

Self-Focused Particle Beam Drivers for Plasma Wakefield Accelerators

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Abstract

Strong radial forces are experienced by the particle beam that drives the wakefield in plasma-based accelerators. These forces may destroy the beam although, under proper arrangements, they can also keep it in radial equilibrium which allows the beam to maintain the wakefield over a large distance and to provide high energy gain for the accelerated particles. This paper demonstrates the existence of acceptable equilibria for the prebunched beams and addresses the issue of optimum bunch spacing, with implications for forthcoming experiments.

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I. Introduction

The concept of plasma-based accelerators has become an area of growing interest in recent years (see, e.g., reviews^{1,2} and references therein). It offers the attractive opportunity of achieving very high accelerating gradients that exceed the limits of conventional accelerators. These promising acceleration schemes involve excitation of large amplitude plasma waves by either laser pulses or relativistic charged particle beams. These waves, with phase velocity close to the speed of light, can then accelerate particles to superhigh energies in future linear colliders.

Several experiments with laser-driven plasma waves have already been initiated. Thus far, the maximum measured energy gain reported is 44 MeV.³ The acceleration occurs in a length of 0.5 mm, which corresponds to an accelerating gradient of 90 GeV/m. More recent experiments⁴ indicate even higher gradients, up to 200 GeV/m. Plasma Wakefield Acceleration (PWFA) experiments, in which particle beams are used to drive the wave, are not as numerous. Yet they are of comparable interest with the laser experiments, for both physical and engineering reasons. A successful proof-of-principal demonstration has been presented,⁵⁻⁸ and now these experiments need to be developed to the stage where the accelerating gradients, which will be in the GeV/m range, are achieved over a macroscopic distance. The latter requires a better quality driving beam than those generally produced by linacs. One possible solution is to use a modulated beam from an electron-positron storage ring, as proposed in.⁹⁻¹¹ The numerical simulations with NOVOCODE^{9,12} have shown that a train of N particle bunches of moderate density $n_b \sim n_p/N$, where n_p is the plasma density, can generate the accelerating electric field E_z with amplitude up to the wave-breaking limit $E_0 = \sqrt{4\pi n_p m c^2}$.

The present paper is a part of theoretical studies related to the experiment proposed

in.⁹⁻¹¹ These studies include the analysis of plasma wave nonlinearity and address the question of whether the beam can maintain its structure in a plasma over a sufficiently long distance without being destroyed by the excited plasma waves. We drop the “rigid” driver assumption used in previous studies⁹⁻¹² and take into account the effect of plasma fields on the driver radial equilibrium. We also consider the problem of bunch sequence optimization for the case of nonlinear plasma response.

There are three different spatial scales involved in the problem. The shortest scale is the wavelength, c/ω_p , of the excited wakefield, where $\omega_p = \sqrt{4\pi n_p e^2/m}$ is the plasma frequency. Next in order is the focusing–defocusing length associated with the radial force, F_r , acting on the driving beam. This force tends to change the beam radius a over the distance $L_F \sim c \sqrt{\gamma m a / |F_r|}$, where $\gamma \gg 1$ is the relativistic factor of the driver. It is natural to choose a to be of the order of c/ω_p . A much greater value of a , at a given beam density, cannot substantially increase the accelerating gradient but it would require a beam of a much greater total current. On the other hand, having $a \ll c/\omega_p$ would make the wave excitation less efficient. With $F_r = eE_0$ and $a = c/\omega_p$, we obtain $L_F \sim \sqrt{\gamma} c/\omega_p \gg c/\omega_p$. Note that, apart from a numerical coefficient, L_F is the beta function of the driver in the plasma. The largest of the three spatial scale lengths is the deceleration length of the driving beam L_d , the distance at which beam particles lose about half of their energy. For $E_z = E_0$ we have

