# On the Radial Profile and Scaling of Ion Thermal Conductivity from Toroidal ITG Mode

J.-Y. Kim and Y. Kishimoto

*Naka Fusion Research, Japan Atomic Energy Research Institute, Japan*

W. Horton and T. Tajima

*Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712*

M. Wakatani

*Plasma Physics Laboratory, Kyoto University, Uji 611, Japan*

(April 29, 1997)

## Abstract

The issue of the physics responsible for the radial increase and scaling problems of the ion thermal conductivity $\chi_i$ is presented within the framework of the toroidal ion temperature gradient (ITG) mode. The observed radial increase of $\chi_i$ with its large value near the edge region, which was difficult to explain from the slab-like ITG mode with the gyro-Bohm type $\chi_i$, can be explained well in terms of the toroidal ITG mode with Bohm-type $\chi_i$. Against several previous arguments, it is shown how such a Bohm-like scaling is possible from the toroidal ITG mode.

Typeset using REVTEX

## I. INTRODUCTION

As a candidate to explain the ion thermal transport in high ion temperature tokamak plasmas, the ion temperature gradient (ITG) mode[1−5] has received steady attention. Numerous works have been performed to explain the anomalous ion thermal transport in terms of the ITG mode. A well-known conclusion from these studies is that the ITG transport model has severe difficulty in explaining the observed radial increase of fluctuation level and the thermal conductivity (see Refs. 6 and 7 for a review of such previous works).

Recently, some progress has been made on this problem. More exact calculations of the linear stability of the toroidal ITG mode using advanced numerical tools[9−15] have indicated that the actual ion temperature profile is near marginal or $\eta_i \sim \eta_c$ in the core region, where $\eta_i \equiv L_n/L_{T_i}$ with the density and ion-temperature scale lengths, $L_n$ and $L_{T_i}$, and $\eta_c$ is its threshold value. The radial increase of the ion thermal conductivity $\chi_i$ may be then simply be explained as a consequence of the radial increase of $\eta_i - \eta_c$ or the growth rate. More specifically, the problem is seen when we estimate the ion thermal conductivity from the usual mixing length formula,

$$\chi_i(r) = \gamma_{\max}(r)/k_r^2, \tag{1}$$

where $\gamma_{\max}$ is the local ($r$) growth rate maximized over $k_\theta \rho_i$ and $k_r$ is its radial wavenumber. From Eq. (1) $\chi_i(r)$ is expected to decrease radially due to the strong temperature dependence; $\chi_i \propto T_i^{3/2}$ when we assume the usual slab-like mode structure $k_r \sim k_\theta \sim 1/\rho_i$, where $k_\theta$ is the poloidal wavenumber and $\rho_i$ is the ion Larmor radius. This radial decrease can be overcome if the growth rate $\gamma_{\max}$ (proportional to $\eta_i - \eta_c$) has a stronger radial increase. Recent advanced calculations now show that the actual ion temperature has such a profile so that a radially increasing $\chi_i(r)$ is possible inside the core region from this transition.[7,9]

Despite this progress, there still remain, at least, two questions which should be addressed for a clear understanding of the radial profile problem. The first is why the ion temperature profile has such a feature, i.e., its deviation from marginality increasing radially. The second is that even though $\chi_i$ can increase radially by the above argument, the magnitude of the increase from this effect is much smaller than the observed value near the edge region when the usual nearly isotropic mixing length formula with $k_r \sim k_\theta$ are used. This is a well-recognized fact from many previous works (see Refs. 6–8). For a complete explanation over all plasma region, including near the edge region, it is thus necessary to seek a different form of $\chi_i$ (or another instability which is strongly unstable near the edge region). Such an example is indeed found in a recent work,[9] where a formula of $\chi_i$ is constructed from intensive numerical simulations of the toroidal ITG mode using a linear gyrokinetic code and nonlinear toroidal gyro-fluid code. Transport simulation based on this formula predicts the central temperature of TFTR L-mode plasma very well and the model is being applied to other major tokamaks as well. Comparing the formula in Ref. 9 with the usual mixing length formula (1) with $k_r \sim k_\theta$, we can see that the formula in Ref. 9 has a coefficient that is approximately one order of magnitude larger than that of the usual formula, while both are the same gyro-Bohm scaling. Due to this difference in the coefficient, the formula in Ref. 9 can yield the large $\chi_i$ near the edge region. However, we note that a critical problem of this model is that the physical origin of the large coefficient and its scaling is not clear.

