Multi-scale aspects of magnetic nozzle modeling

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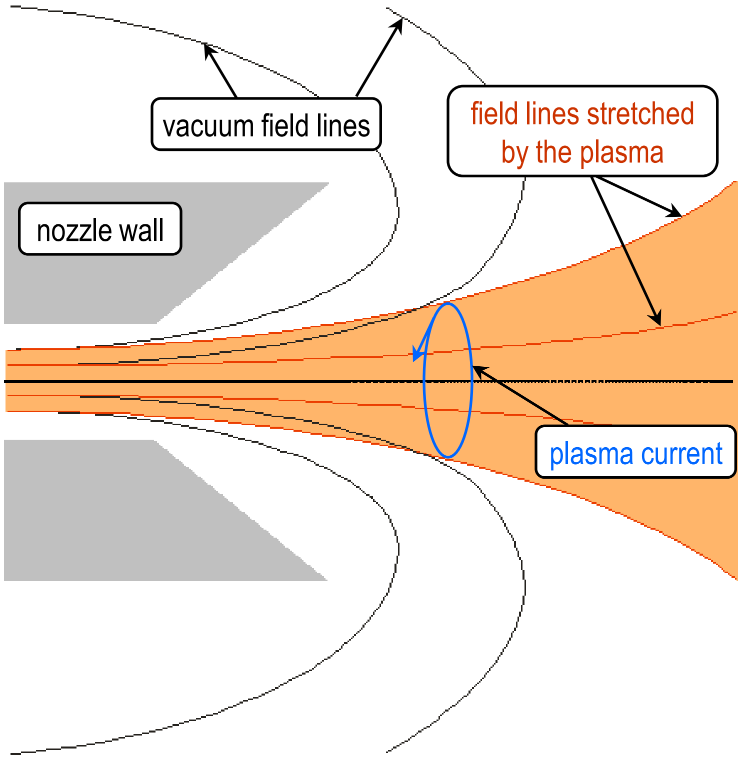
INTRODUCTION

- Plasma-based propulsion systems generate thrust by ejecting directed plasma flow.

- A strong magnetic field is used to guide the plasma towards the nozzle exit.

- The ejected plasma must break free from the nozzle to produce thrust.

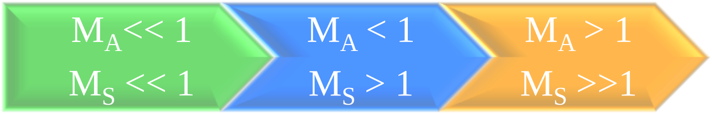
- **MHD detachment scenario:**
  a super-Alfvénic flow detaches together with the magnetic field by stretching the field lines and changing their configuration.



E.N.Parker, Astrophys. J. **128**, 664 (1958)
E.B.Hooper, Journal of Propulsion and Power **9**, 757 (1993)
A. Arefiev and B. Breizman, Phys. Plasmas **12**, 043504 (2005)

MULTI-SCALE ASPECTS OF THE PROBLEM

- Key time scales are:
  - ion time of flight through the nozzle $\tau_i = L / V$
  - electron travel time through the nozzle $\tau_e = L / \mathrm{v}_e$
  - propagation time of magneto-acoustic perturbations across the flow $\tau_A = R / V_A$
  - propagation time of sonic waves along the flow $\tau_S = L / C_S$
- There is a significant change of parameters along the flow:
  - sonic Mach number M_S changes from $M_S << 1$ to $M_S >> 1$
  - Alfvénic Mach number M_A changes from $M_A << 1$ to $M_A > 1$



$M_A << 1$
$M_S << 1$

$M_A < 1$
$M_S > 1$

$M_A > 1$
$M_S >> 1$

- Practical applications require simulation times much longer than τ_i .
- Computational challenges:
  - multiple time scales need to be resolved in a dynamical simulation
  - increasing computational domain due to the flow expansion
  - dramatic change of key physics parameters along the flow

DEALING WITH ISSUES

- A brute-force approach is to employ an existing comprehensive code.
- This option is not practical for an optimization tool that should be able to scan the parameter space. The specific drawbacks are
  - a dynamical MHD simulation requires huge computational resources
  - running time for a single set of parameters is too long to scan the parameter space
- We use the following method to tackle the problem
  - split the problem into components that can be treated independently
  - use the multi-scale aspects to treat some components analytically
  - assemble the components into a multi-scale model tailored to the specific problem
- The sub-problems are
  - electron kinetics
  - ion dynamics along the field lines and the self-consistent magnetic field distortion

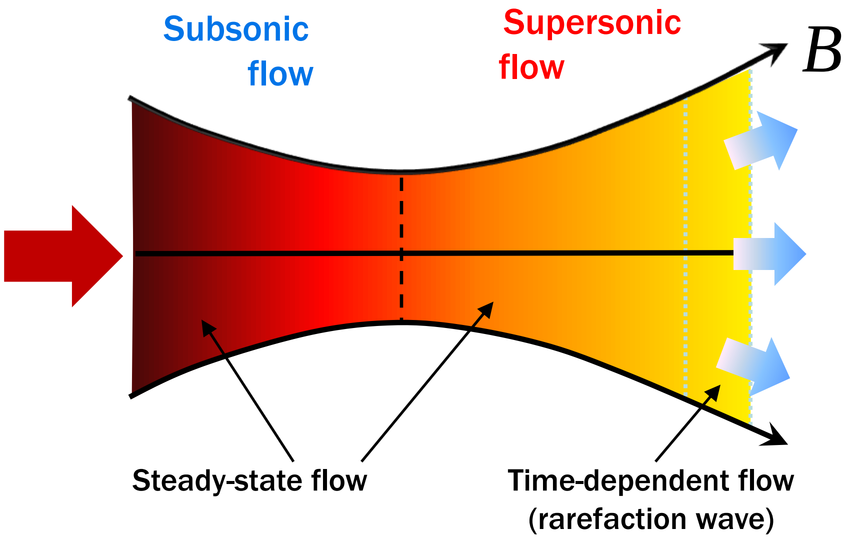
ELECTRON KINETICS

- A magnetic mirror is needed to convert an incoming subsonic plasma flow into a supersonic outgoing flow.
- Electron pressure produces an ambipolar electric field that accelerates plasma ions downstream.
- Collisionless plasma expansion through the mirror distorts an incoming Maxwellian electron distribution.
- Electrons require a fully kinetic description downstream from the mirror.
- The flow expansion is adiabatic, because the rate of the expansion is determined by the ion speed.
- We can then use the conservation of the electron magnetic moment and longitudinal adiabatic invariant to treat a part of the problem analytically.