$$L_d \sim \frac{\gamma c}{\omega_p} \gg L_F \gg \frac{c}{\omega_p}. \quad (1)$$

The rigid beam approximation is valid only on the shortest scale length, c/ω_p . On the focusing scale length, the radial dynamics of the beam becomes important. Depending on the sign of the radial force, the particles are either confined or pushed out and lost. This loss is very fast compared to the energy loss rate. Therefore, it can be treated as instantaneous on the driver deceleration scale length. This raises the question of whether the beam can reach a radial equilibrium such that all the beam particles would experience an inward radial

force while they are slowed down by the self-generated longitudinal field. This would allow the maintenance of a suitable structure of the wakefield over a distance that exceeds the focusing length. Note that such an equilibrium may in fact be reached automatically as a result of the beam self-modulation caused by the radial forces. However, it is not necessary to follow the actual radial dynamics of the beam in order to check the existence of these equilibria or to find them. What can be done instead is a procedure that modifies the beam profile by eliminating the defocused particles and calculates the fields self-consistently.

In this paper we use a modified version of NOVOCODE that includes such a procedure, and we demonstrate the existence of radial equilibria of longitudinally modulated beams. Once a suitable beam density profile is obtained from the calculations, the beam distribution function over the transverse momenta can then be found to match this profile to the shape of the radial potential well. Since there is actually no restriction on the radial profile of the beam other than being reasonably smooth and providing a potential well for the particles, one can construct solutions for which the beam distribution function is compatible with experimental constraints.

The rest of the paper is organized as follows. In Sec. II, we construct the radially self-focused density profiles for the bunched beams. Section III deals with the distribution function of beam particles. In Sec. IV, we analyze the factors that determine the optimum initial modulation of the beam. Section V summarizes the results.

II. Self-focused bunched beam

The original version of NOVOCODE calculates the fields generated in a plasma by a rigid axisymmetric ultrarelativistic driving beam of a given current density profile $\mathbf{j}_b = qn_b c\mathbf{z}$, where q is the beam particle charge, \mathbf{z} is the unit vector along the beam axis, and $n_b(r, z - ct)$

is the beam density. Unless specified otherwise, we choose the beam density to have the form

$$n_b(r, z - ct) = n_b(0, z - ct) \exp\left(-\frac{r^2}{2a^2}\right), \quad (2)$$

where $n_b(0, z - ct)$ is the beam density on the axis. The code solves the Maxwell equations

$$\text{rot}\mathbf{H} = \frac{4\pi}{c} (\mathbf{j}_b - en\mathbf{v}) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (3)$$

$$\text{rot}\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t}, \quad (4)$$

with the fluid equations for plasma electrons

$$\frac{\partial n}{\partial t} + \text{div}(n\mathbf{v}) = 0, \quad (5)$$

$$\frac{\partial \mathbf{p}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{p} = -e\mathbf{E} - \frac{e}{c} [\mathbf{v} \times \mathbf{H}]. \quad (6)$$

Here $-e$ with $e > 0$ is the electron charge; n , \mathbf{v} , and \mathbf{p} are the density, velocity, and momentum of plasma electrons, respectively; and plasma ions are treated as an immobile homogeneous background.

Equations (3)–(6) allow a traveling wave solution moving with the beam so that all quantities depend on $z - ct$ rather than on z and t separately. In this solution, the Lorentz force $q(\mathbf{E} + [\mathbf{z} \times \mathbf{H}])$ acting on a ultrarelativistic driving beam can be represented by a potential Φ as

$$q(\mathbf{E} + [\mathbf{z} \times \mathbf{H}]) = -q\nabla\Phi \quad (7)$$

with $\Phi = 0$ at $r \rightarrow \infty$.