A purpose of the present work is to provide insight into the above-mentioned questions. For the first question, we will show that the radial increase of $\eta_i - \eta_c$ and $\chi_i$ is indeed the feature expected from the steady state dynamics when we assume the transport is governed by the ITG mode. For the second question, we will show that the large $\chi_i$ near the edge region can be explained simply by assuming that $\chi_i$ has Bohm-like form, or *equivalently* $k_r$ is order of $1/\sqrt{L\rho_i}$, where $L$ is the equilibrium scale length. From Eq. (1) it is easy to see that with the Bohm-type, $\chi_i$ indeed becomes much larger than the gyro-Bohm case for a given growth rate due to the intermediate or mesoscale length of the radial correlation length. Radial correlation lengths of this size ($\sqrt{L\rho_i}$) are consistent both with measurements of the turbulence and computer simulations. Note that $\rho_i$ is of order a few millimeters whereas $\sqrt{L\rho_i}$ is of order a few centimeters in the major tokamaks.

A critical point for this model is whether such a Bohm-type diffusivity is actually possible from the toroidal ITG mode. Here, we note that it still remains uncertain whether the $\chi_i$ from the toroidal ITG mode has Bohm or gyro-Bohm scaling. It was well recognized earlier that the toroidal ITG mode has the global mode width of order $\sqrt{L\rho_i}$.[10,4] If we take this width as the radial mixing length the Bohm-type $\chi_i$ is obtained, but this choice has been typically denied in most previous works based on some physical arguments that we explain below. Recently, with the advance in the full toroidal simulation of the toroidal ITG mode,[8,11,12] this problem has received more extensive interest, but still there is no definite conclusion (most global simulation results indicate Bohm-like scaling[13,14] while local simulations in flux tube geometry show gyro-Bohm-like scaling.[9,15]) Meanwhile, experimentally, many recent power balance analyses in the Tokamak Fusion Test Reactor (TFTR),[16] Joint European Torus (JET),[17] DIII-D,[18] and JT-60U[19] indicate that the experimental $\chi_i$ is closer to the Bohm-type, rather than the gyro-Bohm, scaling. Here we present the case for the Bohm-type scaling from direct physical arguments.

## II. RADIAL INCREASE IN $\eta_I - \eta_C$ AND $\chi_I$

From a consideration of steady state dynamics, we first show that the radial increase in $\eta - \eta_c$ and $\chi_i$ is indeed an expected consequence of the ITG mode driven transport (independently of the details of whether the mode is the slab or toroidal ITG). Here, by the steady state dynamics we mean how plasma systems move to a steady state under an external heating ($P_{i,e}$) through the self-consistent time evolution of the ion temperature profile and the fluctuations. For the argument, we assume two well-known basic properties of ITG: that it becomes unstable when $\eta_i > \eta_c$ and that it has the growth rate proportional to $T_i^{1/2}$ (at

$\eta_i > \eta_c$). We first present a quantitative analysis and then a heuristic argument for the dynamics to the steady state.

Let us start from the following model equations for the evolution of fluctuation and ion temperature,

$$\frac{\partial E}{\partial t} = 2\left[\widehat{\gamma}_L(\eta_i - \eta_c) - \widehat{\gamma}_N E\right] E, \tag{2}$$

and

$$\frac{\partial T_i}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\chi_i \frac{\partial T_i}{\partial r}\right) + P_i, \tag{3}$$

where $\sqrt{E} = |e\phi/T_i|$ is a normalized fluctuation amplitude, $\widehat{\gamma}_L$ and $\widehat{\gamma}_N$ represent the coefficients of linear growth rate and nonlinear saturation force terms, respectively, and $P_i(r)$ is the net input power per ion typically $> 100\,\mathrm{kev/s}$. Equation (2) describes the force balance between the linear driving and nonlinear saturation forces, while Eq. (3) is the usual ion heat balance equation. Note that in Eq. (2) we have assumed the linear growth rate form: $\gamma_L = \widehat{\gamma}_L(\eta_i - \eta_c)$, and the nonlinear saturation rate form: $\gamma_N = \widehat{\gamma}_N E$. Equations (2) and (3) consist of a set of coupled equations describing the time evolution of $E$ and $T_i$ (or $\eta_i$), when we neglect the time-variation of the density profile and assume the usual quasilinear relation $\chi_i = \chi_0 E$ between $\chi_i$ and $E$.

