# Symmetries and entropy production of transport in toroidal confinement systems

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A synthesized formulation of the classical, neoclassical, and anomalous transport in toroidal confinement systems with electromagnetic fluctuations and large mean flows is presented. The positive definite entropy production rate and the conjugate flux-force pairs are rigorously defined for each transport process. The Onsager symmetries of the classical and neoclassical transport matrices are derived from the self-adjointness of the linearized collision operator. The linear gyrokinetic equation with given electromagnetic fluctuations determines the anomalous fluxes with the quasilinear anomalous transport matrix which satisfies the Onsager symmetry.

# 1 Introduction

Plasma transport has been one of the most actively researched subjects in fusion science since it is not only an important problem to be understood for achieving controlled nuclear fusion but is also a theoretically interesting phenomenon containing rich physics. In a magnetically confined plasma, transport of particles, momentum, and energy results from Coulomb particle collisions and turbulent fluctuations driven by various instabilities. Classical transport occurs when particle gyromotions around the magnetic field lines are randomly disturbed by collisions. In a hot plasma where the mean free path is comparable to or larger than the system size, the guiding center drift motions combined with collisions cause another type of collisional transport, which depends on the confining magnetic field geometry and is called neoclassical transport. Theories of the classical and neoclassical transport have been systematically well established as seen in several reviews (Braginskii, 1965; Galeev & Sagdeev, 1979; Hinton & Hazeltine, 1976; Hirshman & Sigmar, 1981; Balescu, 1988). Compared to them, anomalous transport, which is driven by plasma turbulence, is more difficult to treat theoretically because it is essentially a nonlinear problem. Various theoretical studies of the anomalous transport based on fluid and kinetic turbulence models have been done (Connor & Wilson, 1994). For example, test particle models are applied to analyses of the anomalous transport in stochastic magnetic fields (Balescu, 1995). Also, modern chaos theories help us understand the onset of anomalous diffusion in drift waves (Horton & Ichikawa, 1996).

Attempts to uniformly describe the classical, neoclassical and anomalous transport have been made by Shaing (1988*a*, 1988*b*), Balescu (1990, 1991), Sugama & Horton (1995, 1997*a*), and Sugama *et al.* (1996). Along the same line as those works, this paper presents the synthesized formulation of the classical, neoclassical and anomalous transport in toroidal confinement systems, and elucidates entropy production and symmetry properties relevant to each transport process. Here we consider a toroidal plasma in which electromagnetic

fluctuations and large mean toroidal flows exist. This will be helpful in treating internal transport barriers found in large tokamaks where significant reduction of heat transport and large sheared toroidal flows are observed (Koide, 1996).

A basic kinetic equation for a turbulent plasma is written as

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla+\frac{e_a}{m_a}\left\{\left(\mathbf{E}+\widehat{\mathbf{E}}\right)+\frac{1}{c}\mathbf{v}\times\left(\mathbf{B}+\widehat{\mathbf{B}}\right)\right\}\cdot\frac{\partial}{\partial\mathbf{v}}\right](f_a+\widehat{f}_a)=C_a(f_a+\widehat{f}_a) \tag{1}$$

where $C_a \equiv \sum_b C_{ab}$ denotes a collision term and the distribution function for species $a$ (the electromagnetic fields) is divided into the ensemble average part $f_a$ ($\mathbf{E}=-\nabla\Phi-c^{-1}\partial\mathbf{A}/\partial t, \mathbf{B}=\nabla\times\mathbf{A}$) and the fluctuating part $\widehat{f}_a$ ($\widehat{\mathbf{E}}=-\nabla\widehat{\phi}-c^{-1}\partial\widehat{\mathbf{A}}/\partial t, \widehat{\mathbf{B}}=\nabla\times\widehat{\mathbf{A}}$). Taking an ensemble average $\langle\cdot\rangle_{\rm ens}$ of (1) gives the kinetic equation for $f_a$ as

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla+\frac{e_a}{m_a}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial\mathbf{v}}\right]f_a=\langle C_a\rangle_{\rm ens}+\mathcal{D}_a \tag{2}$$

where the right-hand side consists of the collision term and the fluctuation-particle interaction term $\mathcal{D}_a$ defined by

$$\mathcal{D}_a=-\frac{e_a}{m_a}\left\langle\left(\widehat{\mathbf{E}}+\frac{1}{c}\mathbf{v}\times\widehat{\mathbf{B}}\right)\cdot\frac{\partial\widehat{f}_a}{\partial\mathbf{v}}\right\rangle_{\rm ens}. \tag{3}$$

The classical and neoclassical transport occur due to collisions ($C_a$) while the anomalous (or turbulent) transport results from the fluctuation-particle interactions ($\mathcal{D}_a$). These transport processes produce entropy and the entropy production rates, which are kinetically defined as functionals of the distribution functions, can be also rewritten in the thermodynamic form as the inner products of conjugate pairs of the transport fluxes and the thermodynamic forces. One of main purposes of transport theories is to obtain the transport equations which connect the transport fluxes to the thermodynamic forces by the transport matrix. The transport matrix has symmetry properties which are deeply related to the self-adjointness of the linearized collision operator (Rosenbluth *et al.*, 1972):

$$T_a\int d^3v\frac{g_{a1}}{f_{a0}}C^L_{ab}(h_{a1},h_{b1})+T_b\int d^3v\frac{g_{b1}}{f_{b0}}C^L_{ba}(h_{b1},h_{a1})$$

$$= T_a \int d^3v \frac{h_{a1}}{f_{a0}} C^L_{ab}(g_{a1}, g_{b1}) + T_b \int d^3v \frac{h_{b1}}{f_{b0}} C^L_{ba}(g_{b1}, g_{a1}) \tag{4}$$

where the linearized collision operator $C^L_a \equiv \sum_b C^L_{ab}$ is defined by

$$C^L_{ab}(f_{a1}, f_{b1}) = C_{ab}(f_{a1}, f_{b0}) + C_{ab}(f_{a0}, f_{b1}). \tag{5}$$

