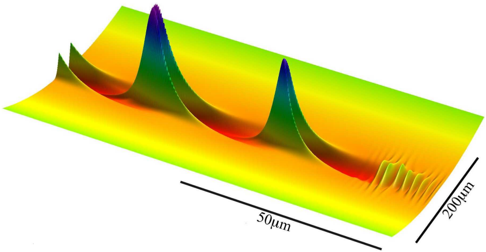

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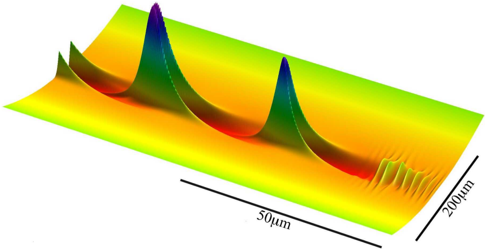

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  Put
  $$\boldsymbol{p} = \tilde{\boldsymbol{P}} + \tilde{\boldsymbol{p}}$$
  where $\boldsymbol{P}_b$ is arbitrary.

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  $$\boldsymbol{p} = \tilde{\boldsymbol{P}} + \tilde{\boldsymbol{p}}$$
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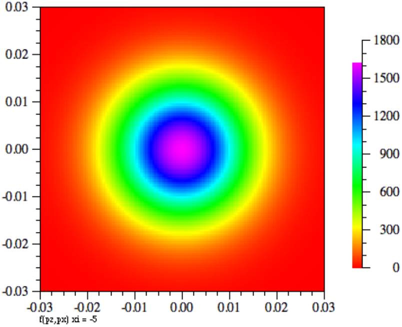
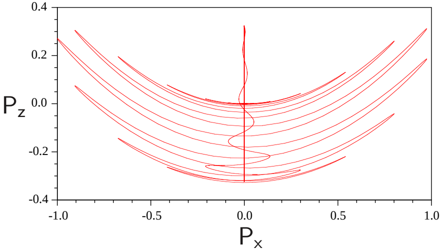
Parameters

$a_0 = 1,\ \omega_0 = 10\,\omega_p,$
$\omega_p\,\tau = 2,\ T_0 = 50\ \text{eV}$

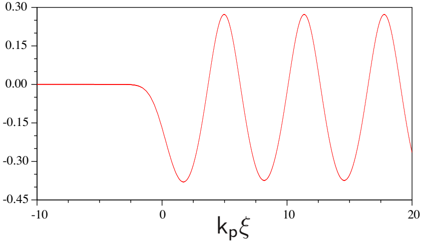
$f(\xi, \tilde{p}_x, \tilde{p}_z)$



Momentum Space Orbit



$\phi(\xi)$



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$$\begin{aligned} k_0\Delta\xi &= 0.0621 \quad (\lambda_0/\Delta\xi \approx 100) \\ \Delta t &= \tfrac{1}{4}\,\Delta\xi \end{aligned}$$

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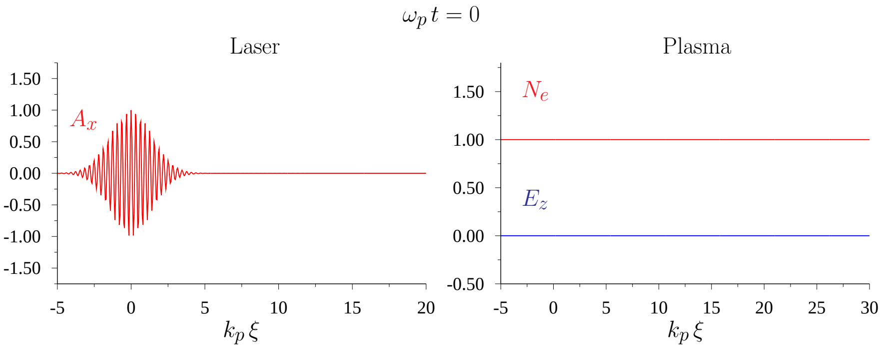

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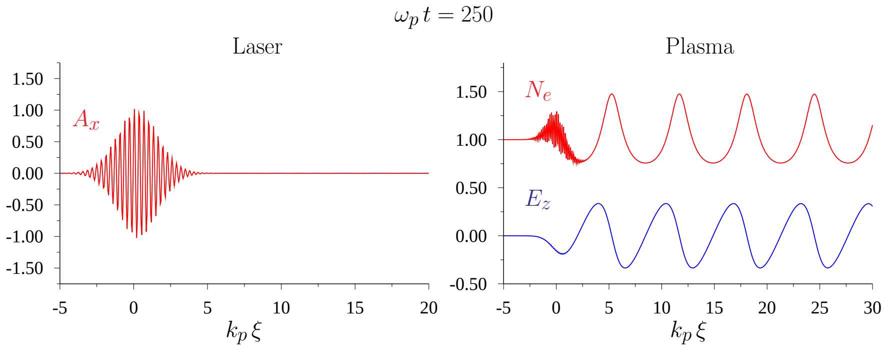

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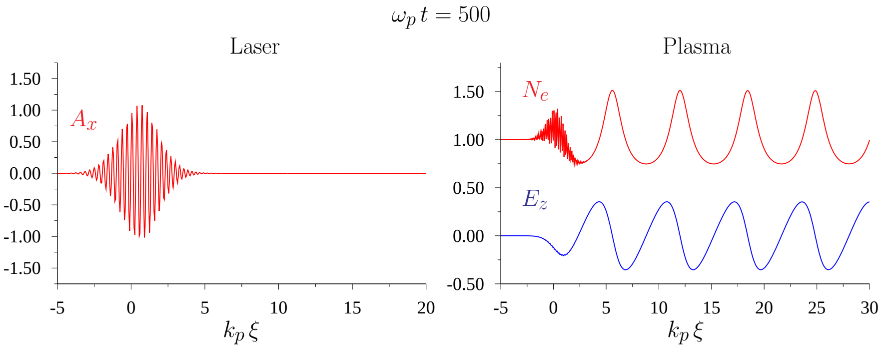

