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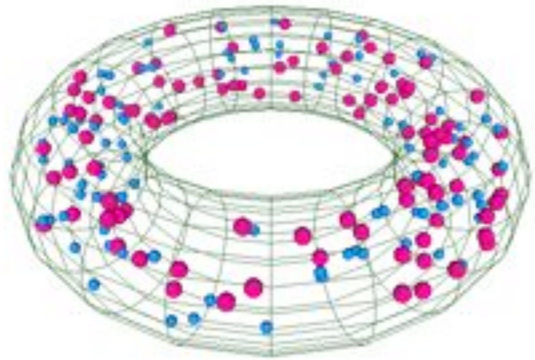

Global Kinetic MHD Simulation by the Gyrokinetic PIC Code

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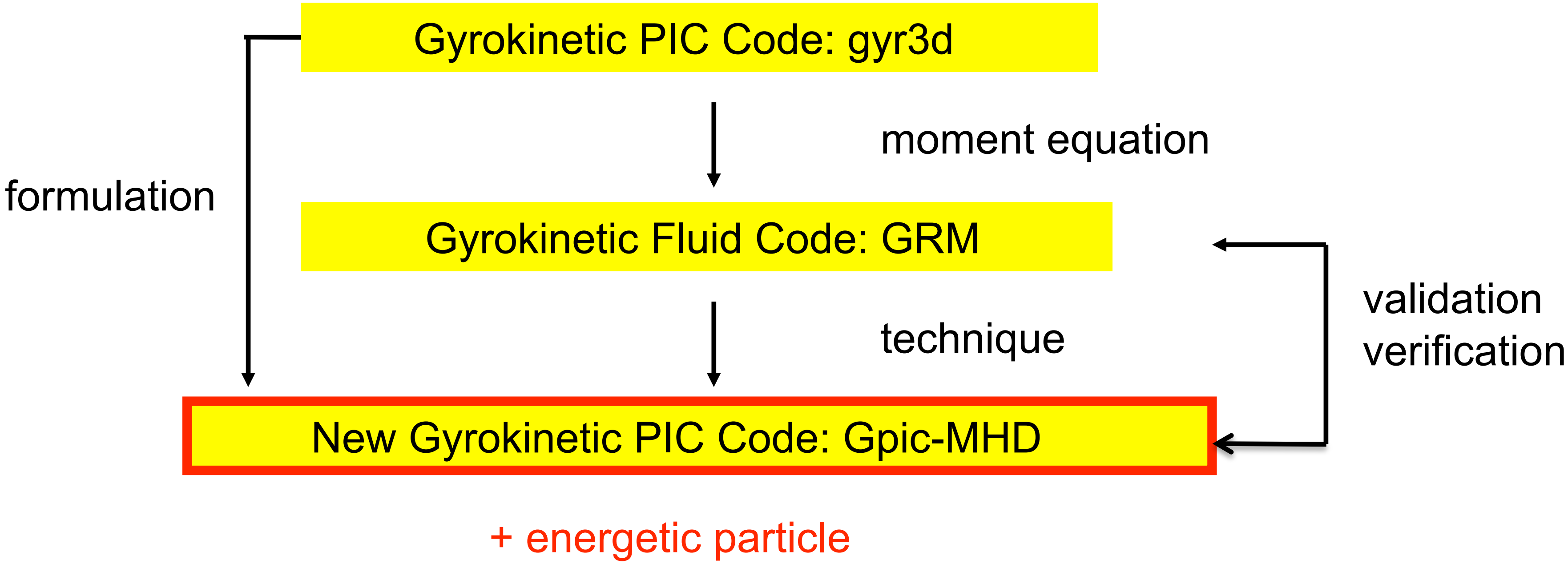
7. Summary

1. Motivation

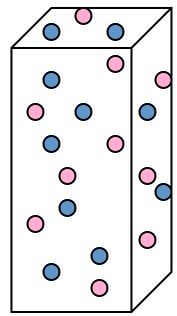
The simulation model, which can treat the kinetic MHD or extended MHD phenomena, is necessary to study the MHD-like phenomena in the present day and future high temperature large tokamaks.



simulation of collisionless (kinetic) internal kink mode

Gyrokinetic Particle Simulation

H. Naitou, K. Tsuda, W.W. Lee, R.D. Sydora,
Phys. Plasmas 2, 4257 (1995)

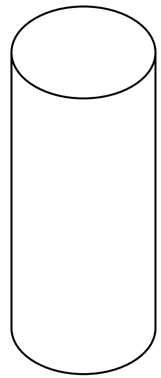


$$L_x \times L_y \times L_z = 16\, d_e \times 16\, d_e \times 8000\, d_e$$

low β

$$d_e / a = 0.125, \quad \rho_s / a = 0.03125 \quad \Rightarrow \quad v_{te} / v_A = 0.25$$

Gyrokinetic Fluid Simulation



Gyro-Reduced-MHD (GRM) simulation

$$d_e / a \approx 0.001, \quad \rho_s / a \approx 0.003 \quad \Rightarrow \quad v_{te} / v_A \approx 3$$

low β + high β

2. Gyro-Reduced-MHD Code

3-field model

- vortex equation
- ohm's law along the magnetic field including
  electron inertia and pressure terms
- electron conservation law
  isothermal model
  $T_i = 0$, $\Gamma_i = 0$ (ion velocity parallel to B)

5-field model

3-field model +

- equation for Γ_i
- equation for T_i (perturbed temp.)
  Linear Ion Landau damping term is added to the equation for the perturbed ion temperature.

$$T_{i\perp} = 0$$

$$T_{i//}^{0} = \mathrm{const}$$

Model Including Ion Landau Damping

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi = -\frac{\boldsymbol{b}\times\nabla\phi}{B_0}\bullet\nabla(\nabla_\perp^2\phi) - v_A^2\boldsymbol{b}^*\bullet\nabla(\nabla_\perp^2 A_z)$$

$$\frac{\partial}{\partial t}A_z = -\boldsymbol{b}^*\bullet\nabla\phi + d_e^2\frac{d}{dt}(\nabla_\perp^2 A_z) + \frac{T_{e0}}{n_{e0}e}\boldsymbol{b}^*\bullet\nabla n_e$$

$$\frac{\partial}{\partial t}n_e = -\frac{\boldsymbol{b}\times\nabla\phi}{B_0}\bullet\nabla n_e - \frac{1}{e\mu_0}\boldsymbol{b}^*\bullet\nabla(\nabla_\perp^2 A_z) - \boldsymbol{b}^*\bullet\nabla\Gamma_i$$