For an arbitrary profile of n_b , the radial dependence $\Phi(r)$ in some of the beam slices may force particles to leave the beam quickly. This typically happens on the focusing scale length, i.e., much faster than the rate at which the particles slow down. Therefore, it is interesting to find the beam profiles such that $n_b = 0$ in the regions where the radial potential is unable

to confine the particles. With this motivation, we use the following procedure to modify the beam. We first choose a set of seed bunches spaced one plasma wavelength apart so that

$$n_b(0, -x) = \begin{cases} n_0, & \text{if } 2\pi nc/\omega_p < x < (2n+1)\pi c/\omega_p, \quad n = 0, 1, \dots; \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

We then calculate the fields generated in the plasma by integrating Eqs. (3)–(6) in time starting from the leading edge of the beam. For every time step, we check whether the radial potential $\Phi(r)$ in the corresponding beam slice confines particles or pushes them out, and we eliminate the unconfined part of the beam slice. The resulting electric field on the beam axis and the areas occupied by the modified beam are shown in Fig. 1 (for $q < 0$). Different shades in Fig. 1a indicate the radial fall-off of the beam density. Note that some particles of the modified beam are in the accelerating phase of the longitudinal field. These particles tend to suppress the wave rather than amplify it. In addition, the partially depleted slices make the radial profile of the beam somewhat artificial from a practical point of view. This suggests the idea of removing both the accelerated particles and the incomplete slices from the beam, which should give a stronger wakefield as well as a more natural beam shape.

In order to construct the improved beam, we start from the seed profile

$$n_b(0, -x) = \begin{cases} 0, & \text{if } x < 0, \\ n_0, & \text{if } x > 0, \end{cases} \quad (9)$$

and, while computing the fields, we totally remove the slices that contain either defocused or accelerated particles, or both. Since some of the particles removed by this procedure cannot leave the beam by themselves, the procedure implies that the beam needs to be pre-shaped by a chopping device of a kind discussed in.^{10,11} Our simulations give an example of how the beam should be chopped in order to generate a desired wakefield that keeps the beam particles focused.

The corresponding profiles for the negatively charged and the positively charged drivers are shown in Fig. 2. This figure also shows the accelerating gradient and the potential Φ on

the beam axis. Note that the wakefield excited by the optimized driver (Fig. 2) is indeed stronger than that shown in Fig. 1.

The two-dimensional plot of $\Phi(r, z - ct)$ for the electron driver (Fig. 3) shows that the excited wave has a regular structure despite considerable nonlinear distortions. As the calculations continue after the last bunch of the driver, the wave eventually breaks. The larger the wave amplitude, the shorter is the interval between the last bunch and the onset of wave-breaking. For the example presented in Fig. 3, this interval is about two plasma wavelengths. Thus, the wakefield bucket next to the last driving bunch can provide acceleration and focusing for properly positioned particles.

III. Kinetic equilibrium of the driver

It has already been pointed out in Sec. 1 that the beam focusing/defocusing length L_F is typically much shorter than the deceleration length L_d . Therefore, the particles that are confined by the focusing field make many radial oscillations over their deceleration length. The particle orbits cross during these oscillations since different particles oscillate with different frequencies. This makes the description of the driving beam an essentially kinetic problem, whereas the fluid model still applies to the plasma. The kinetic equation for the beam distribution function f has the form

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} + \mathbf{v}_\perp \frac{\partial f}{\partial \mathbf{r}} + q \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{H}] \right)_z \frac{\partial f}{\partial p_z} + q \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{H}] \right)_\perp \cdot \frac{\partial f}{\partial \mathbf{p}_\perp} = 0, \quad (10)$$

where z is the coordinate along the beam axis, \mathbf{r} is the two-dimensional vector perpendicular to the axis, and the subscript \perp refers to the perpendicular components of the vectors.

The electric field \mathbf{E} excited by an axisymmetric driver has two nonzero components, E_r and E_z ; the azimuthal component of \mathbf{E} vanishes. The only nonzero component of the corresponding magnetic field is H_φ . The fields E_r , E_z , and H_φ generated by a charged bunch of size c/ω_p (in both the radial and the longitudinal directions) are generally of the same

order of magnitude. This means that, for a beam with a sufficiently small angular spread, one can neglect the magnetic contribution to the longitudinal force in Eq. (10).

