

# Gyrokinetic study of ion temperature gradient instability in vicinity of flux surfaces with reversed magnetic shear

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## Abstract

Ion temperature gradient (ITG) driven instability is investigated in the vicinity of a flux surface where the magnetic shear reverses. The generic properties of the profile of the magnetic shear are taken into account with gyrokinetic stability theory in the local sheared slab geometry integral equation. The stability analysis shows that there are four distinct unstable ITG branches with significantly different eigenvalues and mode structures existing simultaneously exist in the vicinity of the minimum  $q$  layer. The variation of eigenmode structures with magnetic shear is investigated in detail. Mixing length estimation of the induced plasma transport is performed. Detailed numerical results are presented and general correlations with simulations and experiments are noted.

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# I INTRODUCTION

One of the most significant achievements in tokamak experiment in recent years is discovery of internal transport barrier (ITB).<sup>1-3</sup> Plasma configuration with an ITB has been proposed as one of the promising scenarios for steady state advanced tokamak operation because of its potential to provide good confinement, improved stability and large fraction of aligned bootstrap current. A common understanding for the physics mechanism leading to improved confinement in an ITB is the stabilization of a variety of micro-instabilities by a negative central magnetic shear (NCS) and by a perpendicular velocity shear.<sup>4</sup>

An ITB is characterized by steep temperature and density gradients near a flux surface where the magnetic shear reverses. A representative feature of an optimized configuration with an ITB is that, besides strong or moderate negative magnetic shear at small radii, there is very weak magnetic shear (VWS), positive or negative, at intermediate radii. The experimental data clearly show that an ITB region always coincides with vicinity of the flux surface where the magnetic shear reverses. It is also clearly evident from experiments that the highest normalized  $\beta$ (=plasma pressure/pressure of magnetic field) and confinement enhancement are reached in the vicinity of the shear reversed surface as shown in the DIII-D device.<sup>3</sup>

As a part of the effort to understand the mechanism for the formation of an ITB and for the sake of completeness of micro-instability theory, the ion temperature gradient (ITG) modes are studied in the long wavelength approximation in a sheared slab configuration with VWS.<sup>6</sup> The ballooning mode symmetry ( $x \rightarrow x + 1/nq'$ ) fails near the  $q$ -reversal layer and is replaced by local mirror symmetry about the  $q_{\min}$  surface. Thus, we find even and odd parity of the eigenmode. Also, this breaking of the ballooning symmetry suggests that the slab model be used.

Actually, the study of drift-like instabilities on a flux surface where the magnetic shear

reverses and in its vicinity is an important basic topic of magnetically confined plasmas. Wave particle resonant interaction in such a region is essentially different from that in regions with strong or moderate magnetic shear. Therefore, behaviors of the instabilities on such a surface and in its vicinity are significantly different and worthwhile for further investigation.

The ITG driven instability in systems with reversed magnetic shear have been attacked with fluid and gyrokinetic simulations. The simulations<sup>7-10</sup> reveal that the eigenmode structures surrounding the  $q_{\min}$  surface have unique features which have correlation with the confinement improvement observed in experiments. The experimental ion pressure profiles often appear consistent with simulation results with the development of abrupt change in their gradient at, or just to one side, of the reversed shear layer. This feature is called the internal transport barrier although some transport continues, of course.

Ion temperature gradient (ITG or  $\eta_i$ ) modes are studied systematically in the vicinity of a flux surface where the magnetic shear reversal in the present work. Gyrokinetic theory with finite gyro-radius effects included in a sheared slab geometry is employed and the resulting integral eigenvalue equation is solved numerically. The gradient ( $\hat{s}_d$ ) of magnetic shear ( $\hat{s}$ ) which is important on the surface and in the vicinity only, is taken into account. We show that there are four distinct unstable ITG branches with significantly different eigenvalues and mode structures simultaneously exist in the vicinity. Mixing length estimation of the induced plasma transport is performed. Detailed numerical results are presented.

The remainder of this work is organized as follows. The physics models for magnetic configuration are given in Sec. II. The basic equation applied in this work is provided in Sec. III. Numerical results are given in Sec. IV, and Sec. V is devoted to conclusions and discussion.