3-Field Model

$$\Gamma_i = 0$$

$$\frac{\partial}{\partial t}\Gamma_i = -\frac{\boldsymbol{b}\times\nabla\phi}{B_0}\bullet\nabla\Gamma_i - \left(\frac{T_{e0}}{m_i} + \frac{T_{i0}}{m_i}\right)\boldsymbol{b}^*\bullet\nabla n_e + \frac{\varepsilon_0 T_{i0}\omega_{pi}^2}{m_i\omega_{ci}^2}\boldsymbol{b}^*\bullet\nabla\left(\nabla_\perp^2\phi\right) - \frac{n_{i0}}{m_i}\boldsymbol{b}^*\bullet\nabla\tilde{T}_i$$

← added

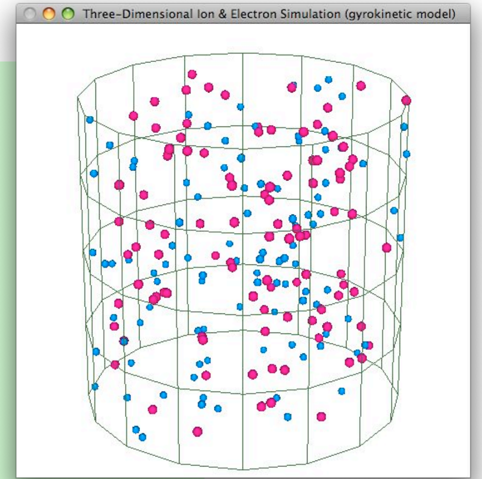
$$\frac{\partial}{\partial t}\tilde{T}_i = -\frac{\boldsymbol{b}\times\nabla\phi}{B_0}\bullet\nabla\tilde{T}_i - T_{i0}\boldsymbol{b}^*\bullet\nabla V_i - \frac{2^{3/2}}{\sqrt{\pi}}|k_{11}|v_{Ti}\tilde{T}_i$$

↑ ion Landau damping

$\tilde{T}_i$: perturbed ion temperature

3. Gpic-MHD code

Gyrokinetic PIC code for MHD simulation



- Cylindrical geometry.
- Same normalization as the GRM-code.
- Helical Symmetry is assumed.
- Limit of $k_\perp \rho_i << .1$
- Benchmark with the results of the GRM-code.
- Utilize techniques developed for the fluid code.

  ------ mesh accumulation, mode selection, etc.

****

- To include new algorithm

  ------- split weight method, etc.

4. Simulation results of kinetic internal kink mode

Single-helicity version

Cylindrical geometry

Nonuniform mesh in the radial direction

Single helicity (2D - code)

Gyrokinetic PIC code with δf scheme

Parallelization by particle decomposition

Benchmark run :

