Plasma Rotation in Tokamaks

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Toroidal rotation impacts a variety of important plasmas physics topics

- Determining the magnitude, profile and evolution of toroidal flow is important in a number of topical areas
  - \( E \times B \) flow shear of anomalous transport
  - Prevention of locked modes --- penetration of resonant field errors
  - Control of edge localized modes via resonant magnetic perturbations
  - Resistive wall mode physics

Garofalo et al, PRL ‘99
Mechanisms have been developed to control/affect plasma rotation

- Toroidal rotation is influenced by:
  - External sources --- neutral beams
  - Intrinsic rotation --- topic of considerable research
    - A number of mechanisms have been proposed for intrinsic rotation --- turbulence, etc.
  - 3-D Magnetic fields
    - Field errors
    - Due to MHD instabilities
    - Applied 3-D fields
      -- both resonant and non-resonant

W. Zhu et al, PRL, 2006
Describing toroidal rotation in tokamaks is a transport problem

- To date, most treatments describing toroidal rotation evolution rely on:
  - Braginskii formulation (collisional plasma $v > v_{th}/qR$) --- in practice, never rigorously applicable to modern tokamaks
  - Additional physics added in ad hoc manner
    - Sources
    - Collisional transport
    - Radial plasma transport due to neoclassical effects
    - Turbulent transport --- often modeled as anomalous diffusion/pinch coefficients
    - 3-D fields
    - Magnetic field transients
    - etc.
A new approach to construct transport equations for tokamak plasmas is derived. [Callen et al, NF 49, 085021 (2009), Callen et al PoP 16, 082504 (2009)].

- Starting point is kinetic equation, not Braginskii
- Multiple timescale equilibration processes
- Toroidal momentum balance (or $E_r$) derived
- Ambipolar particle transport

Momentum balance equation is derived that accounts for classical, neoclassical collisional transport, anomalous transport, sources and sinks, magnetic field transients, and the effects of 3-D fields (neoclassical toroidal viscosity --- NTV).
Outline

• Fluid moment equations
• Expansion procedure
• Multiple timescale approach
  – Radial momentum balance
  – Parallel momentum balance
  – Toroidal momentum balance
• Physics of neoclassical toroidal viscosity (NTV)
  – Theory
  – Experimental validation
• Toroidal rotation equation
• Radial electric field and ambipolarity
Starting point for the calculation is the plasma kinetic equation

- Plasma kinetic equation for $f_s(x,v,t)$.
  \[
  \frac{\partial f_s}{\partial t} + \vec{v} \cdot \nabla f_s + \frac{q_s}{m_s} (\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_s}{\partial \vec{v}} = C(f_s) + S(f_s)
  \]
  - $C(f_s)$ = Fokker-Planck collision operator
  - $S(f_s)$ = Kinetic source --- applied RF fields, neutral beams, etc.

- Fluid moment equations are obtained from velocity-space moments of the plasma kinetic equation
  \[
  \int d^3\vec{v} (1, m_s \vec{v}, \frac{m_s \vec{v}^2}{2}) [\frac{\partial f_s}{\partial t} + \vec{v} \cdot \nabla f_s + \frac{q_s}{m_s} (\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f_s}{\partial \vec{v}} = C(f_s) + S(f_s)]
  \]
  - Evolution equations for low order velocity space moments $(n_s, \nabla_s, p_s)$

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Fluid equations require kinetically determined closure moments

- **Exact fluid equations**
  - **Density**
    \[
    \frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \vec{V}_s) = S_n
    \]
  - **Momentum**
    \[
    \frac{\partial}{\partial t} (m_s n_s \vec{V}_s) + \nabla \cdot (m_s n_s \vec{V}_s \vec{V}_s) = n_s q_s (\vec{E} + \vec{V}_s \times \vec{B}) - \nabla p_s - \nabla \cdot \vec{\pi}_s + \vec{R}_s + \vec{S}_m
    \]
  - **Energy**
    \[
    \frac{3}{2} \frac{\partial p_s}{\partial t} + \nabla \cdot \left( \frac{5}{2} p_s \vec{V}_s + \bar{q}_s \right) = Q_s + \vec{V}_s \cdot \nabla p_s - \vec{\pi}_s : \nabla \vec{V}_s + S_E
    \]

- **Closure moments are required for a closed set of equations**
  - **Heat flux** \( q_s \), **viscous stress tensor** \( \vec{\pi}_s \) determined kinetically
  - **Collision operator physics determines** \( Q_s, \vec{F}_s \)
A number of assumptions are made to make analytic progress