In the steady state limit, in which we are interested here, Eqs. (2) and (3) give

$$E = \frac{\widehat{\gamma}_L}{\widehat{\gamma}_N}(\eta_i - \eta_c), \tag{4}$$

and

$$E\eta_i = \frac{L_n}{r\chi_0 T_i}\int_0^r r' P_i(r')dr', \tag{5}$$

respectively. The radial profiles of the steady state $E(r)$ and $\eta_i - \eta_c$ are then readily obtained as

$$E(r) = \frac{\eta_c(r)}{2\alpha(r)} \left[ -1 + \sqrt{1 + \frac{4\alpha(r) L_n(r) Q_i(r)}{\chi_0(r)\eta_c^2(r) T_i(r)}} \;\right], \tag{6}$$

and

$$\eta_i(r) - \eta_c(r) = \alpha(r) E(r), \tag{7}$$

where $\alpha(r) \equiv \widehat{\gamma}_N(r)/\widehat{\gamma}_L(r)$ and $Q_i(r) \equiv \int_0^r dr' r' P_i(r')/r$. The steady state profile of $T_i(r)$ can be also calculated from Eq. (7) with a boundary (edge) ion temperature.

The fact that the steady state $E(r)$ and $\eta_i - \eta_c$ have radially increasing profiles can now be seen from Eqs. (6) and (7). From Eq. (6), we first have $E(r) \simeq L_n Q_i/\chi_0 \eta_c T_i$ when $\alpha < \overline{\alpha} \equiv \chi_0 \eta_c^2 T_i / 4 L_n Q_i$, while $E(r) \simeq \sqrt{L_n Q_i/\chi_0 \alpha T_i}$ when $\alpha > \overline{\alpha}$. Here, we note the following dependences; $\chi_0 \propto k_y \rho_i v_i L_T \propto T_i^{1/2}$ from the usual quasilinear formula of $\chi_i$ (for example, see Ref. 14) and $\alpha \propto T_i^{-1/2}$ or $T_i^{-1}$ for the toroidal or slab-like regimes where the radial mode widths are $\sqrt{\rho_i L}$ and $\rho_i$, respectively (these will be shown later). Assuming that the radial variation of the other quantities such as $Q_i(r)$, $L_n$, and $\eta_c$ are relatively weak, compared with the temperature variation, we then have

$$E(r) \propto \frac{1}{T_i^{3/2}} \quad \text{for} \;\; \alpha < \overline{\alpha}, \qquad E(r) \propto \frac{1}{T_i^{1/2}} \quad \text{for} \;\; \alpha > \overline{\alpha}, \tag{8}$$

for the toroidal mode, while

$$E(r) \propto \frac{1}{T_i^{3/2}} \quad \text{for} \;\; \alpha < \overline{\alpha}, \qquad E(r) \propto \frac{1}{T_i^{1/4}} \quad \text{for} \;\; \alpha > \overline{\alpha}, \tag{9}$$

for the slab-like mode. (The normalized nonlinear restoring force is $\alpha$ and the normalized ion heating power is $1/\overline{\alpha}$.) From these we can see clearly that the steady state fluctuation amplitude $E(r)$ increases radially for the usual radially decreasing temperature profile [the radial decrease of $T_i(r)$ can be easily seen from Eq. (7) which gives roughly $T_i(r) \sim T_i(0)\exp(-\eta_c r)$ for $\eta_i \sim \eta_c$]. Noting $\alpha \propto T_i^{-1/2}$ or $T_i^{-1}$, a more rapid radial increase is also expected for the $\eta_i - \eta_c$ from Eq. (7).

