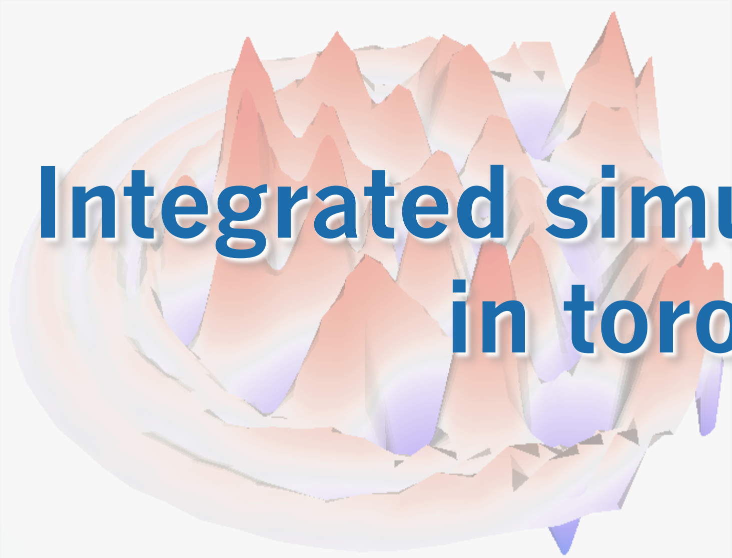
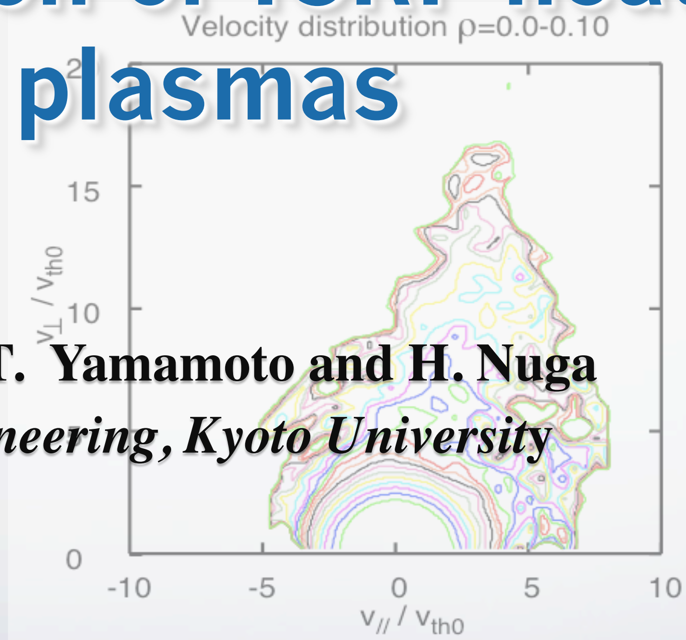
2008 US-Japan Workshop on Progress of Multi-Scale Simulation Models, Dallas, 22 Nov. 2008



Integrated simulation of ICRF heating in toroidal plasmas

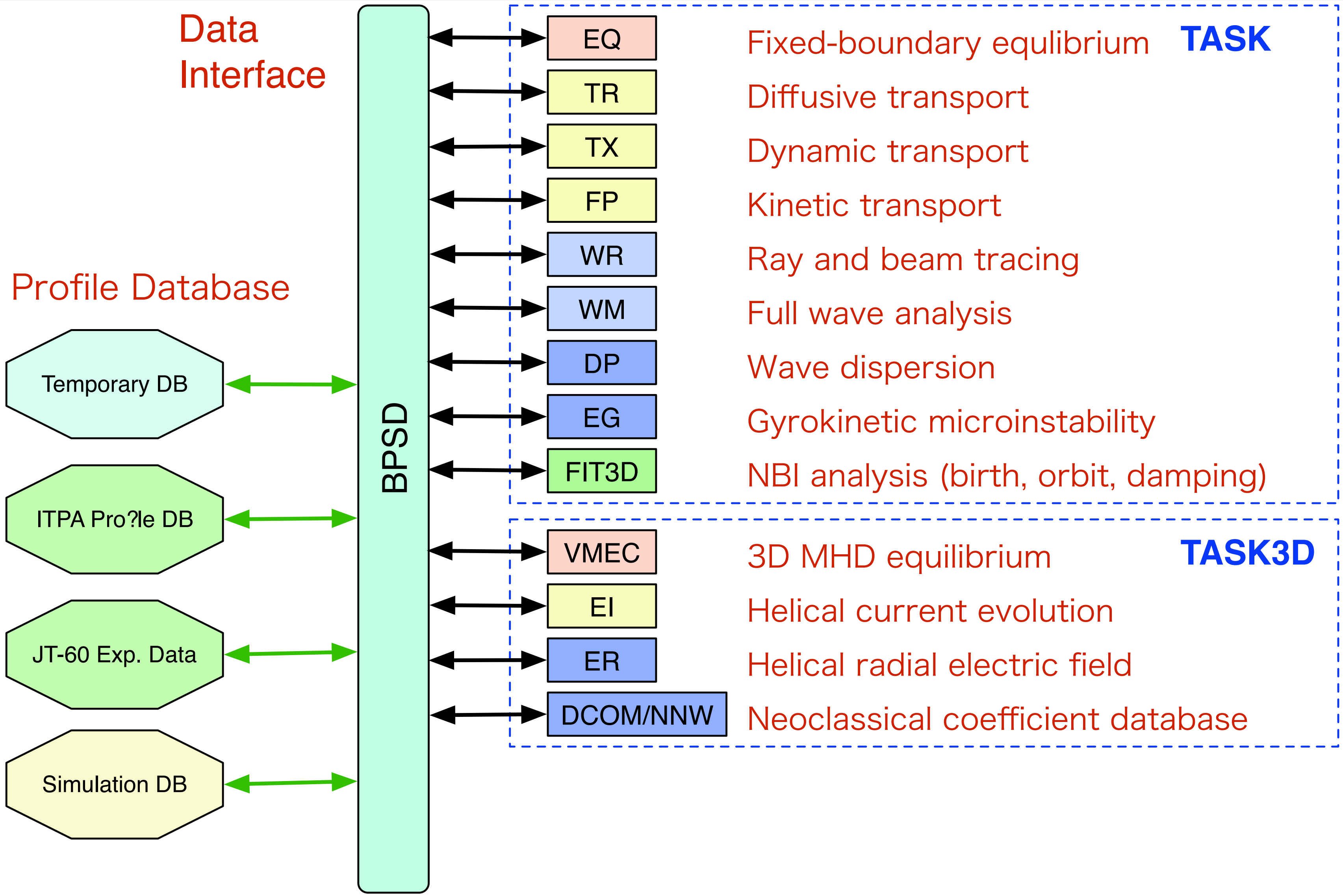


S. Murakami, A. Fukuyama, T. Yamamoto and H. Nuga
Department of Nuclear Engineering, Kyoto University

Issues on ICRF Heating

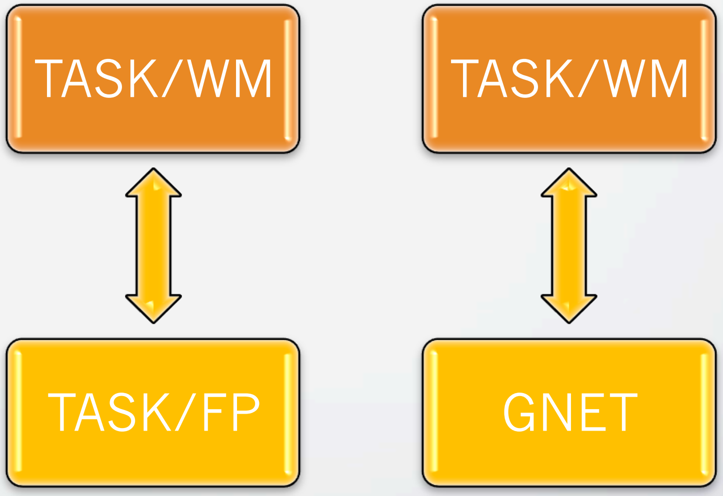
- Self-consistent and accurate modeling of wave-plasma interactions is one of the key issues in producing and sustaining burning plasmas.
- In ICRF heating study different time scale processes should be taking into account.
  - RF wave propagation and wave-particle interaction ($\tau \sim 2\pi/\omega_{ci}$)
  - Energetic ion drift motion ($\tau \sim L/v_{fast}$)
  - Slowing down of energetic ion ($\tau \sim \tau_s$).
- Therefore the integrated modeling is necessary for the ICRF heating simulation.
- We are upgrading wave-related components of the integrated modeling code TASK in order to describe the absorption of ICRF waves by energetic ions.

Integrated Modeling Code : TASK



ICRF Modeling in the TASK Code

- The full wave component TASK/WM was coupled with the Fokker-Planck component TASK/FP and the drift kinetic solover GNET code.
- TASK/FP
  the modification of the multi-species momentum distribution functions.
- GNET:
  the power deposition profile including the finite orbit size effects.
- At present there is no feed back from GNET to TASK/WM.

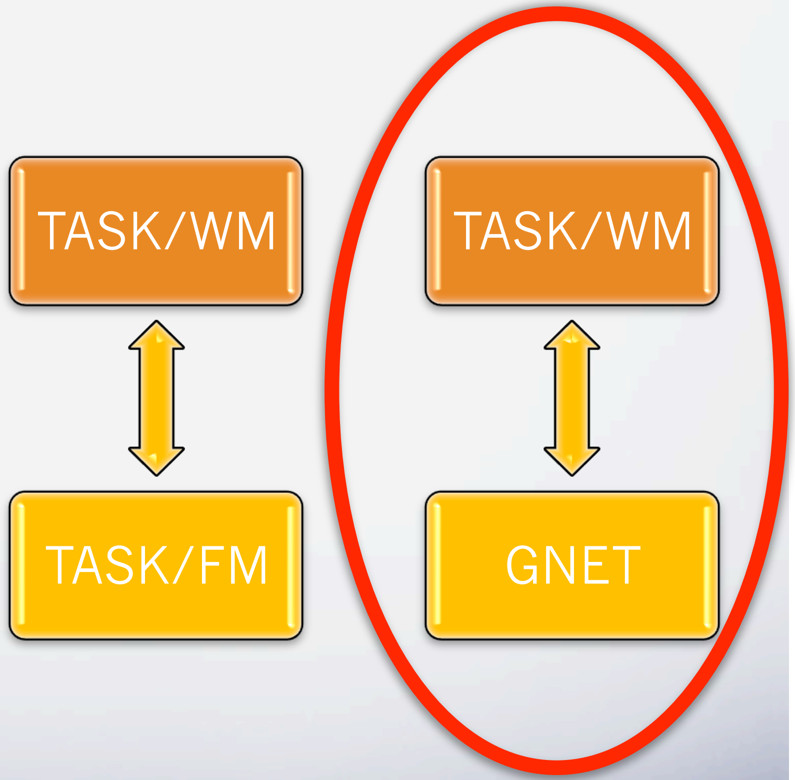


Differences between TASK/FP and GNET

	TASK/FP	GNET
Calculation method	Kinetic fluid	Monte Carlo
Dimension	3	5
Bounce average	Yes	No
Plasma-wave	QL diffusion	MC kick
Collision	Nonlinear	Linear
Collision	Relativistic	Non-relativistic
Orbit width	Zero	Finite
Toroidal inhomogeneity	No	Yes

ICRF Modeling in the TASK Code

- **The full wave component TASK/WM was coupled with the Fokker-Planck component TASK/FP and the drift kinetic solover GNET code.**
- **TASK/FP**
  the modification of the multi-species momentum distribution functions.
- **GNET:**
  the power deposition profile including the finite orbit size effects.