$d_e / a = 0.06, \quad \rho_s / a = 0.06, \quad T_i / T_e = 1.0$

$N_r = 129, \quad N_\theta = 128$

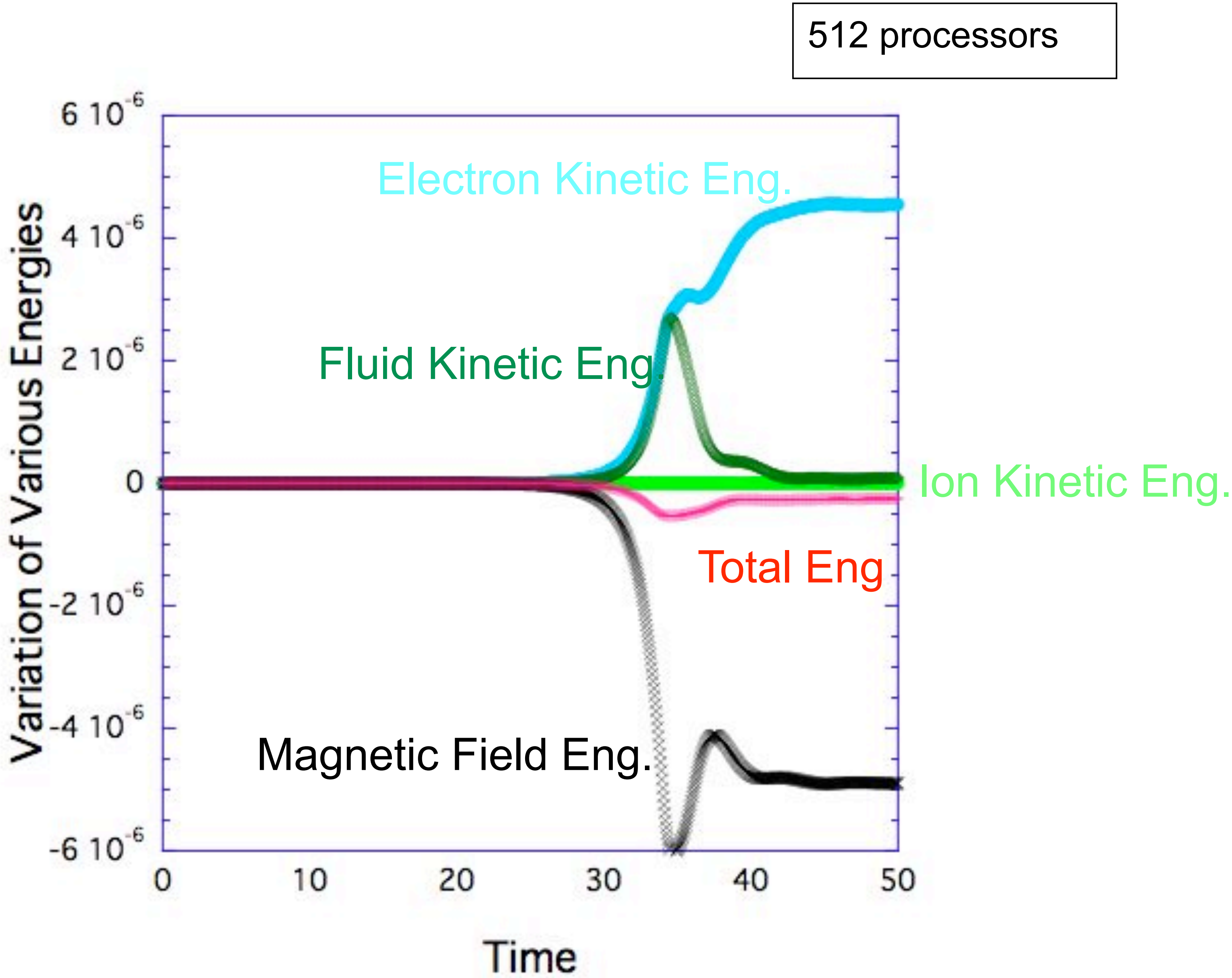
$m \leq 20$

10000 steps

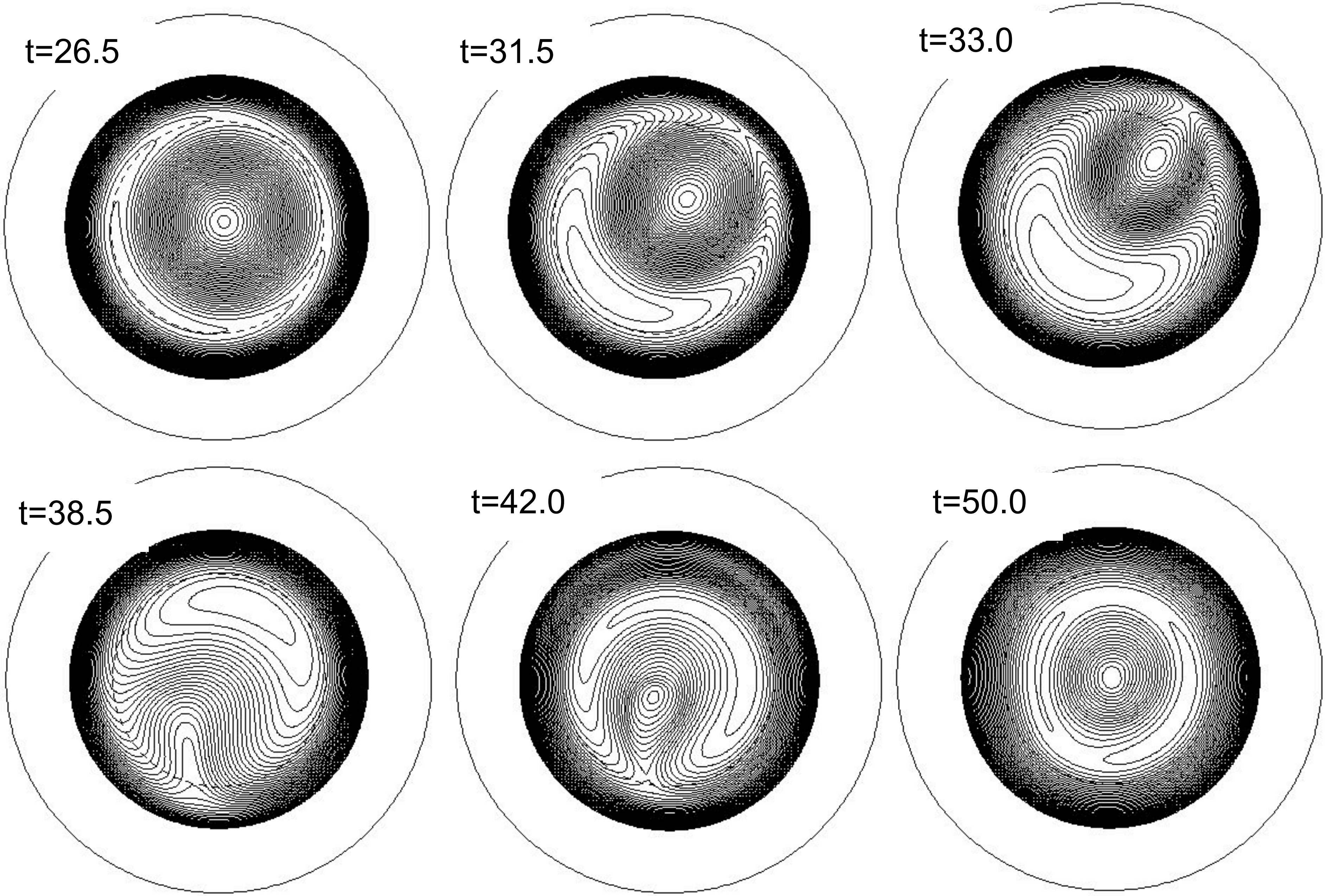
(0.5 million electrons + 0.5 million ions) × number of CPU's

$$q(r) = q_0 \left[1 - 4 \, (1 - q_0) \left(\frac{r}{a}\right)^2 \right]^{-1}$$

$q_0 = 0.85, \; q(0.5a) = 1.0, \; q_a = 2.125$


512 processors
Electron Kinetic Eng.
Fluid Kinetic Eng.
Ion Kinetic Eng.
Total Eng
Magnetic Field Eng.
Variation of Various Energies
6 10-6
4 10-6
2 10-6
0
-2 10-6
-4 10-6
-6 10-6
0
10
20
30
40
50
Time

