Steady State Tokamak Equilibria Without Current Drive

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Abstract

Steady-state tokamak equilibria without current drive are found. This is made possible by including the potato bootstrap current close to the magnetic axis. Tokamaks with this class of equilibria do not need seed current or current drive, and are intrinsically steady state.

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A tokamak is not known to be an intrinsically steady-state plasma confinement scheme in the sense that it needs external current drive, either inductive or non-inductive, or both, to maintain the equilibrium.[1] Although there are high-bootstrap-current-fraction ($\gtrsim 99\%$) tokamak equilibrium with a small amount of the seed current in the region close to the magnetic axis, they are still not intrinsically steady state.[2,3] To find an intrinsically steady-state tokamak equilibrium one needs to find at least a self-generated current in the region close to the magnetic axis. It turns out there does exist such a current.[4,5] The origin of this current is associated with the unique orbit topology in the region close to the magnetic axis.[6,7] Because the variation of the minor radius over the width of the orbit is significant there, the fraction of the trapped particles does not vanish. This is in contrast to the fraction of the trapped banana orbits which vanishes on the magnetic axis. Because trapped particles have the shape of a potato in the region close to the magnetic axis, we call these orbits the potato orbits to distinguish from the well-known banana orbits in the region away from the magnetic axis. It has been known that potato orbits associated with fusion-born alpha particles can drive a bootstrap current.[4] This result is extended to fuel ions and electrons.[5] The magnitude of the bootstrap current for potato electrons and potato ions is larger than that of alpha particles due to higher fuel density. For typical tokamaks, the potato bootstrap current is a significant fraction of the banana bootstrap current. This makes it feasible to have tokamak equilibria with only bootstrap current and diamagnetic current.[5] Here we demonstrate that such equilibria exist.

A convenient expression for the banana bootstrap current is given in Ref. 8. Here, we express the flux surface and radial averaged potato bootstrap current $\left\langle J_{\parallel} B \right\rangle_b$ derived in Ref. 5 in a similar form

$$\left\langle J_{\parallel} B \right\rangle_b = J_0 \frac{\ell^b_{22} \widehat{\mu}^e_{11}}{\ell^{eb}_{11}\ell^{eb}_{22} - (\ell^{eb}_{12})^2} \left\{ \left(1 - \frac{\ell^{eb}_{12}}{\ell^{eb}_{22}} \frac{\widehat{\mu}^e_{12}}{\ell^e_{11}} \right) \left[A^e_1 + \right.\right.$$

$$\left.\left.\frac{1}{Z_i}\,\frac{T_i}{T_e}\left(A_1^i+\alpha_i A_2^i\right)\right]+\frac{\widehat{\mu}_{12}^e}{\widehat{\mu}_{11}^e}\left(1-\frac{\ell_{12}^{eb}}{\ell_{22}^{eb}}\,\frac{\widehat{\mu}_{22}^e}{\widehat{\mu}_{12}^e}\right)A_2^e\right\}, \tag{1}$$

where $J_0=-IcP_e,\ I=RB_t$, c is the speed of light, R is the major radius, B_t is the toroidal magnetic field strength, $P_e=N_eT_e$ is the electron pressure, N_e is the electron density, T_e is the electron temperature, $A_i^e=P_e'/P_e, A_1^i=P_i'/P_i,\ A_2^i=T_i'/T_i,\ A_2^e=T_e'/T_e,\ P_i$ is the ion pressure, T_i is the ion temperature, Z_i is the ion charge, and prime denotes $d/d\psi$ with ψ the poloidal flux function. The electron viscous coefficients $\widehat{\mu}_{ij}^e$ are

$$\widehat{\mu}_{11}^e=\frac{N_eM_e}{\tau_{ee}}\,x_e\left(0.531+0.928Z_i\right), \tag{2}$$

$$\widehat{\mu}_{12}^e=-\frac{N_eM_e}{\tau_{ee}}\,x_e\left(0.542+1.237Z_i\right), \tag{3}$$