PLASMA FLOW CONFIGURATION

- The expanding plume is time-dependent and it consists of two parts:
  - a steady-state supersonic flow adjacent to the mirror
  - a rarefaction wave at the plasma edge

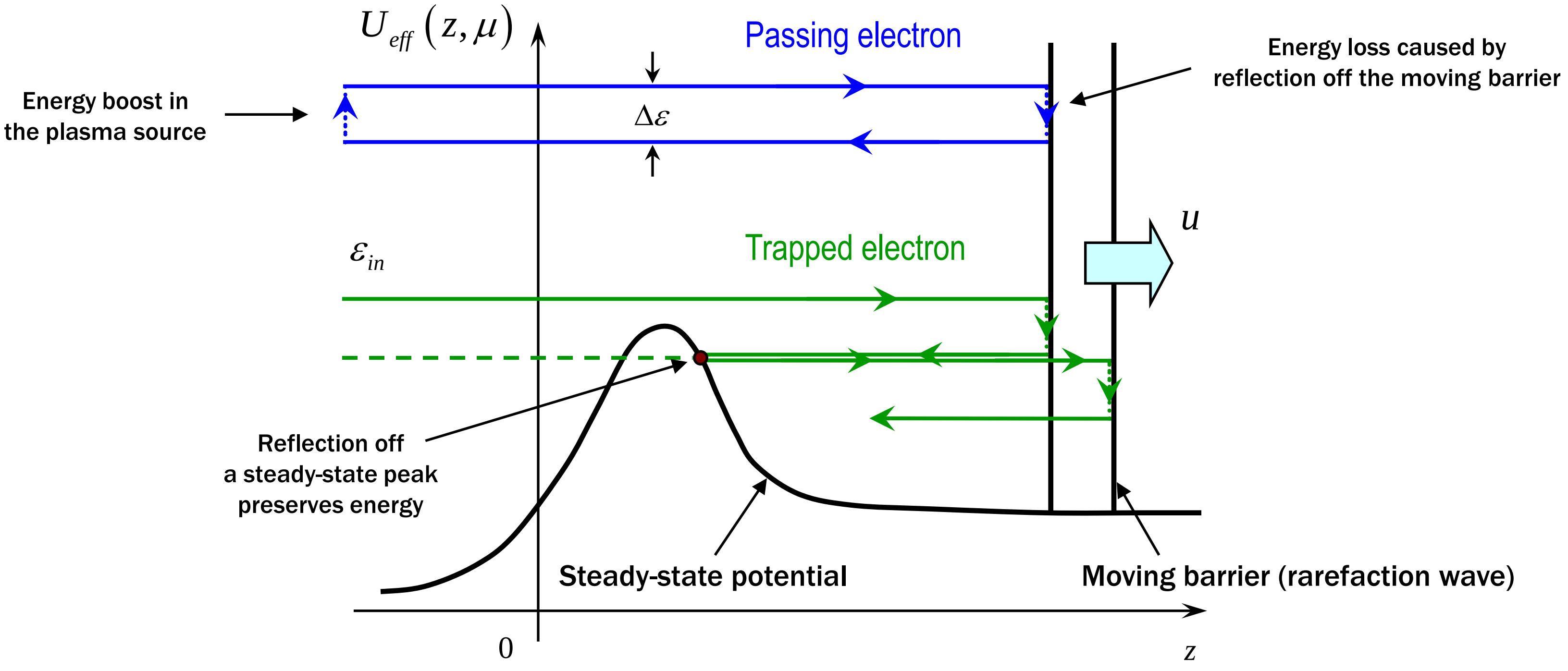
- Most of the flow is in a steady-state, but the rarefaction wave affects the flow globally through collisionless electrons.



ELECTRON DYNAMICS

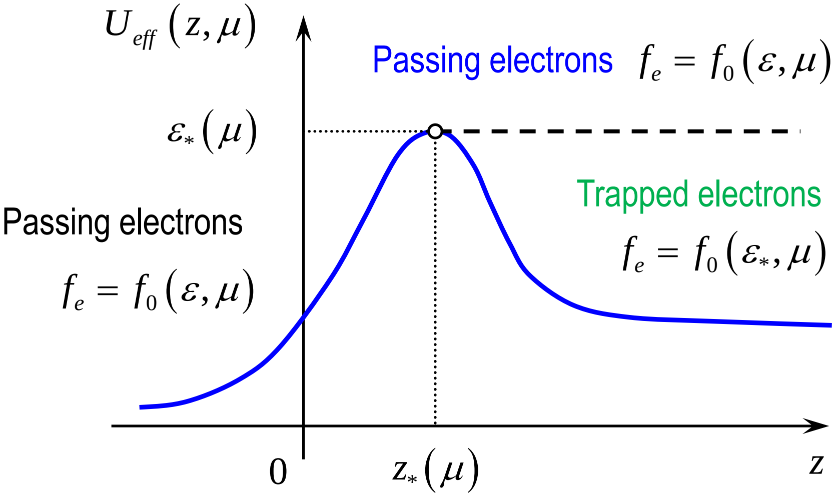
- Electron dynamics is controlled by the ambipolar potential φ(*z*,*t*) and the guiding magnetic field *B*(*z*).

- Electron motion is characterized by $U_{eff}\left(z,\mu,t\right)=\mu B\left(z\right)-\left|e\right|\varphi\left(z,t\right)$, where μ is the conserved magnetic moment.



ELECTRON DISTRIBUTION FUNCTION

- The total energy ε and magnetic moment μ of each passing electron are conserved.
- The longitudinal adiabatic invariant I and magnetic moment μ of each trapped electron are conserved.
- The electron distribution is nearly symmetric with respect to the axial velocity everywhere downstream.