Simulation of ICRF Heating by GNET

- **In order to study the ICRF heating in a 3D magnetic configuration we combine two global simulation codes; a drift kinetic equation solver GNET and a wave field solver TASK/WM.**
  - **GNET: a linearized drift kinetic equation in 5-D phase space**
  - **TASK/WM: Maxwell's equation for RF wave electric field**
- **We apply the simulation code to a simple circular cross section tokamak as a first step.**
- **We make clear the characteristics of energetic ions distribution in the phase-space.**

TASK/WM
RF wave field :
$E_{+}, k_{//}, k_{perp}$

→

GNET
Energetic ion distribution

TASK/WM: Full Wave Field Solver

- **Magnetic surface coordinate**: (ψ, θ, φ)
- **Boundary value problem of Maxwell's equation**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + \mathrm{i}\,\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

- **Kinetic dielectric tensor** : $\overleftrightarrow{\epsilon}$
  - Wave-particle interaction : $Z[(\omega - n\omega_{\mathrm{c}})/k_{||}v_{\mathrm{th}}]$
  - Finite Lamor radius effect : reductive method
- **Toroidal and poloidal direction mode expansion**
- **Radial direction : finite element method**

TASK/WM: Formulation and Boundary Conditions

Wave electric field: $\boldsymbol{E}(\psi_l, \theta, \varphi) = \sum_{mn} \boldsymbol{E}_{mn}(\psi_l) e^{i(m\theta+n\varphi)}$

Metric tensor: $g_{ij}(\psi_l, \theta, \varphi) = \sum_{m'n'} (g_{ij})_{m'n'}(\psi_l) e^{i(m'\theta+n'N_h\varphi)}$

Antenna current:

$$j^{\psi}_{\rm ext} = \sum_{mn} \left(j^{\psi}\right)_{mn} e^{i(m\theta+n\varphi)} \Theta(\psi - \psi_{\rm ant})$$

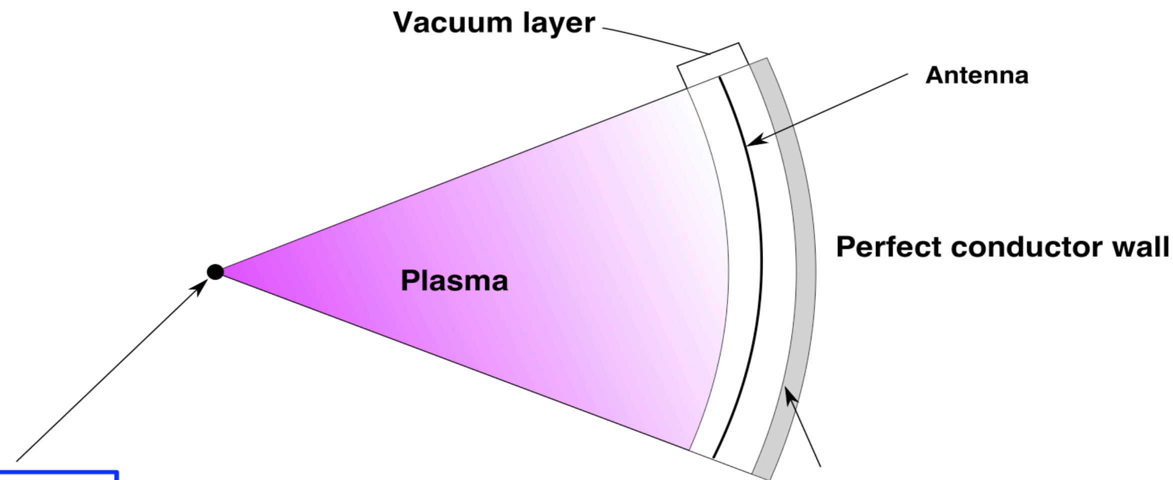
$$j^{\theta,\varphi}_{\rm ext} = \sum_{mn} \left(j^{\theta,\varphi}\right)_{mn} e^{i(m\theta+n\varphi)} \delta(\psi - \psi_{\rm ant})$$

N_h: **pitch number of the helical coil**

$\psi_{\rm ant}$: **radial antenna position**

Θ: **step function**

δ: **delta function**



Magnetic axis

$$\begin{cases} \dfrac{\partial E_{\varphi}^{0n}}{\partial \psi} = 0, & (m = 0) \\ E_{\varphi}^{mn} = 0, & (m \neq 0) \end{cases}$$

$E_{\theta}^{mn} = 0$

Charge density and magnetic field are finite

Vacuum-Perfect conductor wall

$E_{\theta}^{mn} = 0, \quad E_{\varphi}^{mn} = 0$

Tangential electric field is zero

ICH Simulation Model by GNET

- We solve **the drift kinetic equation** as a (time-dependent) initial value problem in 5D phase space based on **the Monte Carlo technique.**

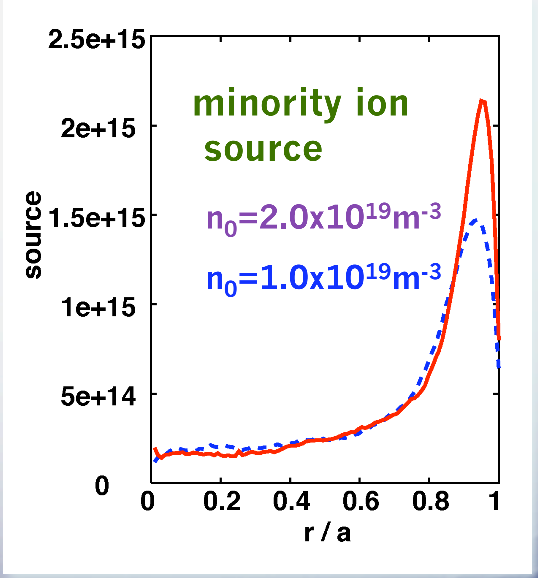
$$\frac{\partial f_{min}}{\partial t} + (\mathbf{v}_{//} + \mathbf{v}_D)\cdot\nabla f_{min} + \mathbf{a}\cdot\nabla_{\mathbf{v}} f_{min} - C(f_{min}) - Q_{ICRF}(f_{min}) - L_{particle} = S_{particle}$$

$C(f)$: linear Clulomb Collision Operator

Q_{ICRF} : ICRF heating term
wave-particle interaction model

$S_{particle}$: particle source
=> by ionization of neutral particle
(AURORA code)

$L_{particle}$: particle sink (loss)
=> Charge exchange loss
=> Orbit loss (outermost flux surface)



- The minority ion distribution f is evaluated through **a convolution of $S_{particle}$** with a characteristic time dependent **Green function**.

Time Dependent Green Function $\mathcal{G}$

- It is convenient to introduce **the Green function $\mathcal{G}$(x,v,t | x',v')** which is defined by **the homogeneous F-P equation**

$$\frac{\partial \mathcal{G}}{\partial t} + (\mathbf{v}_{//} + \mathbf{v}_D)\cdot\nabla\mathcal{G} + \dot{\mathbf{v}}\cdot\nabla_{\mathbf{v}}\mathcal{G} - C(\mathcal{G}) - L(\mathcal{G}) - Q_{ICRF}(\mathcal{G}) = 0$$

with the initial condition
$$\mathcal{G}(\mathbf{x},\mathbf{v},t=0\,|\,\mathbf{x}',\mathbf{v}') = \delta(\mathbf{x}-\mathbf{x}')\delta(\mathbf{v}-\mathbf{v}')$$

- Then, **the solution of the inhomogeneous problem** is given by the **convolution with $\mathcal{G}$**;

$$f_{\min}(\mathbf{x},\ \mathbf{v},\ t) = \int_0^t dt' \int d\mathbf{x}' \int d\mathbf{v}'\ S_{particle}\,\mathcal{G}(\mathbf{x},\ \mathbf{v},\ t-t'\ \,|\,\mathbf{x}',\ \mathbf{v}').$$

- In this approach, only **the Green function $\mathcal{G}$ has to be determined** by the **Monte Carlo technique.**

Monte Carlo Simulation for G

- **Complicated particle motion**
  Eq. of motion in the Boozer coordinates $(\psi,\theta,\phi,\rho_c)$
  3D MHD equilibrium (VMEC+NEWBOZ)

- **Coulomb collisions**
  Liner Monte Carlo collision operator [Boozer and Kuo-Petravic]
  Energy and pitch angle scattering

$$C_s(f) = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2\nu_E^s\left(vf + \frac{T_s}{m}\frac{\partial f}{\partial v}\right)\right] + \frac{\nu_d^s}{2}\frac{\partial}{\partial \lambda}\left(1-\lambda^2\right)\frac{\partial f}{\partial \lambda}$$

ν_E^s: **energy transfer rate**
ν_d^s: **deflection collision frequency by background of s particles**
λ: **pitch angle ($= v_\perp / v_{||}$)**

Monte Carlo Simulation for $\mathcal{G}$

- **The Q_{ICRF} term is modeled by the Monte Carlo method.**

- **When the test particle pass through the resonance layer the perpendicular velocity of this particle is changed by the following amount**

$$\Delta v_{\perp} = \sqrt{\left(v_{\perp 0} + \frac{q}{2m} I\left(E_{+}\left(J_{n-1}(k_{\perp}\rho)\ \cos\phi_r \right)^2 + \frac{q^2}{4m^2} \left\{ I\left(E_{+}\left(J_{n-1}(k_{\perp}\rho)\right\}^2\ \sin^2\phi_r \right.\right.\right.} - v_{\perp 0}$$

$$\approx \frac{q}{2m} I\left(E_{+}\left(J_{n-1}(k_{\perp}\rho)\ \cos\phi_r + \frac{q^2}{8m^2 v_{\perp 0}} \left\{ I\left(E_{+}\left(J_{n-1}(k_{\perp}\rho)\right\}^2\ \sin^2\phi_r \right.\right.\right.$$

$$I = \min\left(\sqrt{2\pi / n\dot{\omega}}\ ,\ 2\pi(n\ddot{\omega}/2)^{-1/3} Ai(0)\ \right)$$

E_+: RF electric field   **ϕ_r: random phase**   **ρ : gyroradius**

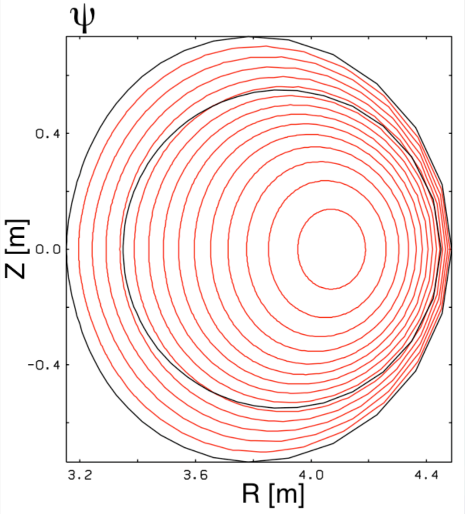
- **Effect of multi $k_{//}$ modes is included by checking the resonance condition for all $k_{//j}$.**