The radial increase in $E(r)$ and $\eta_i - \eta_c$ thus appears as a consequence of the self-consistent steady state limit from the coupled fluctuation and temperature evolution equations. A more physical understanding of this feature follows from the following consideration of the dynamics. Under the external heating, the ion temperature is expected *first* to take a marginal profile, decreasing radially with a boundary temperature. Different behavior is then expected in the ITG mode excitation and the temperature profile evolution between the core and edge regions. In the core region with higher temperature a faster growth of the ITG mode occurs, removing more quickly the external heat load. Meanwhile, in the outer core edge region with a smaller growth rate the heat removal is slower, which will then permit the build-up of a steeper temperature profile or a larger deviation of $\eta_i$ from $\eta_c$. As a result, the edge region develops a larger $\eta_i - \eta_c$ resulting in a steady growth of the ITG fluctuations, until stopped by the nonlinear saturation force. (Meanwhile, in the core region with a larger heat removal ability, the growth of the ITG mode will be stopped soon by the rapid relaxation of the ion temperature profile into near the marginal profile). The heat removal ability of the edge region increases during this second phase of the growth both in the mode amplitude $|\phi|$ and $\eta_i$ [note from Eq. (3) the ITG mode driven heat flux $q_i(r) \propto \chi_i(r)\eta_i(r)$ with $\chi_i(r) \propto |\phi|^2$]. In the *second* stage a steady state will finally be established when the heat removal rates between the inside core and outside edge regions balance each other.

From the above arguments, it is clear that the outer edge region with a lower temperature is driven to have a larger $\eta_i - \eta_c$ and $|\phi|$, in order to make the system arrive at a steady state. This means that the radial increase of $\eta_i - \eta_c$ or $|\phi|$ is indeed a consequence of the self-consistent ITG mode driven transport in the steady state limit (where the power balance analyses of $\chi_i$ are performed).

## III. RATE OF RADIAL INCREASE IN $\eta_I - \eta_C$ AND FLUCTUATION INTENSITY

In the previous section, we have shown that the radial increase of $\eta_i - \eta_c$ and $E(r)$ is a necessary consequence of the steady state limit. We now consider the other problem: what is the rate of radial increase in these quantities and how much is it different between the slab-like and the toroidal modes. From this consideration we will get the actual profiles and magnitudes of $\eta_i - \eta_c$, $E$, and $\chi_i$.

From the earlier steady state dynamics consideration, we can see first that with a stronger nonlinear saturation force the fluctuation is limited to a smaller amplitude so that a larger $\eta_i - \eta_c$ must develop for the system to arrive at a steady state. In other words, this means that with a larger nonlinear saturation force $\alpha$ the radial increase rate will decrease for the saturation amplitude $E(r)$, while increase for the $\eta_i - \eta_c$. This can be seen more quantitatively from Eqs. (6) and (7), which show

$$E \propto \alpha^0 \quad \text{for} \quad \alpha < \overline{\alpha}, \qquad E \propto \alpha^{-1/2} \quad \text{for} \quad \alpha > \overline{\alpha}, \tag{10}$$

and

$$\eta_i - \eta_c \propto \alpha \quad \text{for} \quad \alpha < \overline{\alpha}, \qquad \eta_i - \eta_c \propto \alpha^{1/2} \quad \text{for} \quad \alpha > \overline{\alpha}. \tag{11}$$

We see clearly that with increasing $\alpha$, $E$ decreases while $\eta_i - \eta_c$ increases.

The radial rates of increase of $\eta_i - \eta_c$ and $E(r)$ are thus closely related to the strength of the nonlinear saturation force $\alpha$. Thus we need to present a simple summary of the nonlinear saturation force physics. Especially, we will concentrate on how the nonlinear force is different between the slab-like mode with the gyro-Bohm $\chi_i$ and the toroidal mode which has the Bohm-type $\chi_i$ (in Sec. IV we will discuss how the Bohm-type $\chi_i$ is possible from the toroidal ITG mode).

For this purpose, it is enough to note that the nonlinear saturation force $\gamma_N$ has the following $k_r$ dependence,

$$\gamma_N \propto k_r^2. \tag{12}$$

The origin of this $k_r$ dependence can be easily understood from the $\mathbf{E} \times \mathbf{B}$ convective nonlinear interaction form of the drift waves,

$$\mathbf{b} \times \nabla\phi \cdot \nabla f, \tag{13}$$

which indicates that the nonlinear interaction rate through beat waves is proportional to

$$\left(\frac{\partial}{\partial r}\right)^2 \sim k_r^2, \tag{14}$$

where $\mathbf{b} = \mathbf{B}/\mathrm{B}$, $\phi$ is the perturbed electrostatic potential, and $f$ is some fluctuating quantity. The same $k_r$ dependence of $\gamma_N$ is also implied from the form of the mixing length formula (1), which can be obtained from the quasilinear relation $\chi_i = \chi_0 E$, with the saturation amplitude (4), if we recognize that