- Small gyroradius expansion \( \frac{\rho}{L} \ll 1 \)
  - Consequences for how we describe flows
    - Lowest order --- MHD force balance
    - First order flows are within flux surfaces
    - Second order “transport” fluxes across surfaces
- Lowest order axisymmetric magnetic fields --- nested toroidal flux surfaces
- Small 3-D non-axisymmetric magnetic fields --- no magnetic islands
  - In practice, many 3-D fields of interest
    \[ \frac{\tilde{B}}{B_o} \sim 10^{-3} - 10^{-4} \]
- Small plasma fluctuations \( \frac{\tilde{n}_s}{n_o} \sim \frac{\tilde{T}_s}{T_o} \ll 1 \)
- Slow magnetic field transients

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Multi-stage strategy is used to determine transport equations

- Average the density, momentum and energy equations over fluctuations (average over toroidal angle) and then average over the flux surfaces

\[ <Q> = \frac{\int \int \int d\psi d\theta \frac{dQ}{d\psi d\theta}}{\int \int d\psi d\theta} \]

- Sequentially consider specific components of the equilibrium force balance equation and their consequences
  - Radial --- zeroth order radial force balance enforced by compressional Alfven waves to obtain relation between flows, electric field and pressure gradients
  - Parallel --- first order poloidal flows, heat fluxes within magnetic surface
  - Toroidal --- require net radial current from all particle fluxes to vanish --> establishes flux surface averaged toroidal momentum balance equation

- Use results of the net second order ambipolar fluxes back into flux surface averaged transport equations to obtain comprehensive “radial” transport equations for \( n_s \), \( T_s \) and toroidal rotation
Small gyroradius expansion is used

- Gyroradius expansion: order terms and physical processes such as equilibrium, Pfirsch-Schluter flows, non-axisymmetries and fluctuations

\[ p = p(\psi) + \delta[\bar{p}(\psi) + \tilde{p}(\psi, \theta, \zeta)] + O(\delta^2) \quad \delta \sim \rho_i/a \ll 1 \]

Equilibrium

Pfirsch-Schluter variations

- Magnetic field is the sum of an axisymmetric magnetic field and small 3-D fluctuations

\[ \vec{B} = \vec{B}_o + \delta \tilde{B} = q(\psi)\vec{\nabla}\psi \times \vec{\nabla}\theta + \vec{\nabla}\zeta \times \nabla\psi + \delta \tilde{B} \]

\[ |B| = B_o(\psi, \theta) + \delta B_{\parallel}(\psi, \theta, \zeta) \]

- Fluctuation derivatives are large perpendicular to \( \mathbf{B} \) --- ballooning-like ordering

\[ \nabla_p \tilde{p} \sim \frac{1}{\delta} \delta \sim 1, \quad \nabla_{\parallel} \tilde{p} \sim \delta^0 \delta \sim \delta \]

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Transport equations for density and pressure are obtained by flux surface averaging

- Flux surface averaging density and energy equations with $V' = dV/d\psi$

Density

\[
\frac{\partial n_{0s}}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial \psi} (V' \Gamma_s) = < S_n > \\
\Gamma_s = < (n_{0s} \tilde{V}_2 + \tilde{n}_{st} \tilde{V}_{st}) \cdot \nabla \psi >
\]

Energy

\[
\frac{3}{2} \frac{\partial p_{s0}}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial \psi} (V' < \tilde{q}_{2s} + \frac{5}{2} (p_{s0} \tilde{V}_2 + \tilde{p}_{1s} \tilde{V}_{1s}) >) \cdot \nabla \psi >
\]

\[
= < Q_s > - < \tilde{R}_{1s} \cdot \tilde{V}_{1s} > - < \tilde{R}_{1s} \cdot \tilde{V}_{1s} > + < \tilde{V}_{2s} \cdot \nabla p_{0s} > \tilde{V}_{1s} \cdot \nabla \tilde{p}_{1s} > - < \tilde{p}_{1s} : \nabla \tilde{V}_{1s} > + < S_E >
\]

- Cross-field particle/heat fluxes due to collisional and fluctuation processes
Flux surface average of the momentum balance equation has three components

- Convenient to consider the “radial” \( \langle \mathbf{e}_\psi \cdot (\text{momentum balance}) \rangle \), parallel \( \langle \mathbf{B}_0 \cdot (\text{momentum balance}) \rangle \), and toroidal \( \langle \mathbf{e}_\zeta \cdot (\text{momentum balance}) \rangle \) projections

Radial \( \mathcal{O}(\delta^0) \)
\[
m_s n_0 s \frac{\partial \tilde{V}_s}{\partial t} = n_s q_s (\tilde{E} + \tilde{V}_s \times \tilde{B}) - \nabla p_s
\]