Temporal Behavior of the Magnetic Field Structure

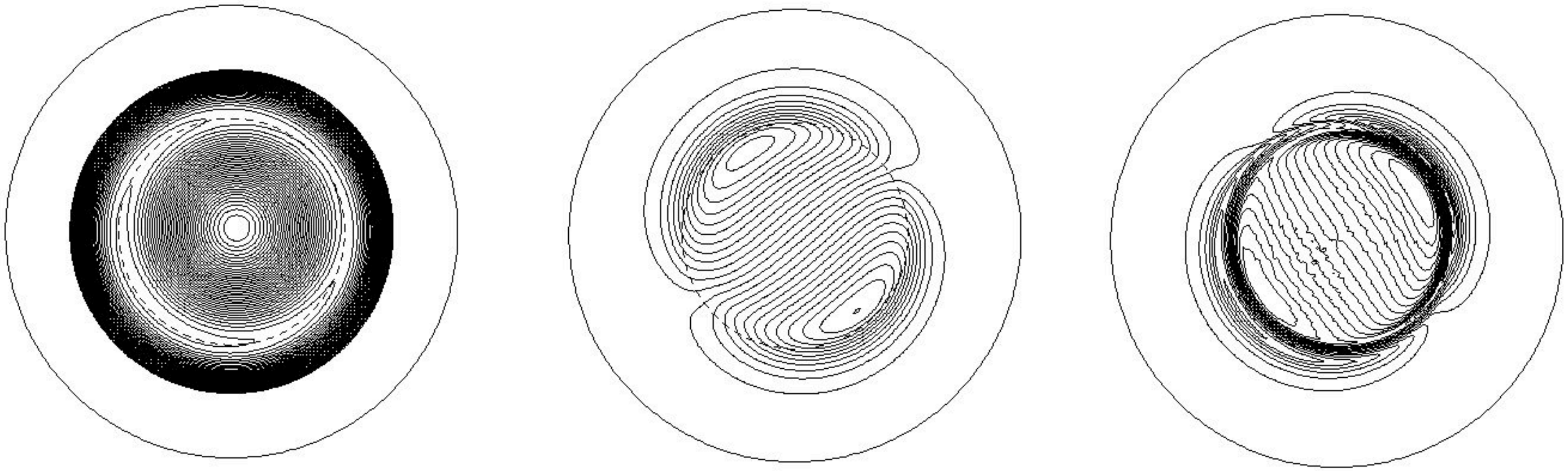


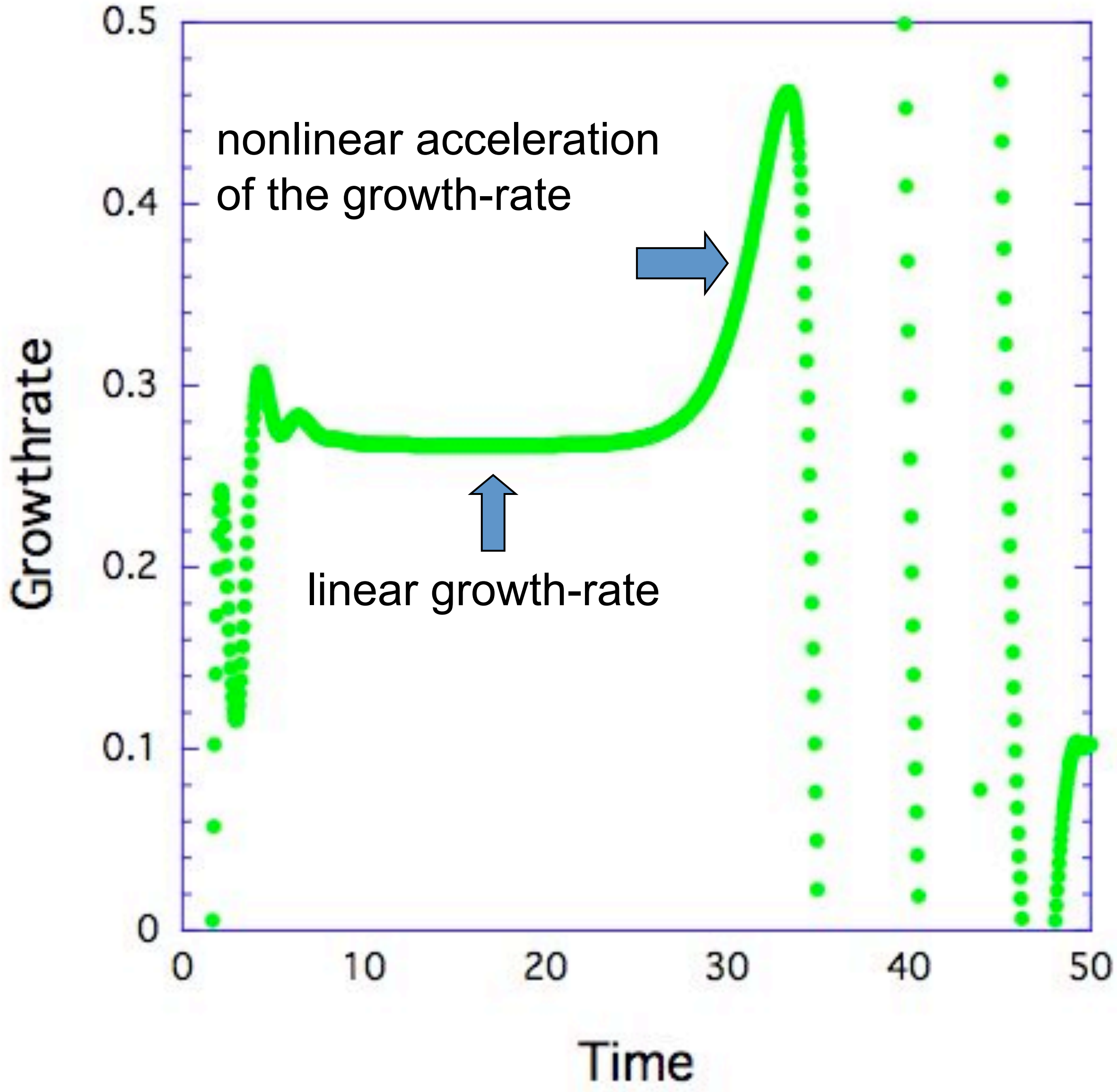
t = 25

(a) Magnetic Field Structure

(b) Electrostatic Potential Profile

(c) Current Density Profile

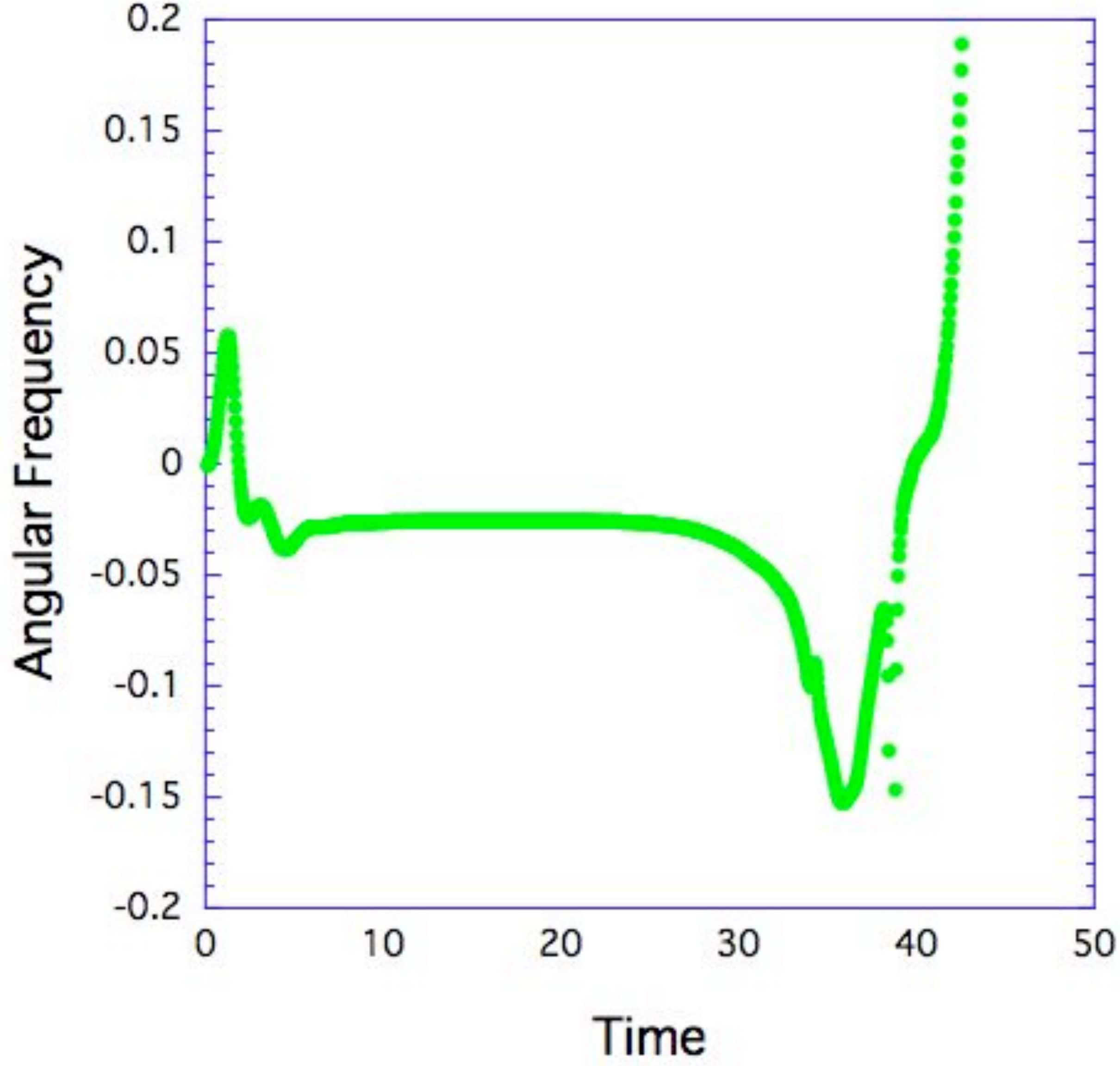




GRM code

$\gamma = 0.318$

Plasma is slightly Rotating.
Reason: ?