MODEL EQUATIONS

- The dynamical behavior of quasineutral flow is governed by the following equations:

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial z}=-\frac{|e|}{m_i}\frac{\partial \varphi}{\partial z},$$

$$\frac{\partial n}{\partial t}+B\frac{\partial}{\partial z}\left(\frac{nV}{B}\right)=0,$$

$$n=\frac{2^{5/2}\pi}{m_e^{3/2}}\frac{2}{3}\left[\int\limits_{|e|B/B'-|e|\varphi}^{\infty} f_0'(\varepsilon)\left[\varepsilon+|e|\varphi-\mu_*(\varepsilon)B\right]^{3/2}d\varepsilon-\int\limits_{-|e|\varphi}^{\infty} f_0'(\varepsilon)\left[\varepsilon+|e|\varphi\right]^{3/2}d\varepsilon\right].$$

- The function $\mu_*(\varepsilon)$ is determined by the conditions

$$\varepsilon=|e|\left[B/B'-\varphi\right],$$
$$\mu_*=|e|/B'.$$

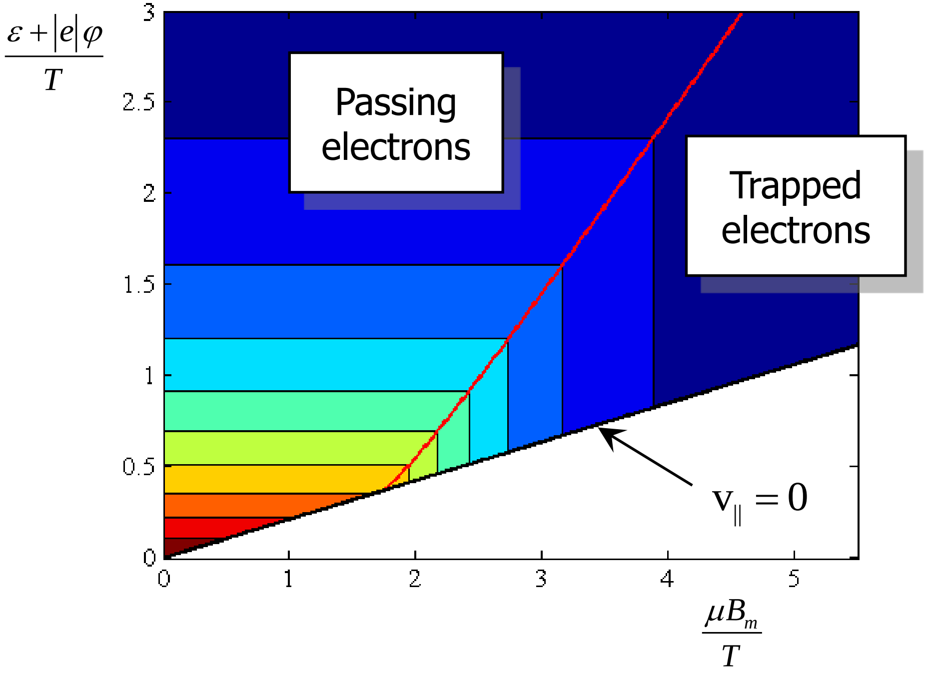
- In the steady-state portion of the flow, the system reduces to an integral equation that relates B and φ.

DISTORTION OF MAXWELLIAN DISTRIBUTION

- The phase space below the separatrix (red line) is under populated during the collisionless expansion, compared to the collisional regime.

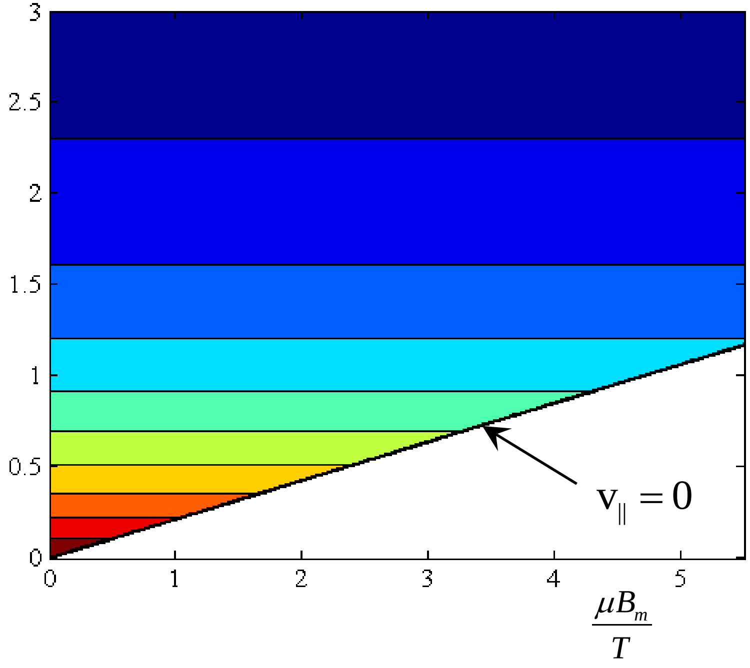
Collisionless expansion

[contour plot of EDF]