Here $f_{a0}$ and $f_{b0}$ represent the Maxwellian distribution functions satisfying $C_{ab}(f_{a0}, f_{b0}) = 0$ while $f_{a1}$, $f_{b1}$, $g_{a1}$, $g_{b1}$, $h_{a1}$, and $h_{b1}$ are arbitrary deviations from the Maxwellian distribution. The positive definiteness of the collisional entropy production is represented by the inequality associated with the full nonlinear collision operator $-\sum_{a,b} \int d^3v (\ln f_a)$ $C_{ab}(f_a, f_b) \geq 0$, which reduces in terms of the linearized collision operator to

$$-T_a \int d^3v \frac{g_{a1}}{f_{a0}} C^L_{ab}(g_{a1}, g_{b1}) - T_b \int d^3v \frac{g_{b1}}{f_{b0}} C^L_{ba}(g_{b1}, g_{a1}) \geq 0. \tag{6}$$

When $m_a/m_b \ll 1$ or $m_b/m_a \ll 1$, slow collisional heat exchange between particle species $a$ and $b$ allows different temperatures $T_a \neq T_b$, for which (4) is valid to the lowest order in $(m_a/m_b)^{1/2}$ or $(m_a/m_b)^{1/2}$.

In order to show in detail the symmetry properties of the plasma transport processes, we consider in this work an axisymmetric toroidal system in which large toroidal flows on the order of the ion thermal velocity are allowed to exist. Then, the magnetic field is given by

$$\mathbf{B} = I(\Psi)\nabla\zeta + \nabla\zeta \times \nabla\Psi \tag{7}$$

where $\zeta$ is the toroidal angle, $\Psi$ represents the poloidal flux, and $I(\Psi) = RB_T$. We employ the drift ordering parameter defined by $\delta \equiv \rho_a/L$ ($\rho_a \equiv v_{Ta}/\Omega_a$: the thermal gyroradius, $L$: the equilibrium scale length) to expand the distribution functions as

$$f_a = f_{a0} + f_{a1} + f_{a2} + \cdots, \qquad \widehat{f}_a = \widehat{f}_{a1} + \widehat{f}_{a2} + \cdots \tag{8}$$

where the averaged part and the fluctuating part are assumed to be expanded by the same ordering parameter $\delta$. The lowest-order flow velocity $\mathbf{V}_0$ is in the toroidal direction (Hinton

& Wong, 1985) and is written as

$$\mathbf{V}_0 = V_0\widehat{\boldsymbol{\zeta}}, \quad V_0 = RV^\zeta = -Rc\frac{\partial\Phi_0(\Psi)}{\partial\Psi} \tag{9}$$

where $\Phi_0(\Psi)$ denotes the lowest-order electrostatic potential in $\delta$. Then, it is convenient to introduce the phase space variables $(\mathbf{x}, \varepsilon, \mu, \xi)$ in which the particle position $\mathbf{x}$ is observed from the laboratory frame while the particle energy $\varepsilon$, the magnetic moment $\mu$, and the gyrophase $\xi$ are defined in terms of the velocity $\mathbf{v}' \equiv \mathbf{v} - \mathbf{V}_0$ in the moving frame as

$$\varepsilon = \frac{1}{2}m_a(v')^2 + \Xi_a, \quad \mu = \frac{m_a(v'_\perp)^2}{2B}, \quad \frac{\mathbf{v}'_\perp}{v'_\perp} = \mathbf{e}_1\cos\xi + \mathbf{e}_2\sin\xi \tag{10}$$

where $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{b} \equiv \mathbf{B}/B)$ are unit vectors which forms a right-handed orthogonal system at each point, and $\mathbf{v}' = v'_\parallel\mathbf{b} + \mathbf{v}'_\perp$ with $v'_\parallel = \mathbf{v}' \cdot \mathbf{b}$. In the energy variable $\varepsilon$, $\Xi_a \equiv e_a\widetilde{\Phi}_1 - \frac{1}{2}m_aV_0^2$ represents the sum of the poloidal-angle-dependent part of the electrostatic potential $\widetilde{\Phi}_1 \equiv \Phi_1 - \langle\Phi_1\rangle[= \mathcal{O}(\delta)]$ and the potential energy due to the centrifugal force. Here the magnetic flux surface average is denoted by $\langle\cdot\rangle$. The particle energy $\varepsilon$ and the magnetic moment $\mu$ defined by (10) are conserved along the lowest-order guiding center orbit: $\overline{(d\varepsilon/dt)}_0 = \overline{(d\mu/dt)}_0 = 0$ where $\bar{\cdot} \equiv \oint d\xi/2\pi$ represents the gyrophase average. The lowest-order distribution function is given by the Maxwellian which satisfies $\overline{(df_{a0}/dt)}_0 \equiv (\mathbf{V}_0 + v'_\parallel\mathbf{b}) \cdot \nabla f_{a0} = 0$ [$\partial f_{a0}/\partial t = \mathcal{O}(\delta^2)$ is neglected by the transport ordering] and is written as

$$f_{a0} = n_a\left(\frac{m_a}{2\pi T_a}\right)^{3/2}\exp\left(-\frac{m_a(v')^2}{2T_a}\right) = N_a\left(\frac{m_a}{2\pi T_a}\right)^{3/2}\exp\left(-\frac{\varepsilon}{T_a}\right) \tag{11}$$

where the temperature $T_a = T_a(\Psi)$ and $N_a = N_a(\Psi)$ are flux-surface functions although generally the density $n_a$ depends on the poloidal angle $\theta$ through $\Xi_a$ and is given by $n_a = N_a\exp(-\Xi_a/T_a)$. It is seen from (11) that, in the lowest-order, the state of the toroidally rotating plasma is described by the three flux-surface functions $N_a(\Psi)$, $T_a(\Psi)$, and $V^\zeta(\Psi)$. Then, spatio-temporal dependences of these state variables $(N_a, T_a, V^\zeta)$ are governed by