$$\widehat{\gamma}_N = \chi_0 k_r^2. \tag{15}$$

From the above $k_r$ dependence of $\gamma_N$, we can now get easily an estimate of the difference in the nonlinear saturation force between the slab-like mode with the gyro-Bohm $\chi_i$ or $k_r \sim k_\theta \sim 1/\rho_i$ and the toroidal mode assumed to have the Bohm-type $\chi_i$ or $k_r \sim 1/\sqrt{\rho_i L}$. We see that the nonlinear saturation force is smaller, roughly by $\rho_i/L$, for the toroidal mode, compared with the slab mode. This means that the toroidal mode has a much weaker nonlinear saturation force than the slab mode.

The above result leads to two important occurrences:

(i) That the actual magnitudes or profiles of the steady state $\eta_i - \eta_c$ and $E(r)$ will be significantly different between the slab and toroidal modes. For the toroidal mode with a weaker nonlinear saturation force we can expect a much slower radial increase in $\eta_i - \eta_c$, while a more rapid increase in $E(r)$ [see Eqs. (10) and (11)]. Here, let us recall that the slab-like mode with the gyro-Bohm $\chi_i$ has severe difficulty in explaining the large $\chi_i$ near the edge region. A basic reason of this failure can now be explained as due to the nonlinear saturation force of the slab-like mode is too strong (relative to $\widehat{\gamma}_L = d\widehat{\gamma}_L/d\eta_i$) and this makes the steady state $\eta_i - \eta_c$ ($E(r)$) too large (too small) compared with the observed values. To explain the observed relatively small $\eta_i - \eta_c$ but a large $E(r)$, there must be a mode which has a much weaker nonlinear saturation force. We now see that the toroidal mode plays the role of such a mode. To support this more explicitly, in Fig. 1 we present a comparison of the $\chi_i$ values calculated from the gyro-Bohm and Bohm type $\chi_i$ for the TFTR supershot 44669A discharge. We can see clearly that the $\chi_i$ near the edge region, which was smaller by about one order in the gyro-Bohm case, becomes comparable to the observed value in the Bohm case.

(ii) The second feature expected from the very weak nonlinear saturation force of the toroidal ITG mode is that the steady state ion temperature profile will now have a strong near-marginality over a larger region in the inside core region. Note from the above argument that the toroidal mode has a much weaker radial increase rate of $\eta_i - \eta_c$. More specifically, if we use the explicit form $\alpha = \widehat{\gamma}_N/\widehat{\gamma}_L = \chi_0 k_r^2/\widehat{\gamma}_L \propto T_i^{-1/2}$ for the toroidal mode and take $E(r)$ in the small $\alpha$ limit from Eq. (6), we can derive

$$\delta(r) \equiv \eta_i - \eta_c \simeq \alpha(r)Q_i(r)/\chi_0(r)\eta_c(r)T_i(r) \sim \delta(a)rT_i^2(a)/aT_i^2(r) \tag{16}$$

from Eq. (7). Assuming $\delta(a) \sim 1-2$ in the edge boundary, we see that $\delta(r)$ approaches rapidly to zero when we move to the core region. As mentioned in the Introduction, recent advanced calculations indicate that the core region is very near to marginality. The above result now provides an explanation for the physical origin of such a strong near-marginality and the associated stiffness of the temperature profiles.

A consequence of the strong near-marginality is that the ion energy confinement quality in the core region will now be directly linked through the $\eta_c$ to the boundary ion temperature. Noting that a substantial fraction of ion energy is confined in the inside core region, this means that the confinement time scaling of total ion energy will be closely related to the property of $\eta_c$. This emphasizes the importance of knowing the exact parameter dependence of the $\eta_c$ for the toroidal ITG mode as previously emphasized by the complex parameterization of $\eta_c$ given by Kotschenreuther *et al.*[9] In addition, it suggests that some confinement scalings, which were difficult to explain from the usual consideration of $\chi_i$, may be understood in terms of the properties of $\eta_c$.