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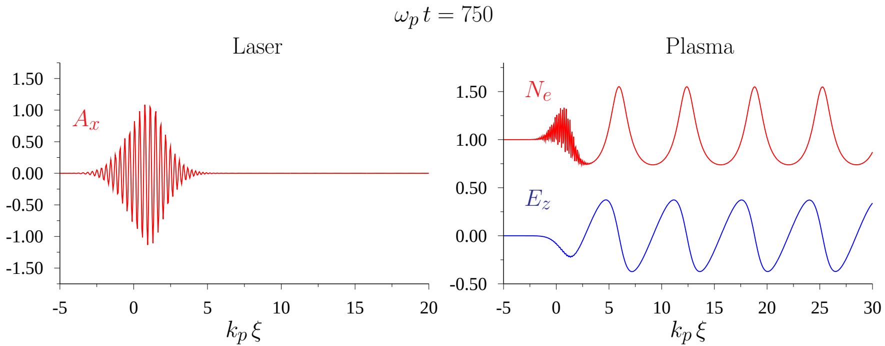

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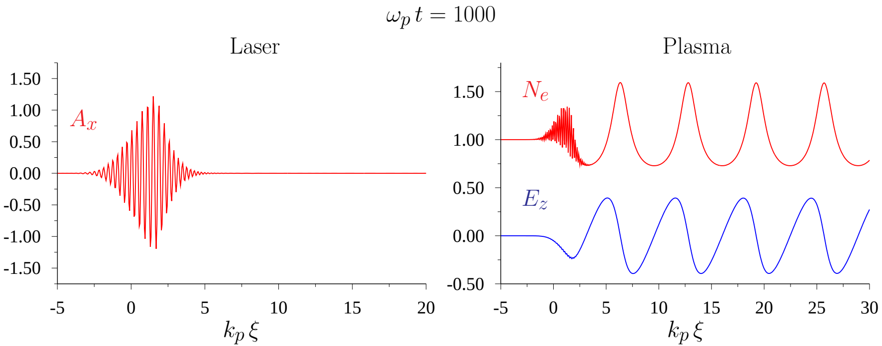

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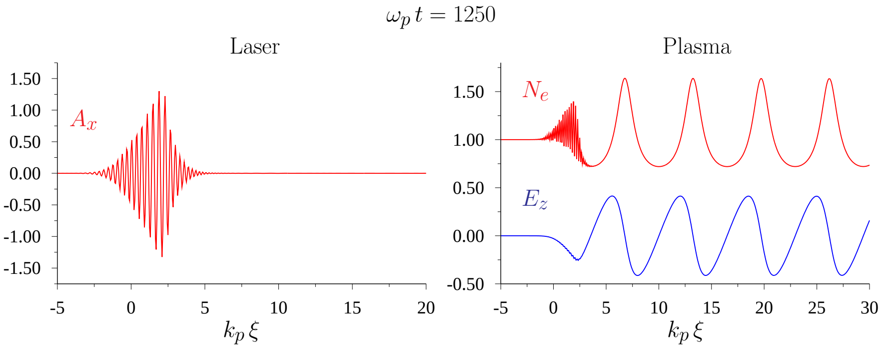

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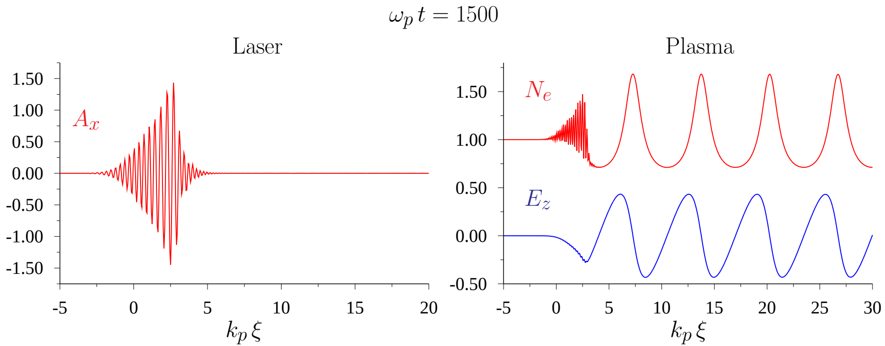

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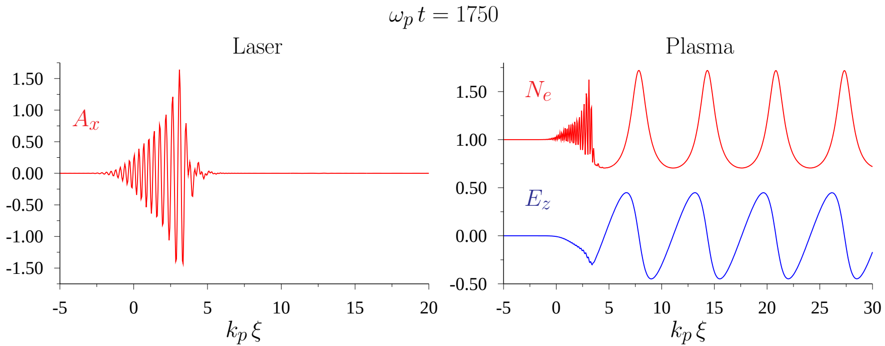

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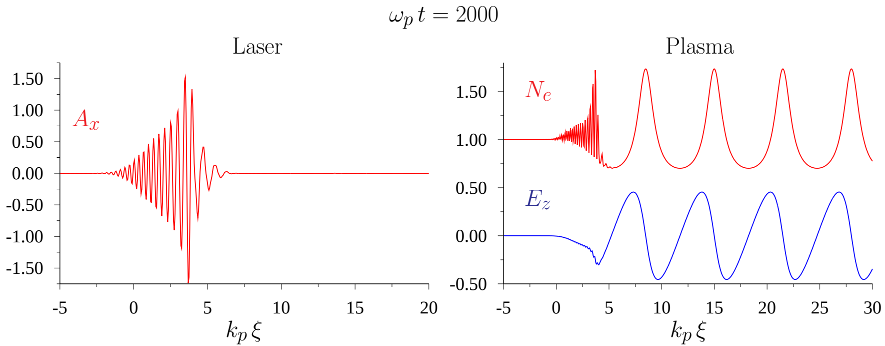