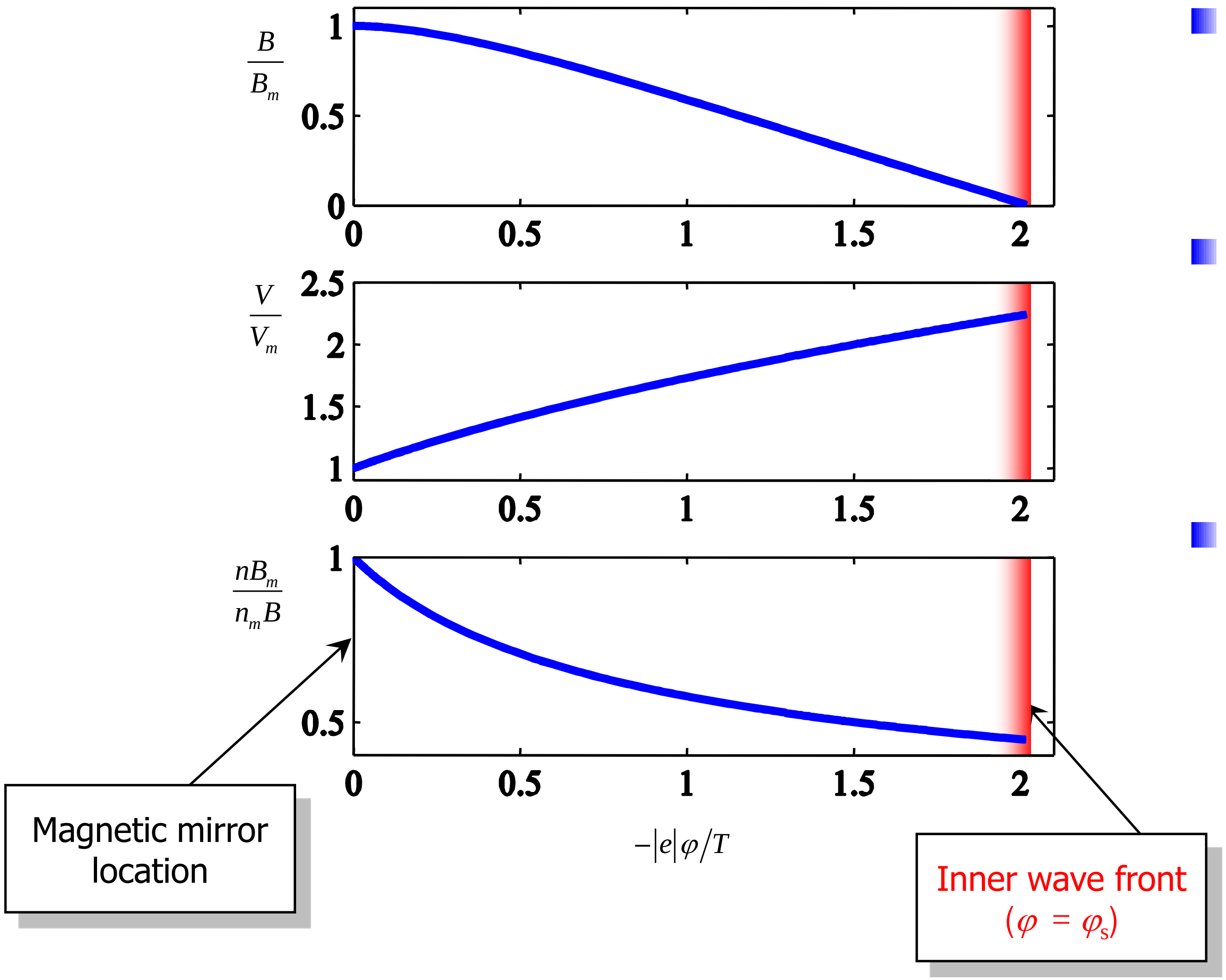


Collisional expansion

[contour plot of EDF]



STEADY-STATE PART OF THE FLOW



Magnetic mirror location

Inner wave front ($\varphi = \varphi_s$)

- The potential drop in the steady-state part is only $\Delta|e|\varphi \approx -2T$
- A rarefaction wave at the plasma edge reflects energetic electrons back.
- The distortion of the Maxwellian distribution limits the ion energy gain and makes it finite.

SUMMARY - I

- A supersonic flow downstream from the mirror consists of a steady-state part and a rarefaction wave at the leading edge of the expanding plasma.

- The magnetic mirror, together with the rarefaction wave, produce a population of trapped electrons.

- The model allows us to solve the problem quantitatively for a given distribution of incoming electrons.

- The model for the electrons allows us to eliminate the electron time scale from consideration.

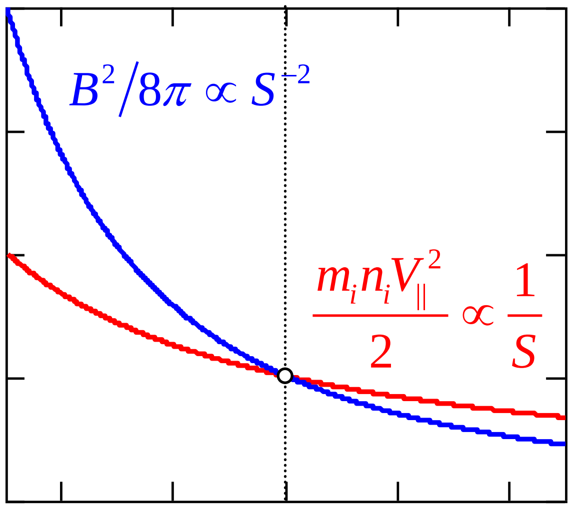
MHD DETACHMENT CONCEPT

Conserved quantities:

Magnetic flux $(BS) = const$

Flow velocity $(V_\parallel) = const$

Plasma flux $(n_i V_\parallel S) = const$



S (nozzle cross-section)

- Magnetic energy decreases downstream faster than plasma kinetic energy.
- An initially sub-Alfvénic flow becomes super-Alfvénic downstream.
- Plasma flow can stretch the magnetic field lines after $B^2/8\pi$ drops below $m_i n_i V_\parallel^2/2$.