5. Performance on the parallel computer

1. Single-helicity version of Gpic-MHD
   - 2-dimensional code
   - use replicas for field quantities
2. Multi-helicity version of Gpic-MHD
   - 3-dimensional code
   - domain decomposition in *z* + replicas

Parallel computer

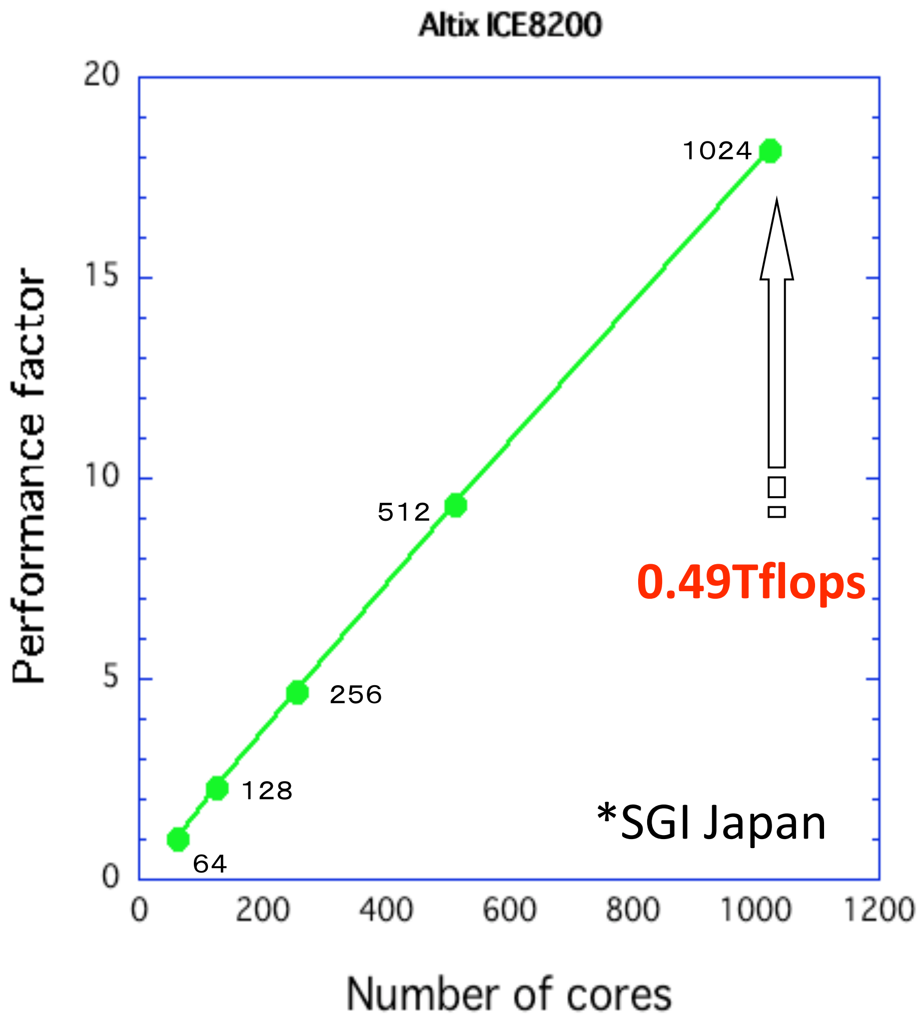
- JAEA Altix3700Bx2
- Intel Itanium2 (1.6GHz, 6.4Gflops)
- NUMALink4
- 2048 cores (128 cores / node, 16 nodes)
- 13.1 T-Flops

1 million particles / core

Performance of Gpic-MHD on Altix ICE8200

Intel Xeon CPU X5365
(3.0GHz, 12 Gflops)
1024 cores
4xDDR InfiniBand

- million particles / core
- 2000 steps
- 129x128x128 meshes
- 3D code with domain decomposition in z (64 segments) + replicas
- normalized by the case with 64 cores



6. Revisit the Formulation of Split-Weight Scheme

- v_z-version

  include electron thermodynamic response

  lower noise

  large time step size

- p_z-version

  +

  generalized potential

  solve cancellation problem

  This version seems very complicated, but ....

gyrokinetic Vlasov equation

$$p_z = v_z + \frac{q_s}{m_s} A_z$$

v_z - version

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + \frac{d\mathbf{x}_s}{dt} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{dv_{zs}}{dt} \frac{\partial f_s}{\partial v_z}$$

p_z - version

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + \frac{d\mathbf{x}_s}{dt} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{dp_{zs}}{dt} \frac{\partial f_s}{\partial p_z}$$

where

$$\frac{d\mathbf{x}_s}{dt} = -\frac{\nabla \phi \times \mathbf{b}}{B_0} + v_z \mathbf{b}^*$$

$$\frac{dv_{zs}}{dt} = -\frac{q_s}{m_s} \left(\mathbf{b}^* \cdot \nabla \phi + \underline{\underline{\frac{\partial A_z}{\partial t}}} \right)$$

$$\frac{dp_{zs}}{dt} = -\frac{q_s}{m_s} \left(\mathbf{b} \cdot \nabla \phi - v_z \mathbf{b} \cdot \nabla A_z \right)$$

and

$$\mathbf{b}^* = \mathbf{b} + \frac{\nabla A_z \times \mathbf{b}}{B_0}$$

Choices of f_0

For simplicity, temporary neglect the thermodynamic response.

decomposition in δf method

$$f = f_0 + \delta f$$

choices of equilibrium verocity distribution fufnction : f_0

conventional δf v_z-version

1. $$f_{0e}(v_z) = n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e v_z^2}{2T_e}\right)$$

split-weight-scheme p_z-version

2. $$f_{0e}(p_z,\mathbf{x},t) = n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e(p_z + \frac{e}{m_e}A_z)^2}{2T_e}\right)$$

$$= n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e p_z^2}{2T_e}\right)\exp\left(\frac{e}{T_e}\Psi\right)$$

$$p_z = v_z - \tfrac{e}{m_e}A_z$$

where $\Psi = -p_z A_z - \frac{e}{2m_e}A_z^2$

conventional δf p_z-version

3. $$f_{0e}(p_z) = n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e p_z^2}{2T_e}\right)$$