the three surface-averaged diffusion-type equations (or transport equations) which contain the divergence terms of the surface-averaged radial fluxes of the particles, heat, and toroidal momentum. These radial particle, heat, and toroidal-momentum flux are defined in terms of the *ensemble-averaged* distribution function $f_a$ as $\Gamma_a \equiv \langle \int d^3v\ f_a \mathbf{v}\cdot\nabla\Psi\rangle$, $q_a \equiv \left\langle \int d^3v\ f_a \left(\varepsilon - \frac{5}{2}T_a\right)\mathbf{v}\cdot\nabla\Psi\right\rangle$, and $\Pi_a \equiv \langle \int d^3v\ f_a m_a v_\zeta \mathbf{v}\cdot\nabla\Psi\rangle$, respectively, where the poloidal flux $\Psi$ is used as a radial coordinate and $v_\zeta \equiv R\widehat{\zeta}\cdot\mathbf{v} = R^2 V^\zeta + (I/B)v'_\parallel + R\widehat{\zeta}\cdot\mathbf{v}'_\perp$ represents the covariant toroidal component of the particle velocity in the laboratory frame. Thus, in order to obtain a closed system of equations describing the toroidally rotating plasma, we need to derive explicit thermodynamic expressions for these radial fluxes as well as the averaged parallel current defined by $J_E \equiv \left\langle BJ_\parallel\right\rangle / \langle B^2\rangle^{1/2} \equiv \sum_a e_a \left\langle \int d^3v\ f_a v_\parallel\right\rangle$. It is shown later from the thermodynamic expressions of the entropy production rates that the thermodynamic forces conjugate to the transport fluxes $\Gamma_a$, $q_a/T_a$, $\Pi_a$, and $J_E$ are given by

$$X_{a1} \equiv -\frac{1}{N_a}\frac{\partial(N_a T_a)}{\partial\Psi} - e_a\frac{\partial\langle\Phi_1\rangle}{\partial\Psi}, \quad X_{a2} \equiv -\frac{\partial T_a}{\partial\Psi},$$

$$X_V \equiv -\frac{\partial V^\zeta}{\partial\Psi} = c\frac{\partial^2\Phi_0}{\partial\Psi^2}, \quad X_E \equiv \frac{\left\langle BE_\parallel^{(A)}\right\rangle}{\langle B^2\rangle^{1/2}}, \tag{12}$$

respectively.

We find from the definitions that the radial transport fluxes require only the gyrophase-dependent part of the distribution function $\widetilde{f}_a \equiv f_a - \overline{f}_a$ while the parallel current requires only the gyrophase-averaged part $\overline{f}_a$. Also, it is easily found that the lowest-order distribution function $f_{a0}$ makes no contributions to these four transport fluxes. The higher-order distribution functions $f_{a1}$, $f_{a2}$, $\cdots$ are obtained by recursively solving the kinetic equation (2) which is rewritten by

$$\Omega_a \frac{\partial \widetilde{f}_a}{\partial\xi} = \mathcal{L}(\overline{f}_a + \widetilde{f}_a) - \langle C_a\rangle_{\rm ens} - \mathcal{D}_a \tag{13}$$

where the differential operator $\mathcal{L} \equiv \frac{d}{dt} + \Omega_a\frac{\partial}{\partial\xi} \equiv \dfrac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \dot{\varepsilon}\dfrac{\partial}{\partial\varepsilon} + \dot{\mu}\dfrac{\partial}{\partial\mu} + (\dot{\xi} + \Omega_a)\dfrac{\partial}{\partial\xi}$ represents the time derivative along the particle orbit with the $\mathcal{O}(\delta^{-1})$ contribution from the

rapid gyromotion dropped (Hazeltine, 1973). From (13), we obtain the gyrophase-dependent parts of the distribution function up to $\mathcal{O}(\delta^2)$ as

$$\frac{1}{\Omega_a}\int^{\xi} d\xi\, \widetilde{\mathcal{L}\overline{f}}_a = \frac{1}{\Omega_a}\int^{\xi} d\xi\, (\widetilde{\mathcal{L}f_{a0}} + \widetilde{\mathcal{L}\overline{f}}_{a1}) \equiv \widetilde{f}_{a1} + \widetilde{f}_a^N$$

$$\frac{1}{\Omega_a}\int^{\xi} d\xi\, [\widetilde{\mathcal{L}\widetilde{f}_{a1}} - C_a^L(\widetilde{f}_{a1}) - \widetilde{\mathcal{D}}_a] \equiv \widetilde{f}_a^H + \widetilde{f}_a^C + \widetilde{f}_a^A \tag{14}$$

where $\widetilde{f}_a^N$, $\widetilde{f}_a^H$, $\widetilde{f}_a^C$, and $\widetilde{f}_a^A$ are of $\mathcal{O}(\delta^2)$. As is shown later, the $\mathcal{O}(\delta)$ gyrophase-dependent distribution function $\widetilde{f}_{a1}$ gives the particle, heat and toroidal momentum flows, which are tangential to the flux surface and do not contribute to the radial transport. Thus, the radial transport fluxes $\Gamma_a$, $q_a$, and $\Pi_a$ are of $\mathcal{O}(\delta^2)$.

The classical transport fluxes are defined by the $\mathbf{v}'_\perp$-moment of $\widetilde{f}_a^C$, which is derived from the gyrophase-integral of the the gyrophase-dependent part of the collision term $C_a^L(\widetilde{f}_{a1})$ as in (14). This definition seems natural because the classical transport is due to particles' gyromotions ($\widetilde{f}_{a1}$) with collisions ($C_a$). Analogously, the anomalous transport fluxes are defined from $\widetilde{f}_a^A$, which results from the gyrophase-dependent fluctuation-particle interaction term $\widetilde{\mathcal{D}}_a$. The neoclassical transport is obtained from the $\mathbf{v}'_\perp$-moment of $\widetilde{f}_a^N$. This is physically understandable by noticing that $\widetilde{f}_a^N$ is defined from $\widetilde{\mathcal{L}\overline{f}}_{a1}$ and hence contains the neoclassical orbital effects ($\mathcal{L}$) of the non-Maxwellian guiding-centers $\overline{f}_{a1}$. The residual function $\widetilde{f}_a^H$ is shown to give the nondissipative transport fluxes caused by the parallel gyroviscosity (Sugama & Horton, 1997*a*). More details of the present analysis may be found in Sugama & Horton (1997*a*, *b*).

In the following sections, we find detailed expressions of the transport fluxes and the entropy production rates, from which conjugate flux-force pairs are specified and symmetry properties of the transport matrices are shown. The classical, neoclassical, and anomalous transport are treated in §2–4, respectively. Summary of our results are given in §5.