## IV. ON THE POSSIBILITY OF BOHM-TYPE DIFFUSIVITY

In the previous section, we have shown that with the Bohm-type diffusivity or the radial mixing length of order $\sqrt{L\rho_i}$ the observed radial profile of $\chi_i$ can be explained to near the edge region. Here, we briefly present the argument for the occurrence of this radial fluctuation scale. It was well-recognized earlier[10,4] that the mesoscale radial mode width of the toroidal mode is of order $\sqrt{L\rho_i}$. This is also true for the longer wavelength trapped ion mode, and there has been a nonlinear calculation which led to Bohm-type scaling as well.[20] More recently, the same mode width has been shown to occur also for a more general class of the toroidal modes in the linearly varying equilibrium profile.[21,22] If we simply take this global width as the mixing length, i.e.,

$$k_r^2 = 1/L\rho_i, \tag{17}$$

$\chi_i$ then becomes the Bohm-type. However, as mentioned in the Introduction, this choice has been denied in some previous works based on some physical arguments. In this section we check more carefully these arguments, and will show that most of them are indeed problematic so there is a good possibility of the Bohm-type diffusivity from the toroidal ITG mode.

(i) One argument against the choice (17) is that the global mode width is valid only over a very small $\theta$ region in the poloidal space and when averaged over all $\theta$ region it becomes much smaller. In fact, the typical eigenfunction form of the toroidal ITG mode,

$$\phi(x,\theta) \propto e^{-\epsilon x^2 + isk_\theta x(\theta-\theta_0) + im\theta - \sigma(\theta-\theta_0)^2}, \tag{18}$$

indicates that the large mode width or small $k_r \sim \epsilon^{1/2}$ is valid only near $\theta \sim \theta_0$ region, where $\epsilon \sim 1/L\rho_i$, $x = r - r_0$ with the mode center $r_0$, $s$ is shear, $\theta_0$ is the Bloch shift parameter, $m$ is the main poloidal mode number, and $\sigma \sim 1$. Due to the second term in the exponent, which describes the twisted streamline structure of the toroidal mode (see Fig. 2), the value of $k_r$ averaged over $\theta$ indeed becomes[10]

$$\bar{k}_r = \left\langle k_r^2 \right\rangle^{1/2} \sim sk_\theta \Delta\theta, \tag{19}$$

with $\Delta\theta = 1/\sqrt{\sigma}$, the mode width in the ballooning $\theta$ space. This wavenumber is much larger than the value (17) based on the global width and actually the same order as the slab value (note typically $\Delta\theta \sim 1$). This explains a basic reason why many previous works on the toroidal ITG mode did not use the form (17) as the effective mixing length.

The point missed by this simple argument for the use of $k_r$ in Eq. (19) rather than that in Eq. (17) is that it is the entire Poisson bracket $\widehat{\mathbf{b}}\cdot\nabla_\perp\phi\times\nabla_\perp f$ from the $\mathbf{E}\times\mathbf{B}$ convection that determines the nonlinear saturation force. This convective nonlinearity is invariant to the rotation of convective cells with anisotropic structure. In other words, the nonlinear interaction rate does not vary, even when the streamline direction or $k_r$ of anisotropic convective cells changes.[23] We then note that the typical linear eigenmode structure of the toroidal ITG mode, as shown in Fig. 2, corresponds almost to this case where the local streamline direction is changed (or twisted) with increasing $|\theta - \theta_0|$, but its degree of anisotropy is nearly kept. This feature of the toroidal ITG mode is also seen more quantitatively from the eigenfunction form (18). Note that the second term in the exponent which was responsible for the large $k_r$ in Eq. (19) can be made to disappear, to lowest order, by the local rotation of the coordinate system $(x, \theta-\theta_0)$ by $\delta\theta$, given by

$$x = x' + \delta\theta(\theta-\theta_0)',$$

$$\theta - \theta_0 = (\theta - \theta_0)' - \delta\theta x',$$

where $x$ is normalized by $\rho_i$. In the rotated coordinate system $[x', (\theta - \theta_0)']$ the radial wavenumber ($k_r' \sim \partial/\partial x'$) thus becomes of order $\epsilon^{1/2}$ or $1/\sqrt{L\rho_i}$, same as the rate at $\theta \sim \theta_0$, indicating that the degree of anisotropy is almost invariant even though the streamline direction varies with $(\theta - \theta_0)$. As stated above, this then means that the local nonlinear interaction rate does not vary with $(\theta - \theta_0)$ but is almost the same as the value at $\theta \sim \theta_0$ estimated using the width (17). We can thus conclude that the wavenumber (17) can be used effectively over all $\theta$ region. We need just to redefine it as the wavenumber along the local streamline direction (i.e., as $k_r'$, see Fig. 2), rather than the fixed radial direction.