Usually, $\Psi = \phi - p_z A_z - \frac{e}{2m_e}A_z^2$

p_z-version: split-weight scheme

lower order moments for electrons :

$$\int f_{0e} dp_z = n_{0e}$$

$$\int \left(p_z + \tfrac{e}{m_e} A_z\right) f_{0e} dp_z = 0$$

$$\int \left(p_z + \tfrac{e}{m_e} A_z\right)^2 f_{0e} dp_z = n_{0e} v_{te}^2$$

gyrokinetic Poisson's equation :

$$\frac{\omega_{pi}^2}{\omega_{ci}^2} \nabla_\perp^2 \phi = \frac{e}{\varepsilon_0} \int \delta f_e dp_z - \frac{e}{\varepsilon_0} \int \delta f_i dp_z$$

Ampere's law :

$$\nabla_\perp^2 A_z = \mu_0 e \int \left(p_z + \tfrac{e}{m_e} A_z\right) \delta f_e dp_z - \mu_0 e \int \left(p_z - \tfrac{e}{m_i} A_z\right) \delta f_i dp_z$$

where

$$\delta f = \delta h$$

p_z-version: conventional δf

lower order moments for electrons :

$$\int f_{0e} dp_z = n_{0e}$$

$$\int \left(p_z + \tfrac{e}{m_e} A_z\right) f_{0e} dp_z = \tfrac{e}{m_e} n_{0e} A_z$$

$$\int \left(p_z + \tfrac{e}{m_e} A_z\right)^2 f_{0e} dp_z = n_{0e} v_{te}^2 + \left(\tfrac{e}{m_e}\right)^2 A_z^2$$

gyrokinetic Poisson's equation :

$$\frac{\omega_{pi}^2}{\omega_{ci}^2} \nabla_\perp^2 \phi = \frac{e}{\varepsilon_0} \int \delta f_e dp_z - \frac{e}{\varepsilon_0} \int \delta f_i dp_z$$

Ampere's law :

$$\nabla_\perp^2 A_z = \frac{n_{0e} \mu_0 e^2}{m_e} A_z + \mu_0 e \int \left(p_z + \tfrac{e}{m_e} A_z\right) \delta f_e dp_z$$

$$+ \frac{n_{0i} \mu_0 e^2}{m_e} A_z - \mu_0 e \int \left(p_z - \tfrac{e}{m_i} A_z\right) \delta f_i dp_z$$

$$\therefore \nabla_\perp^2 A_z - \left(\frac{\omega_{pe}^2}{c^2} + \frac{\omega_{pi}^2}{c^2}\right) A_z = \mu_0 e \int \left(p_z + \tfrac{e}{m_e} A_z\right) \delta f_e dp_z - \mu_0 e \int \left(p_z - \tfrac{e}{m_i} A_z\right) \delta f_i dp_z$$

Problem of cancellation

p_z-version with 0-th order current (1)

v_{De} : electron drift velocity along B - field

0 - th order electron velocity distribution :

$$f_{0e}(p_z,\mathbf{x},t) = n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e(p_z+\frac{e}{m_e}A_z - v_{De})^2}{2T_e}\right)$$

$$= n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e(p_z - v_{De})^2}{2T_e}\right)\exp\left(\frac{e}{T_e}\Psi_e\right)$$

where $\Psi_e = -(p_z - v_{De})A_z - \frac{e}{2m_e}A_z^2$

0 - th order ion velocity distribution :

$$f_{0i}(p_z,\mathbf{x},t) = n_{0i}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e(p_z+\frac{e}{m_e}A_z)^2}{2T_e}\right)$$

$$= n_{0i}\sqrt{\frac{m_i}{2\pi T_i}}\exp\left(-\frac{m_e p_z^2}{2T_i}\right)\exp\left(\frac{e}{T_e}\Psi_i\right)$$

where $\Psi_i = -p_z A_z - \frac{e}{2m_e}A_z^2$

p_z-version with 0-th order current (2)

lower order moments for electrons :

$$\int f_{0e} dp_z = n_{0e}$$

$$\int \left(p_z + \tfrac{e}{m_e} A_z\right) f_{0e} dp_z = n_{0e} v_{De}$$

$$\int \left(p_z + \tfrac{e}{m_e} A_z\right)^2 f_{0e} dp_z = n_{0e} v_{te}^2 + n_{0e} v_{De}^2$$

gyrokinetic Poisson's equation :

$$\frac{\omega_{pi}^2}{\omega_{ci}^2} \nabla_\perp^2 \phi = \frac{e}{\varepsilon_0} \int \delta f_e dp_z - \frac{e}{\varepsilon_0} \int \delta f_i dp_z$$

Ampere's law :

$$\nabla_\perp^2 \delta A_z = \mu_0 e \int \left(p_z + \tfrac{e}{m_e} A_z\right) \delta f_e dp_z - \mu_0 e \int \left(p_z - \tfrac{e}{m_i} A_z\right) \delta f_i dp_z$$

where

$$\delta f = \delta h$$

$$\delta A_z = A_z - A_{z0}$$

$$\nabla_\perp A_{z0} = \mu_0 e v_{De}$$

p_z-version with 0-th order current (3)

Summary of δf method

δf (= split - weight scheme) :

$$f = f_0 + \delta f$$

$$\delta f = wf$$

$$\therefore \frac{1}{f} = \frac{1-w}{f_0}$$

gyrokinetic Vlasov equation :

$$\frac{df}{dt} = \frac{df_0}{dt} + w\frac{df}{dt} + \frac{dw}{dt}f = 0$$

$$\therefore \frac{dw}{dt} = -\frac{1}{f}\frac{df_0}{dt} = -(1-w)\frac{1}{f_0}\frac{df_0}{dt}$$

For marker particles,

$$\delta f = \sum w_j \delta(\mathbf{x} - \mathbf{x}_j)\delta(p - p_j)$$

$$\frac{dw_j}{dt} = -(1-w_j)\frac{1}{f_0}\frac{df_0}{dt}\bigg|_{\mathbf{x}=\mathbf{x}_j,\, p=p_j}$$

p_z-version with 0-th order current (4)

equation for weights of electrons :

$$f_{0e}(p_z,\mathbf{x},t) = n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e(p_z+\frac{e}{m_e}A_z-v_{De})^2}{2T_e}\right)$$

$$\frac{df_{0e}}{dt} = \frac{\partial f_{0e}}{\partial t} + \frac{d\mathbf{x}_e}{dt}\cdot\frac{\partial f_{0e}}{\partial \mathbf{x}} + \frac{dp_{ze}}{dt}\frac{\partial f_{0e}}{\partial p_z}$$