Parallel \( \mathcal{O}(\delta) \)
\[
m_s n_0 s \frac{\partial < \tilde{B}_0 \cdot \tilde{V}_s >}{\partial t} = n_s q_s < \tilde{B}_0 \cdot \tilde{E} > - < \tilde{B}_0 \cdot \nabla \cdot \tilde{\pi}_s > + < \tilde{B}_0 \cdot \tilde{R}_s > - m_s n_s < \tilde{B}_0 \cdot \tilde{V}_{s1} \cdot \nabla \tilde{V}_{s1} > + ...
\]

Toroidal \( \mathcal{O}(\delta^2) \)
\[
\frac{\partial}{\partial t} \langle \tilde{e}_\zeta \cdot m_s n_0 s \tilde{V}_s \rangle = q_s \Gamma_s - \langle \tilde{e}_\zeta \cdot \nabla \cdot \tilde{\pi}_s \rangle - < \nabla \cdot (m_s n_0 s (\tilde{e}_\zeta \cdot \tilde{V}_{s1}) \tilde{V}_{s1} > + ...
\]

Radial particle flux enters toroidal momentum balance equation

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The different orders of the momentum balance equation refer to different timescales

- To leading order in $\delta$, MHD force balance
  - Summing radial momentum balance $\vec{J}_0 \times \vec{B}_0 = \nabla p_0$
  - Radial force balance produces relationship between toroidal, poloidal flows, $E_r$ and pressure gradient

$$0 = \vec{e}_\psi \cdot [n_i q_i (\vec{E} + \vec{V} \times \vec{B}) - \nabla p_i]$$

$$\Omega_i = \vec{V} \cdot \nabla \zeta = \vec{V} \cdot \nabla \xi = -\left( \frac{d\Phi}{d\psi} + \frac{1}{n_i q_i} \frac{dp_i}{d\psi} - q \vec{V} \cdot \nabla \theta \right)$$

$$V_i = \frac{E_r}{B_0^2} - \frac{1}{n_i q_i} \frac{dp_i}{d\psi} + \frac{B_p}{B_0} V_p$$

- First order flows are on magnetic surfaces $V_1 \sim \delta$

$$\vec{V}_1 = \vec{e}_\theta \vec{V} \cdot \nabla \theta + \vec{e}_\zeta \vec{V} \cdot \nabla \xi = V_\parallel \frac{\vec{B}_0}{B_0} + \vec{V}_\perp$$

$$\vec{V}_\perp = \frac{\vec{B}_0 \times \nabla \psi}{B_0^2} \left( \frac{d\Phi}{d\psi} + \frac{1}{n_i q_i} \frac{dp_i}{d\psi} \right)$$

- Radial flows perpendicular to flux surfaces are second order

$$\vec{V}_{1s} \cdot \nabla \psi = 0, \quad \vec{V}_{2s} \cdot \nabla \psi \neq 0$$

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Poloidal flow is obtained from parallel force balance

- Summing the parallel force balance over species

\[ m_i n_{i0} \frac{\partial < B_0 \cdot \tilde{V}_i >}{\partial t} = - \sum_s < B_0 \cdot \nabla \cdot \tilde{\pi}_s > - m_i n_{i0} < B_0 \cdot \tilde{V}_i \cdot \nabla \tilde{V}_i > + < B_0 \cdot \tilde{J} \times \tilde{B} > + \sum_s < B_0 \cdot \tilde{S}_s > \]

  - The poloidal flow is determined mainly by the parallel viscous force
  - Parallel viscosity is calculated from kinetic theory --- collisional process, accounts for the speed dependence of the Fokker-Planck collision operator

\[ < B_0 \cdot \nabla \cdot \tilde{\pi}_{ii} > \equiv m_i n_{i0} \left[ \mu_{i00} U_{i0} + \mu_{i01} \frac{-2}{5 p_i} Q_{i\theta} + ... \right] < B^2 > \mu_{i00}, \mu_{i01} \sim \sqrt{\epsilon} V_i \]

  - On times longer than the ion collision time

\[ U_{i0}^{0} \equiv \frac{\tilde{V} \cdot \nabla \theta}{B \cdot \nabla \theta} = - \frac{\mu_{i01}}{5 p_i} \frac{-2}{\mu_{i00}} Q_{i\theta} \equiv \frac{c_p I}{q_i < B^2 > d\psi} \Rightarrow V_p \equiv \frac{1.17}{q_i B} \frac{dT_i}{dr} \]

\[ U_{i\theta} = U_{i0}^{0} + S_\theta \quad S_0 = \text{Sum of sources and turbulent Maxwell/Reynolds stress} \]

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Viscous damping occurs in the direction of asymmetry

- Viscous stress tensor can be written in CGL form
  \[ \tilde{\Pi}_{sll} = \frac{1}{3} (p_\parallel - p_\perp)(\hat{b}\hat{b} - \mathbf{I}) \]