$$\widehat{\mu}_{22}^e=\frac{N_eM_e}{\tau_{ee}}\,x_e\left(1.282+2.732Z_i\right), \tag{4}$$

where M_e is the electron mass, τ_{ee} is the electron-electron collision time, $x_e=2.2(I_0v_{te0}/\Omega_{e0})^{1/2}\cdot[q_0/(\delta_0I_0R_0)]^{1/3}$, v_{te0} is the electron speed, Ω_{e0} is the electron gyrofrequency, q_0 is the safety factor, δ_0 is the elongation parameter, and the subscript "0" indicates the quantities are evaluated at the magnetic axis.

The parameter α_i is defined as

$$\alpha_i=\frac{\ell_{22}^i\left(\widehat{\mu}_{22}^i/\widehat{\mu}_{11}^i\right)}{\widehat{\mu}_{22}^i+\ell_{22}^i-(\widehat{\mu}_{12}^i)^2/\widehat{\mu}_{11}^i}, \tag{5}$$

where the ion viscous coefficients are

$$\widehat{\mu}_{11}^i=\frac{\sqrt{2}\,N_iM_i}{\tau_{ii}}\,0.376\,x_i, \tag{6}$$

$$\widehat{\mu}_{12}^i=-\frac{\sqrt{2}\,N_iM_i}{\tau_{ii}}\,0.383\,x_i, \tag{7}$$

$$\widehat{\mu}_{22}^i=\frac{\sqrt{2}\,N_iM_i}{\tau_{ii}}\,0.907\,x_i, \tag{8}$$

M_i is the ion mass, N_i is the ion density, τ_{ii} is the ion-ion collision time, $x_i = 2.2\left(I_0 v_{ti0}/\Omega_{i0}\right)^{1/3} \cdot \left(q_0/\delta_0 I_0 R_0\right)^{1/3}$, v_{ti} is the ion thermal speed, and Ω_i is the ion gyrofrequency. The quantities ℓ_{ij}^{eb} are defined as $\ell_{ij}^{eb} = \ell_{ij}^{e} + \widehat{\mu}_{ij}^{e}$ where $\ell_{11}^{e} = N_e M_e/\tau_{ei}$, $\ell_{12}^{e} = -1.5\ell_{11}^{e}$, and $\ell_{22}^{e} = \left(13/4 + \sqrt{2}/Z_i\right)\ell_{11}^{e}$.

For the computational purpose, we have to connect potato asymptotic limit to the banana asymptotic limit. There is no unique way to accomplish this goal. Here, we simply join the viscosity coefficient $\widehat{\mu}_{ij}^{e}$ and $\widehat{\mu}_{ij}^{i}$ in the banana and potato limits by the following simple formula

$$\widehat{\mu}_{ij}^{e} = \left[\left(\widehat{\mu}_{ij}^{e}\right)_p^3 + \left(\widehat{\mu}_{ij}^{e}\right)_b^3\right]^{1/3}. \tag{9}$$

where $\left(\widehat{\mu}_{ij}^{e}\right)_p$ are potato viscosity coefficients given in Eqs. (2)–(4) and Eqs. (6)–(7) and $\left(\widehat{\mu}_{ij}^{e}\right)_b$ are banana viscosity coefficients given in Ref. 8. The connection formula in Eq. (9) is motivated by the observation that the potato modification on the standard banana orbit is of the order of $\left[\left(I_0 v_{tj0}/\Omega_{j0}\right)^{1/3} \Big/ \sqrt{\epsilon}\right]^3$, where ϵ is the inverse aspect ratio. Or in terms of the fraction of trapped particles f_t, $\left(f_t^p/f_t^b\right)^3$ where f_t^p is the fraction of the trapped potato which is proportional to x_j and f_t^b is the fraction of the trapped bananas which is proportional to $\sqrt{\epsilon}$. The correction of the banana orbit effects on the potato orbits is $\left(f_t^b/f_t^p\right)^4$ as shown in Ref. 7. Thus, our connection formula overestimates slightly in the transition region. One could construct a more sophisticated connection formula to account for this asymmetric asymptotic behaviors. But the resultant formula is more complicated and not necessarily more accurate.