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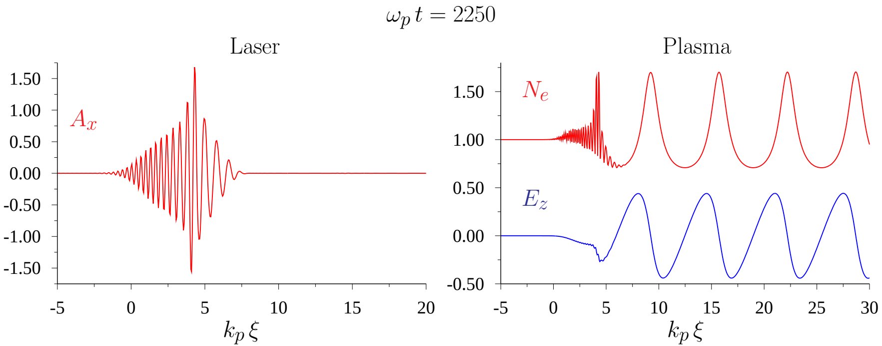

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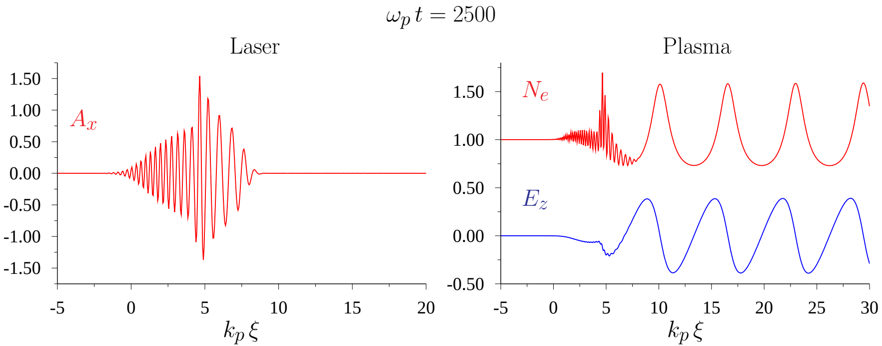

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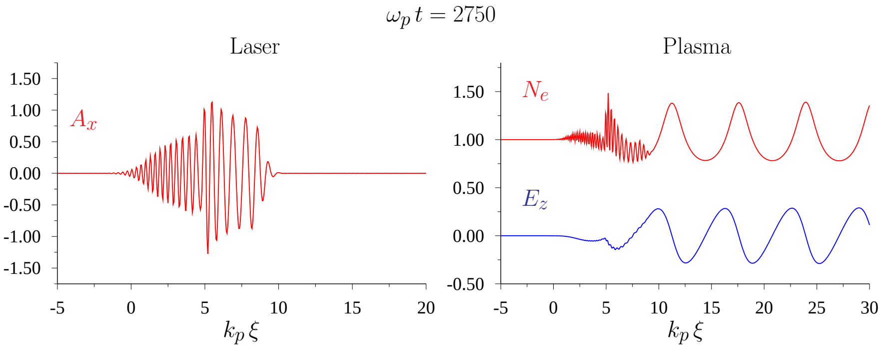

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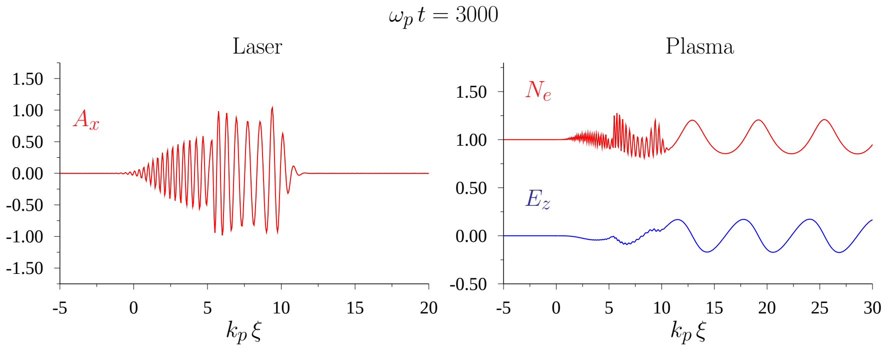

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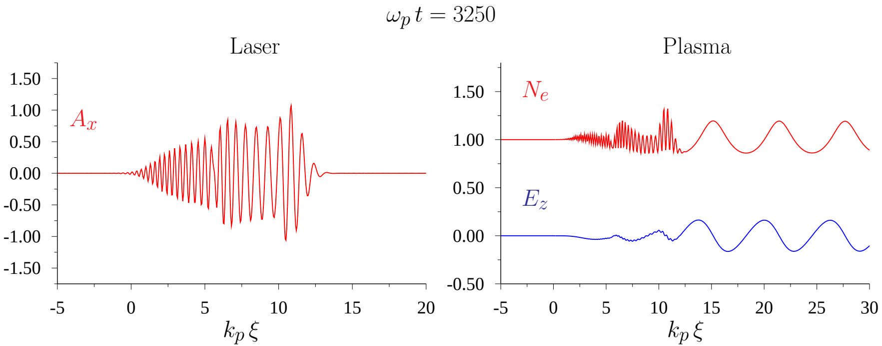

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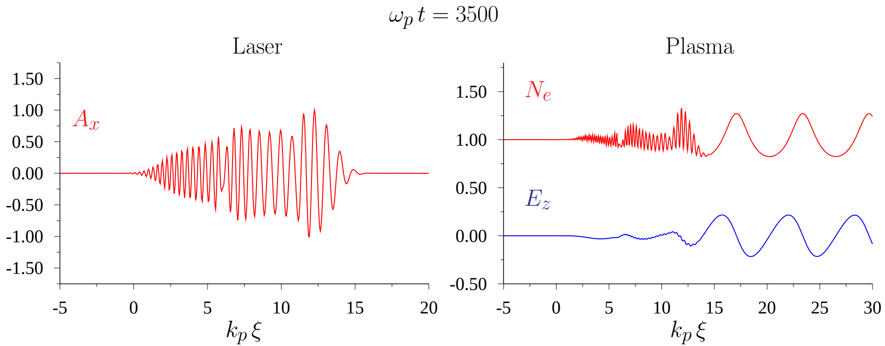