- Pressure anisotropies are driven by flows/heat fluxes in the direction of \( \text{Grad} |\mathbf{B}| \)
  \[ p_\parallel - p_\perp \sim \mu \vec{V} \cdot \nabla \mathbf{B}, \quad \mu \vec{Q} \cdot \nabla \mathbf{B} \]
  - To leading order, axisymmetric magnetic fields
  - Lowest order flows are within magnetic flux surfaces

  ---\> Damping in the poloidal direction

\[ B = B(\psi, \theta) \]
\[ \vec{V}_{sl} \cdot \nabla \psi = 0 \rightarrow \vec{V}_{sl} \cdot \nabla B \sim \vec{V}_{sl} \cdot \nabla \theta \frac{\partial B}{\partial \theta} \]
After determining the poloidal flow, there is a unique relationship between $E_r$ and the toroidal rotation

- Recalling the radial force balance relationship

$$\Omega_t \equiv \vec{V} \cdot \nabla \zeta = -\left(\frac{d\Phi}{d\psi} + \frac{1}{n_i q_i} \frac{dp_i}{d\psi} - q \vec{V} \cdot \nabla \theta\right)$$

- Using parallel momentum balance result $<\vec{B}_0 \cdot \nabla \cdot \vec{\pi}_i > \equiv 0$

- Relationship between radial electric field and toroidal flow

$$V_t \equiv \frac{E_r}{B_p} - \frac{1}{n_i q_i B_p} \frac{dp_i}{dr} + \frac{1.17}{q_i B_p} \frac{dT_i}{dr}$$

Pressure/temperature profiles determined by transport processes

- Poloidal flow damping produces parallel plasma flow

$$\vec{V}_1 = e_\theta \vec{V} \cdot \nabla \theta + e_\zeta \vec{V} \cdot \nabla \zeta = \vec{V}_\parallel \frac{\vec{B}_0^2}{B_0} + \vec{V}_\perp$$

$$\vec{V}_\perp = \frac{\vec{B}_0}{B_0^2} \left( \frac{d\Phi}{d\psi} + \frac{1}{n_i q_i} \frac{dp_i}{d\psi} \right)$$

$$V_\parallel \equiv -R \left( \frac{d\Phi}{d\psi} + \frac{1}{n_i q_i} \frac{dp_i}{d\psi} \right) + \frac{1.17 R_0^2}{q_i R} \frac{dT_i}{d\psi}$$
Electron parallel momentum balance produces parallel Ohm’s law

- Following the same logic for the parallel electron momentum balance equation

\[
0 = -n_e e < \vec{E} \cdot \vec{B} > - < \vec{B}_0 \cdot \nabla \cdot \vec{\pi}_e > + < \vec{B} \cdot \vec{R}_e > + < \vec{B}_0 \cdot \vec{S}_{em} > \\
-m_e n_{e0} < \vec{B}_0 \cdot \vec{V}_e \cdot \nabla \vec{V}_e > - n_{e0} e < \vec{B}_0 \cdot \vec{V}_e \times \vec{B} > 
\]

- Using collision friction and neoclassical closure from kinetic theory

\[
\dot{\vec{B}}_0 \cdot \vec{R}_e \approx \eta_h n_e e \vec{J} \cdot \vec{B}_0 \\
< \vec{B}_0 \cdot \nabla \cdot \vec{\pi}_{e||} >= m_e n_{e0} < B^2 > (\mu_{e00} U_{e\theta} + \mu_{e01} Q_{e\theta}) 
\]

- Parallel Ohm’s law

\[
< \vec{E} \cdot \vec{B}_0 >= \eta_{hc}^{nc} < \vec{J} \cdot \vec{B}_0 > - \eta_h [ < \vec{J} \cdot \vec{B}_{BS} > + < \vec{J} \cdot \vec{B}_{CD} > + < \vec{J} \cdot \vec{B}_{dyn} > ] 
\]

Bootstrap current, current drive from external sources, dynamo due to fluctuations

\[
\eta_{hc}^{nc} = \eta_h (1 + \frac{\nu_{e00}}{\nu_{e\Psi}}) \quad < \vec{J} \cdot \vec{B}_{BS} >= - \frac{\eta_h \mu_{e00}}{\nu_{e\Psi}} (\frac{dp}{d\psi} - n_e e U_{i\theta} < B^2 >) \\
< \vec{J} \cdot \vec{B}_{CD} >= - \frac{< \vec{B} \cdot \vec{S}_{em} >}{n_e e \nu_h} \\
< \vec{J} \cdot \vec{B}_{dyn} >= \frac{1}{\eta_h} (< \vec{B}_0 \cdot \vec{V}_e \times \vec{B} > + \frac{m_e}{e} < \vec{B}_0 \cdot \vec{V}_e \cdot \nabla \vec{V}_e > ) 
\]
Toroidal torque from force balance gives radial flows