For simplicity, we assume $T_e = T_i$ and assume that $\gamma = L_p/L_t = 0.5$. Here L_p and L_t are temperature and pressure gradient scale length. With these assumptions, parallel plasma current is completely determined by the pressure gradient. The quantity II' in Grad-Shafranov equation is determined by

$$\left\langle J_{\|} B\right\rangle = IcP' - \frac{c\langle B^2\rangle}{4\pi} I', \tag{10}$$

with the vacuum value $I = RB_t$ as the boundary condition. The $\left\langle J_{\|} B \right\rangle$ on the left-hand side of Eq. (10) is given in Eq. (1) for our equilibrium calculations. Note that because there is no other current source besides the pressure gradient driven current, our equilibrium current density profile is exactly the same as the bootstrap current density profile. This is not an assumption, but is a natural consequence of a complete pressure-gradient-driven-current tokamak.

The fact that equilibrium exist follows from a theorem in Ref. 9. It is shown that as long as pressure P and current I are analytic functions of ψ and that plasma current does not vanish on the magnetic axis, tokamak equilibrium exist. The potato bootstrap current is approximately a constant in ψ if pressure is a parabolic function of minor radius r or a linear function in ψ in the region close to the magnetic axis. Thus, the property of the potato bootstrap current is consistent with the existence theorem of the equilibria.

To demonstrate the existence of the equilibria explicitly, we solve Grad-Shafranov equation numerically with a fixed boundary code (TOQ).[3] We have found equilibria in the parameter space we have searched. Here, we only show a typical one with an aspect ratio $A = 1.4$. The vacuum magnetic field on the axis is $2T$. In Fig. 1, we show the flux surface of this particular equilibrium which has an elongation parameter $\delta = 3.0$ and a triangularity parameter $\kappa = 0.522$ at the edge. The plasma beta β on the magnetic axis is $\beta_0 = 52\%$ and the average β is 32.2%. Note that the safety factor q profile is reversed as shown in Fig. 2. The reversed q profile is natural to this class of equilibria. In fact, all the equilibria we have found so far have reversed q profile. This does not imply, however, that there are no equilibria with monotonic increasing q profiles. It is just that we have not searched for them. The pressure gradient P' profile employed is shown in Fig. 3. Note that the increasing in the magnitude of P' in the region close to the magnetic axis is to reduce the q value on the magnetic axis. The pressure profile is shown in Fig. 4. The toroidal current density profile is shown in Fig. 5 with a total current of 9.6 MA. This particular equilibrium is stable against high-n ballooning mode checked by the BALOO code.[3]. We have not studied the kink stability property, which is beyond the scope of the present paper. However, we do plan to study the kink stability for this class of equilibria in the future. We would like to note that if kink modes are unstable, they could be stabilized by a close fitting wall.

In conclusion, we have found a class of steady-state tokamak equilibria without current drive. This class of equilibria has natural reversed q profile, and is stable against high-n ballooning mode. Tokamak can, therefore, in principle, be operated in this intrinsically steady-state mode in certain parameter space.

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FIGURE CAPTIONS

FIG. 1. Flux surface of a steady-state tokamak equilibrium with $A = 1.4, \delta = 3.0$, and $\kappa = 0.522$.

FIG. 2. Safety factor q profile as a function of normalized radius $\sqrt{\psi}$.

FIG. 3. Pressure gradient profile P' as a function of $\sqrt{\psi}$.

FIG. 4. Pressure profile as a function of major radius R.

FIG. 5. Toroidal current density as a function of major radius R.