We then introduce the function

$$F = \int f dp_z, \quad (11)$$

which is the beam distribution over transverse momenta, and integrate Eq. (10) over p_z , assuming that the beam spread in p_z is sufficiently small. This integration reduces Eq. (10) to the following equation for F :

$$\frac{\partial F}{\partial t} + c \frac{\partial F}{\partial z} + \frac{c}{p_0} \mathbf{p}_\perp \cdot \frac{\partial F}{\partial \mathbf{r}} - q \nabla \Phi \cdot \frac{\partial F}{\partial \mathbf{p}_\perp} = 0, \quad (12)$$

where p_0 is the average z -component of the particle momentum and Φ is the potential defined by Eq. (7). In this equation, we have neglected the difference between the average z -velocity of the beam and the speed of light.

We now take into account Eq. (1) and consider a z -interval that is much longer than L_F but much shorter than L_d . This allows us to construct a radial equilibrium in the cylindrical coordinate system $(r, \varphi, z - ct)$. The equilibrium distribution satisfies the equation

$$\frac{c}{p_0} \mathbf{p}_\perp \cdot \frac{\partial F}{\partial \mathbf{r}} - q \nabla \Phi \cdot \frac{\partial F}{\partial \mathbf{p}_\perp} = 0. \quad (13)$$

It follows from Eq. (13) that the equilibrium distribution function is generally a function of two constants of motion: the perpendicular energy

$$W = \frac{cp_\perp^2}{2p_0} + q\Phi \quad (14)$$

and the z -component of the angular momentum, $M_z = [\mathbf{r} \times \mathbf{p}]_z$. In order to simplify the problem, we assume that the distribution function has no M_z dependence and also that F vanishes for $W \geq 0$, which means that all the beam particles are trapped in the radial potential well formed by the focusing force. We now use this distribution to calculate the

beam current density j_z :

$$j_z = 2\pi q p_0 \int_{q\Phi(r)}^0 F(W) dW. \quad (15)$$

In addition to Eq. (15), Φ is related to j_z by the solution of Eqs. (3)–(6). Once this additional relationship is found, subject to the constraints that all the beam particles experience radial focusing and that both j_z and Φ are monotonic functions of r , one can use Eq. (15) to obtain the equilibrium distribution function

$$F(q\Phi) = -\frac{1}{2\pi q^2 p_0} \left(\frac{\partial j_z}{\partial \Phi} \right)_{z-ct}. \quad (16)$$

This result shows the existence of many radial equilibria for longitudinally modulated beams for a broad range of beam current profiles. If applied to the optimized beam of Fig. 2 described in Sec. II, Eq. (16) indicates that the beam emittance must change along the bunch as the square root of Φ in order to keep the bunch radius constant.

Such a distribution may self-establish dynamically when the prebunched beam has a very low initial emittance. The excited wave forces the beam particles to oscillate in the radial potential well. Particle orbits in phase space tend to mix during these oscillations. This mechanism can increase the beam emittance to an equilibrium level. Depending on initial conditions, different equilibria can be reached as a result of the radial dynamics. A particle code is now under development to solve the dynamical problem, which will allow checking whether the beam can indeed relax to the described equilibrium.

Since the beam continuously loses its energy as it propagates through the plasma, its equilibrium distribution $F(W; M_z)$ has to evolve on a slow time scale. The perpendicular energy W will not be a conserved quantity in this case, but instead a radial adiabatic invariant, J , can be introduced, so that F becomes a function of J and M_z . The generalization of our results to this case is straightforward.

IV. Optimum bunch spacing

The results presented in Fig. 3 show that the lengths of the oscillation cycles of the potential Φ differ from the natural period $2\pi c/\omega_p$ with some variations along the beam. We find that this deviation stems from three effects.