- Toroidal force balance produces an equation for radial particle flux

\[ \mathbf{e}_\zeta \cdot \vec{V}_s \times \vec{B} = -\vec{V}_s \cdot \mathbf{e}_\zeta \times \vec{B} = \vec{V}_s \cdot \nabla \psi \]

- Particle flux is induced by toroidal torques on the plasma

\[ \mathbf{e}_\zeta \cdot (n_s q_s \vec{V}_s \times \vec{B} + \sum_j \vec{F}_j) = 0 \quad q_s \Gamma_s = -\sum_j \vec{e}_j \cdot \vec{F}_j = -\sum_j T_{j\zeta} \]

- Flux surface averaged toroidal momentum balance equation produces equation for particle flux

\[ q_s \Gamma_s = -\left< \mathbf{e}_\zeta \cdot \vec{R} \right> + \left< \mathbf{e}_\zeta \cdot \nabla \cdot \vec{\pi}_s \right> - n_o \left< \mathbf{e}_\zeta \cdot \vec{E} \right> - \left< \mathbf{e}_\zeta \cdot \vec{S}_m \right> \]
\[ + \frac{\partial}{\partial t} (m_s n_s \left< \mathbf{e}_\zeta \cdot \vec{V}_s \right>) - \left< \mathbf{e}_\zeta \cdot n_o \vec{V} \times \vec{B} \right> + \left< \nabla \cdot m_s n_s (\mathbf{e}_\zeta \cdot \vec{V}) \vec{V} \right> \]

Collisions, sources, Inertia, fluctuations
Parallel Ohm’s law is used to describe collisional ambipolar particle flux

- Consider the particle fluxes from collisional friction

\[
\Gamma_s^a = -\frac{1}{q_s} < \vec{e}_\xi \cdot \vec{R}_s > -n_0 < \vec{e}_\xi \cdot \vec{E} > \quad \vec{R}_e \equiv n_e (\eta_{HJ} \vec{J}_\parallel + \eta_{kJ} \vec{J}_\perp) = -\vec{R}_i
\]

- Vector identity used to facilitate analysis

\[
\vec{e}_\xi = \frac{I}{B_0} \frac{\vec{B}_0 \times \nabla \psi}{B_0^2}
\]

- Collisional-friction can be decomposed into parallel and perpendicular contributions

\[
\frac{1}{q_s} < \vec{e}_\xi \cdot \vec{R}_s > = -n_{eH} \eta_{H} < \frac{\vec{J} \cdot \vec{B}_0}{B_0^2} > + n_{eK} \eta_{K} < \frac{1}{B_0^2} \nabla \psi^2 > \frac{dp_0}{d\psi}
\]

Classical transport

\[
\frac{\vec{J} \cdot \vec{B}_0}{B_0^2} \geq \frac{\vec{J} \cdot \vec{B}_0}{B_0^2} + \frac{\vec{J} \cdot \vec{B}_0 (\frac{1}{B_0^2} - \frac{1}{B_0^2})}{B_0^2}
\]

Pfirsch-Schluter

Calculated from parallel Ohm’s law
Particle flux has many contributions --- six ambipolar components

- The intrinsically ambipolar contributions to particle fluxes can be identified

\[
\Gamma^a = \Gamma_{cl} + \Gamma_{PS} + \Gamma_{bp} + \Gamma_{CD} + \Gamma_{dyn} + \Gamma_E \\
\Gamma_{cl} = -n_e\eta_1 \frac{1}{B_0^2} \frac{\left| \nabla \psi \right|^2}{B_0^2} \frac{dp_0}{d\psi} \quad D_{cl} \equiv v_e \rho_e^2 \\
\Gamma_{PS} = -n_e I_{||} I_{||} \frac{1}{B_0^2} \left( 1 - \frac{B_0^2}{B_0^2} \right)^2 \frac{dp_0}{d\psi} \quad D_{PS} \sim q^2 D_{cl} \\
\Gamma_{bp} = \frac{I}{e} \left( \frac{1}{B_0^2} \right) \frac{\left< \vec{B}_0 \cdot \nabla \cdot \vec{\Pi}_e \right>}{\left< \vec{B}_0 \cdot \nabla \cdot \vec{\Pi}_e \right>} \quad D_{bp} \equiv \mu_e \rho_{ep}^2 \sim \frac{q^2}{\varepsilon^{3/2}} D_{cl} \\
\Gamma_{CD} + \Gamma_{dyn} = \frac{n_e I_{||}}{B_0^2} \left< \vec{J} \cdot \vec{B}_{CD} + \vec{J} \cdot \vec{B}_{dyn} \right> \\
\Gamma_E = -n_e \left< \vec{e}_e \cdot \vec{E} \right> \left( 1 - \frac{I^2 \left< \frac{R^2}{B_0^2} \right>}{\left< B_0^2 \right>} - \frac{I \left< \vec{B}_p \cdot \vec{E} \right>}{\left< B_0^2 \right>} \right)
\]