## 2 Classical transport

The classical transport fluxes of particles, heat and toroidal momentum are derived from the $\mathcal{O}(\delta^2)$ gyrophase-dependent distribution function $\widetilde{f}_a^C$ in (14) and is given by

$$\Gamma_a^{\rm cl} \equiv \left\langle \int d^3v\ \widetilde{f}_a^C \mathbf{v}\cdot\nabla\Psi \right\rangle = -\frac{m_a c}{e_a}\left\langle \int d^3v\ C_a^L(\widetilde{f}_{a1})\ \mathbf{v}'_\perp\cdot(R\widehat{\boldsymbol{\zeta}}) \right\rangle$$

$$\frac{1}{T_a}q_a^{\rm cl} \equiv \left\langle \int d^3v\ \widetilde{f}_a^C \left(\frac{\varepsilon}{T_a}-\frac{5}{2}\right)\mathbf{v}\cdot\nabla\Psi \right\rangle = -\frac{m_a c}{e_a}\left\langle \int d^3v\ C_a^L(\widetilde{f}_{a1})\ \left(\frac{\varepsilon}{T_a}-\frac{5}{2}\right)\mathbf{v}'_\perp\cdot(R\widehat{\boldsymbol{\zeta}}) \right\rangle$$

$$\Pi_a^{\rm cl} \equiv \left\langle \int d^3v\ \widetilde{f}_a^C m_a v_\zeta \mathbf{v}\cdot\nabla\Psi \right\rangle = -\frac{m_a^2 c}{e_a}\left\langle \int d^3v\ C_a^L(\widetilde{f}_{a1})\ \frac{1}{2}\widetilde{(v_\zeta^2)} \right\rangle \tag{15}$$

where a partial integral in $\xi$ is taken and $\mathbf{v}'_\perp\cdot(R\widehat{\boldsymbol{\zeta}}) = -B^{-1}(\mathbf{v}'\times\mathbf{b})\cdot\nabla\Psi$ and $\frac{1}{2}\widetilde{(v_\zeta^2)} \equiv \frac{1}{2}\left[v_\zeta^2 - \overline{(v_\zeta^2)}\right] = \left(R^2V^\zeta + \frac{I}{B}v'_\parallel\right)\mathbf{v}'_\perp\cdot(R\widehat{\boldsymbol{\zeta}}) + \frac{1}{2}\widetilde{(\mathbf{v}'_\perp\mathbf{v}'_\perp)} : (R\widehat{\boldsymbol{\zeta}})(R\widehat{\boldsymbol{\zeta}})$ are used. The $\mathcal{O}(\delta)$ gyrophase-dependent distribution function $\widetilde{f}_{a1}$ is immediately obtained from (11) and (14) as

$$\widetilde{f}_{a1} = f_{a0}\frac{m_a}{T_a}\left[\mathbf{u}_{\perp a}\cdot\mathbf{v}'_\perp + \frac{2}{5}\left(\frac{\mathbf{q}_{\perp a}}{p_a} - \frac{\Xi_a}{T_a}\mathbf{u}_{\perp a}\right)\cdot\mathbf{v}'_\perp\left(\frac{m_a(v')^2}{2T_a}-\frac{5}{2}\right)\right.$$
$$\left. + \frac{1}{2p_a}\boldsymbol{\pi}_a^{\rm gyro} : \left(\mathbf{v}'\mathbf{v}' - \frac{(v')^2}{3}\mathsf{I}\right)\right] \tag{16}$$

where the perpendicular flows and the gyroviscosity are given in terms of the thermodynamic forces in (12) as

$$n_a\mathbf{u}_{\perp a} \equiv \int d^3v\ \widetilde{f}_{a1}\mathbf{v}'_\perp = \frac{c}{e_aB}\left(X_{a1} + \frac{\Xi_a}{T_a}X_{a2} + m_aR^2V^\zeta X_V\right)\nabla\Psi\times\mathbf{b}$$

$$\frac{\mathbf{q}_{\perp a}}{T_a} \equiv \frac{1}{T_a}\int d^3v\ \widetilde{f}_{a1}\left(\varepsilon-\frac{5}{2}\right)\mathbf{v}'_\perp = \frac{5}{2}\frac{cX_{a2}}{e_aB}\nabla\Psi\times\mathbf{b} + \frac{\Xi_a}{T_a}n_a\mathbf{u}_{\perp a}$$

$$\boldsymbol{\pi}_a^{\rm gyro} \equiv \int d^3v\ \widetilde{f}_{a1}m_a\left(\mathbf{v}'\mathbf{v}' - \frac{(v')^2}{3}\mathsf{I}\right)$$
$$= \frac{p_aX_V}{2B\Omega_a}\left[-(\nabla\Psi)(\nabla\Psi) + (\nabla\Psi\times\mathbf{b})(\nabla\Psi\times\mathbf{b}) + 2I\{\mathbf{b}(\nabla\Psi\times\mathbf{b}) + (\nabla\Psi\times\mathbf{b})\mathbf{b}\}\right].$$

Then, the classical transport equations are obtained by substituting (16) with (17) into (15) as

$$\Gamma_a^{\rm cl} = \sum_b [(L^{\rm cl})_{11}{}^{ab} X_{b1} + (L^{\rm cl})_{12}{}^{ab} X_{b2}] + (L^{\rm cl})_{1V}{}^{a} X_V$$

$$\frac{1}{T_a} q_a^{\rm cl} = \sum_b [(L^{\rm cl})_{21}{}^{ab} X_{b1} + (L^{\rm cl})_{22}{}^{ab} X_{b2}] + (L^{\rm cl})_{2V}{}^{a} X_V$$

$$\sum_a \Pi_a^{\rm cl} = \sum_b [(L^{\rm cl})_{V1}{}^{b} X_{b1} + (L^{\rm cl})_{V2}{}^{b} X_{b2}] + (L^{\rm cl})_{VV} X_{b2}. \tag{17}$$

