

Variational Symplectic Integrator for the Guiding Center Motion of Charged Particles for Long Time Simulations in General Magnetic Fields

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A variational symplectic integrator for the guiding center motion of charged particles in general magnetic fields is developed for long time simulation studies of magnetized plasmas [1]. Instead of discretizing the differential equations of the guiding center motion, the action of the guiding center motion is discretized and minimized to obtain the iteration rules for advancing the dynamics. Standard integrators only guarantee the error to be small in each time-step. The errors at different time-steps often accumulate coherently, and result in a large error over a large number of time-steps. The variational symplectic integrator conserves exactly a discrete Lagrangian symplectic structure, and has better numerical properties over long integration time, compared with standard integrators, such as the standard and variable time-step 4th order Runge-Kutta methods. The symplectic integrator conserves the symplectic structure exactly, and guarantees that the energy error is bounded by a small number for all the time-steps. To construct the symplectic algorithm, it is necessary to adopt the variational approach because standard symplectic integrators are only valid for canonical Hamiltonian systems, and the guiding-center dynamics in general magnetic field does not possess a (global) canonical symplectic structure. Numerical examples with more than 25 million time-steps are given to demonstrate the superiority of the variational symplectic integrator. This significant improvement in long term simulation capability of gyrokinetics is a direct, otherwise-impossible result of the geometric formulation of the gyrokinetic theory using the modern language of differential geometry [2, 3].

[1] H. Qin and X. Guan, *Physical Review Letters* **100**, 035006 (2008).

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[3] H. Qin, *Fields Institute Communications* **46**, Topics in Kinetic Theory, American Mathematical Society, 171 (2004).