Collisional transport from classical, Pfirsch-Schluter, banana-plateau. current drive dynamo and electric field pinch contributions
Plasma fluctuations influence particle flux/toroidal momentum balance

- At $O(\delta^2)$, plasma fluctuation effects enter into the toroidal momentum balance
  - Microturbulence effects --- turbulent Reynolds/Maxwell stresses
  - 3-D magnetic fields --- error fields, applied 3-D coils
    - Resonant magnetic perturbations ---> localized electromagnetic torques
    - Non-resonant magnetic perturbations ---> Neoclassical toroidal viscosity
- In general, these effects are not intrinsically ambipolar and hence will affect toroidal momentum balance.
Resonant magnetic fields produce localized electromagnetic torques at rational surfaces

- Inherent magnetic field errors or applied 3-D magnetic coils may have components that are resonant in the plasma

\[
\vec{B} \sim e^{im\theta - in\xi} \Rightarrow \frac{m}{n} = q(\psi)
\]

- Two asymptotic limits
  - Fully penetrated - radial magnetic field produces a magnetic island at the rational surface
  - Fully shielded - eddy currents flow in a resistive layer at \( q = \frac{m}{n} \), magnetic perturbation does not penetrate, current sheet

- Sufficient plasma rotation relative to the 3-D field source provides effective shielding --- rotation sustains current sheet
  - Produces localized electromagnetic perturbation (Fitzpatrick and Hender ‘91)

\[
< \vec{J} \times \vec{B} > \sim \delta(r - r_{mn}) \frac{B_{rmn}^2}{B_0^2} \frac{\omega}{\Delta^2 + (\omega \tau_L)^2} \hat{b} \times \hat{n}
\]

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3-D magnetic fields produce neoclassical toroidal viscous forces (NTV) throughout the plasma

- In an axisymmetric magnetic field, the toroidal component of the parallel viscous stress tensor is zero ($\mu dB/d\zeta = 0$)

$$<\vec{e}_\zeta \cdot \nabla \cdot \pi_{\parallel s}^A> = 0$$

- However, in the presence of 3-D magnetic fields, toroidal torques on toroidally flowing plasmas are generated.

$$<\vec{e}_\zeta \cdot \nabla \cdot \pi_{\parallel s} > = m_i n_i \mu_{ii} \frac{\tilde{B}_{eff}^2}{B_0^2} [< R^2 \Omega_t > - < R^2 \Omega_s >]$$

- Physics --- transit-time magnetic pumping, banana-drift, ripple-trapping effects
- Generally, the ion component dominates (the ion root of stellarator physics)
- Ion viscous damping coefficient $\mu_{it}$ depends on collisionality, $E_r$
- $B_{eff}^2$ is a weighted average sum over all m and n.

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The NTV force is felt throughout the plasma

- Unlike torques due to resonant 3-D magnetic fields, the NTV force is global
  - Applied 3-D fields on NSTX demonstrated the damping effect of toroidal flow (Zhu et al, PRL ‘06)
- Favorable comparison to analytic predictions
- NTV physics has been seen on NSTX, DIII-D, MAST, JET
Another unusual aspect of NTV is the appearance of an off-set rotation frequency

- NTV force \(\sim \mu_i B_{\text{eff}}^2(\Omega - \Omega^*)\)
  - Offset velocity \(\Omega^*\) is a diamagnetic-type toroidal rotation frequency proportional to ion temperature gradient
    \[
    \Omega^* = \frac{c_p + c_i}{q_i} \frac{dT_i}{d\psi}
    \]
- Physics of the offset is due to ions of different energy having different radial drift speeds --- produces \(c_t\). Poloidal flow damping coefficient \(c_p\) due to parallel viscosity.
Experiments on DIII-D have demonstrated the presence of the NTV offset velocity

- Off-set rotation velocity observed on DIII-D (Garofalo et al ‘08)

Initially, slowly rotating Plasmas sped up to the Offset NTV velocity when 3-D fields are applied
Recent experiments on DIII-D have demonstrated a peak in the NTV force at zero radial electric field

- The toroidal damping rate ($\mu_{ti}$) is sensitive to the value of the radial electric field
  - Damping rate corresponds to different collisionality regimes of stellarator neoclassical transport
  - Smoothed formula constructed to model different collisionality regimes (Cole et al, ’10)

$$
\mu_{ti} \sim \frac{nv_i^2 \sqrt{\epsilon \hat{V}}}{<R^2>[0.3 |\omega_{||}|_B ]\hat{V} + 0.04\hat{v}^{3/2} + |\omega_E|^{3/2}}
$$