$$\therefore \frac{1}{f_0}\frac{df_{0e}}{dt} = \frac{m_e}{T_e}\left(p_z+\tfrac{e}{m_e}A_z-v_{De}\right)\left[\underline{-\frac{e}{m_e}\frac{\partial A_z}{\partial t}+\frac{d\mathbf{x}_e}{dt}\cdot\left(-\frac{e}{m_e}\nabla A_z+\nabla v_{De}\right)-\frac{dp_{ze}}{dt}}\right]$$

$$\boxed{\frac{dw_{ej}}{dt} = -(1-w_{ej})\frac{m_e}{T_e}\left(p_{zej}+\tfrac{e}{m_e}A_z-v_{De}\right)\left[-\frac{e}{m_e}\frac{\partial A_z}{\partial t}+\frac{d\mathbf{x}_{ej}}{dt}\cdot\left(-\frac{e}{m_e}\nabla A_z+\nabla v_{De}\right)-\frac{dp_{zej}}{dt}\right]}$$

equation for weights of ions :

$$\frac{dw_{ij}}{dt} = -(1-w_{ij})\frac{m_e}{T_e}\left(p_{zij}-\tfrac{e}{m_i}A_z\right)\left[\frac{e}{m_i}\frac{\partial A_z}{\partial t}+\frac{d\mathbf{x}_{ij}}{dt}\cdot\frac{e}{m_i}\nabla A_z-\frac{dp_{zij}}{dt}\right]$$

p_z-version with 0-th order current (5)

Comparison with v_z-version (1):

equation for weights of electrons in v_z formulation :

$$f_{0e}(v_z,\mathbf{x}) = n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e(v_z - v_{De})^2}{2T_e}\right)$$

$$\frac{df_{0e}}{dt} = \frac{d\mathbf{x}_e}{dt}\cdot\frac{\partial f_{0e}}{\partial\mathbf{x}} + \frac{dv_{ze}}{dt}\frac{\partial f_{0e}}{\partial v_z}$$

$$\therefore \frac{1}{f_0}\frac{df_{0e}}{dt} = \frac{m_e}{T_e}(v_z - v_{De})\left(\underline{\frac{d\mathbf{x}_e}{dt}\cdot\nabla v_{De} - \frac{dv_{ze}}{dt}}\right)$$

$$= \frac{1}{f_0}\frac{df_{0e}}{dt}\bigg|_{\mathrm{pz-version}}$$

$\because$ see next slide

p_z-version with 0-th order current (6)

Comparison with v_z-version (2):

$$\because \quad -\frac{e}{m_e}\frac{\partial A_z}{\partial t}+\frac{d\mathbf{x}_e}{dt}\cdot\left(-\frac{e}{m_e}\nabla A_z+\nabla v_{De}\right)-\frac{dp_{ze}}{dt}$$

$$=\frac{d\mathbf{x}_e}{dt}\cdot\nabla v_{De}-\frac{e}{m_e}\frac{\partial A_z}{\partial t}-\frac{e}{m_e}\left(-\frac{\nabla\phi\times\mathbf{b}}{B_0}+v_z\mathbf{b}^*\right)\cdot\nabla A_z-\frac{e}{m_e}\left(\mathbf{b}\cdot\nabla\phi-v_z\mathbf{b}\cdot\nabla A_z\right)$$

canceled out

$$=\frac{d\mathbf{x}_e}{dt}\cdot\nabla v_{De}-\frac{e}{m_e}\left(\mathbf{b}^*\cdot\nabla\phi+\frac{\partial A_z}{\partial t}\right)$$

$$=\frac{d\mathbf{x}_e}{dt}\cdot\nabla v_{De}-\frac{dv_z}{dt}$$

where

$$\mathbf{b}^*=\mathbf{b}+\frac{\nabla A_z\times\mathbf{b}}{B_0}$$

p_z-version with 0-th order current and ϕ (1)

decomposition in δf method

$$f = f_0 + \delta f$$

0 - th order electron velocity distribution :

$$f_{0e}(p_z, \mathbf{x}, t) = n_{0e}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(\frac{e\phi}{T_e} - \frac{m_e(p_z + \frac{e}{m_e}A_z - v_{De})^2}{2T_e}\right)$$

0 - th order ion velocity distribution :

$$f_{0i}(p_z, \mathbf{x}, t) = n_{0i}\sqrt{\frac{m_e}{2\pi T_e}}\exp\left(-\frac{m_e(p_z + \frac{e}{m_e}A_z)^2}{2T_e}\right)$$

p_z-version with 0-th order current and ϕ (2)

lower order moments for electrons :

$$\int f_{0e} dp_z = n_{0e} \underline{\exp(e\phi/T_e)}$$

$$\int \left(p_z + \tfrac{e}{m_e} A_z\right) f_{0e} dp_z = n_{0e} v_{De} \underline{\exp(e\phi/T_e)}$$

$$\int \left(p_z + \tfrac{e}{m_e} A_z\right)^2 f_{0e} dp_z = n_{0e} v_{te}^2 \underline{\exp(e\phi/T_e)} + n_{0e} v_{De}^2 \underline{\exp(e\phi/T_e)}$$

gyrokinetic Poisson's equation :

$$\frac{\omega_{pi}^2}{\omega_{ci}^2} \nabla_\perp^2 \phi = \underline{\frac{e n_{0e}}{\varepsilon_0} \left[\exp(e\phi/T_e) - 1\right]} + \frac{e}{\varepsilon_0} \int \delta f_e dp_z - \frac{e}{\varepsilon_0} \int \delta f_i dp_z$$

Ampere's law :

$$\nabla_\perp^2 \delta A_z = \underline{\mu_0 e n_{0e} v_{De} \left[\exp(e\phi/T_e) - 1\right]} + \mu_0 e \int \left(p_z + \tfrac{e}{m_e} A_z\right) \delta f_e dp_z$$
$$- \mu_0 e \int \left(p_z - \tfrac{e}{m_i} A_z\right) \delta f_i dp_z$$

where

$$\delta A_z = A_z - A_{z0}$$

$$\nabla_\perp A_{z0} = \mu_0 e v_{De}$$

p_z-version with 0-th order current and ϕ (3)

equation for weights of electrons :

$$\therefore \frac{dw_{ej}}{dt} = -(1-w_{ej})\frac{1}{f_0}\frac{df_{0e}}{dt}$$

$$= -(1-w_{ej})\left\{\frac{e}{T_e}\left(\frac{\partial\phi}{\partial t} + v_z \mathbf{b}^* \cdot \nabla\phi\right)\right.$$

$$\left. + \frac{m_e}{T_e}\left(p_z + \tfrac{e}{m_e}A_z - v_{De}\right)\left[-\frac{e}{m_e}\frac{\partial A_z}{\partial t} + \frac{d\mathbf{x}_e}{dt}\cdot\left(-\frac{e}{m_e}\nabla A_z + \nabla v_{De}\right) - \frac{dp_{ze}}{dt}\right]\right\}_{\mathbf{x}=\mathbf{x}_j,\, p=p_j}$$

where we have used the following relation

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \frac{d\mathbf{x}_e}{dt}\cdot\nabla\phi = \frac{\partial\phi}{\partial t} + \left(-\frac{\nabla\phi\times\mathbf{b}}{B_0} + v_z\mathbf{b}^*\right)\cdot\nabla\phi = \frac{\partial\phi}{\partial t} + v_z\mathbf{b}^*\cdot\nabla\phi$$

equation for weights of ions :

$$\frac{dw_{ij}}{dt} = -(1-w_{ij})\frac{m_e}{T_e}\left(p_z - \tfrac{e}{m_i}A_z\right)\left[\frac{e}{m_i}\frac{\partial A_z}{\partial t} + \frac{d\mathbf{x}_i}{dt}\cdot\frac{e}{m_i}\nabla A_z - \frac{dp_{zi}}{dt}\right]_{\mathbf{x}=\mathbf{x}_j,\, p=p_j}$$