## II PHYSICS MODELS

A sheared slab configuration model is found to be sufficient for the considerations developed here. The specific effects from the toroidal mode coupling are developed in the theory and simulations of Kishimoto *et al.*<sup>7,8</sup> The magnetic shear approaches zero at the  $q_{\min}$  flux surface, where the magnetic shear reverses, and is close to zero in the vicinity of the critical surface. Therefore, the gradient of the magnetic shear, i.e. the second derivative of the safety factor with respect to the radial variable, play the dominant role in the wave-particle interaction, and has to be taken into account in studies of the driftwave instabilities in such a region. Then, the model is extended as follows. The shearing magnetic field is given by

$$\mathbf{B} = B_0[\hat{\mathbf{z}} + (\frac{x}{L_s} + \frac{x^2}{L_{s2}})\hat{\mathbf{y}}], \quad (1)$$

where

$$\frac{1}{L_s} = -\frac{r_0 q_0'}{Rq_0^2} = -\frac{s}{Rq_0}, \quad (2)$$

$$\frac{1}{L_{s2}^2} = -\frac{r_0}{2Rq_0^2} \frac{d^2 q}{dr^2} + \frac{r_0}{Rq_0^3} \left(\frac{dq}{dr}\right)^2 = -\frac{\hat{s}_d}{Rq_0}, \quad (3)$$

$x = r - r_0$ ,  $R$  is the major radius,  $q_0 = q(0)$  is the safety factor at the surface of  $r = r_0$  for tokamak plasmas. In addition, all the derivatives are also taken at the surface of  $r = r_0$ .

The wave perturbations are assumed in the form

$$\tilde{f}(x, y, t) \sim \tilde{f}(x)e^{-i\omega t + ik_z z + ik_y y},$$

and the parallel component of the wave vector is

$$k_{\parallel} = \frac{\mathbf{B} \cdot \mathbf{k}}{|\mathbf{B}|} = k_z - \frac{sk_y}{Rq_0}x - \frac{\hat{s}_d k_y}{Rq_0}x^2. \quad (4)$$

The positions of mode rational surfaces defined by  $k_{\parallel} = 0$  are given as

$$x_{\pm} = \frac{Rq_0}{2\hat{s}_d k_y} \left[ -\frac{sk_y}{Rq_0} \pm \sqrt{\left(\frac{sk_y}{Rq_0}\right)^2 + 4\frac{\hat{s}_d k_y k_z}{Rq_0}} \right]. \quad (5)$$

The effects introduced by the  $\widehat{s}_d$  term are that the condition  $k_{\parallel} = 0$  turns out to be a second order equation instead of a linear equation as it is for  $\widehat{s}_d = 0$ , and, therefore, there appears a second rational surface for each  $k_y, k_z$ . For  $k_z = 0$ , it is clear that the two rational surfaces are located at  $x = x_+ = 0$  and  $x = x_- = -s/\widehat{s}_d$ . For  $k_z \neq 0$ , however, they are located at two sides of the  $x = 0$  surface, respectively, and none of them coincide with the  $x = 0$  surface.

A common understanding is that the equilibrium quantities are constant, equal to the values at the mode rational surface, over the whole mode structure in studies of micro-instabilities. On the other hand, as mentioned above, the values are taken at the surface of  $x = 0$  in the model employed. As a result, there is a limitation on the value of  $k_z$ . The mode rational surfaces are far away from the surface of  $x = 0$  and the mode structure does not cover the latter at all if  $k_z$  is too large. In this case, it is hard to identify that the properties obtained from the studies are valid at the rational surface or at that of  $x = 0$ , especially, when the equilibrium parameters vary dramatically in the radial direction as they are in an ITB. In addition, a small  $k_z$  would introduce a minor shift of the mode rational surface for  $\widehat{s} \neq 0$ . Furthermore, the most important parameter which governs interaction between the two eigenmodes centered at the two rational surfaces is the distance between them,  $\Delta = x_+ - x_-$ . There would not be any significant differences between  $k_z = 0$  and  $k_z \neq 0$  as long as the distance is same. Therefore,  $k_z = 0$  cases are investigated only in this work.

Gyrokinetic theory is employed for ions. Electrons are adiabatic. Finite  $\beta$  effects are neglected and electrostatic perturbations are considered only.