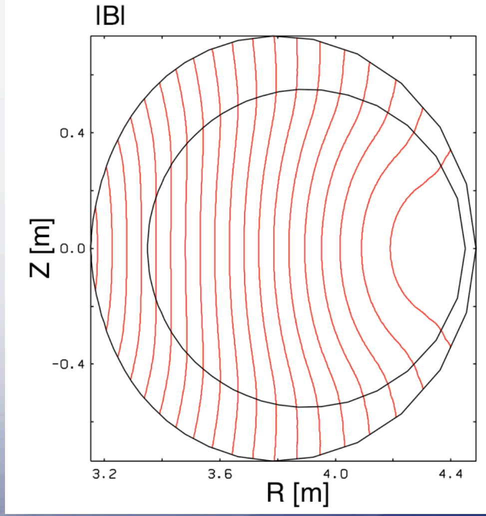
$$\omega = k_{//\,j} v_{//} - n\omega_c$$

Simulation results by TASK/WM

- **Plasma parameters**

Plasma major radius	R_0	3.6 m
Plasma minor radius	a	0.6 m
Magnetic field at magnetic axis	B_0	3.0 T
Temperature at magnetic axis	T_0	3.0 keV
Temperature on plasma boundary	T_s	0.3 keV
Density at magnetic axis	n_0	$1.0 \times 10^{20}/\mathrm{m}^3$
Density on plasma boundary	n_s	$0.1 \times 10^{20}/\mathrm{m}^3$
Antenna current density	j_{ext}	1.0 A/m
Wave frequency	f_{RF}	42, 45 MHz
Minority ion ratio	$\frac{n_{\mathrm{H}}}{n_{\mathrm{D}}+n_{\mathrm{H}}}$	5 %
Collision frequency	ν_s/ω	0.003





TASK/WM: Full wave field solver

- **Maxwell's equations for stationary wave electric field**

$$\nabla \times \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \overleftrightarrow{\epsilon} \cdot \boldsymbol{E} + i\omega\mu_0 \boldsymbol{j}_{\mathrm{ext}}$$

$\boldsymbol{E}$: electric field

$\overleftrightarrow{\epsilon}$: dielectric tensor

$\boldsymbol{j}_{\mathrm{ext}}$: antenna current

c: light velocity

μ_0: magnetic permeability in vacuum

ω: wave angular frequency

- **Various dielectric tensor model are available in TASK.**
  In the present analysis, we assume a simple collisional cold plasma model.

$$\overleftrightarrow{\epsilon} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

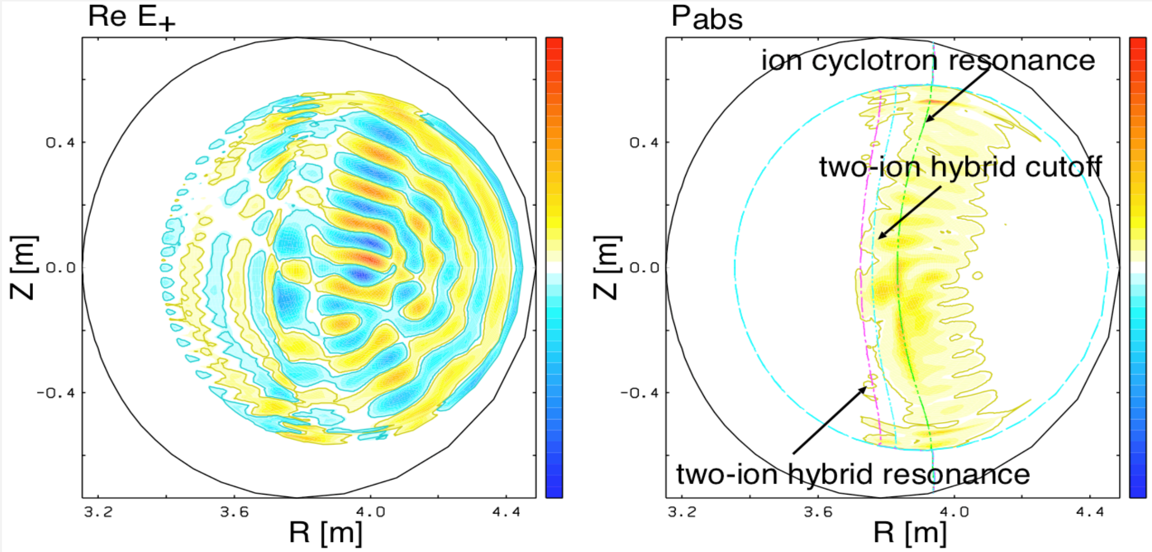
$$S = 1 - \sum_s \frac{\omega_{\mathrm{p}s}^2}{\omega} \frac{\omega + i\nu_s}{(\omega + i\nu_s)^2 - \Omega_s^2}$$

$$D = \sum_s \frac{\omega_{\mathrm{p}s}^2}{\omega} \frac{\Omega_s}{(\omega + i\nu_s)^2 - \Omega_s^2}$$

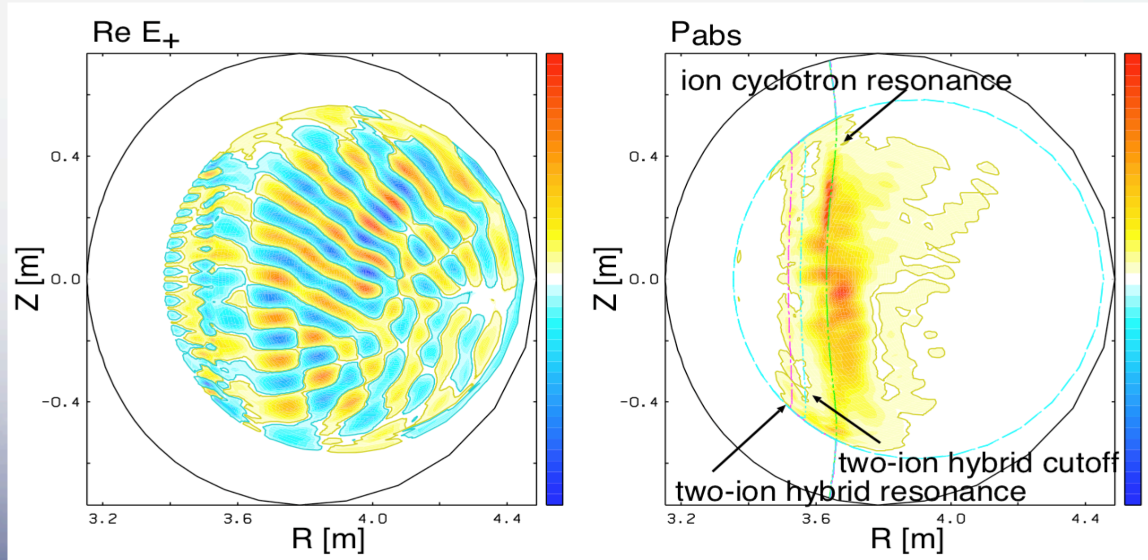
$$P = 1 - \sum_s \frac{\omega_{\mathrm{p}s}^2}{\omega} \frac{1}{\omega + i\nu_s}$$

Full wave analysis by TASK/WM

- **On axis heating (42MHz)**



- **Off axis heating (45MHz)**



RF Electric Field for GNET

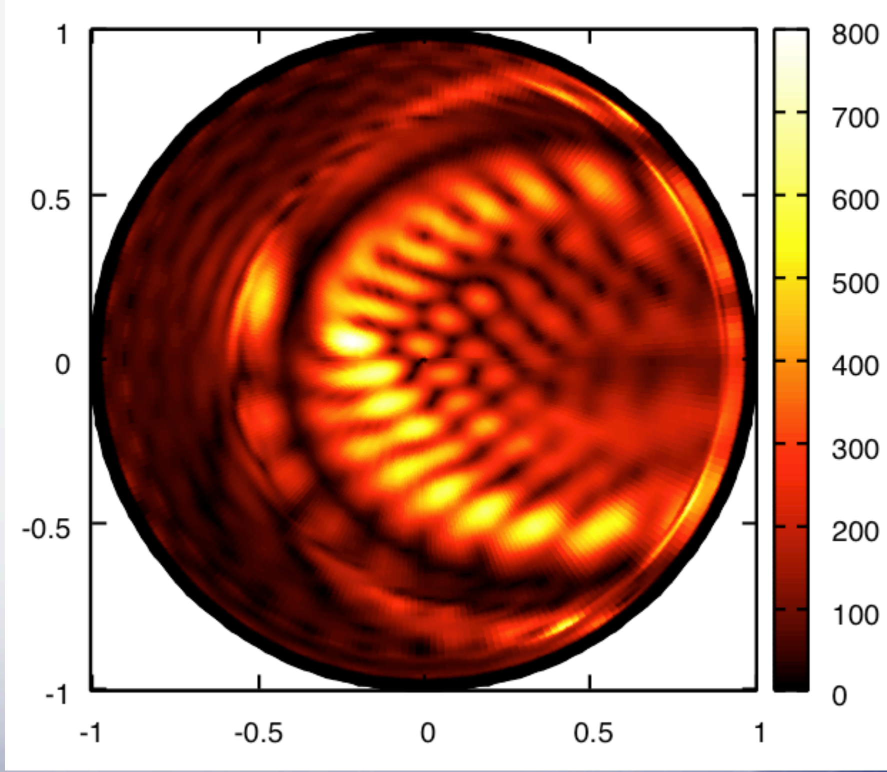
- Considering only **fundamental ion cyclotron resonance** and assuming $k_{perp}\ \rho \ll 1$ is small, then