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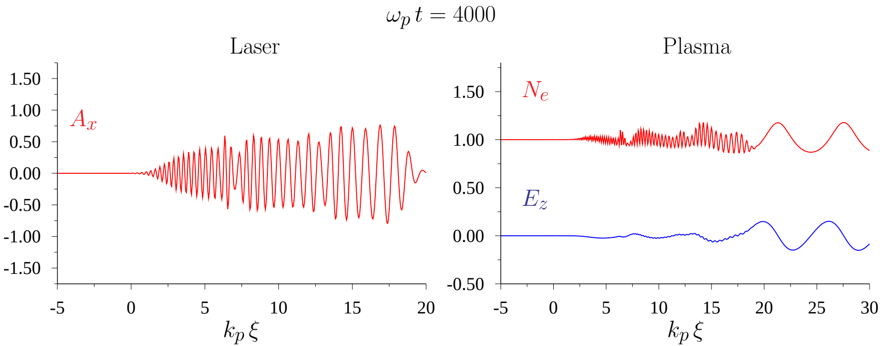

Laser Evolution

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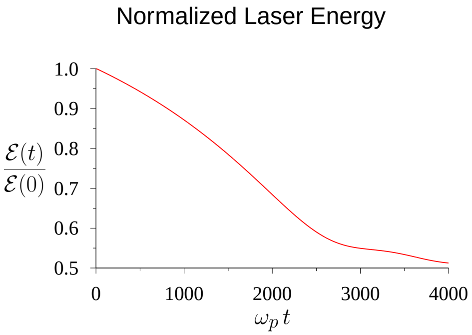

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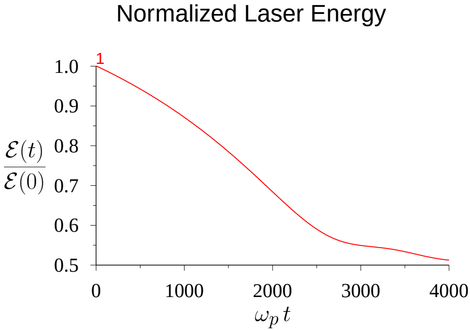

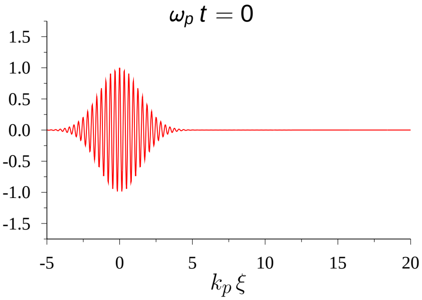

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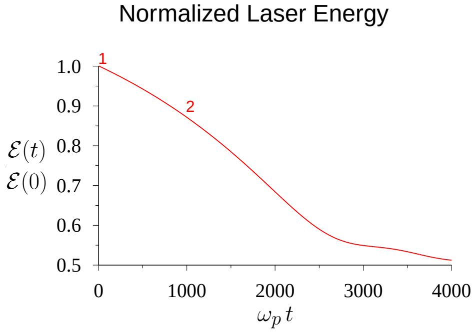
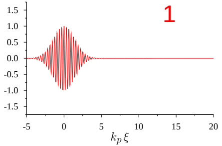
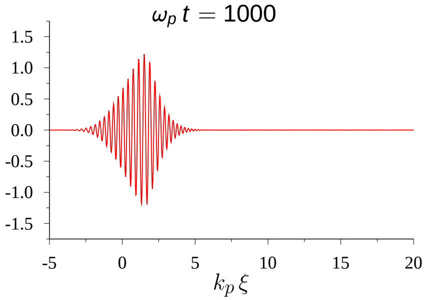
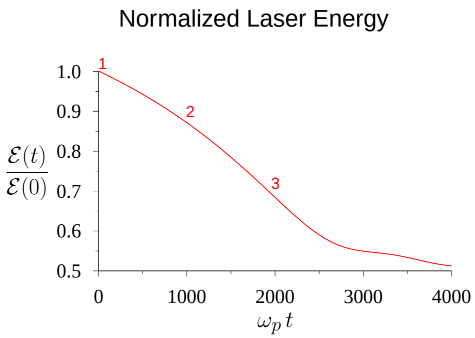


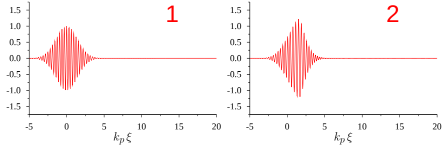




2: Minimum pulse length.

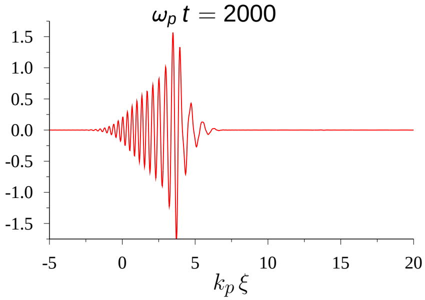
Laser Evolution





2: Minimum pulse length.

3: Maximum wake amplitude.



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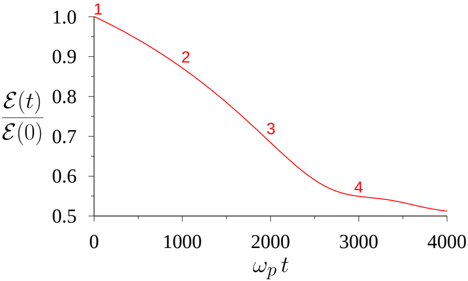
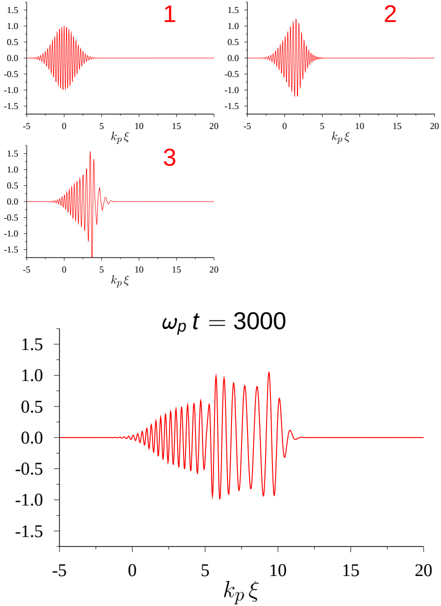




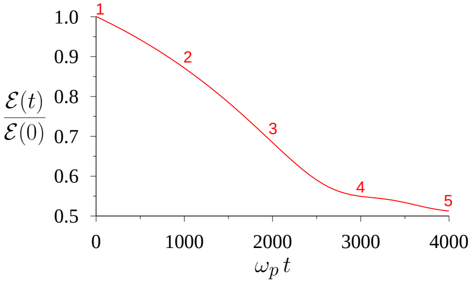
2: Minimum pulse length.

