Control of resistive wall modes and error fields

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A tokamak plasma has a wide range of sensitivities to externally produced magnetic perturbations. A non-axisymmetric external perturbation helically distorts the magnetic surfaces, which produces a helical component to the equilibrium plasma current and can amplify the original perturbation. If the amplification factor goes to infinity the plasma is marginally stable. The closer a stable plasma is to marginal stability, the more the incipient instability amplifies an external perturbation that couples to it. The perturbation to which the plasma has the greatest sensitivity drives what would become a resistive wall mode (RWM) at a sufficiently high plasma pressure. An RWM is an ideal kink instability, which would be unstable if there were no conducting structures surrounding the plasma but would be stable if these structures, such as the chamber wall, were perfect conductors. The RWM growth rate is proportional to the resistivity of the surrounding structures. Even small field errors, which means unintentional perturbations to the external magnetic field, $\delta B/B{\sim}10^{-4}$, can strongly damp toroidal rotation by splitting the magnetic surfaces with the formation of stationery magnetic islands or by breaking the toroidal symmetry of the magnetic field strength. Tokamak plasmas that do not rotate frequently disrupt, and strong rotation damping can affect the zonal flows that limit microturbulent transport. Field errors also modify the location of the plasma strike points on divertor structures, which can place large heat loads on structures not designed to take them.

External magnetic control of RWM's and error fields are in principle the same, except RWM's tend to rotate on the resistive time scale of the wall, so coils with a faster response and a more sophisticated sensor system are required. The perturbations to a tokamak can be taken to have a definite toroidal mode number N and come in degenerate pairs, which are rotated toroidally by $\pi/2N$ relative to each other. To design an effective control system, one needs the spectrum of error fields to which the plasma is most sensitive. An error field is defined by the spatial distribution of its normal magnetic field, $\delta\vec{B}\cdot\hat{n}$, on a surface just outside the plasma. On this control surface, the external error fields can be decomposed using the set of orthogonal distributions, $\oint \delta\vec{B}_i\cdot\hat{n}\delta\vec{B}_j\cdot\hat{n}da = 0$, that are ordered by plasma sensitivity. The distribution to which the plasma is most sensitive produces an unacceptable effect on the plasma at the smallest amplitude, $\sqrt{\oint(\delta\vec{B}_1\cdot\hat{n})^2 da}$, on the control surface. If b_i is the amplitude of the i^{th} error field distribution divided by the amplitude required for an unacceptable effect, $b_i = b_i^{(e)} + \Sigma_j M_{ij} J_j$, where $b_i^{(e)}$ is the intrinsic field error and the J_j are the currents in the control coils. The accuracy with which the tokamak is built determines I_b, the number of $b_i^{(e)}$ that have unacceptable amplitudes and require control. The number of control coils, J_c, must be satisfy $J_c \geq I_b$. The matrix M_{ij} relating the I_b fields requiring control and the J_c control coil currents has I_b singular values. The ratio of the largest to the smallest singular value, called the condition number, gives the accuracy with which the coil currents must be specified to control all I_b external error fields that require control. A general feature is that the condition number increases exponentially with I_b, so the difficulties of sensing and control increase exponentially. Existing error field control systems focus on only a single error field distribution, and little work has been done to identify the error fields of secondary importance. Until this is done, it is difficult to clarify the adequacy of the ITER error field control system or quantify the accuracy with which ITER must be built.

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