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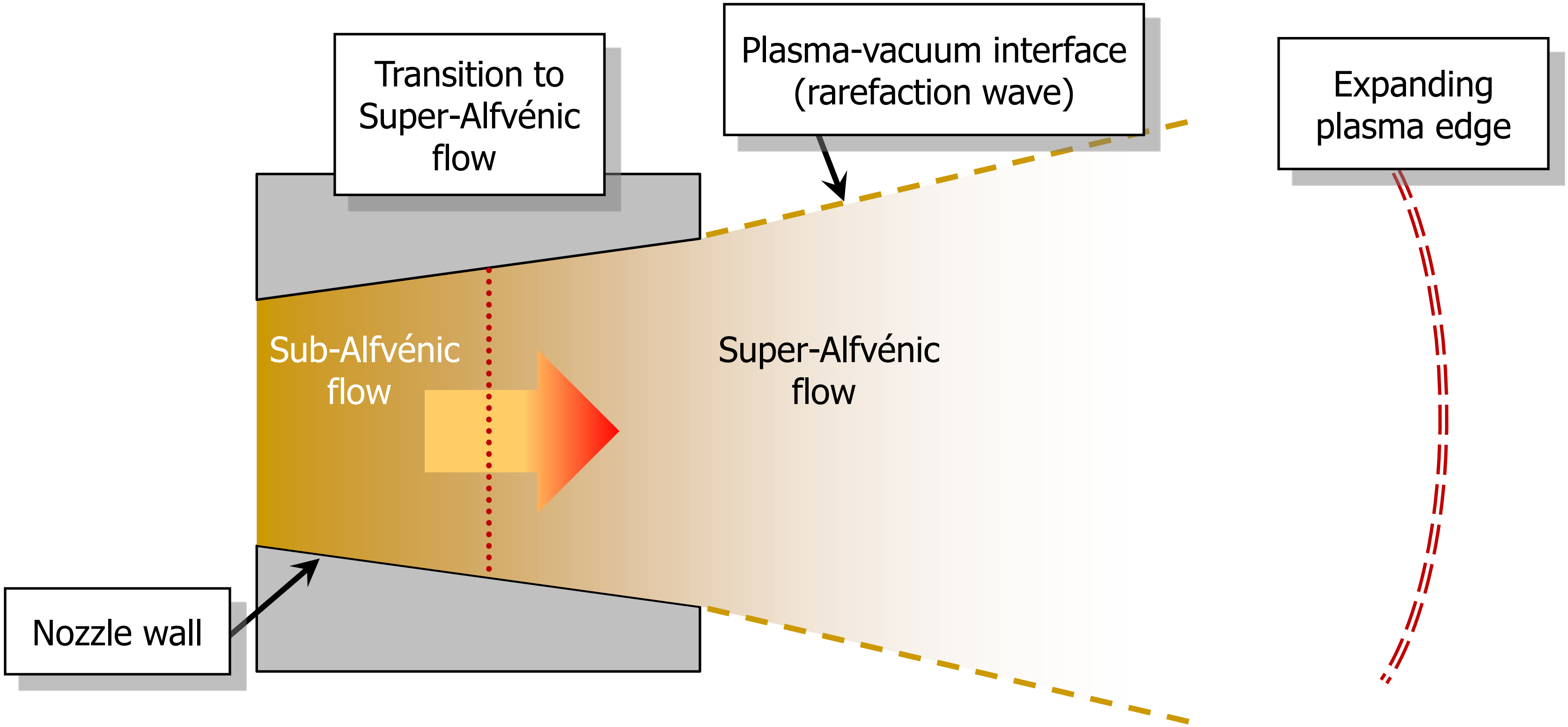
MAGNETIC NOZZLE LAYOUT



- The nozzle is required to be weakly diverging to produce a well-directed plume.
- The flow adjacent to the nozzle reaches a steady-state regime.
- The expanding plasma edge is highly super-Alfvénic, so that magnetic field perturbations are unable to reach the nozzle.

MODEL EQUATIONS

- The steady-state flow is described by MHD-like equations

$$\left(\frac{\partial}{\partial z}+\frac{B_r}{B_z}\frac{\partial}{\partial r}\right)B_z=-B_z\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{B_r}{B_z}\right)$$

$$\left(\frac{\partial}{\partial z}+\frac{B_r}{B_z}\frac{\partial}{\partial r}\right)\frac{B_r}{B_z}=-\frac{1}{\Pi_{\parallel}}\frac{\partial}{\partial r}\left(\Pi_{\perp}+\frac{B_z^2}{8\pi}\right)$$

$$\left(\frac{\partial}{\partial z}+\frac{B_r}{B_z}\frac{\partial}{\partial r}\right)\Pi_{\parallel}=-\left(\Pi_{\parallel}-\Pi_{\perp}\right)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{B_r}{B_z}\right)$$

$$\Pi_{\perp}=\int\left\langle f_i\right\rangle(a,\mu,\varepsilon)\frac{4\pi\mu B_z^2}{m_i^2 v_{\parallel}}\sqrt{\frac{m_i}{2\left(\varepsilon-\mu B_z\right)}}\,d\mu\,d\varepsilon$$

$\langle f_i \rangle$ = given input function
a = magnetic flux surface label
ε = total ion energy
μ = magnetic moment

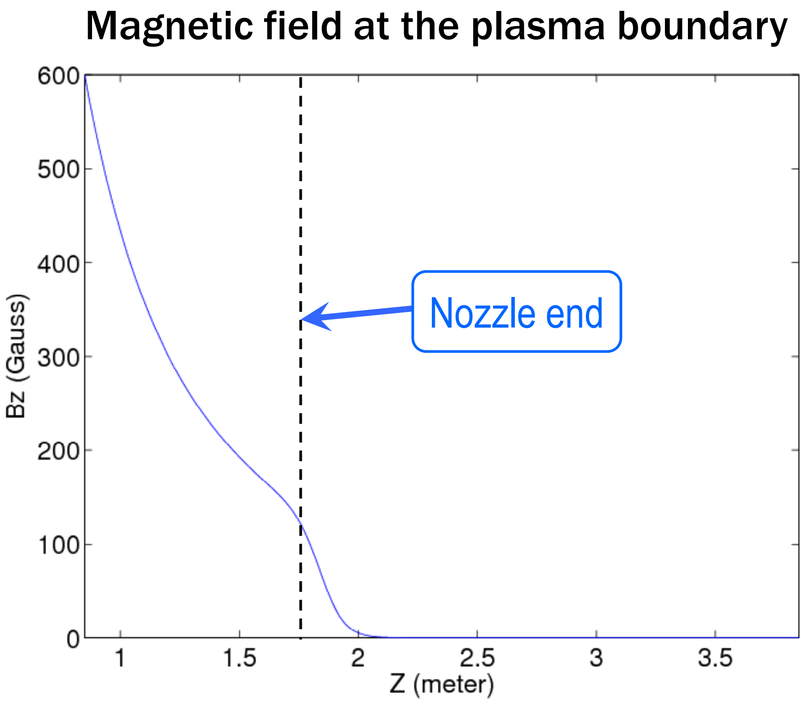
- The flow is assumed to be axisymmetric and paraxial.

- We neglect ion ambipolar acceleration, by assuming that electrons are cold.