Here the classical transport coefficients are given by

$$\begin{bmatrix} (L^{\rm cl})_{11}{}^{ab} & (L^{\rm cl})_{12}{}^{ab} \\ (L^{\rm cl})_{21}{}^{ab} & (L^{\rm cl})_{22}{}^{ab} \end{bmatrix} = \left\langle \frac{c^2 R^2 B_P^2}{e_a e_b B^2} \begin{bmatrix} 1 & 0 \\ \frac{\Xi_a}{T_a} & 1 \end{bmatrix} \begin{bmatrix} -l_{11}^{ab} & l_{12}^{ab} \\ l_{21}^{ab} & -l_{22}^{ab} \end{bmatrix} \begin{bmatrix} 1 & \frac{\Xi_b}{T_b} \\ 0 & 1 \end{bmatrix} \right\rangle$$

$$\begin{bmatrix} (L^{\rm cl})_{1V}^{a} \\ (L^{\rm cl})_{2V}^{a} \end{bmatrix} = \begin{bmatrix} (L^{\rm cl})_{V1}^{a} \\ (L^{\rm cl})_{V2}^{a} \end{bmatrix} = \sum_b m_b R^2 V^\zeta \frac{c^2 R^2 B_P^2}{e_a e_b B^2} \begin{bmatrix} 1 & 0 \\ \frac{\Xi_a}{T_a} & 1 \end{bmatrix} \begin{bmatrix} -l_{11}^{ab} \\ l_{21}^{ab} \end{bmatrix}$$

$$(L^{\rm cl})_{VV} = -\sum_{a,b} \frac{m_a m_b c^2}{e_a e_b} \left\langle \frac{R^2 B_P^2}{B^2} \left( \frac{(R^2 B_P^2 + 4I^2)}{4B^2} l_V^{ab} + R^4 (V^\zeta)^2 l_{11}^{ab} \right) \right\rangle. \tag{18}$$

where the coefficients $l_{jk}^{ab}$ and $l_V^{ab}$ are defined by

$$l_{jk}^{ab} = \delta_{ab} \frac{m_a^2}{T_a} \sum_{a'} \int d^3v \; v_\parallel L_{j-1}^{(3/2)}(x_a^2) C_{aa'}[v_\parallel L_{k-1}^{(3/2)}(x_a^2) f_{a0}, f_{b0}]$$

$$+ \frac{m_a m_b}{T_b} \int d^3v \; v_\parallel L_{j-1}^{(3/2)}(x_a^2) C_{ab}[f_{a0}, v_\parallel L_{k-1}^{(3/2)}(x_b^2) f_{b0}]$$

$$l_V^{ab} \equiv \delta_{ab} \frac{m_a^2}{15 T_a} \sum_{a'} \int d^3v \; (v')^2 C_{aa'}[(v')^2 f_{a0}, \; f_{a'0}] + \frac{m_a m_b}{15 T_b} \int d^3v \; (v')^2 C_{ab}[f_{a0}, \; (v')^2 f_{b0}],$$

respectively, where the Laguerre polynomials $L_j^{(3/2)}(x_a^2)$ $[x_a^2 \equiv m_a v^2/2T_a, \; L_0^{(3/2)(x_a^2)} \equiv 1, \; L_1^{3/2}(x_a^2) \equiv \frac{5}{2} - x_a^2, \cdots]$ are used. From the self-adjointness (4) of the linearized collision operator with (19), we have

$$l_{jk}^{ab} = l_{kj}^{ba}, \quad l_V^{ab} = l_V^{ba}. \tag{19}$$

Using (18) and (19) and taking account of the coefficients' parity with respect to $V^\zeta$, we obtain the Onsager symmetry (Onsager, 1931; de Groot & Mazur, 1962) for the classical

transport coefficients which is written as

$$(L^{\rm cl})_{mn}{}^{ab}(V^\zeta) = (L^{\rm cl})_{mn}{}^{ab}(-V^\zeta) = (L^{\rm cl})_{nm}{}^{ba}(V^\zeta) \qquad (m,n=1,2)$$

$$(L^{\rm cl})_{mV}{}^{a}(V^\zeta) = -(L^{\rm cl})_{mV}{}^{a}(-V^\zeta) = (L^{\rm cl})_{Vm}{}^{a}(V^\zeta) \qquad (m=1,2)$$

$$(L^{\rm cl})_{VV}(V^\zeta) = (L^{\rm cl})_{VV}(-V^\zeta). \tag{20}$$

The collisional entropy production rate is kinetically defined by the quadratic form of the $\mathcal{O}(\delta)$ distribution function associated with the linearized collision operator as $\sigma_a \equiv -\int d^3v\ f_{a0}^{-1} f_{a1} \sum_b C_{ab}^L(f_{a1}, f_{b1})$ for species $a$. The contribution of the classical transport to the entropy production rate is defined by the quadratic form of the gyrophase-dependent part of the distribution function and is rewritten in the surface-averaged thermodynamic form as

$$T_a\Big\langle \sigma_a^{\rm cl}\Big\rangle \equiv -T_a\left\langle \int d^3v \frac{\widetilde{f}_{a1}}{f_{a0}} C_a^L(\widetilde{f}_{a1})\right\rangle = \Gamma_a^{\rm cl} X_{a1} + \frac{1}{T_a} q_a^{\rm cl} X_{a2} + \Pi_a^{\rm cl} X_V \tag{21}$$

where (15)–(17) are used. Thus, the classical entropy production is given by the inner product of the conjugate pair of the classical fluxes $(\Gamma_a^{\rm cl}, q_a^{\rm cl}/T_a, \Pi_a^{\rm cl})$ and the thermodynamic forces $(X_{a1}, X_{a2}, X_V)$. The second law of thermodynamics for the classical transport process $\sum_a T_a\Big\langle \sigma_a^{\rm cl}\Big\rangle \geq 0$ and accordingly the positive definiteness of the classical transport matrix are guaranteed by (6) and (21).