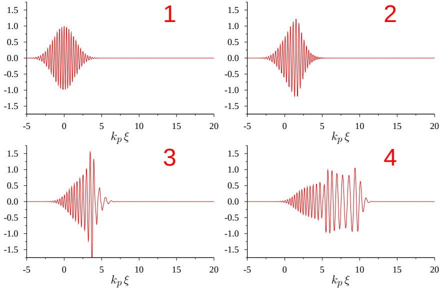
3: Maximum wake amplitude.

4: Rapid pulse lengthening.

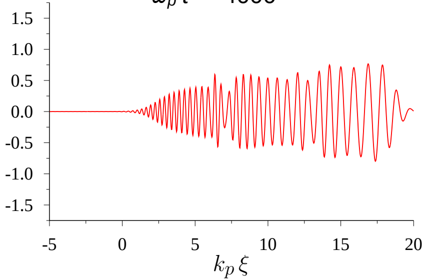
Laser Evolution

Normalized Laser Energy





$\omega_p t = 4000$



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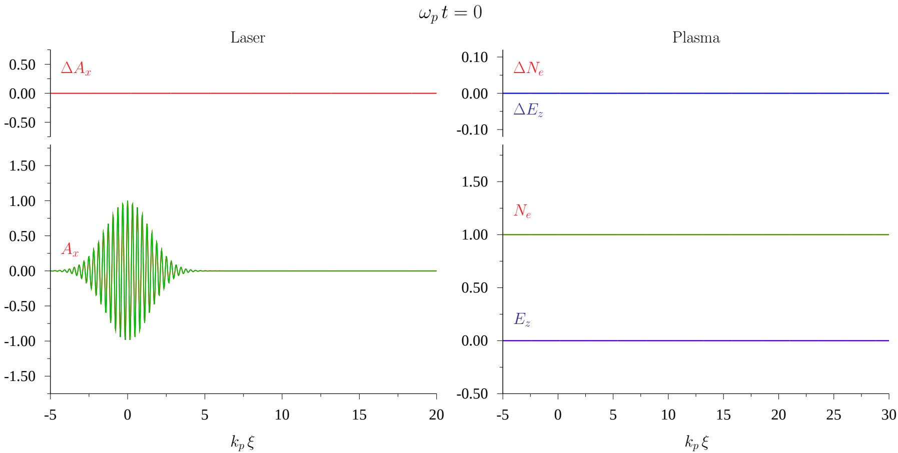
Quasi-Static Reduced Wave Operator, Averaged



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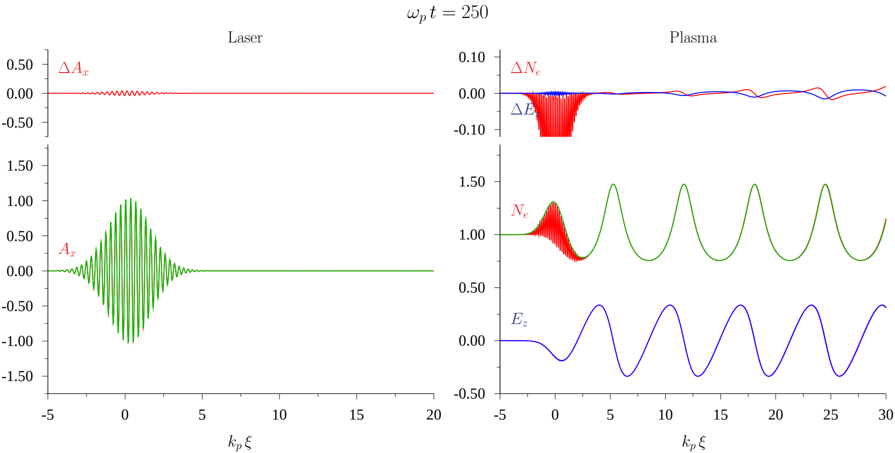

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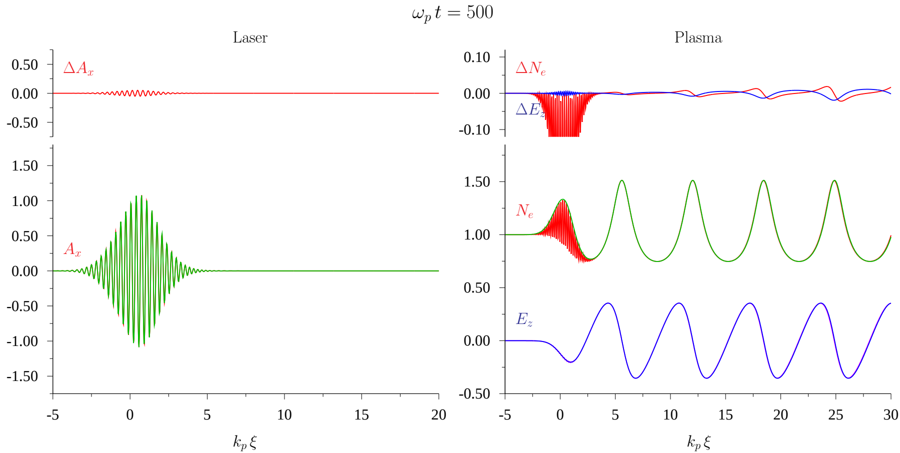

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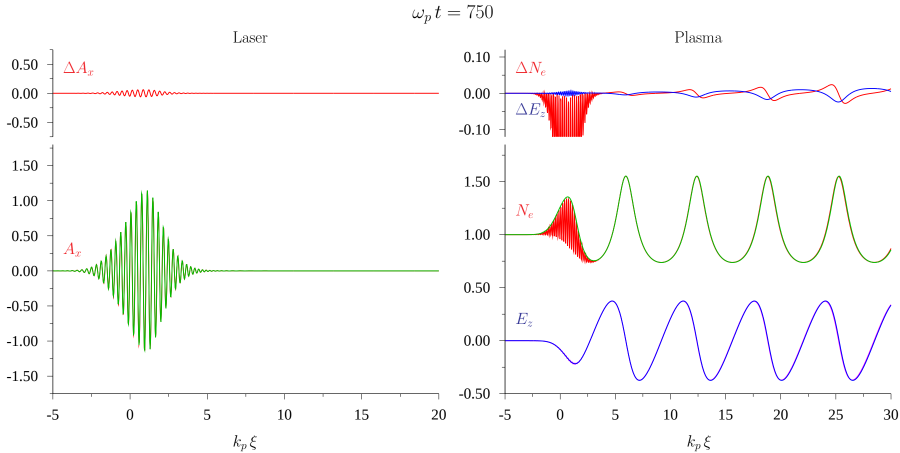