Peaks at $\omega_E \sim 0$

Recent experiment on DIII-D Demonstrates peak NTV at $\omega_E \sim 0$
Particle flux has 8 non-ambipolar contributions

- Not intrinsically ambipolar

\[ \Gamma_{na} = \Gamma_{\pi\parallel}^{NA} + \Gamma_{\pi\perp} + \Gamma_{pol} + \Gamma_{Rey} + \Gamma_{Max} + \Gamma_{JxB} + \Gamma_{\psi_p} + \Gamma_{S} \]

\[ \Gamma_{\pi\parallel}^{NA} = \frac{1}{q_s} \left< \vec{e}_\zeta \cdot \nabla \cdot \tilde{\pi}_{\pi\parallel} \right> \geq \frac{m_n n_{\parallel l} \tilde{B}_{\text{eff}}}{q_i} \left< R^2 \Omega_r - 3 \left< R^3 \Omega_s \right> \right> \]

\[ \Gamma_{\pi\perp} = \frac{1}{q_s} \left< \vec{e}_\zeta \cdot \nabla \cdot \pi_{\pi\perp} \right> \sim -\chi_t \nabla^2 \Omega_t \quad \chi_t \sim (1 + q^2) \nu_p \rho_i^2 \]

\[ \Gamma_{pol} = \frac{\partial}{\partial t} \left( \frac{m_n n_{so}}{q_s} \left< \vec{e}_\zeta \cdot \tilde{V}_s \right> \right) \]

\[ \Gamma_{Rey} = \frac{1}{q_s V_s} \frac{\partial}{\partial \psi} \left< V_s \Pi_{s\psi\zeta} \right> \quad \Pi_{s\psi\zeta} = m_n s \left< \tilde{V}_s \cdot \nabla \psi \tilde{V}_s \cdot \vec{e}_\zeta \right> + \left< \nabla \psi \cdot \tilde{\pi}_{s\parallel} \vec{e}_\zeta \right> \]

\[ \Gamma_{Max} = -\left< \vec{e}_\zeta \cdot n_s \tilde{V}_s \times \tilde{B} \right> \]

\[ \Gamma_{JxB} = \frac{1}{e} \left< \vec{e}_\zeta \cdot \tilde{J}_\parallel \times \tilde{B} \right> \sim \delta(r - r_{mn}) \frac{c_{\Omega} m_n n_{\parallel l} R}{e} \frac{\omega}{\Delta^2 + (\omega \rho_i)^2} \frac{\tilde{B}_{rmn}}{B_0^2} \]

\[ \Gamma_{\psi_p} = \frac{\psi_p}{q_s} \frac{\partial}{\partial \psi} \left( m_n n_{s0} \left< \vec{e}_\zeta \cdot \tilde{V}_s \right> \right) \]

\[ \Gamma_{S} = -\frac{1}{q_s} \left< \vec{e}_\zeta \cdot \tilde{S}_{sm} \right> \]

Additional particle fluxes from neoclassical toroidal viscosity (NTV), cross field viscosity, polarization flow, microturbulence induced Reynolds and Maxwell stresses, field-error induced localized EM torques, poloidal field transients and momentum sources.

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Zero radial current produces torque balance relation

- Summing radial species currents to obtain net radial plasma current

\[
< \tilde{J} \cdot \nabla \psi > = \sum_s q_s \Gamma_s = \sum_s q_s \Gamma_s^{NA}
\]

- Charge continuity requires no net radial current

\[
\frac{\partial}{\partial t} < \rho_q > = - \frac{1}{V'} \frac{d}{d\psi} V' < \tilde{J} \cdot \nabla \psi > = 0
\]

- Setting radial current equal to zero produces a comprehensive toroidal torque balance relation

\[
L_i \equiv m_i n_{i0} < R^2 \Omega_i >
\]

\[
\frac{1}{V'} \frac{\partial}{\partial t} (V' L_i) = - < \tilde{e}_z \cdot \nabla \cdot \pi_{i0}^{NA} > - < \tilde{e}_z \cdot \nabla \cdot \pi_{i0} > - \frac{1}{V'} \frac{\partial}{\partial \psi} (V' \Pi_{i\psi\psi}) + < \tilde{e}_z \cdot \tilde{J} \times \tilde{B} > - \psi_p \frac{\partial L}{\partial \psi} + < \tilde{e}_z \cdot \sum_s \tilde{S}_{sm} >
\]

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Toroidal rotation equation includes many different effects