### III INTEGRAL DISPERSION EQUATION

Finite Larmor radius effects, that are neglected in Ref. 6, are included here. The gyrokinetic integral eigenvalue equation is obtained from quasineutrality condition as<sup>9</sup>

$$\left[ 1 + \tau - \left( \tau + \frac{1}{\hat{\omega}} \right) \Gamma_0(k_\perp) + \frac{\eta_i k_\perp^2}{2\tau\hat{\omega}} (\Gamma_0(k_\perp) - \Gamma_1(k_\perp)) \right] \hat{\phi}(k) + \int_{-\infty}^{+\infty} \frac{dk'}{\sqrt{2\pi}} K(k, k') \hat{\phi}(k') = 0 \quad (6)$$

where

$$K(k, k') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx e^{i(k'-k)x} \times \left\{ \left[ \left( \tau + \frac{1}{\hat{\omega}} \right) \Gamma_0(k_\perp, k'_\perp) + \frac{\eta_i}{\hat{\omega}} (b\Gamma_1(k_\perp, k'_\perp) - b_1\Gamma_0(k_\perp, k'_\perp)) \right] \right. \\ \left. \times (1 + \zeta_i Z(\zeta_i)) + \frac{\eta_i}{\hat{\omega}} \left[ \zeta_i^2 + \left( \zeta_i^2 - \frac{1}{2} \right) \zeta_i Z(\zeta_i) \right] \Gamma_0(k_\perp, k'_\perp) \right\}, \quad (7)$$

$$\zeta_i = \frac{\omega}{\sqrt{2}|k_\parallel|v_{ti}} = \frac{\sqrt{\tau}}{2} \frac{\hat{\omega}}{|x(\hat{s} + s_2x)|}, \quad (8)$$

$\hat{\phi}(k)$  is the Fourier component of the perturbed electrostatic potential  $\tilde{\phi}(x)$ ,  $Z(\zeta_i)$  is the plasma dispersion function, and

$$b = \frac{k_\perp k'_\perp}{2\tau}, \quad b_1 = \frac{k_\perp^2 + k'^2_\perp}{4\tau}, \quad k_\perp^2 = k^2 + k_y^2, \quad k'^2_\perp = k'^2 + k_y^2, \quad (9)$$

$$\hat{s} = \frac{L_n}{L_s}, \quad s_2 = \frac{L_n \rho_i}{L_{s2}^2}, \quad \tau = \frac{T_e}{T_i}, \quad \eta_i = L_n L_{Ti}, \quad (10)$$

$$\hat{\omega} = \frac{\omega}{\omega_{*e}}, \quad \omega_{*e} = \frac{ck_y T_e}{eBL_n},$$

where  $L_n$  and  $L_{Ti}$  are density and temperature gradient scale lengths, respectively,  $k$ ,  $k'$  and  $k_y$  are normalized to  $\rho_s^{-1} = eB/c\sqrt{2T_e m_i}$ ,  $x$  is normalized to  $\rho_s$ . The finite ion gyroradius effects are contained in the functions

$$\Gamma_j(k_\perp, k'_\perp) = I_j \left( \frac{k_\perp k'_\perp}{2\tau} \right) \exp \left[ -\frac{(k_\perp^2 + k'^2_\perp)}{4\tau} \right], \quad \Gamma_j(k_\perp) = I_j \left( \frac{k_\perp^2}{2\tau} \right) \exp \left[ -\frac{k_\perp^2}{2\tau} \right], \quad (11)$$

where  $I_j$  is the modified Bessel function of order  $j = (0, 1)$ .

It is easy to note from Eq. (8) that the eigenvalue solutions of Eq. (6) are independent of the signs of  $\hat{s}$ ,  $s_2$  and  $\hat{s}/s_2$  as it should for a slab model. The structure of the eigenfunction depends on the sign of  $\hat{s}/s_2$  since the second rational surface is located at  $x = x_- = -\hat{s}/s_2$ . However, the eigenfunction for  $\hat{s}/s_2 > 0$  is a mirror image of that for  $\hat{s}/s_2 < 0$  with respect to the  $x = 0$  surface only. Therefore, the sign of  $\hat{s}/s_2$  would not introduce any essential effects on the results either. In conclusion, the signs of  $\hat{s}$ ,  $s_2$  and  $\hat{s}/s_2$  would not have any essential influence on the eigenvalue, eigenfunction structure and turbulent transport induced in the slab limit. In other words, the results described in following sections are valid for both negative and positive magnetic shears  $\hat{s}$  as well as its gradient  $s_2$ .