$$|E_+| J_{n-1}(k_\perp \rho) \approx |E_+|$$

- Also asuuming $B_\varphi \gg B_\theta$,

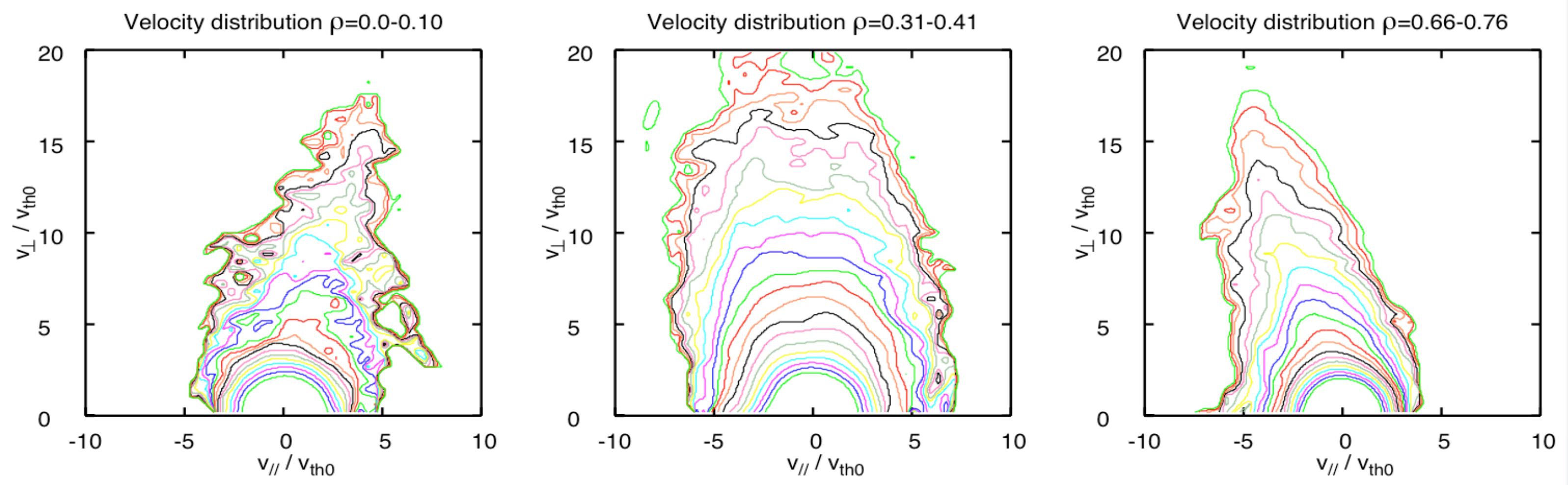
$$k_{||} \approx \frac{n}{R}$$

$|E_+|$ in the Boozer coordinates

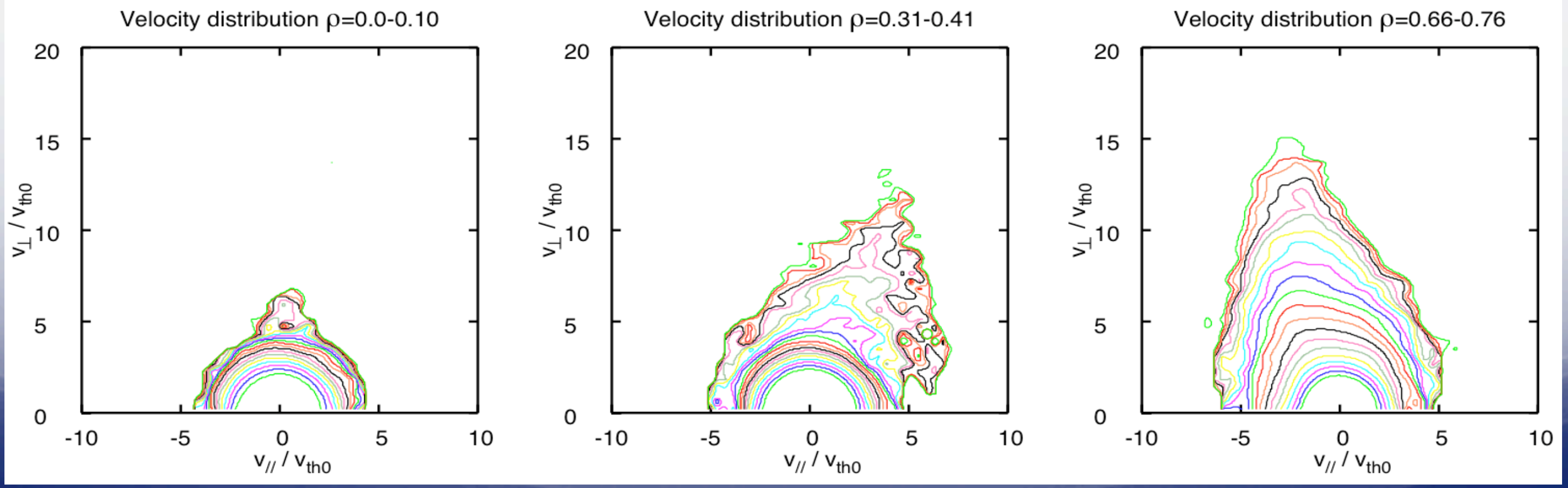


Energetic Ion Distribution (On/Off Axis Heating)

- **On axis heating (f=42MHz, Pabs=5.17MW)**

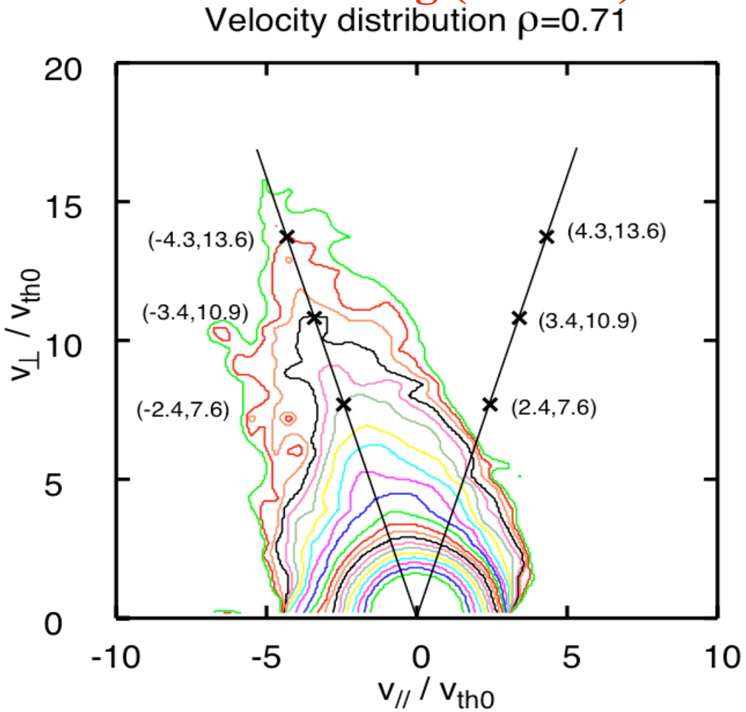


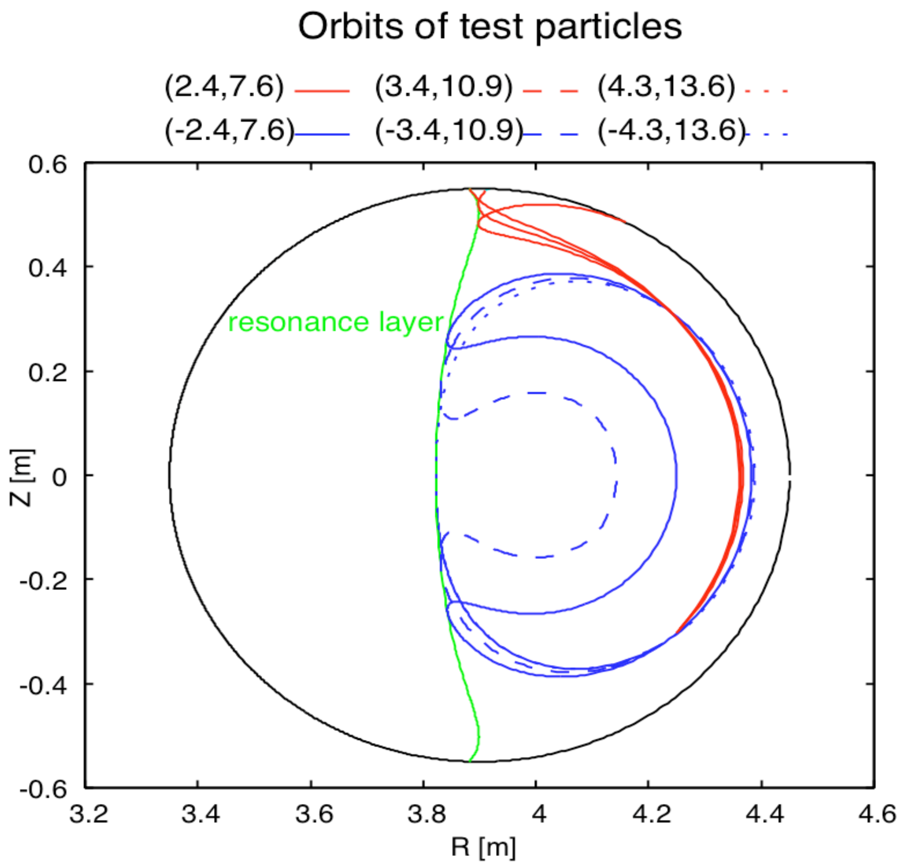
- **Off axis heating (f=45MHz, Pabs=5.26MW)**



Orbits of test particles

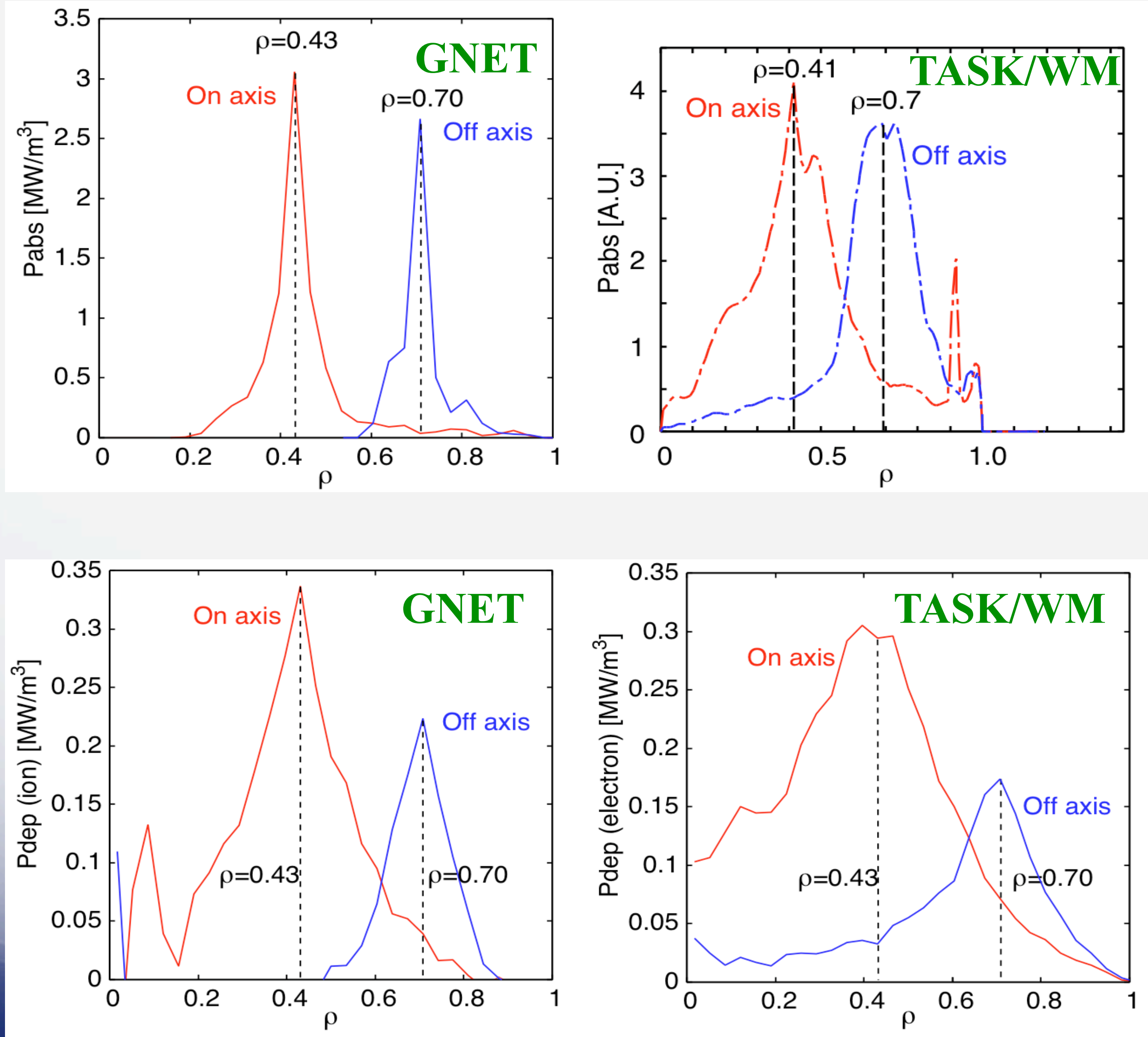
On axis heating (42MHz)