# 3 Neoclassical transport

The neoclassical transport fluxes are included in the perpendicular-velocity moments of the $\mathcal{O}(\delta^2)$ gyrophase-dependent part $\widetilde{f}_a^N$ of the distribution function in (14), which are written as

$$\left\langle \int d^3v\ \widetilde{f}_a^N \mathbf{v}\cdot\nabla\Psi\right\rangle = \Gamma_a^{\rm ncl} + \Gamma_a^{(E)}$$

$$T_a\left\langle \int d^3v\ \widetilde{f}_a^N \left(\frac{\varepsilon}{T_a} - \frac{5}{2}\right)\mathbf{v}\cdot\nabla\Psi\right\rangle = q_a^{\rm ncl} + q_a^{(E)}$$

$$\left\langle \int d^3v \; \widetilde{f}_a^N m_a v_\zeta \mathbf{v} \cdot \nabla \Psi \right\rangle = \Pi_a^{\rm ncl} + \Pi_a^{(E)}. \tag{22}$$

Here the fluxes $\Gamma_a^{(E)}$, $q_a^{(E)}$, and $\Pi_a^{(E)}$ represent the parts driven by the inductive electric field $\mathbf{E}^{(A)} = -c^{-1}\partial \mathbf{A}/\partial t$ (Sugama & Horton, 1997*a*). The neoclassical transport fluxes $\Gamma_a^{\rm ncl}$, $q_a^{\rm ncl}$, $\Pi_a^{\rm ncl}$, and the parallel current $J_E$ are given by

$$\Gamma_a^{\rm ncl} \equiv \left\langle \int d^3v \; \overline{g}_a W_{a1} \right\rangle, \qquad \frac{1}{T_a} q_a^{\rm ncl} \equiv \left\langle \int d^3v \; \overline{g}_a W_{a2} \right\rangle$$

$$\Pi_a^{\rm ncl} \equiv \left\langle \int d^3v \; \overline{g}_a W_{aV} \right\rangle, \qquad J_E \equiv \frac{\left\langle B J_\parallel \right\rangle}{\left\langle B^2 \right\rangle^{1/2}} \equiv \sum_a \left\langle \int d^3v \; \overline{g}_a W_{aE} \right\rangle \tag{23}$$

where $\overline{g}_a$ is defined in terms of the $\mathcal{O}(\delta)$ gyrophase-averaged distribution function $\overline{f}_{a1}$ as

$$\overline{g}_a \equiv \overline{f}_{a1} - f_{a0} \frac{e_a}{T_a} \int^l \frac{dl}{B} \left( B E_\parallel^{(2)} - \frac{B^2}{\left\langle B^2 \right\rangle} \left\langle B E_\parallel^{(2)} \right\rangle \right) \tag{24}$$

and the functions $(W_{a1}, W_{a2}, W_{aV}, W_{aE})$ are defined by

$$W_{a1} \equiv \frac{m_a c}{e_a} v_\parallel' \mathbf{b} \cdot \nabla \left( R^2 V^\zeta + \frac{I}{B} v_\parallel' \right), \qquad W_{a2} \equiv W_{a1} \left( \frac{\varepsilon}{T_a} - \frac{5}{2} \right)$$

$$W_{aV} \equiv \frac{m_a c}{2 e_a} v_\parallel' \mathbf{b} \cdot \nabla \left[ m_a \left( R^2 V^\zeta + \frac{I}{B} v_\parallel' \right)^2 + \mu \frac{R^2 B_P^2}{B} \right], \qquad W_{aE} \equiv \frac{e_a v_\parallel' B}{\left\langle B^2 \right\rangle^{1/2}}. \tag{25}$$

In (24), $\int^l dl$ denotes the integral along the magnetic field line, and $E_\parallel^{(2)} \equiv \mathbf{b} \cdot (-\nabla \Phi^{(2)} - c^{-1} \partial \mathbf{A}/\partial t)$ is the $\mathcal{O}(\delta^2)$ parallel electric field.

In order to derive the neoclassical transport equations, we need to solve the linearized drift kinetic equation for $\overline{g}_a$, which is derived from the $\mathcal{O}(\delta)$ part of (2) (Hinton & Wong, 1985; Catto *et al.*, 1987; Sugama & Horton, 1997*a*) and is written as

$$v_\parallel' \mathbf{b} \cdot \nabla \overline{g}_a - C_a^L(\overline{g}_a) = \frac{1}{T_a} f_{a0} \left( W_{a1} X_{a1} + W_{a2} X_{a2} + W_{aV} X_V + W_{aE} X_E \right). \tag{26}$$

The solution of the linearized drift kinetic equation (26) is generally written in the linear form of the thermodynamic forces as

$$\overline{g}_a = \sum_b (G_{ab1} X_{b1} + G_{ab2} X_{b2}) + G_{aV} X_V + G_{aE} X_E. \tag{27}$$

Here $G_{ab1}$ is defined as the solution of (26) with no thermodynamic forces except for $X_{b1}$ given as the unity. In the similar way, $G_{ab2}$, $G_{aV}$, and $G_{aE}$ are defined. Substituting (27) into (23), we obtain the neoclassical transport equations as

$$\Gamma_a^{\rm ncl} = \sum_b (L_{11}^{ab} X_{b1} + L_{12}^{ab} X_{b2}) + L_{1V}^a X_V + L_{1E}^a X_E$$

$$\frac{1}{T_a} q_a^{\rm ncl} = \sum_b (L_{21}^{ab} X_{b1} + L_{22}^{ab} X_{b2}) + L_{2V}^a X_V + L_{2E}^a X_E$$

$$\sum_a \Pi_a^{\rm ncl} = \sum_b (L_{V1}^b X_{b1} + L_{V2}^b X_{b2}) + L_{VV} X_V + L_{VE} X_E$$

$$J_E = \sum_b (L_{E1}^b X_{b1} + L_{E2}^b X_{b2}) + L_{EV} X_V + L_{EE} X_E \tag{28}$$

with the neoclassical transport coefficients given by

$$L_{mn}^{ab} = \left\langle \int d^3v \; W_{am} G_{abn} \right\rangle, \qquad L_{mM}^a = \left\langle \int d^3v \; W_{am} G_{aM} \right\rangle$$

$$L_{Mm}^b = \sum_a \left\langle \int d^3v \; W_{aM} G_{abm} \right\rangle, \qquad L_{MN} = \sum_a \left\langle \int d^3v \; W_{aM} G_{aN} \right\rangle \tag{29}$$

where $m, n = 1, 2$ and $M, N = V, E$. Here the transport coefficients are dependent on the the toroidal angular velocity $V^\zeta = -c\partial\Phi_0/\partial\Psi$. By using the self-adjointness of the linearized collision operator (4) and taking account of the symmetry properties of the functions $W_{am}$, $W_{aM}$, $G_{abm}$, and $G_{aM}$ $(m = 1, 2; M = V, E)$ with respect to the variable transformations $v'_\parallel \to -v'_\parallel$ and $V^\zeta \to -V^\zeta$, we can prove that the neoclassical transport coefficients satisfy the Onsager symmetry (Sugama & Horton, 1997*b*) which is given by

$$L_{mn}^{ab}(V^\zeta) = L_{nm}^{ba}(-V^\zeta) \qquad (m, n = 1, 2)$$

$$L_{MN}(V^\zeta) = L_{NM}(-V^\zeta) \qquad (M, N = V, E)$$

$$L_{mM}^a(V^\zeta) = -L_{Mm}^a(-V^\zeta) \qquad (m = 1, 2; M = V, E). \tag{30}$$