- Equation for toroidal angular momentum density $L_t = m_i n_{i0} <R^2\Omega_t >$
  \[
  \frac{1}{V'} \frac{\partial}{\partial t} (V' L_t) = -< \bar{\epsilon}_{\zeta} \cdot \nabla \cdot \pi_{\parallel}^{NA} > - < \bar{\epsilon}_{\zeta} \cdot \nabla \cdot \pi_{\perp} > - \frac{1}{V'} \frac{\partial}{\partial \psi} (V' \Pi_{i\psi \zeta}) + < \bar{\epsilon}_{\zeta} \cdot \tilde{J} \times \tilde{B} > - \tilde{\psi}_p \frac{\partial L}{\partial \psi} + < \bar{\epsilon}_{\zeta} \cdot \sum_s \tilde{S}_{sm} >
  \]
  - NTV damping by 3-D magnetic fields
    \[
    - < \bar{\epsilon}_{\zeta} \cdot \nabla \cdot \pi_{s\parallel} > = -m_i n_i u_{ii} \frac{\tilde{B}^2_{eff}}{B_0^2} [ < R^2 \Omega_s > - < R^2 \Omega_s > ] \quad \Omega_s = \frac{c_p + c_t}{q_i} \frac{dT_i}{d\psi}
    \]
  - Collision damping
    \[
    < \bar{\epsilon}_{\zeta} \cdot \nabla \cdot \pi_{s\perp} > \sim -\chi_t \nabla^2 \Omega_t \quad \chi_t \sim (1 + q^2) \nu_i \rho_i^2
    \]
  - Microturbulence-induced ion Reynolds stresses causes radial transport of $L_t$ (diffusion, pinch, residual stress)
    \[
    \Pi_{s\psi \zeta} = m_s n_s < \tilde{V}_s \cdot \nabla \psi \tilde{V} \cdot \bar{\epsilon}_{\zeta} > + < \nabla \psi \cdot \bar{\pi}_s \cdot \bar{\epsilon}_{\zeta} >
    \]
    \[
    \sim -\chi_t \nabla L_t + L_t V_{\text{pinch}} + \Pi^{RS}
    \]

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Toroidal rotation determines radial electric field required for net ambipolar particle flux

- From toroidal rotation equation, radial electric field is determined
  \[ E_r = -\nabla \psi \frac{d\phi}{d\psi} = -\nabla \psi \langle \Omega_r \rangle + \frac{1}{n_i q_i} \frac{d p_i}{d\psi} - \frac{c_p}{q_i} \frac{d T_i}{d\psi} \]

- The resultant electric field causes the electron and ion non-ambipolar radial particle fluxes to be equal (ambipolar)
  - Hence, the net ambipolar particle flux is the sum of \( \Gamma^a + \Gamma^{NA}(E_r) \), which is easiest to evaluate in ion root \([\Gamma^{NA}(E_r) \sim 0]\)
  \[ \Gamma^{net}_e = \Gamma^A_e + \Gamma^{NA}_e (E_r) = \Gamma^{net}_i \]
  
  - Procedure is familiar to stellarator researchers
    - Nonlinear dependences of \( \Gamma_s \) on \( E_r \) can lead to different ‘roots’, transition barriers, etc.

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This is approach is different than Braginskii-like approach and has some consequences

- **Key differences in this new approach for plasma transport equations**
  - First solve for flows of electrons, ions in flux surfaces --- Ohm’s law, poloidal ion flow
  - Derivation of particle flux and toroidal flow are naturally joined
  - Simultaneously solve transport equations for $n$, $T$, $\Omega_t$
  - Effects of micro-turbulence are all included self-consistently
  - Fluctuation induced particle flux is determined from Reynolds/Maxwell stresses
  - Source effects, poloidal field transients are included
  - Net transport equations follow naturally from extended two-fluid moment equations --- consistent with formulations of extended MHD code frameworks (NIMROD, M3D)
Summary

- Comprehensive transport equations for $n$, $T$, $\Omega_t$ have been derived
- Radial, parallel and toroidal components of force balance are considered
  - Radial force balance --- relationship between $V_\nu$, $V_p$, $E_r$ and $dp_i/d\psi$
  - Parallel viscous damping determines neoclassical Ohm’s law and poloidal ion flow
  - Radial particle fluxes arise from average toroidal torques on the plasma
- Radial particle flux has many contributions --- ambipolar and non-ambipolar
- Requiring ambipolar particle flux yields evolution equation for toroidal angular momentum density
Summary

- 3-D magnetic fields have an important effect of flow evolution
  - Localized EM torques from resonant magnetic fields
  - Neoclassical toroidal viscosity (NTV) from variations in |B|
- Many aspects of NTV theory are being tested against experiments
  - Global damping of toroidal flow profile
  - Appearance of an offset rotation $\sim dT_i/d\psi$
  - Peak of NTV torque near $E_r \sim 0$