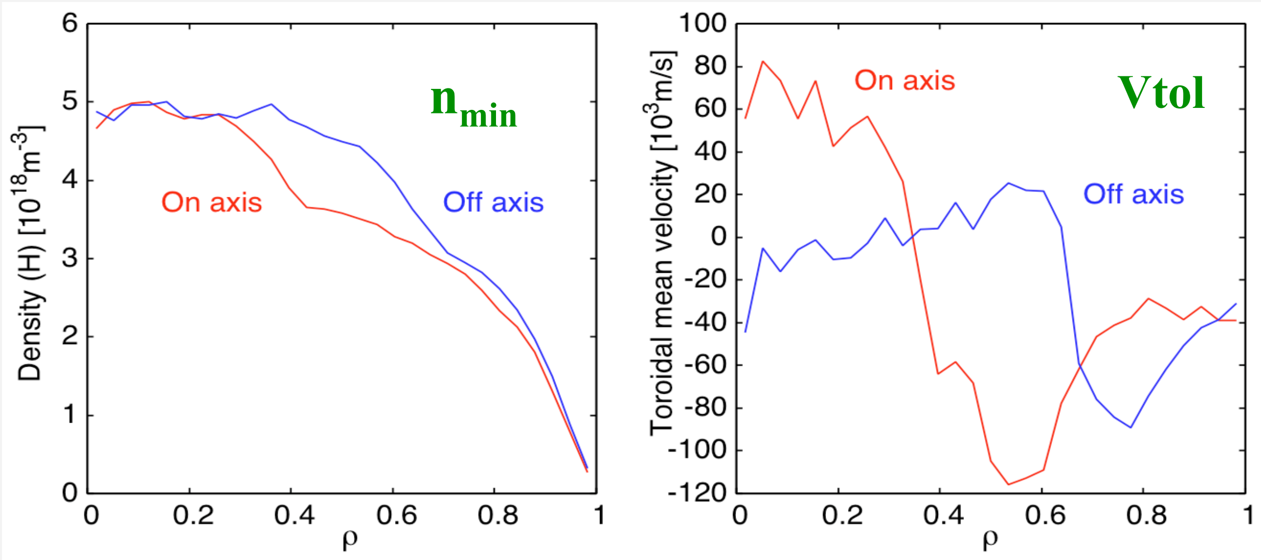
- The **asymmetry in the parallel direction** of the velocity distribution function indicates the **effects of finite orbit width** of trapped particle.

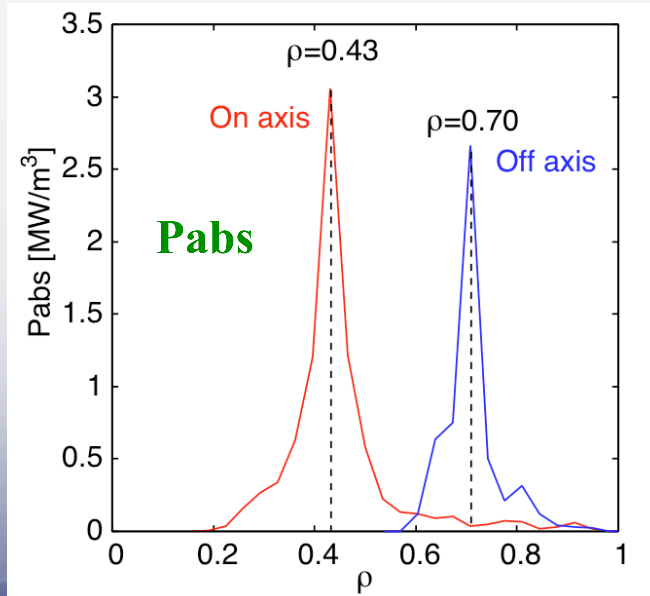
Comparisons of GNET and TASK/WM Results



- **Radial positions of the Pabs and Pdep peaks show good agreements.**
- **More peaked profiles are obtained in the GNET results.**

Density and Toroidal mean velocity





- **Reduction of the minority ion densities** can be seen **near the peak position** of Pabs.
- The **toroidal shear flow** is generated **near the peak position** of Pabs.

Evaluation of Wave Vector k

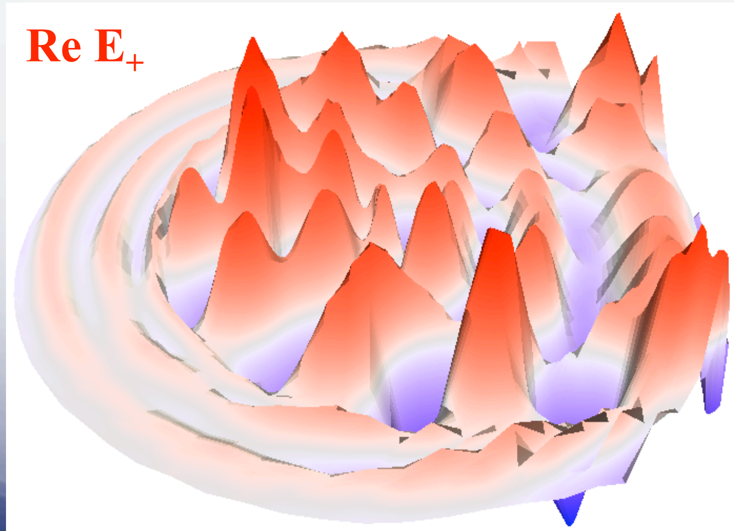
- **Local eikonal approximation**

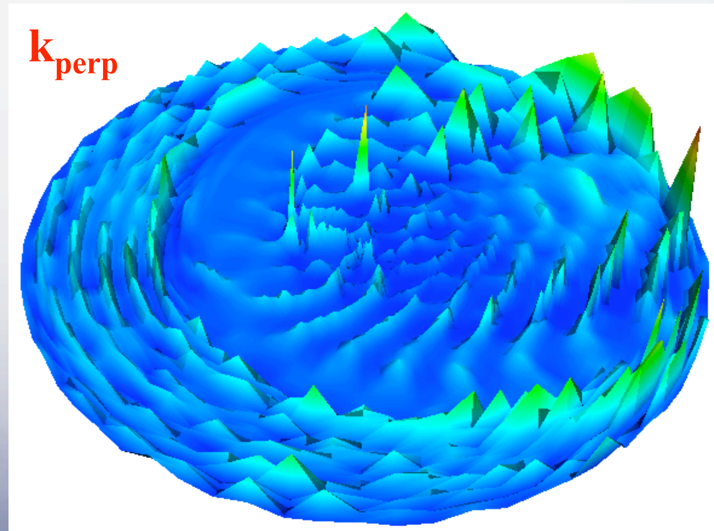
$$\boldsymbol{E}(\rho,\chi,\zeta) = \tilde{\boldsymbol{E}} \exp(i\boldsymbol{k}\cdot\boldsymbol{r})$$

- **Wave vector**

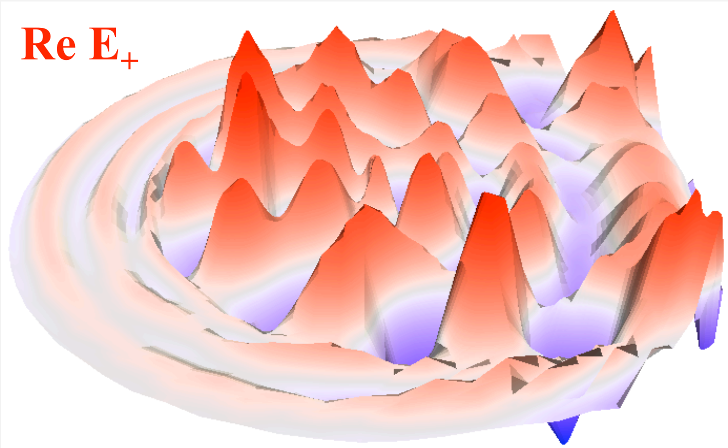
$$k_j = -\mathrm{i}\frac{\frac{\partial}{\partial u^j}E_i(u^1,u^2,u^3)}{E_i(u^1,u^2,u^3)}$$

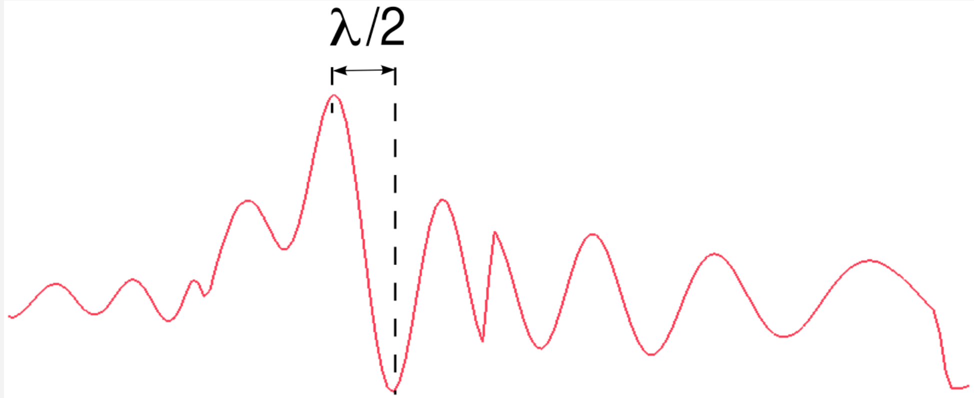
- **Stationary waves have two local wave vector k and -k, therefore the eikonal approximation dose not work.**



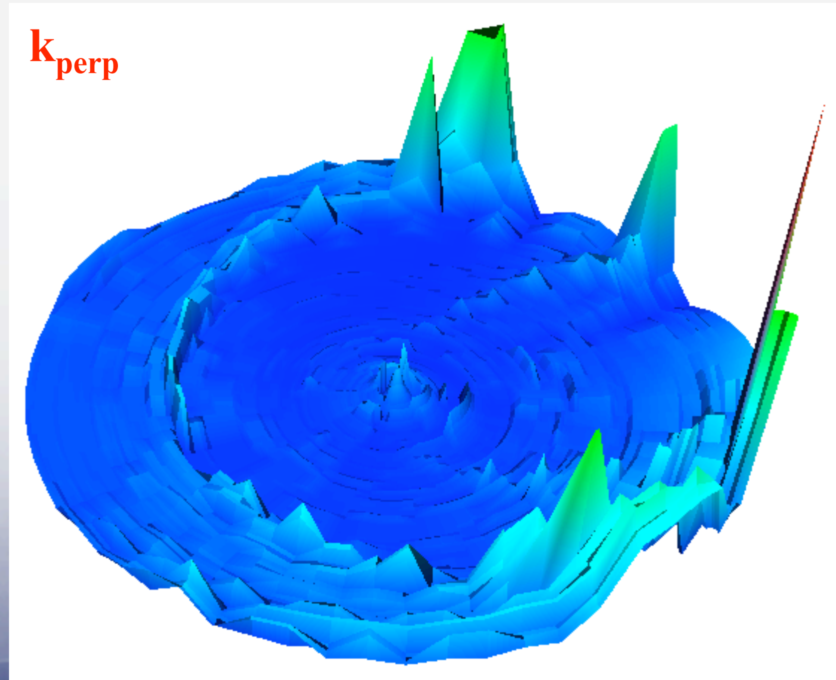


Evaluation of Wave Vector k



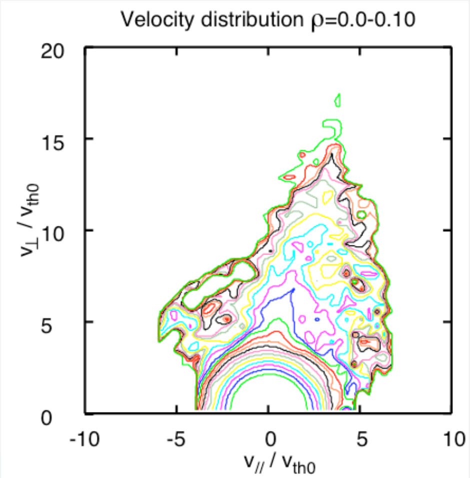


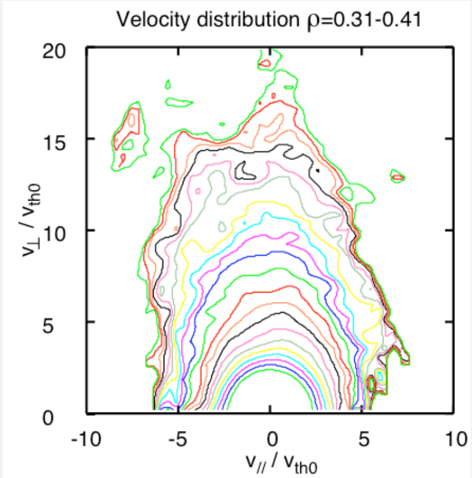
- We **directly evaluate the wave length** and estimate the wave number.
- The obtained results show **relatively good behavior** than that of the eikonal approximation case.