## IV NUMERICAL RESULTS

The integral equation, Eq. (6), has to be solved numerically following the methods developed earlier. The reference parameters for the numerical results are  $\eta_A = 4$ ,  $\tau = 1$ ,  $k_y = 0.35$ , unless otherwise stated. Mixing length estimates for the transport,  $\gamma\langle l^2 \rangle$ , are calculated with

$$\langle l^2 \rangle = \frac{\int (x - x_0)^2 |\phi(x)|^2 dx}{\int |\phi(x)|^2 dx}. \quad (12)$$

and

$$x_0 = \frac{\int x |\phi(x)|^2 dx}{\int |\phi(x)|^2 dx}. \quad (13)$$

The units of  $\gamma\langle l^2 \rangle$  are gyroBohm  $(\rho_s/L_n)(cT_e/eB)(k_y\rho_s)$  diffusivity. As is discussed in last section, the signs of  $\hat{s}$ ,  $s_2$  and  $\hat{s}/s_2$  do not introduce essential effects into the subject. Therefore,  $\hat{s} > 0$  ( $s < 0$ ) and  $s_2 < 0$  ( $\hat{s}_d > 0$ ) are considered only in the following numerical investigations.

The normalized mode growth rate (a) and real frequency (b) are shown in Fig. 1 as functions of magnetic shear  $\hat{s}$  for  $s_2 = -0.01$ . For a given set of plasma parameters, four distinct unstable branches emerge in the vicinity of a flux surface where the magnetic shear

reverses, where the shear is very weak ( $\hat{s} \lesssim 0.1$ ). The modes are titled as  $D$  (the line without symbol),  $D'$  (the line with squares),  $G$  (the line with diamonds) and  $G'$  (the line with crosses) branches, respectively, in this work.

The eigenfunctions  $\tilde{\phi}(x)$  of the four branches are shown in Fig. 2 for  $s_2 = -0.01$  and  $\hat{s} = 0.01$ . The heavy and light lines are the real and imaginary parts, respectively.

As is discussed in Sec. 3, there are two rational surfaces at  $x = x_+$  and  $x = x_-$  when the gradient of the magnetic shear,  $s_2$ , is included in the consideration. In addition, there are two unstable modes centered at each rational surfaces, one of which has even parity and the other has odd parity. The modes located at  $x = x_+$  are strongly coupled with that located at  $x = x_-$  when the magnetic shear is weak enough such that  $x_- \lesssim$  wavelength of the modes. The four branches result from different couplings between the four modes. The  $D$  (Fig. 2a) and  $G$  (Fig. 2b) branches result from the coupling of the two even and two odd modes centered at  $x = x_+$  and  $x = x_-$ , respectively. Meanwhile, the  $D'$  (Fig. 2c) and  $G'$  (Fig. 2d) branches stem from the coupling between one even and one odd mode centered at the two rational surfaces, respectively. As a result, it is clear that  $D$  (Fig. 2a) and  $G$  (Fig. 2b) modes have even parity, while  $D'$  (Fig. 2c) and  $G'$  (Fig. 2d) modes have odd parity in the low shear regime.

In Fig. 1, the branches  $D$  and  $D'$  merge to one branch at  $\hat{s} \gtrsim 0.15$  where the effects of the gradient of magnetic shear are negligible,  $|s_2/\hat{s}| \ll 1$ . In such a case, the mode located at  $x = x_- = -\hat{s}/s_2$  is far away and decoupled from the mode at  $x = x_+ = 0$ . The merged branch is the lowest even parity modes and, therefore, the continuation of the branch  $D$ . On the other hand, the branch  $D'$  is introduced by the effects of  $s_2$  and, therefore, disappears for  $|s_2/\hat{s}| \ll 1$ .

A similar discussion holds for the branches  $G$  and  $G'$ . However, the merged branch has odd parity. The reason is that the odd mode located at  $x = x_- = -\hat{s}/s_2$  is far away and decoupled from the odd mode at  $x = x_+ = 0$  and each mode keeps its own original parity

as there is not the other one when magnetic shear is strong,  $|s_2/\hat{s}| \ll 1$ . In addition, the growth rate of the branch  $G'$  is higher than that of branch  $G$  while it is the opposite for branches  $D$  and  $D'$ . The odd and even modes have larger and smaller values of the average  $k_{\parallel}$  which changes the strength of ion-wave resonance and the growth rate.