FINDING THE STEADY-STATE

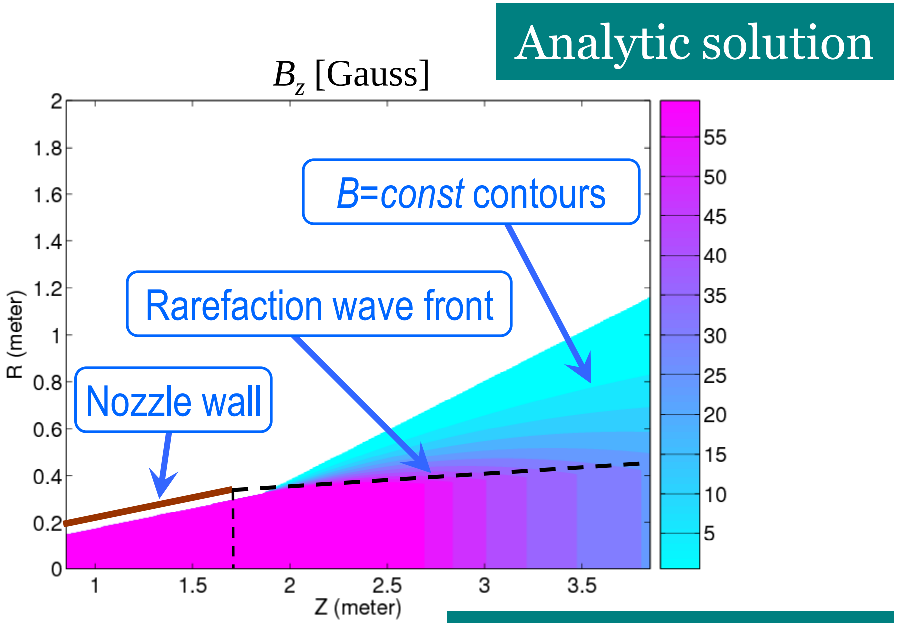
- The steady-state problem is a boundary value problem in *r* and *z* that involves the vacuum field.
- A straightforward approach is challenging, so we use the difference in scales to our advantage to reformulate the problem.
- We solve the equations as an initial value problem with *z* playing the role of time.
- The vacuum field boundary condition is calculated prior to solving the equations and can then be corrected iteratively.
- The reason why this method works is explained after the following numerical example for a “conical” magnetic nozzle.

NOZZLE CODE VERIFICATION



Magnetic field at the plasma boundary

Nozzle end



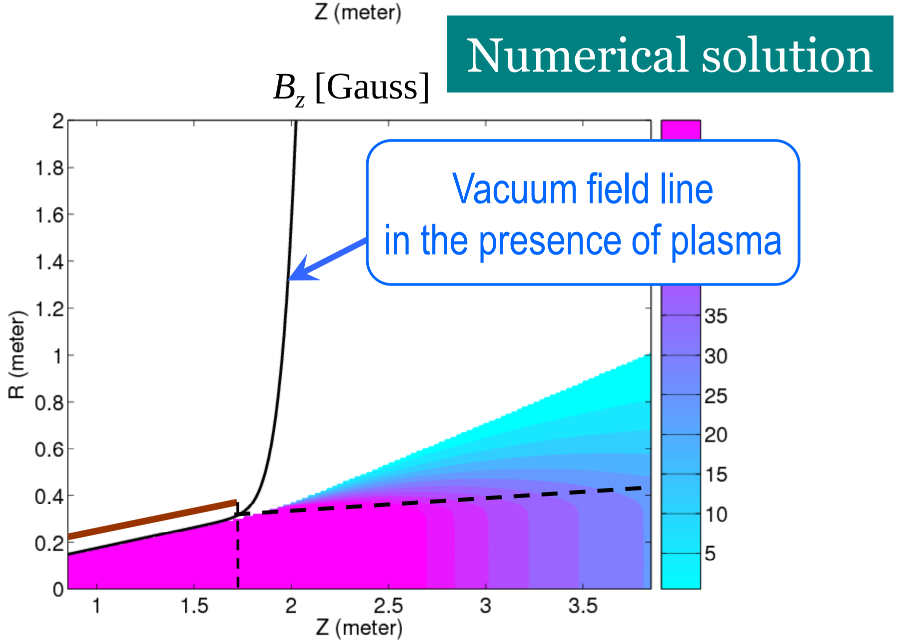
Analytic solution

B_z [Gauss]

B=const contours

Rarefaction wave front

Nozzle wall



Numerical solution

B_z [Gauss]

Vacuum field line in the presence of plasma

Incoming flow parameters

Ion energy: $\varepsilon_{\parallel} = 250$ eV

Plasma density: $n = 5.0 \cdot 10^{14}$ cm^{-3}

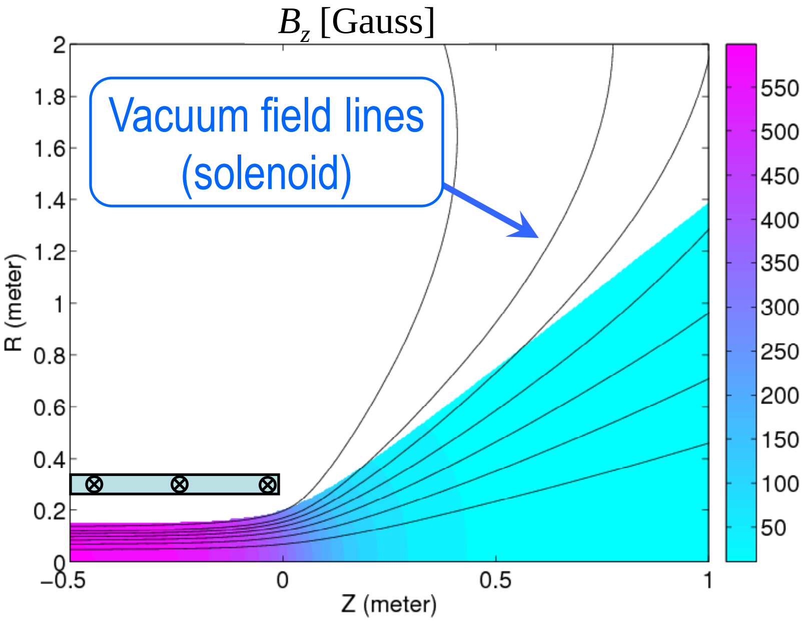
The code accurately reproduces the analytic solution.