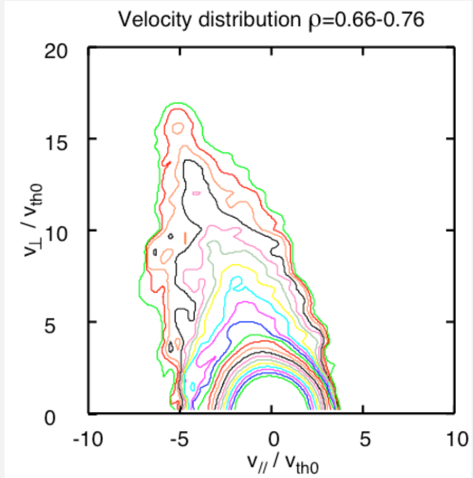


Effect of finite k_{perp}

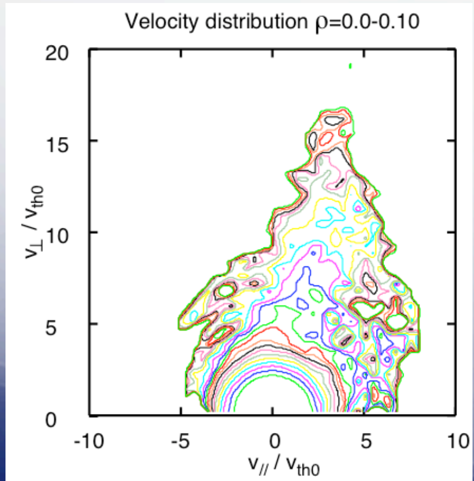
- **On axis (42MHz, Pabs=4.08MW, max|E_+|=4.5kV/m)**

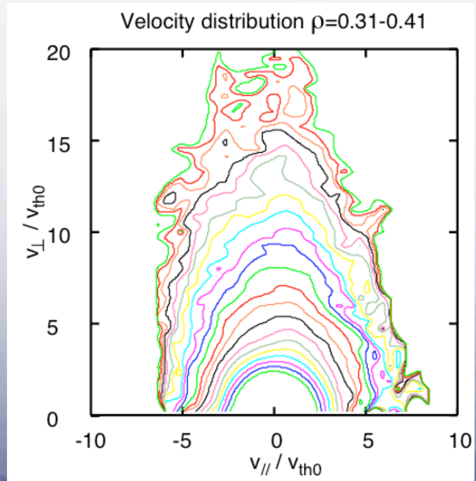


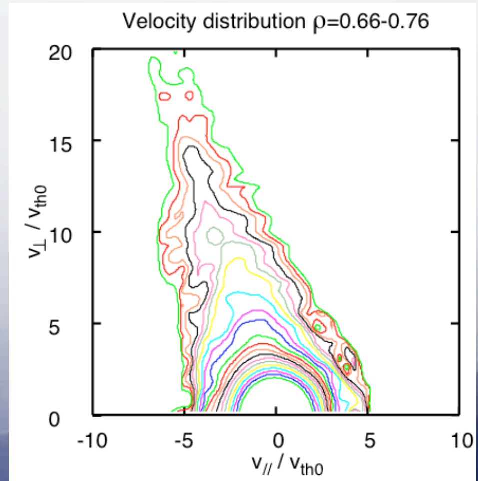




- **On axis (42MHz, Pabs=3.60MW, max|E_+|=4.5kV/m, k_{perp}~0)**







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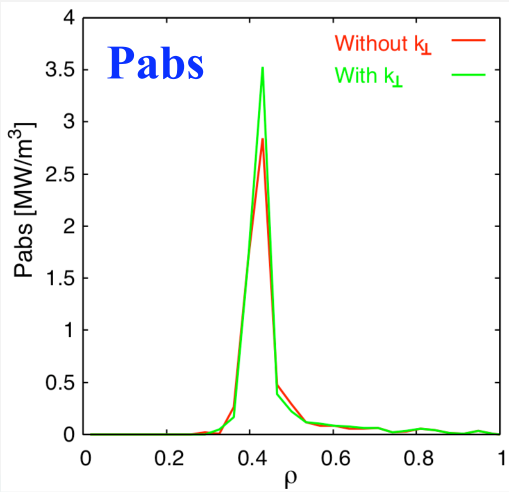
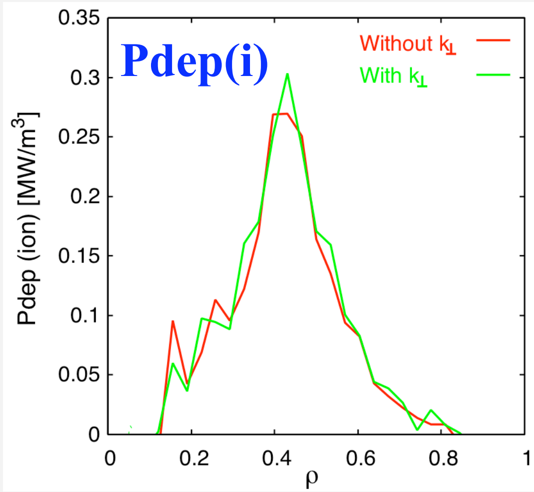
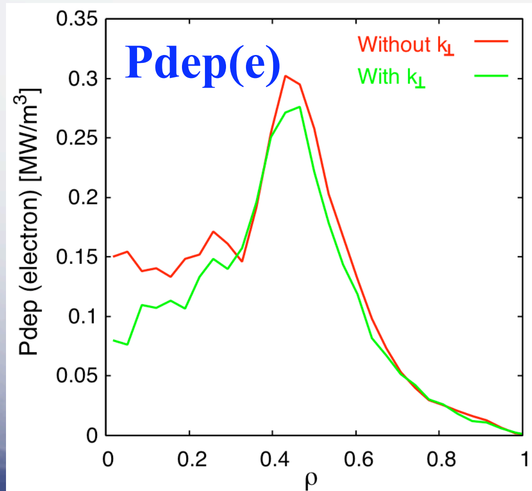
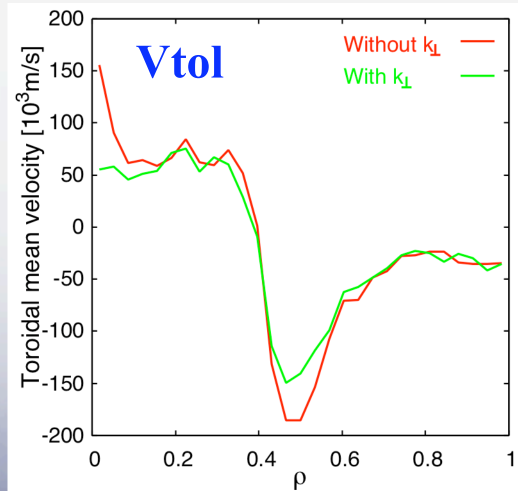