The first effect is independent of the driver density. It results from the system's memory about initial conditions. This can be illustrated on a simple model of a linear oscillator driven by an external force:

$$\ddot{x} + x = f(x, \dot{x}), \quad (17)$$

where

$$f(x, \dot{x}) = \begin{cases} 1, & \text{if } x > 0 \text{ and } \dot{x} > 0, \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

Equations (17) and (18) allow a straightforward analytical solution. Assuming zero initial conditions for x and \dot{x} , we find the following sequence of cycles for $x(t)$:

$$x_n = \begin{cases} 1 - \cos(t - t_n) + R_n \sin(t - t_n), & t_n < t < t_n + \varphi_n + \pi/2, \\ (1 + \sqrt{1 + R_n^2}) \cos(t - t_n - \varphi_n - \pi/2), & t_n + \varphi_n + \pi/2 < t < t_n + \varphi_n + 2\pi, \end{cases} \quad (19)$$

where $\varphi_n = \arctan(1/R_n)$, and t_n and R_n satisfy the following recursion equations:

$$R_0 = 0, \quad R_{n+1} = 1 + \sqrt{1 + R_n^2}, \quad (20)$$

$$t_0 = 0, \quad t_{n+1} = t_n + \arctan(1/R_n) + 2\pi. \quad (21)$$

This solution shows that every cycle is longer than the natural period 2π by the phase shift φ_n . At $n \rightarrow \infty$ the cycle length asymptotically approaches 2π . It is also interesting to note that these forced cycles accumulate a logarithmically growing total phase shift with respect to the phase of a free oscillator.

The second factor that affects the wave period is the flow of plasma electrons neutralizing the average current of the driving beam. The corresponding drift velocity can be roughly

estimated as

$$|\bar{v}_z| \simeq c \frac{\bar{n}_b}{n_p}, \quad (22)$$

where \bar{n}_b is the average driver density and we assume $a \sim c/\omega_p$. The electron flow results in a Doppler shift of the wave frequency, which has opposite signs for positively charged and negatively charged drivers. This explains why the wave periods for the positive driver (Fig. 2b) are systematically shorter than those for the negative one (Fig. 2a). The average electron flow is also seen in the contour plots of the perturbed electron density (Fig. 4). Note that the contours have opposite tilts for positive and negative drivers. The electron drift is generally inhomogeneous, which distorts the wave profile compared to that predicted by the linear theory. A consequence of such a distortion is a non-monotonic radial profile of the potential Φ at some z -locations.

The third effect is that the wave period changes due to the relativistic reduction of the plasma frequency in the nonlinear wave.¹³ This mechanism should increase the period towards the end of the driving pulse. However, in our calculations, the resulting correction to the period is roughly an order of magnitude smaller than the other two corrections.

An interesting manifestation of wave nonlinearity is that the regions of favorable focusing for a positively charged driver are shorter than those for a negatively charged one (Fig. 2). A positive driver has to be localized within the narrow peaks of the plasma electron density in Fig. 5, while a negative driver can occupy the broad regions of density depletion. Therefore, for the same values of n_0 and a , the wake-field is somewhat stronger for a negative driver.

An important practical question from the experimental point of view is how sensitive are the amplitude and the phase of the wakefield to variations of the plasma density. In order to get an idea of this sensitivity, we take the equilibrium beam profiles presented in Fig. 2 and use them as seed profiles in the calculations with somewhat different plasma density. Similar to the run that led to Fig. 1, we modify the seed beam by eliminating the defocused particles

and calculate the resulting accelerating gradient at a given distance from the leading edge of the beam. The dependence of this gradient on the plasma density is shown in Fig. 6 by solid curves. We also plot the maximum accelerating gradient after the fifth beam pulse (dotted curves). Figure 6 indicates that the phase of the excited wave is more sensitive to density variations than the wave amplitude. The asymmetry of the solid curves in Fig. 6 suggests that it might be useful to choose the operational point for the experiment on the gentle slope of the curve rather than at the maximum. This will make the accelerating gradient less susceptible to possible errors.