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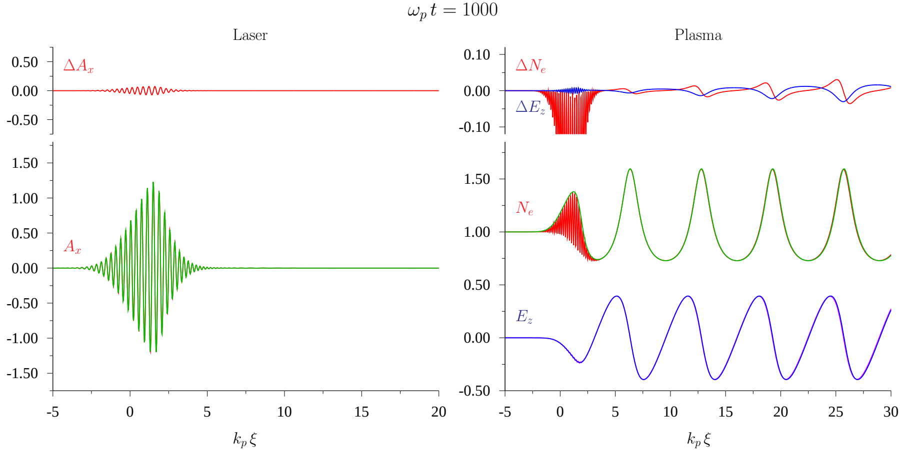

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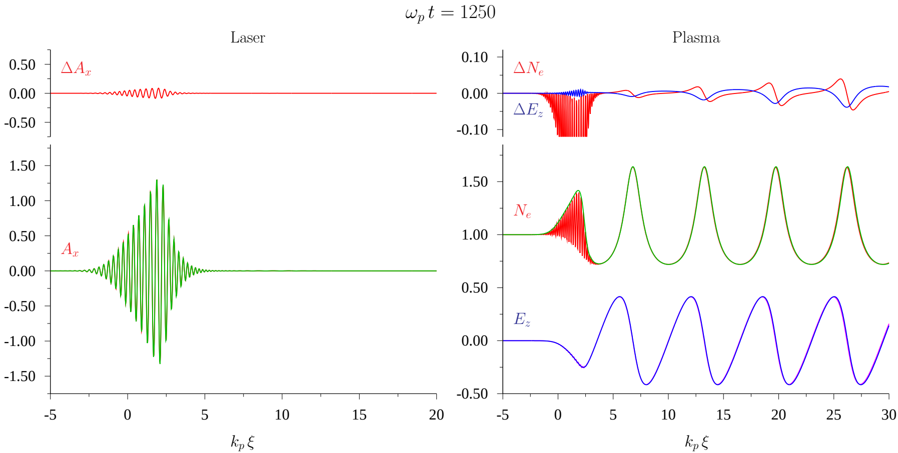

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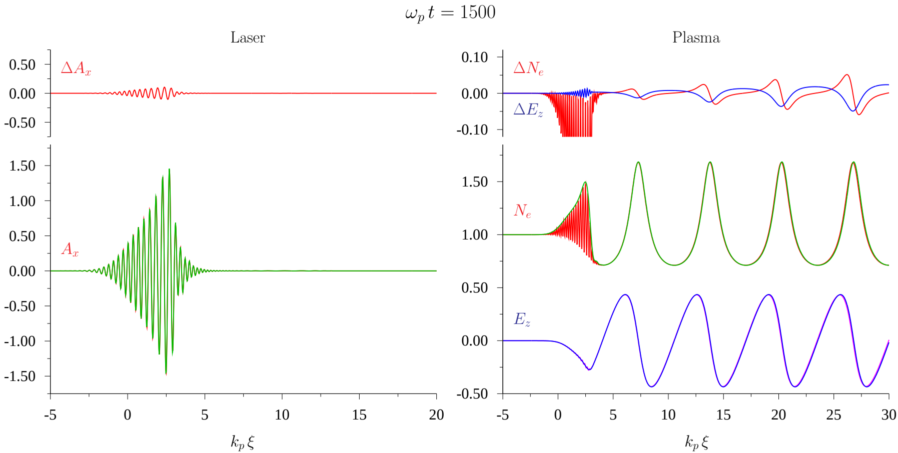

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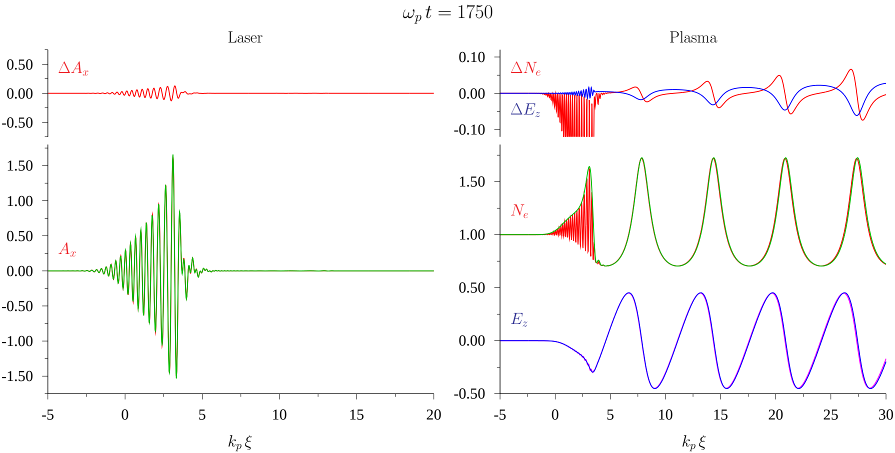