If the system has up-down symmetry $B(\theta) = B(-\theta)$ ($\theta$: a poloidal angle defined such that $\theta = 0$ on the plane of reflection symmetry), we find that $W_{aV}$ and $G_{aV}$ are symmetric

while $W_{am}(m=1,2), W_{aE}, G_{abm}$ $(m=1,2)$ and $G_{aE}$ are antisymmetric with respect to the transformation $(v'_{\|}, \theta, V^{\zeta}) \to (-v'_{\|}, -\theta, -V^{\zeta})$. Then, we see from (29) that $L^{ab}_{mn}$, $L^{a}_{mE}$, $L_{VV}$, and $L_{EE}$ are even while $L^{a}_{mV}$ and $L_{VE}$ are odd in $V_{\zeta}$. Thus, the restricted forms of the Onsager relations for the system with up-down symmetry are written as

$$
\begin{aligned}
& L^{ab}_{mn}(V^{\zeta}) = L^{ab}_{mn}(-V^{\zeta}) = L^{ba}_{nm}(V^{\zeta}) \qquad (m,n=1,2) \\
& L^{a}_{mV}(V^{\zeta}) = -L^{a}_{mV}(-V^{\zeta}) = L^{a}_{Vm}(V^{\zeta}) \qquad (m=1,2) \\
& L^{a}_{mE}(V^{\zeta}) = L^{a}_{mE}(-V^{\zeta}) = -L^{a}_{Em}(V^{\zeta}) \qquad (m=1,2) \\
& L_{VE}(V^{\zeta}) = -L_{VE}(-V^{\zeta}) = -L_{EV}(V^{\zeta}) \\
& L_{VV}(V^{\zeta}) = L_{VV}(-V^{\zeta}), \qquad L_{EE}(V^{\zeta}) = L_{EE}(-V^{\zeta}).
\end{aligned} \tag{31}
$$

Detailed expressions of the neoclassical transport coefficients for the toroidally rotating plasma are given by Hinton & Wong (1985), Catto *et al.* (1987), and Sugama & Horton (1997*b*).

In the same way as in (21), the neoclassical entropy production rate is kinetically defined by the quadratic form of the gyrophase-averaged distribution function and is rewritten in the surface-averaged thermodynamic form as

$$
\sum_a T_a \left\langle \sigma_a^{\rm ncl} \right\rangle \equiv -\sum_a T_a \left\langle \int d^3v \frac{\overline{f}_{a1}}{f_{a0}} C_a^L(\overline{f}_{a1}) \right\rangle = \sum_a \left( \Gamma_a^{\rm ncl} X_{a1} + \frac{1}{T_a} q_a^{\rm ncl} X_{a2} + \Pi_a^{\rm ncl} X_V \right) + J_E X_E \tag{32}
$$

where (23), (24), and (26) are used. In (32), the product of the parallel current $J_E$ and the conjugate force $X_E$ is included as a part of the neoclassical entropy production since we here regard the transport due to guiding center motion described by $\overline{f}_{a1}$ as the neoclassical transport. We also obtain $\sum_a T_a \left\langle \sigma_a^{\rm ncl} \right\rangle \geq 0$ and the positive definiteness of the neoclassical transport matrix from (6), (28), and (32).

# 4 Anomalous transport

The anomalous transport fluxes of particles, heat and toroidal momentum are derived from the $\mathcal{O}(\delta^2)$ gyrophase-dependent distribution function $\widetilde{f}_a^A$ in (14) and is written in the analogous way to the classical fluxes in (15) as

$$\Gamma_a^A \equiv \left\langle \int d^3v\ \widetilde{f}_a^A \mathbf{v}\cdot\nabla\Psi \right\rangle = -\frac{m_a c}{e_a}\left\langle \int d^3v\ \widetilde{D}_a\ \mathbf{v}'_\perp\cdot(R\widehat{\zeta}) \right\rangle$$

$$\frac{1}{T_a}q_a^A \equiv \left\langle \int d^3v\ \widetilde{f}_a^A \left(\frac{\varepsilon}{T_a}-\frac{5}{2}\right)\mathbf{v}\cdot\nabla\Psi \right\rangle + \frac{e_a}{T_a}\left\langle\left\langle \int d^3v\ \widehat{f}_a\left(\widehat{\phi}-\frac{1}{c}\mathbf{V}_0\cdot\widehat{\mathbf{A}}\right)\mathbf{v}\cdot\nabla\Psi \right\rangle\right\rangle$$

$$= -\frac{m_a c}{e_a}\left\langle \int d^3v\ \widetilde{D}_a\ \left(\frac{\varepsilon}{T_a}-\frac{5}{2}\right)\mathbf{v}'_\perp\cdot(R\widehat{\boldsymbol{\zeta}}) \right\rangle + \frac{e_a}{T_a}\left\langle\left\langle \int d^3v\ \widehat{f}_a\left(\widehat{\phi}-\frac{1}{c}\mathbf{V}_0\cdot\widehat{\mathbf{A}}\right)\mathbf{v}\cdot\nabla\Psi \right\rangle\right\rangle$$

$$\Pi_a^A \equiv \left\langle \int d^3v\ \widetilde{f}_a^A m_a v_\zeta \mathbf{v}\cdot\nabla\Psi \right\rangle + \frac{e_a}{c}\left\langle\left\langle \int d^3v\ \widehat{f}_a\widehat{\mathbf{A}}\cdot(R\widehat{\boldsymbol{\zeta}})\mathbf{v}\cdot\nabla\Psi \right\rangle\right\rangle$$

$$= -\frac{m_a^2 c}{e_a}\left\langle \int d^3v\ \widetilde{D}_a\ \frac{1}{2}\widetilde{(v_\zeta^2)} \right\rangle + \frac{e_a}{c}\left\langle\left\langle \int d^3v\ \widehat{f}_a\widehat{\mathbf{A}}\cdot(R\widehat{\boldsymbol{\zeta}})\mathbf{v}\cdot\nabla\Psi \right\rangle\right\rangle. \tag{33}$$