A. Arefiev and B. Breizman, Phys. Plasmas **12**, 043504 (2005)

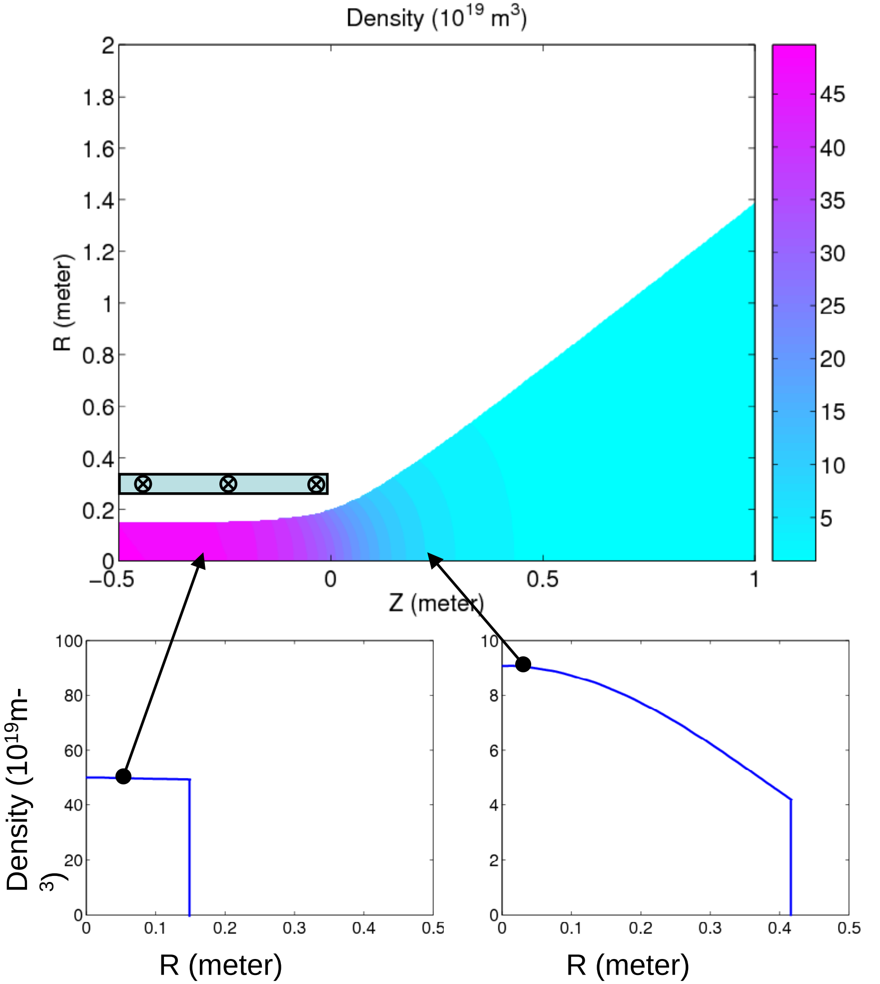
WHY DOES THE METHOD WORK?

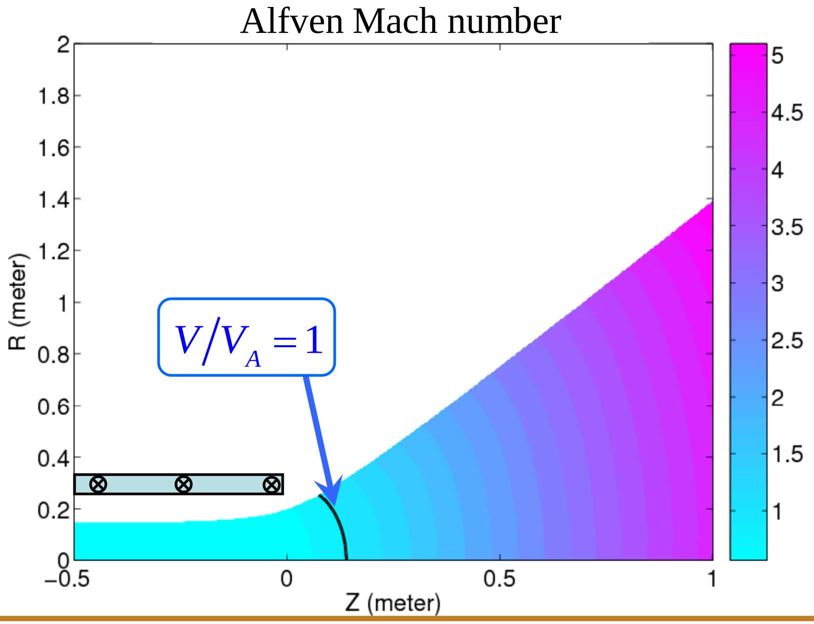
- The weakly diverging nozzle design produces
  - a slowly changing vacuum field inside the nozzle
  - a sharply decreasing vacuum field outside of the nozzle
- The weakly diverging plasma flow can be treated as an ideally conducting cylinder to find the vacuum field prior to solving the flow equations.
- Inside the nozzle, magneto-acoustic waves make the plasma field almost uniform in the flow cross-section.
- What matters is the transverse propagation and that is why our method recovers the solution correctly.
- The flow is super-Alfvénic at the nozzle end, such that magnetic field perturbations propagate only downstream.
- In a super-Alfvénic flow, the equations should be solved in the downstream direction, as done in our method.