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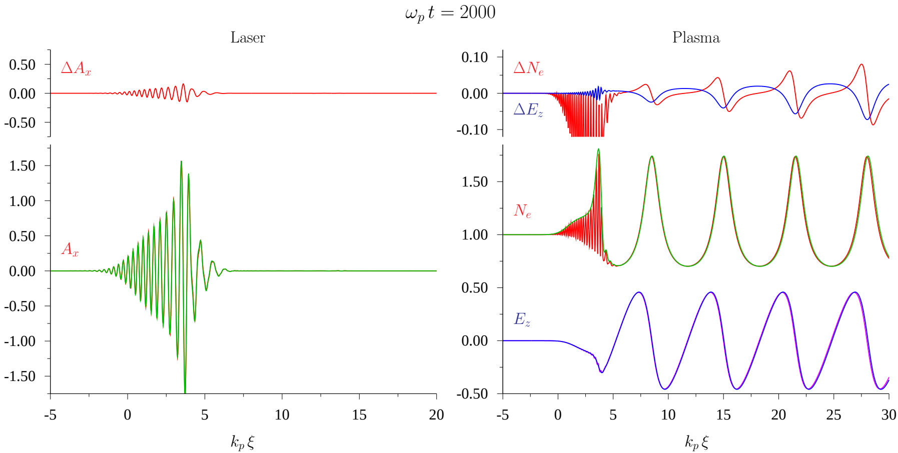

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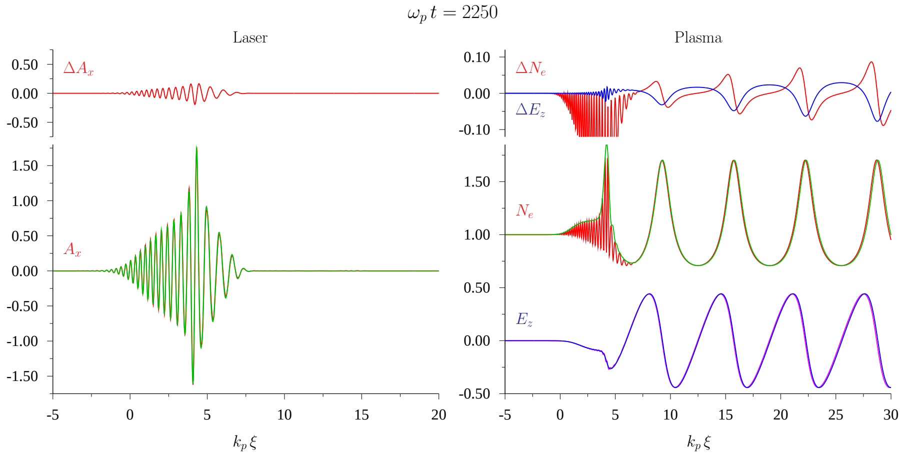

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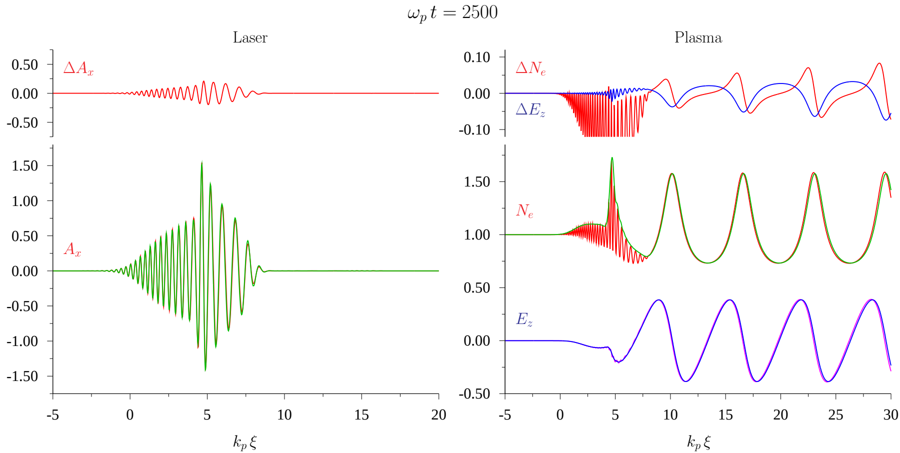

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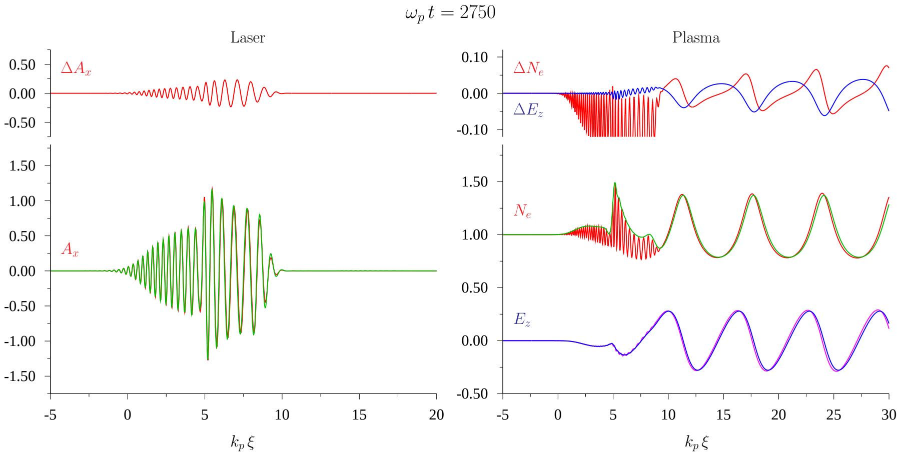

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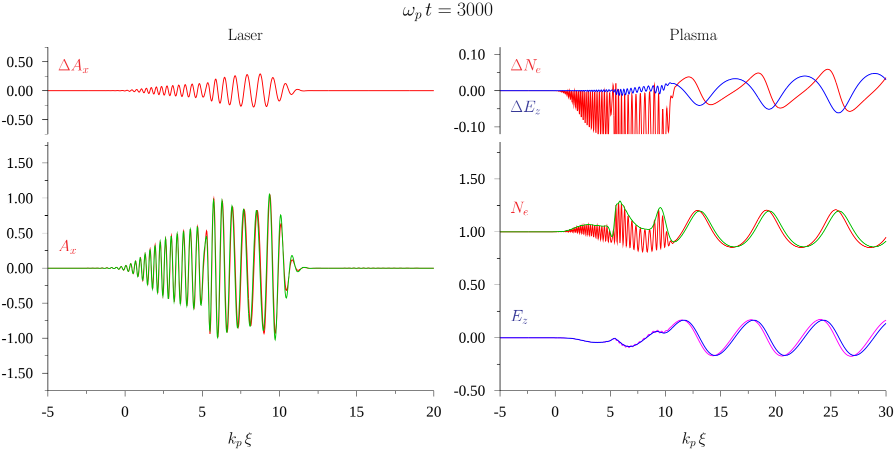