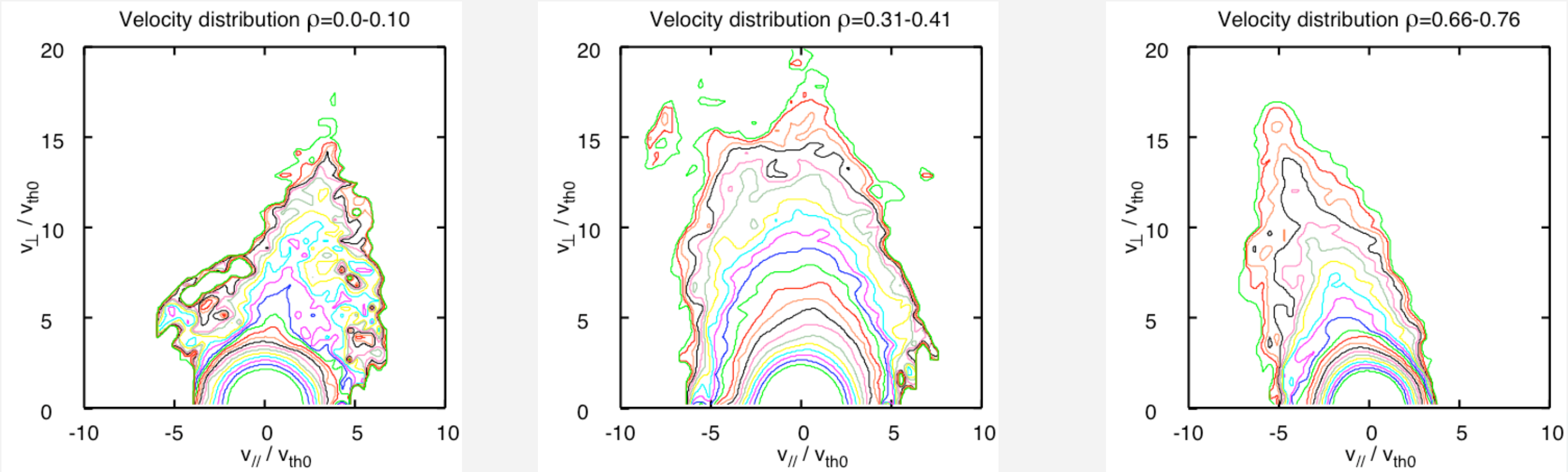




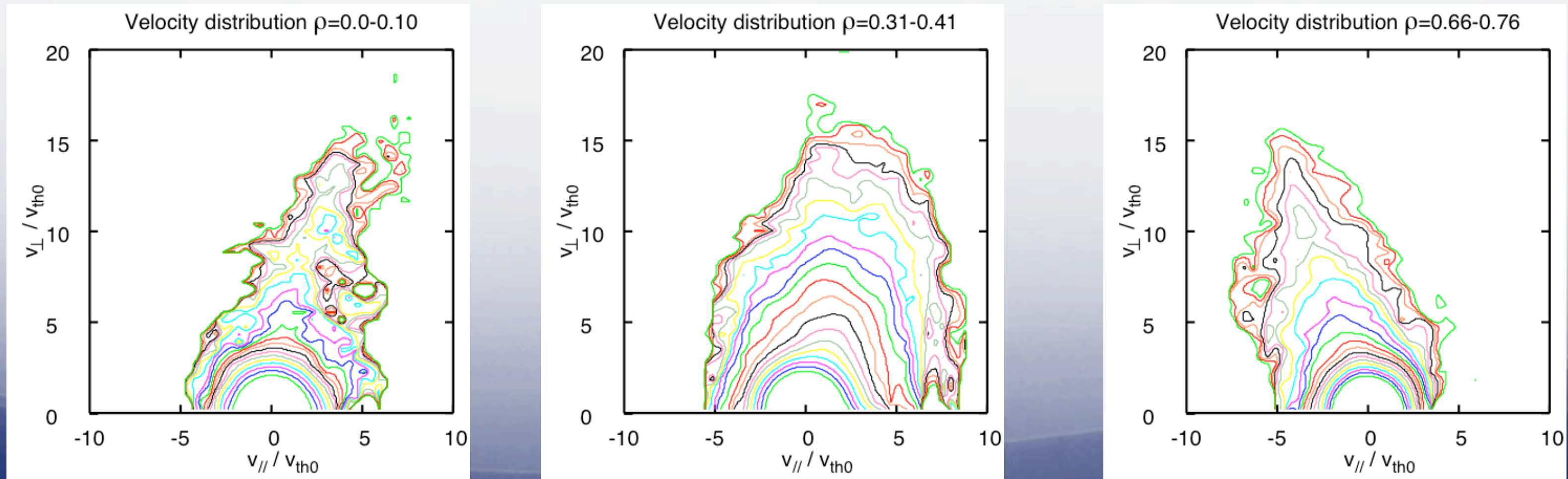
- **No clear difference in the raidal profile of Pabs , Pdep and Vtol of with and without the k_{perp} modeling.**

Dependence of the $k_{//}$ Sign

- **On axis (42MHz, Pabs=4.13MW, max|E_+|=4.5kV/m, $k_{//}$=2.2m^{-1})**

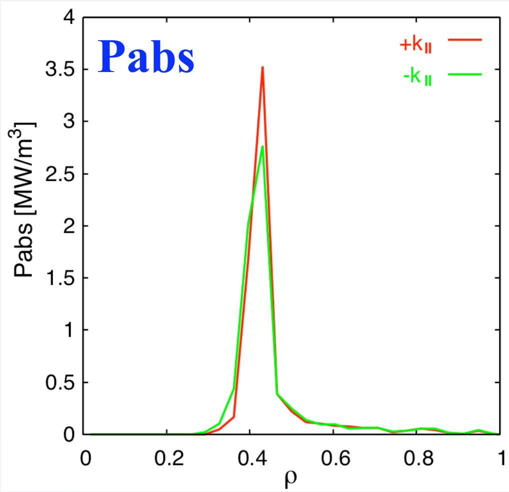


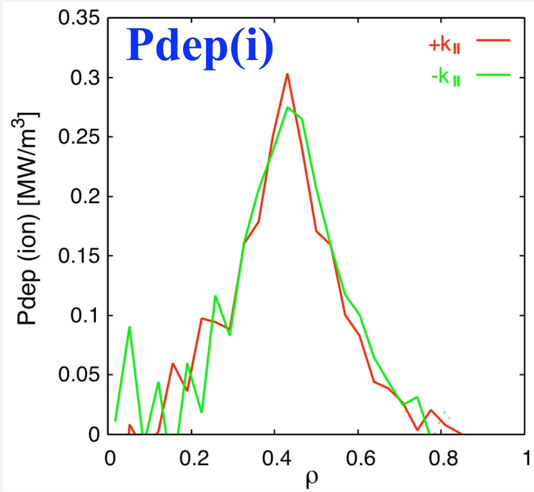
- **On axis (42MHz, Pabs=4.10MW, max|E_+|=4.5kV/m, $k_{//}$=-2.2m^{-1})**

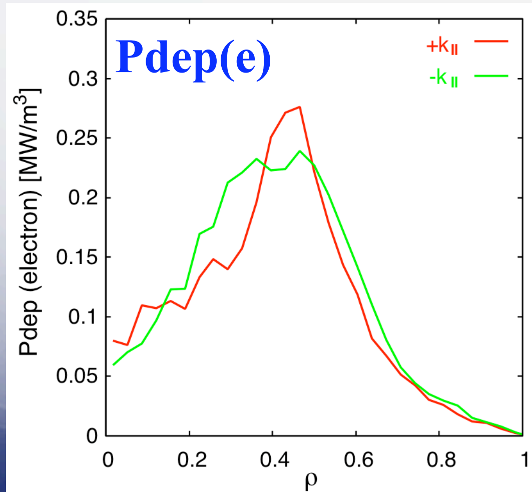


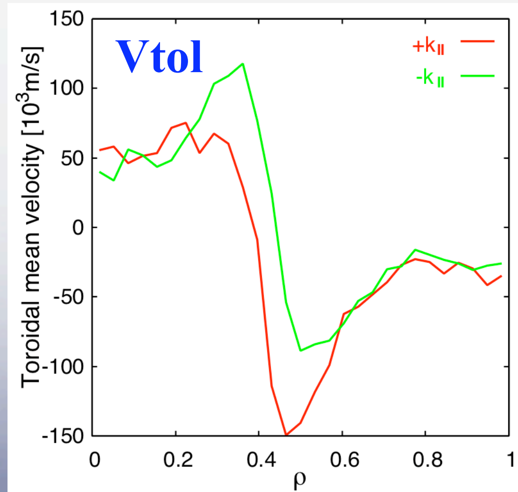
Dependence of the $k_{//}$ Sign

- **On axis (42MHz, max$|E_+|$=3.0kV/m, $|k_{//}|$=2.2m^{-1})**







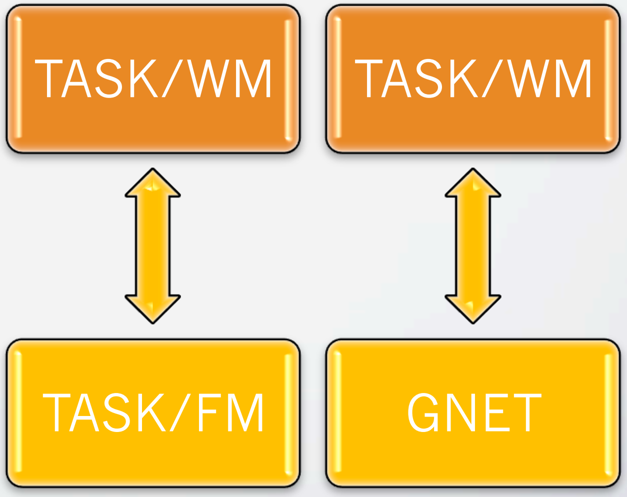


- **No clear difference in the Pabs and Pdep.**
- **We can see a clear difference in the toroidal mean flow generation.**
- **The more positive flow is enhanced in the case of the negative k.**

Summary

- We have upgraded wave-related components of the integrated modeling code TASK in order to describe the absorption of ICRF waves by energetic ions.

- The full wave component was coupled with the Fokker-Planck component and the GNET code to describe the modification of the multi-species momentum distribution functions and to calculate the power deposition profile including the finite orbit size effects.



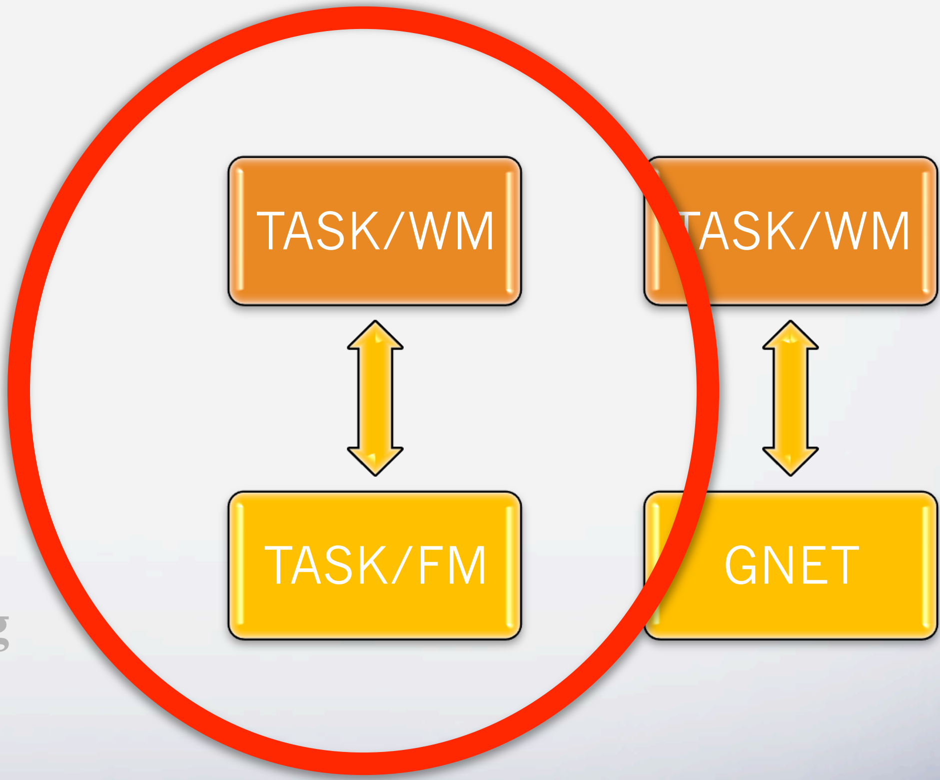
- These advanced modeling provides more accurate evaluation of the efficiency of wave heating and current drive in tokamaks and helical configurations.

ICRF Modeling in the TASK Code

- **The full wave component TASK/WM was coupled with the Fokker-Planck component TASK/FP and the drift kinetic solover GNET code.**

- **TASK/FP**
  the modification of the multi-species momentum distribution functions.

- **GNET:**
  the power deposition profile including the finite orbit size effects.