SUB- TO SUPER-ALFVÉNIC TRANSITION


B_z [Gauss]

Vacuum field lines (solenoid)


Density (10^{19} m^3)


Alfven Mach number

$V/V_A = 1$

Incoming flow parameters

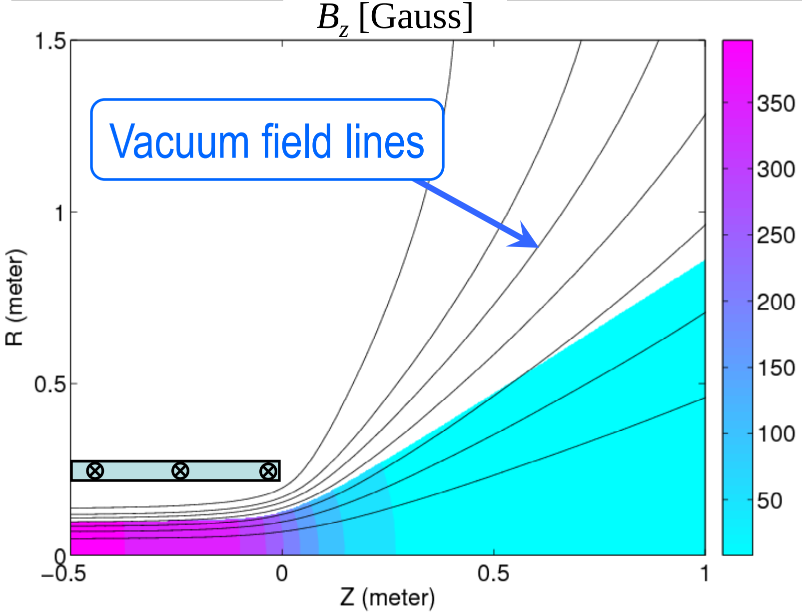
Ion energy: $\varepsilon_{\parallel} = 10\ eV$

Plasma density: $n = 5.0 \cdot 10^{14}\ cm^{-3}$

Thrust $= 114\ N$

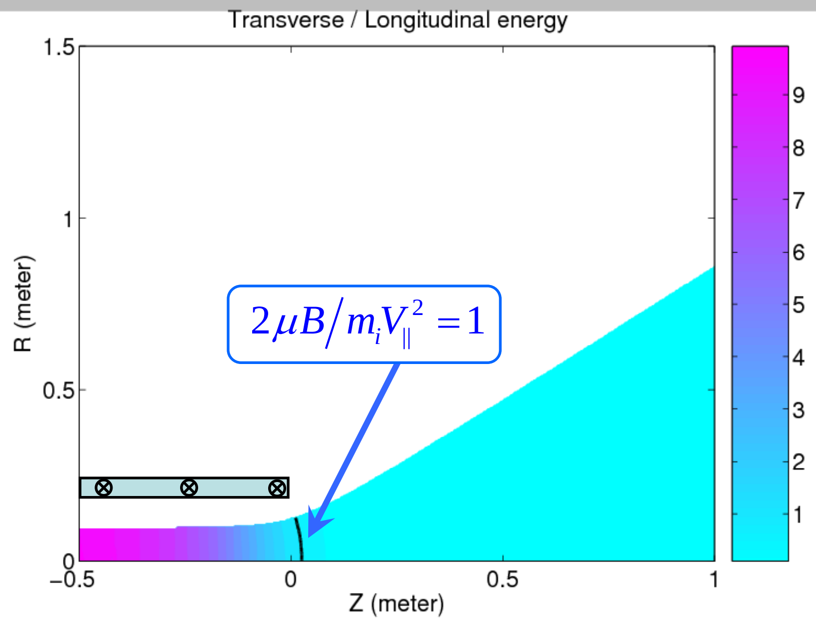
Power $= 400\ kW\ \ (Ar)$

PLASMA FLOW WITH ION GYROMOTION



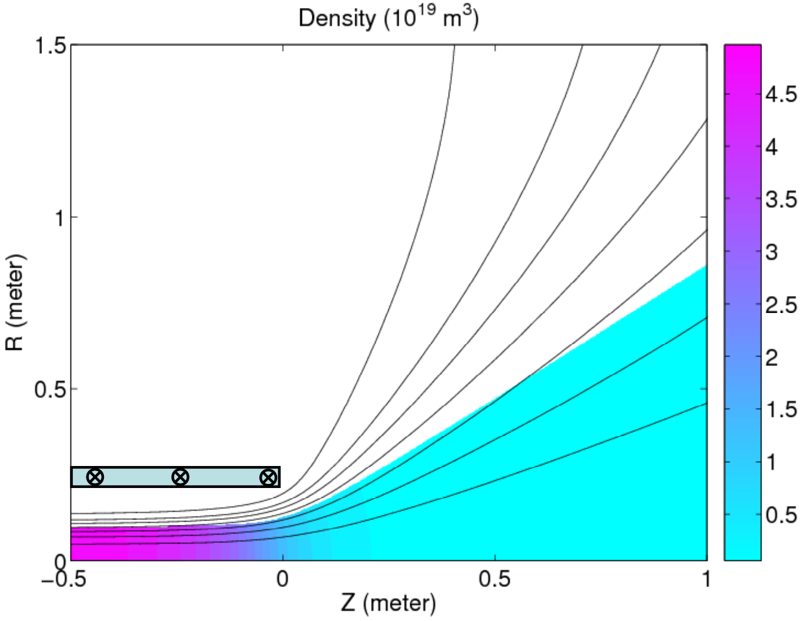
B_z [Gauss]

Vacuum field lines



Transverse / Longitudinal energy

$2\mu B / m_i V_{\parallel}^2 = 1$



Density (10^{19} m^3)

Incoming flow parameters

Ion gyroenergy: $\varepsilon_{\perp} = 100\ eV$

Axial energy: $\varepsilon_{\parallel} = 10\ eV$

Plasma density: $n = 5.0 \cdot 10^{13}\ cm^{-3}$

Plasma radius: $R_p = 10\ cm$

$$\varepsilon_{\parallel} = \frac{m_i V_{\parallel}^2}{2}$$

$$\varepsilon_{\perp} = \frac{m_i V_{\perp}^2}{2} = \mu B$$

$$\text{Thrust} = 16\ N$$

$$\text{Power} = 193\ kW\ \ (Ar)$$

SUMMARY - II

- A properly shaped paraxial magnetic nozzle generates a well-directed detached super-Alfvénic plume.
- The detachment problem reduces to a steady-state problem for applications with continuous operation.
- The developed steady-state Lagrangian code enables broad parameter scans in detachment modeling with modest computational requirements (single work station).
- The next step is to combine the electron and ion modules into an integrated model.