Equations for $\partial\phi/\partial t$ and $\partial A_z/\partial t$ (1)

Taking partial time derivative of gyrokinetic Poisson's equation and Ampere's law, we have

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi = -v_A^2\mathbf{b}^*\cdot\nabla\left(\nabla_\perp^2 A_z\right) - \frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla\left(\nabla_\perp^2\phi\right)$$

: vortex equation

$$\frac{\partial}{\partial t}\nabla_\perp^2 A_z = \mu_0 e^2\left(\frac{n_e}{m_e}+\frac{n_i}{m_i}\right)\left(\frac{\partial A_z}{\partial t}+\mathbf{b}^*\cdot\nabla\phi\right) - \frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla\left(\nabla_\perp^2 A_z\right)$$
$$-\mu_0 e\mathbf{b}^*\cdot\nabla\int v^2 f_e dp + \mu_0 e\mathbf{b}^*\cdot\nabla\int v^2 f_i dp$$

: Ohm's law along the magnetic field

Equations for $\partial\phi/\partial t$ and $\partial A_z/\partial t$ (2)

$$\mu_0 e^2\left(\frac{n_e}{m_e}+\frac{n_i}{m_i}\right)=\left(\frac{1}{d_e^2}+\frac{1}{d_i^2}\right)+\left(\frac{1}{d_e^2}\frac{\delta n_e}{n_{0e}}+\frac{1}{d_i^2}\frac{\delta n_i}{n_{0i}}\right)$$

$$\therefore\left[\left(\frac{1}{d_e^2}+\frac{1}{d_i^2}\right)-\nabla_\perp^2\right]\left(\frac{\partial}{\partial t}A_z\right)=-\left(\frac{1}{d_e^2}\frac{\delta n_e}{n_{0e}}+\frac{1}{d_i^2}\frac{\delta n_i}{n_{0i}}\right)\left(\frac{\partial}{\partial t}A_z\right)$$

$$-\left(\frac{1}{d_e^2}\frac{n_e}{n_{0e}}+\frac{1}{d_i^2}\frac{n_i}{n_{0i}}\right)\mathbf{b}^*\cdot\nabla\phi$$

$$+\frac{\mathbf{b}\times\nabla\phi}{B_0}\cdot\nabla\left(\nabla_\perp^2 A_z\right)+\mu_0 e\mathbf{b}^*\cdot\nabla\int v^2 f_e dp-\mu_0 e\mathbf{b}^*\cdot\nabla\int v^2 f_i dp$$

Assuming

$$m_e/m_i<<1,\quad \delta n_e<<n_{e0},\quad \delta n_i<<n_{i0}$$

$$\int v^2 f_e dp=\frac{P_e}{m_e}=\frac{n_e T_{e0}}{m_e},\qquad \int v^2 f_i dp=\frac{P_i}{m_i}=\frac{n_i T_{i0}}{m_i}$$

we have the Ohm's low along the magnetic field in GRM.

$$\frac{\partial}{\partial t}A_z=-\boldsymbol{b}^*\bullet\nabla\phi+d_e^2\frac{d}{dt}(\nabla_\perp^2 A_z)+\frac{T_{e0}}{n_{e0}e}\boldsymbol{b}^*\bullet\nabla n_e$$

Moment integrals for δf

0 - th order moment :

$$\int \delta f_s dp_z = \int \delta f_s dp_z$$
$$= \sum w_{sj} \delta\left(\mathbf{x} - \mathbf{x}_{sj}\right)$$

1st order moment :

$$\int v_z \delta f_s dp_z = \int \left(p_z - \tfrac{e_s}{m_s} A_z\right) \delta f_s dp_z$$
$$= \sum p_{zsj} w_{sj} \delta\left(\mathbf{x} - \mathbf{x}_{sj}\right) - \tfrac{e_s}{m_s} A_z \sum w_{sj} \delta\left(\mathbf{x} - \mathbf{x}_{sj}\right)$$

2nd order moment :

$$\int {v_z}^2 \delta f_s dp_z = \int \left(p_z - \tfrac{e_s}{m_s} A_z\right)^2 \delta f_s dp_z$$
$$= \sum p_{zsj}^2 w_{sj} \delta\left(\mathbf{x} - \mathbf{x}_{sj}\right) - 2 \tfrac{e_s}{m_s} A_z \sum p_{zsj} w_{sj} \delta\left(\mathbf{x} - \mathbf{x}_{sj}\right) + \left(\tfrac{e_s}{m_s}\right)^2 A_z^2 \sum w_{sj} \delta\left(\mathbf{x} - \mathbf{x}_{sj}\right)$$

Alternative :

(1) Temporary calculate v_{sj} from p_{sj} and advance v_{sj} for a time step as well as p_{sj}.

(2) Use v_{sj} to estimate moment integrals.

(3) Calculate field quantities

(3) Go to (1) with $t \rightarrow t + \Delta t$.

Important!! With this alternative, we can calculate A_z directry by Ampere's law without iterations.

6. Summary

1. The Gpic-MHD code is developed in addition to the GRM code.
   - cylindrical geometry
   - non-uniform mesh in radial direction
   - use only standard techniques
   - single- and multi-helicity versions
2. Kinetic internal kink mode simulation was done successfully.
3. Comparison between Gpic-MHD and GRM is done.
4. Possible to do physically meaningful runs even on single PC or PC cluster.
5. Engineeringly meaningful runs need massive-parallel computer.
6. Performance on the Altix3700Bx2 parallel computer and Altix ICE8200 of SGI is investigated and no saturation is observed up to 1024 cores.
8. The formulation of split-weight scheme is revisited.
   - p_z version and v_z version are the same mathematically
   - possibility of another advanced method

Future Plan

1. Investigate scaling up to about 10000 cores.
2. Verify the numerically sound parameter range.
3. Inclusion of advanced techniques is ongoing.
   split-weight-scheme etc.
4. Toroidal version.