Multi-Species Fokker-Planck Analysis

- **Fokker-Planck equation**
  for **momentum distribution function** : $f_s(p_{\|}, p_\perp, \rho, t)$

$$\frac{\partial f_s}{\partial t} = E(f_s) + C(f_s) + Q(f_s) + L(f_s) + S$$

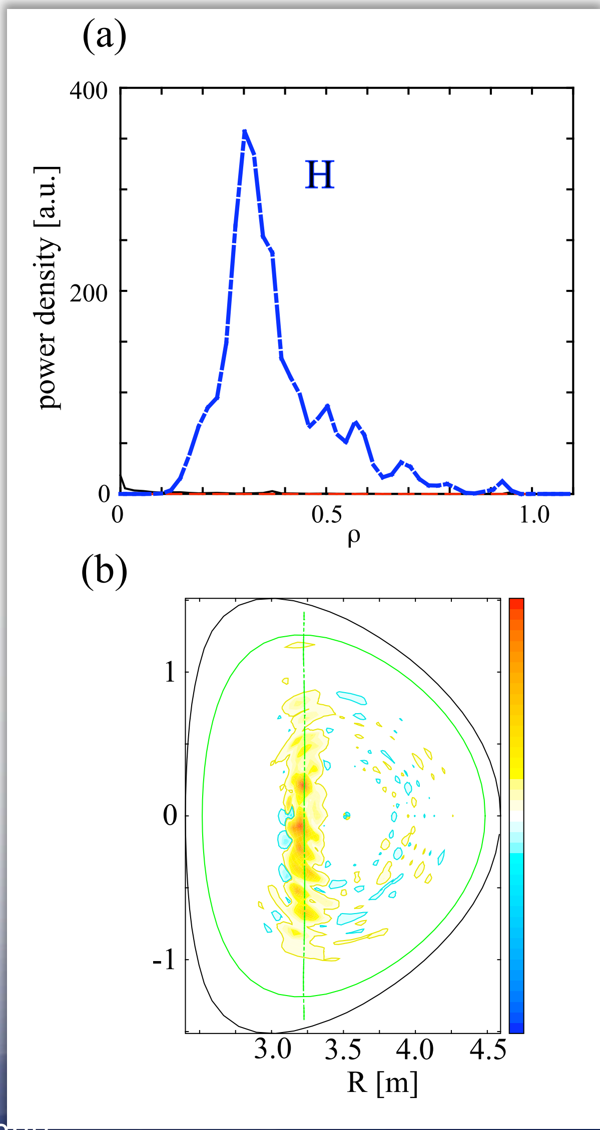
  - $E(f_s)$: Acceleration term due to direct electric field
  - $C(f_s)$: Collisional diffusion and friction term
  - $Q(f_s)$: Quasi-linear diffusion term due to wave-plasma interaction
  - $L(f_s)$ and S : Spatial diffusion term and source/sink term

- Including these features
  - **Multi species**: collisional interaction between species
  - **Bounce average**: trapped particle effect
  - **Weak relativistic**: momentum $\boldsymbol{p}$, collision term
  - **Non-linear collision**: momentum and energy conservation

Simulation Results of TASK/WM

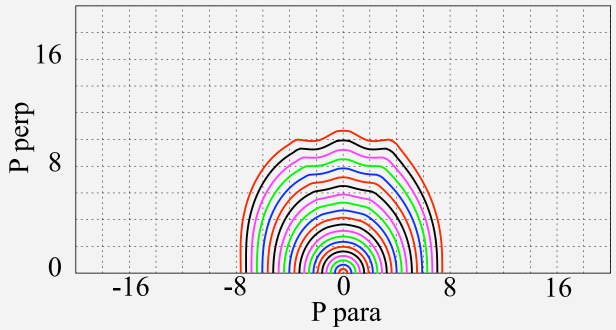
Plasma parameter (JT-60U) | wave absorption profiles (WM)

major radius	R_0	3.5m
minor radius	a	0.98m
elongation	κ	1.28
triangularity	δ	0.31
magnetic field on axis	B_0	3.3T
temperature on axis	T_0	4.0keV
temperature on surface	T_s	0.4keV
deisity on axis	n_0	$3.0\times10^{19}/m^3$
density on surface	n_s	$3.0\times10^{18}m^3$
minority ion ratio	n_H/n_e	5%
frequency	f_{RF}	55MHz
toroidal mode number	n_ϕ	16 or 24
number of poloidal mode number	N_θ	32 ~ 64

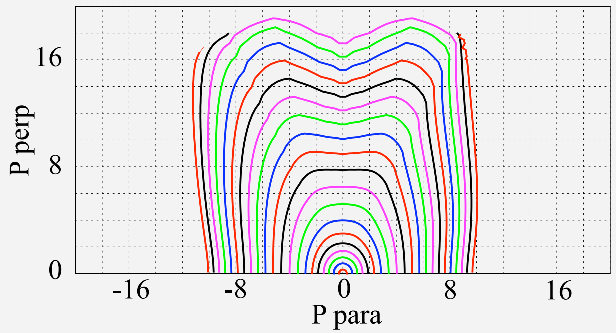


Distribution Function of H at Various Radial Points

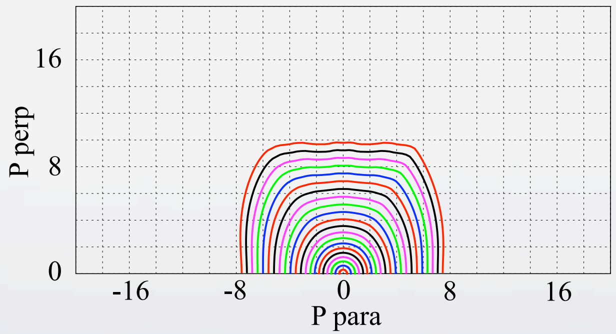
10 msec after the onset of wave heating.



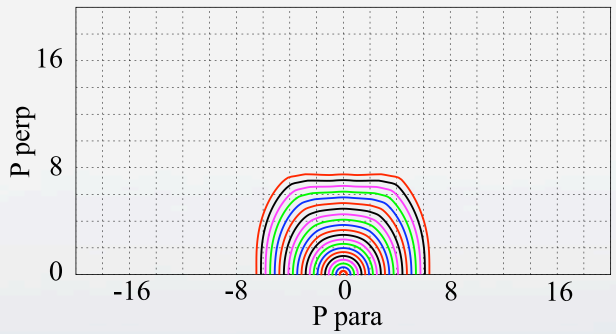
$\rho = 0.25$



$\rho = 0.35$



$\rho = 0.45$

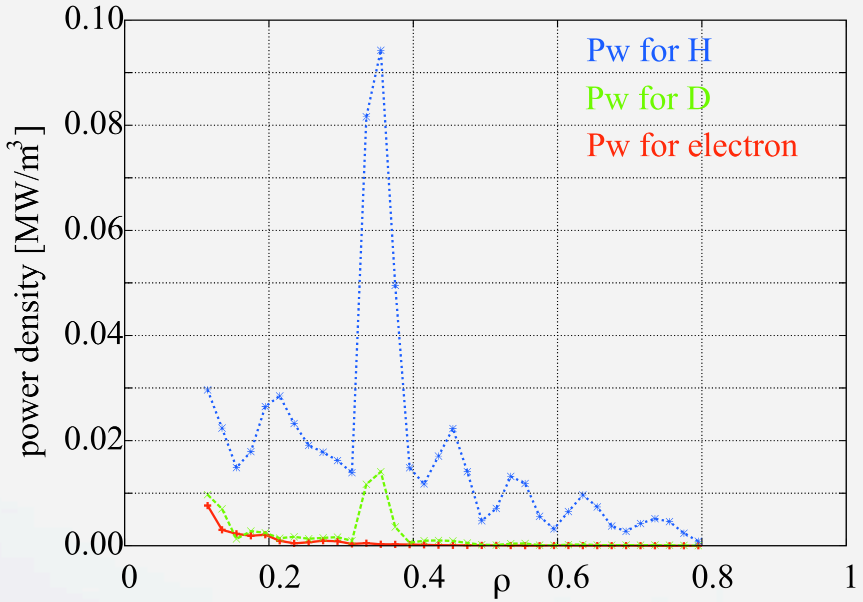


$\rho = 0.55$

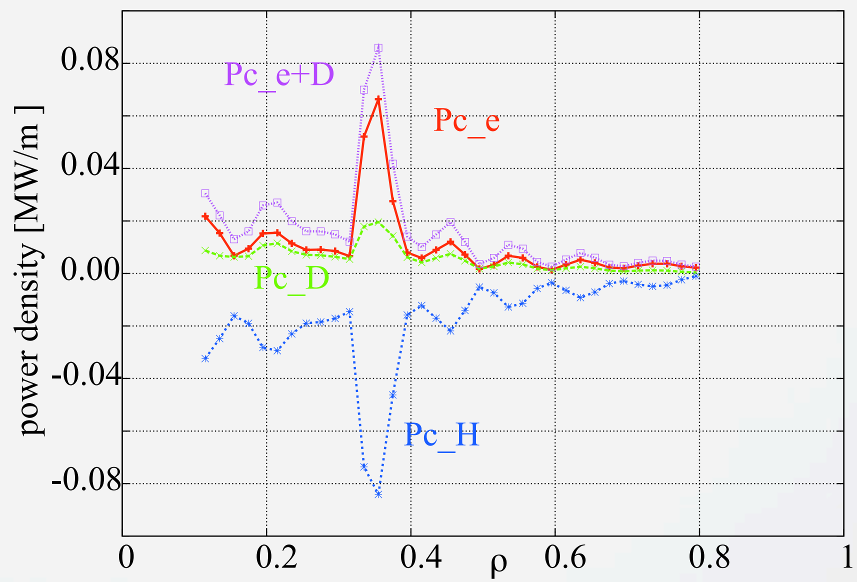
Distribution function has two tips because protons are accelerated where the particle orbits are tangential to cyclotron resonance surface.

Radial Profiles Calculated by TASK/FP

wave absorption



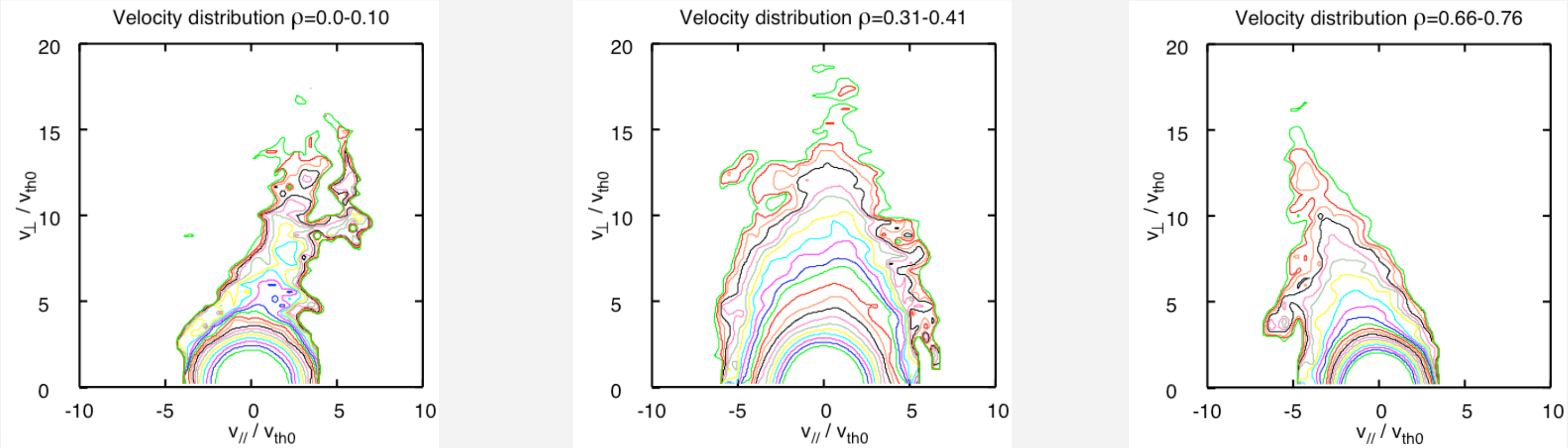
collisional power transfer



- **Wave absorption has a peak at $\rho = 0.35$.**
- **Protons are heated by ICRF waves mainly.**
- **Deuterons are heated by waves through FLR effects and collision with accelerated protons.**
- **Electrons are heated bt collision with protons**

Distribution with finite k_{perp}

- On axis (42MHz, Pabs=2.64MW, max$|E_+|$=3.0kV/m)



- On axis (42MHz, Pabs=4.08MW, max$|E_+|$=4.5kV/m)