(ii) Another argument against the choice (17) is that the linear width may be valid only in the linear or quasilinear regime, but not in the strong turbulent regime where the global structure of toroidal mode can be disintegrated into the slab mode-like structures. We agree that once the elongated cells disintegrate into isotropic cells, the nonlinear saturation force will increase rapidly to that of the slab mode with the strong nonlinear force limiting the fluctuations to small levels. An important point to note here is, however, that by causality this nonlinear disintegration can occur only after the linear mode has grown to a sufficient amplitude that the critical nonlinear interaction for fragmentation becomes possible. The large fluctuation amplitude, as expected from the width (17), is thus still possible in the time-averaged sense, even when the disintegration occurs. (Note in this case the steady state fluctuation may have an oscillating feature with a linear growth to a large amplitude and then a rapid dissipation by disintegration.)

(iii) Thirdly, we consider the argument that the growth to a large amplitude of a single toroidal mode with twisted eddy structure might be strongly limited by the secondary instability, which is generated by the rapid poloidal variation of twisted eddy, trying to disintegrate the eddy structure to the slab-like vortices.[23,24] For this possibility, we note first that there is still no definite observation of the secondary instability. Recent particle simulations[11−14] show the clear excitation of the toroidal mode with twisted eddy structure and the sustainment of the structure up to a saturation point with a large saturation amplitude. The important point here is that the saturation occurs mainly due to the quasilinear relaxation of the background temperature profile, rather than the disintegration. These results thus suggest that the excitation of the secondary mode may be actually much more difficult (than the simple estimate given in Ref. 24), or that the toroidal mode is free from the instability. In addition, we note that due to their slab-like mode structures the secondary modes will have a strong nonlinear interaction between themselves which will then limit their growth to a very small amplitude. Once the prime mode can grow over this maximum amplitude of the secondary modes, a continuous growth to a much larger amplitude will then be possible.

In summary, in this section we have discussed several arguments based on which the Bohm-type scaling from the toroidal ITG mode has been denied in previous works. We have shown that most of them are problematic. Even though a more complete check is still necessary, for example, using global toroidal particle simulation codes over a wide range of $a/\rho_i$ and $\eta_i - \eta_c$, the arguments presented suggest that the Bohm-type scaling can be obtained from the toroidal ITG mode, which then enables us to simultaneously resolve the radial profile problem of $\chi_i$ and also offers a plausible explanation of the observation of Bohm scaling $\chi_i$ from major tokamaks.[16–19] Furthermore, it is encouraging to note the existence of both linear and nonlinear theoretical considerations[25] which indicate that the addition of $\mathbf{E} \times \mathbf{B}$ shear flow to this large radial mode width instabilities can offer a possible explanation of gyro-Bohm $\chi_i$ observed in H-mode plasmas.[26]

## V. CONCLUSION

It has been shown first that the radial increase of $\chi_i$ and also $\eta_i - \eta_c$ is indeed a feature expected well from the steady state dynamics when the transport is assumed to occur by the ITG mode. The large value of $\chi_i$ near the edge region, which has been difficult to obtain from the usual gyro-Bohm type $\chi_i$ of the slab-like mode, is obtained from the toroidal ITG mode when one assumes the mode has the Bohm-type scaling or, equivalently, the radial mixing length is of order $\sqrt{L\rho_i}$. To check the possibility of such a Bohm-type scaling from the toroidal ITG mode, several previous arguments which discouraged that choice of radial mixing length have been critically examined. It has been shown that these arguments are problematic and there is indeed good reason for the Bohm-type scaling to appear in the toroidal confinement experiments.

**Acknowledgment**

We would like to acknowledge Dr. Azumi for useful discussions, and Drs. T. Hirayama and H. Kishimoto for their support of the present work. This work has been also supported by Japanese Science and Technology Agency and the U.S. Dept. of Energy contract No. DE-FG03-96ER-54346.

# REFERENCES


[1] L.I. Rudakov and R.Z. Sagdeev, Sov. Phys. Dokl. **6**, 415 (1961).

[2] B. Coppi, M.N. Rosenbluth, and R.Z. Sagdeev, Phys. Fluids **10**, 582 (1967).

[3] B. Coppi and F. Pegoraro, Nucl. Fusion **17**, 969 (1977).