V. Summary

In this paper, we have shown that a modulated driving beam can efficiently excite non-linear plasma waves with amplitude up to the wavebreaking limit without being destroyed by the radial defocusing forces. We have found self-consistent radial equilibria that allow the beam to propagate a distance on the order of its slowing down length. In this case, a single stage of acceleration in the beam wakefield should give an energy gain (per particle) that is roughly equal to the particle energy in the driver. For the proposed Novosibirsk plasma wakefield acceleration experiment⁹⁻¹¹ we expect a gain of about 0.5 GeV at the distance of 0.5 meter. Although it is conceivable that the driving beam will reach a self-modulated equilibrium state in the plasma automatically, an appropriate pre-modulation of the driver can make the outcome much more predictable, which is very important for achieving the maximum energy gain and for being able to control the fields acting on the accelerated particles. It can also be important to pre-arrange the driver emittance in accordance with the desired equilibrium profile in order to minimize uncontrollable radial oscillations. Both the modulation and the emittance adjustment can be done with the same chopper. The results presented in this paper allow optimization of the chopper design for the experiments. This optimization will take into account the beam emittance requirements and the phase shifts

of the excited nonlinear wave with respect to the linear plasma wave. Our results also allow determination of a tolerable range for the plasma density variations in the experiment.

The accelerated bunch needs to be placed in the region where there is a radial potential well. Large radial forces in this region generally raise concerns about the bunch emittance. A possible way to keep the emittance low is to make the bunch radius much smaller than the wavelength (without degrading the efficiency). The length of the bunch should also be small compared to the wavelength (in order to reduce the energy spread of the accelerated particles), but it can still be much greater than the bunch radius. There can be fairly strong local perturbations of plasma electrons near this narrow bunch, which may enhance the bunch energy losses. However, a separate analysis shows that the losses cannot compete with the maximum accelerating gradient in the plasma wave as long as the total charge of the bunch is much smaller than the total charge of plasma electrons per one cubic wavelength.

Another important question is whether the equilibria found in this paper are stable with respect to nonaxisymmetric perturbations and whether similar equilibria exist for nonaxisymmetric drivers. More work is needed to address this issue quantitatively. This involves development of a hybrid code that will combine the fluid description of the background plasma with particle simulations of the beam dynamics.

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FIGURE CAPTIONS

FIG. 1. Modification of a set of equidistant rectangular bunches, due to particle defocusing by the generated plasma wave. (a) Final configuration of the beam after all defocused particles are lost. Labels f and d mark the areas of focusing and defocusing, respectively. (b) On-axis electric field generated by the modified beam.

FIG. 2. On-axis electric field E_z and potential Φ generated by chopped self-focused drivers with optimized spacing. The drivers have a Gaussian radial profile with the peak density $0.2 n_p$ and the radius $a = c/\omega_p$: (a) negatively charged driver; (b) positively charged driver.

FIG. 3. Wakefield potential $\Phi(r, z - ct)$ for the run presented in Fig. 2a.

FIG. 4. Contour plots of plasma electron density $n(r, z - ct)$ for the runs presented in Fig. 2a: (a) negatively charged driver (Fig. 2a), (b) positively charged driver (Fig. 2b).

FIG. 5. Perturbed plasma electron density $n(r, z - ct)$ for the run presented in Fig. 2a.

FIG. 6. Effect of plasma density mismatch $\delta n_p/n_p$ on the wakefield generated by the optimized beams of Fig. 2. Solid curves show the accelerating gradient at a given distance after the last bunch, dotted curves the maximum gradient after the last bunch. (a) negatively charged driver (Fig. 2a), (b) positively charged driver (Fig. 2b).