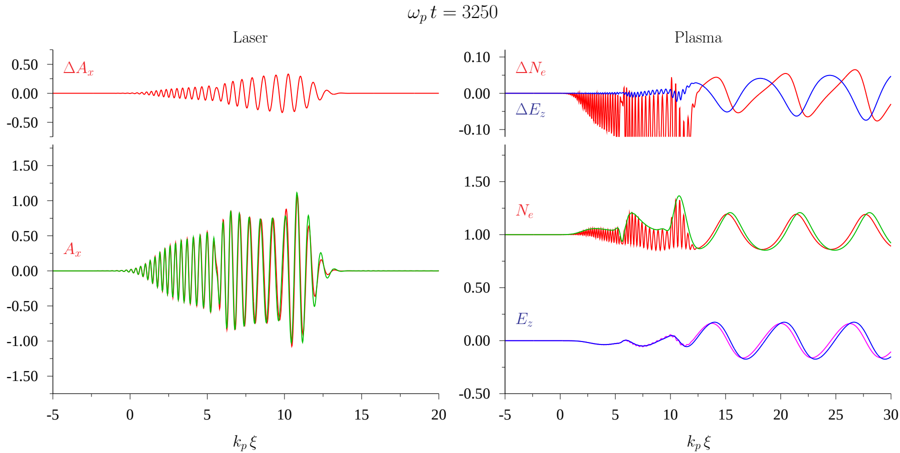

$\omega_p t = 3250$
Laser
Plasma
ΔA_x
A_x
ΔN_e
ΔE_z
N_e
E_z
0.50
0.00
-0.50
0.10
0.00
-0.10
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-5
0
5
10
15
20
25
30
$k_p \xi$

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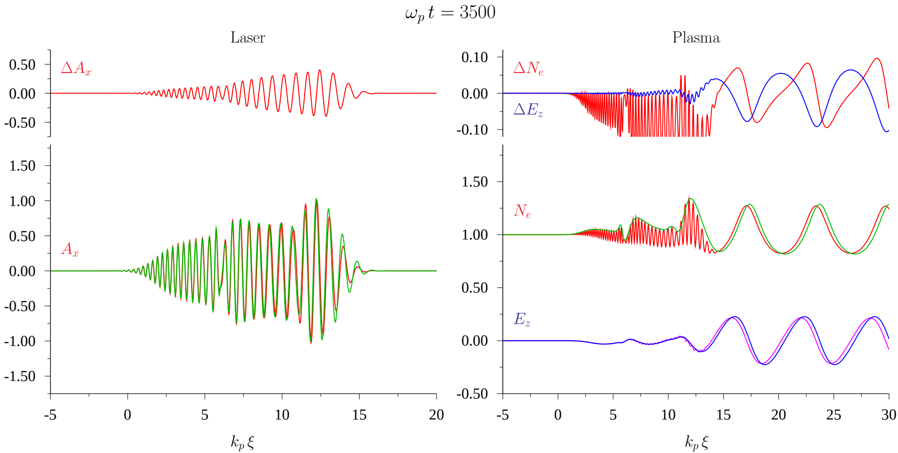

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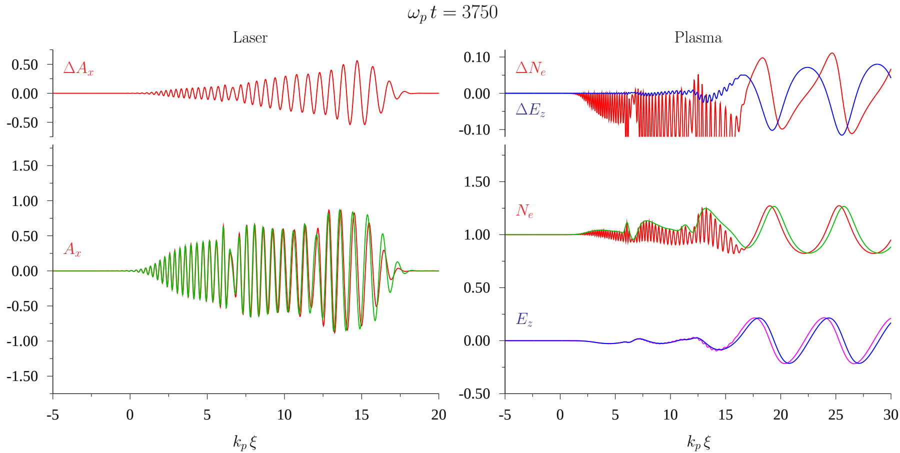

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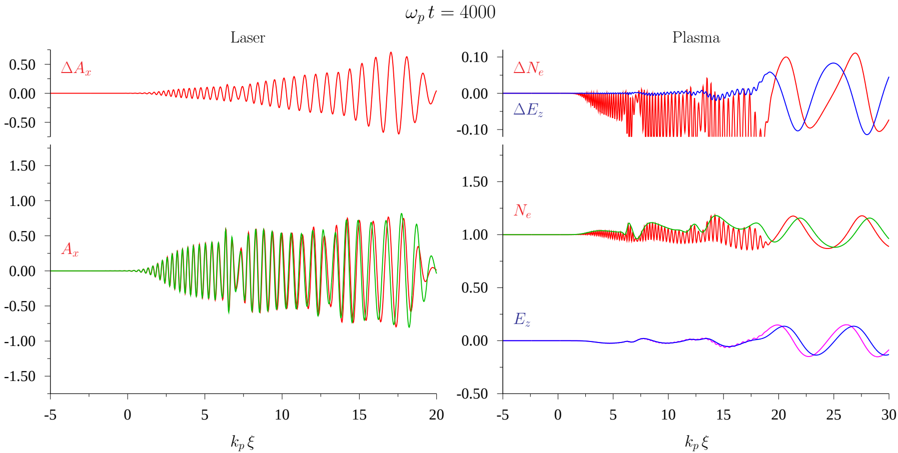

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