[4] W. Horton, D.-I. Choi, and W.M. Tang, Phys. Fluids **24**, 1077 (1981).

[5] F. Romanelli, Phys. Fluids B **1**, 1018 (1989).

[6] J.W. Connor and H.R. Wilson, Plasma Phys. Control. Fusion **36**, 719 (1994).

[7] W. Horton, D. Lindberg, J.-Y. Kim, J.Q. Dong, G.W. Hammett, S.D. Scott, M.C. Zarnstorff, and S. Hamaguchi, Phys. Fluids B **4**, 953 (1992).

[8] W. Horton, M. Wakatani, and A.J. Wootton, *Ion Temperature Gradient Driven Turbulent Transport* (American Institute Physics Conference Proceedings No. 284, New York, 1994).

[9] M. Kotschenreuther, W. Dorland, M. Beer, and G.W. Hammett, Phys. Plasmas **2**, 2381 (1995).

[10] D.-I. Choi and W. Horton, Phys. Fluids **23**, 356 (1980).

[11] M.J. LeBrun, T. Tajima, M. Gray, G. Furnish, and W. Horton, Phys. Fluids B **5**, 752 (1993).

[12] S.E. Parker, W.W. Lee, and R.A. Santoro, Phys. Rev. Lett. **71**, 2042 (1993).

[13] H.E. Mynick and S.E. Parker, Phys. Plasmas **2**, 1217; *ibid*, 2231 (1995).

[14] Y. Kishimoto, T. Tajima, W. Horton, M.J. LeBrun, and J.-Y. Kim, Phys. Plasmas **3**, 1289 (1996).

[15] R.E. Waltz, G.D. Kerbel, and J. Milovich, Phys. Plasmas **1**, 2229 (1994).

[16] F. Perkins, C.W. Barnes, D.W. Johnson, S.D. Scott, M.C. Zarnstorff, M.G. Bell, R.E. Bell, C.E. Bush, B. Grek, K.W. Hill, D.K. Mansfield, H. Park, A.T. Ramsey, J. Schivell, B.C. Stratton, and E. Synakowski, Phys. Fluids B **5**, 477 (1993).

[17] J.P. Christiansen, P.M. Stufferfield, J.G. Cordey, C. Gormezano, C.W. Gowers, J.O'Rourke, D. Stork, A Taroni, and C.D. Challis, Nucl. Fusion **33**, 863 (1993).

[18] C.C. Petty, T.C. Luce, R.I. Pinsker, K.H. Burrell, S.C. Chiu, P. Gohil, R.A. James, and D. Wroblewski, Phys. Rev. Lett. **74**, 1763 (1995).

[19] H. Shirai, T. Takizuka, O. Naito, and M. Sato, J. Phys. Soc. Jpn. **64**, 4209 (1995).

[20] T.S. Hahm and W.M. Tang, Phys. Plasmas **3**, 242 (1996).

[21] J.Y. Kim, Y. Kishimoto, M. Wakatani, and T. Tajima, Phys. Plasmas **3**, 3689 (1996).

[22] J.B. Taylor and H.R. Wilson, *International Conf. on Plasma Phys.* (American Institute Physics Conference Proceedings No. 345, New York 1994) p. 282.

[23] W. Horton, T. Tajima, J.-Y. Kim, Y. Kishimoto, and M. Ottaviani, J. Plasma Phys. **56**, 605 (1996).

[24] S.C. Cowley, R.M. Kulsrud, and R.N. Sudan, Phys. Fluids B **3**, 2767 (1991).

[25] T.S. Hahm and K.H. Burrell, Phys. Plasmas **3**, 427 (1996).

[26] C.C. Petty, T.C. Luce, K.H. Burrell, S.C. Chiu, J.S. deGrassie, C.B. Forest, P. Gohil, C.M. Greenfield, R.J. Groebner, R.W. Harvey, R.I. Pinsker, A. Prater, and R.E. Waltz, Phys. Plasmas **2**, 2342 (1995).

## FIGURE CAPTIONS

FIG. 1. Comparison of the calculated ion thermal conductivities from the gyro-Bohm and Bohm type of $\chi_i$ for the TFTR supershot 44669A discharge.

FIG. 2. The typical linear eigenmode structure of the toroidal ITG mode, obtained from the toroidal particle simulation code by Kishimoto *et al.* in Ref. 14.