It should be noted that the anomalous heat flux $q_a^A$ and the anomalous toroidal momentum flux $\Pi_a^A$ contain the fluctuating potential energy transport $e_a\left\langle\left\langle \int d^3v\ \widehat{f}_a(\widehat{\phi}-c^{-1}\mathbf{V}_0\cdot\widehat{\mathbf{A}})\mathbf{v}\cdot\nabla\Psi \right\rangle\right\rangle$ and the toroidal momentum transport due to the fluctuating vector potential $(e_a/c)\left\langle\left\langle \int d^3v\ \widehat{f}_a\widehat{\mathbf{A}}\cdot(R\widehat{\boldsymbol{\zeta}})\mathbf{v}\cdot\nabla\right.$ respectively. Here $\langle\cdot\rangle$ denotes a double average over the magnetic surface and the ensemble. We also define the anomalous heat transfer rate $Q_a^A$ by

$$Q_a^A \equiv e_a\left\langle \int d^3v\ \widehat{f}_a\left(\frac{\partial}{\partial t}+\mathbf{V}_0\cdot\nabla\right)\left(\widehat{\phi}-\frac{1}{c}\mathbf{v}\cdot\widehat{\mathbf{A}}\right) \right\rangle. \tag{34}$$

In order to evaluate these anomalous fluxes $(\Gamma_a^A, q_a^A/T_a, \Pi_a^A, Q_a^A)$, we need to solve the nonlinear gyrokinetic equation (Sugama & Horton, 1997*a*) for $\widehat{f}_a$ with the Maxwell equations for $\widehat{\phi}$ although they are too complex to obtain the analytical solution. However, without finding the solution, we can derive from the gyrokinetic equation the following relation for the anomalous entropy production rate:

$$\left\langle\left\langle \sigma_a^A \right\rangle\right\rangle \equiv \Gamma_a^A X_{a1}^A + \frac{1}{T_a}q_a^A X_{a2}^A + \Pi_a^A X_{aV}^A + Q_a^A X_{aT} = -\left\langle\left\langle \int d^3v \frac{1}{f_{a0}}\widehat{f}_a C_a^L(\widehat{f}_a) \right\rangle\right\rangle \tag{35}$$

where $X_{a1}^A \equiv X_{a1}/T_a$, $X_{a2}^A \equiv X_{a2}/T_a$, $X_{aV}^A \equiv X_V/T_a$, and $X_{aT}^A \equiv 1/T_a$. In contrast to the classical and neoclassical entropy production rates given by (21) and (32), the anomalous entropy production rate contains the product of the anomalous heat transfer rate $Q_a^A$ and the inverse temperature $X_a^A \equiv 1/T_a$ as the conjugate force. In the same way as $\sum_a T_a \left\langle \sigma_a^{\rm cl} \right\rangle \geq 0$ and $\sum_a T_a \left\langle \sigma_a^{\rm ncl} \right\rangle \geq 0$, the positive definiteness of the anomalous entropy production $\sum_a T_a \langle \sigma_a^a \rangle \geq 0$ is derived from (6) and (35).

Since the distribution function $\widehat{f}_a$ and the electromagnetic fields $(\widehat{\phi}, \widehat{\mathbf{A}})$ obtained by solving the nonlinear gyrokinetic equation with the Maxwell equations are generally nonlinear functions of the thermodynamic forces, the anomalous transport fluxes determined from the correlations between $\widehat{f}_a$ and $(\widehat{\phi}, \widehat{\mathbf{A}})$ are considered to be highly nonlinear functions of the forces (Balescu, 1992), which is a contrast to the linear thermodynamic forms of the classical and neoclassical transport fluxes. However, the symmetry property is shown for the quasilinear anomalous transport matrix which is obtained by the *linear* gyrokinetic equation with the fluctuating electromagnetic fields regarded as given *a priori*. In this case, the quasilinear anomalous transport matrix is considered to be a functional of the fluctuation spectrum $\{\widehat{\phi}, \widehat{\mathbf{A}}\}$ and linearly connect the anomalous fluxes $(\Gamma_a^A, q_a^A/T_a, \Pi_a^A, Q_a^A)$ to the conjugate forces $(X_{a1}^A, X_{a2}^A, X_{aV}^A, X_{aT}^A)$ although the self-consistent dependences of the electromagnetic fields on the forces due to the Maxwell equations remain to be solved. Under these restrictions, we can derive the Onsager symmetry of the quasilinear anomalous transport matrix from the linear gyrokinetic equation with the self-adjointness of the collision operator (4) in the same manner as in Sugama & Horton (1997*a*).

Thus while the transport matrix relating the anomalous fluxes to the forces has a symmetric structure insuring that $\sigma_a^A \geq 0$, the fluctuation spectrum itself evolves through the mode coupling equations. In the renormalized weak turbulence theory limit where the wave kinetic equation applies, there is a positive definite entropy production functional (Horton, 1986) associated with the fluctuation dynamics. The existence of these entropy production

functionals is related to the thermodynamic stability of the systems (Nicolis & Prigogine, 1977; Horton, 1980).

# 5 Summary

In this work, starting from the Boltzmann-type kinetic equation and utilizing the perturbative expansion in the small gyroradius parameter, we have investigated transport processes in toroidal confinement systems with electromagnetic fluctuations and large mean flows. For each of the classical, neoclassical and anomalous transport processes, the positive definite entropy production rate is separately given in the kinetic and thermodynamic forms, from which the conjugate flux-force pairs are clearly defined. For the classical and neoclassical transport matrices, which linearly relate the transport fluxes to the conjugate thermodynamic forces, the Onsager symmetries are derived from the self-adjointness of the linear collision operator. Under given electromagnetic fluctuations, the Onsager symmetry is also satisfied by the quasilinear anomalous transport matrix obtained from the linear gyrokinetic equation.

Work towards establishing a complete description of the self-consistent fluctuation dynamics and particle transport has been an underlying theme of the plasma research of Balescu and his group for many years. His elegant works have made significant contributions to development of statistical plasma physics and deepened our understanding of plasma transport processes. Transport theories for turbulent plasmas are still developing and promise to remain an open, but exciting, frontier far into the future.

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