The mixing length estimation for the transport,  $\gamma\langle l^2 \rangle$ , versus magnetic shear  $\hat{s}$  is given in Fig. 3 for the branches  $D$ ,  $G$ , and  $D'$ . The parameters are the same as that in Fig. 1. The results for branch  $G'$  are similar. It is evident that the transport in the vicinity of the surface where the magnetic shear reverses, where the shear is very low, is almost one order of magnitude lower than the maximum value for  $\hat{s} \sim 0.2$ . The reduction comes from both the decrease of the mode growth rate and the shrink of the mode structures for the  $D$  branch. However, the reduction comes mainly from the latter for  $D'$  and  $G$  branches as can easily be seen in comparison with shown in Fig. 1.

The sharp reduction of the transport at  $\hat{s} \sim 0.2$  shown in Fig. 3 reveals the dramatic variation of the eigenmode structures at this point when magnetic shear changes. Shown in Fig. 4 are the eigenfunctions  $\tilde{\phi}(x)$  of the branch  $D$  for  $\hat{s} = 0.01$  (*a*), 0.05 (*b*), 0.1 (*c*), 0.15 (*d*), 0.2 (*e*) and 0.25 (*f*). The other parameters are the same as that in Fig. 1. The heavy and light lines are the real and imaginary parts, respectively. The two modes centered at  $x = x_+$  and  $x = x_-$  are coupled so strongly for  $\hat{s} = 0.01$  and 0.05 that it is almost impossible to distinguish one from the other. For  $\hat{s} = 0.1$ –0.15, although the two modes are distinguishable, they still couple together strongly, creating a broad eigenmode structure, and introduce a rather high turbulent transport. The results clearly demonstrate that the effects of the gradient of magnetic shear are important for  $|\hat{s}| \lesssim 0.15$  when  $s_2 = \pm 0.01$ . For  $\hat{s} \geq 0.2$ , the two modes separate from each other completely. In addition, both modes shrink into narrower regions and lead to a rather low transport. It is worthwhile to point out that there would be another mode centered at  $x = x_- = 25$  for  $\hat{s} = 0.25$  if the box in the  $x$ -space is large enough and that the latter would be found only if the box is chosen properly, say

from  $x = 15$  to  $x = 30$ . More interestingly, the eigenvalues are the same for the three cases: one peak at  $x = 0$ , one peak at  $x = 25$  or two peaks at these two surfaces, respectively.

By the way, these results obtained here for  $\hat{s} \gtrsim 0.15$  are in contrast with that in Ref. 10 where one peak, at  $x = x_+$  or  $x = x_-$ , is found only for a given  $k_y$  when  $k_z \neq 0$ .

The mode growth rate (*a*) and real frequency (*b*) of the branches  $D$  (the line with squares) and  $D'$  (the line with crosses) versus the gradient of magnetic shear,  $s_2$ , are shown for  $\hat{s} = 0.01$  in Fig. 5. The results for  $\hat{s} = 0.15$  (the line without symbols) are also shown for comparison. The growth rate of  $D'$  branch approaches zero and the mode disappears for  $s_2 = 0$ . The growth rate increases dramatically and becomes comparable with that for  $\hat{s} = 0.15$  when  $|s_2| \gtrsim 0.02$ . In this regime, the  $D'$  branch is the dominant. On the other hand,  $s_2$  has slight destabilization effects on the  $D$  branch. The growth rate and the real frequency of the  $D$  branch for  $\hat{s} = 0.01$  are both much lower than that for  $\hat{s} = 0.15$ .

The same as that in Fig. 5 but for the branch  $G$  (the line with squares) and  $G'$  (the line with crosses) are shown in Fig. 6. The results for  $\hat{s} = 0.15$  (the line without symbols) are also shown for comparison. The growth rate of the  $G'$  branch becomes zero and the mode disappears when  $s_2$  approaches 0. This is similar with that for branch  $D'$ . However, the gradient of the magnetic shear,  $s_2$ , has a rather strong stabilizing effect on both  $G$  and  $G'$  branches in contrast with that for  $D$  and  $D'$  branches. The growth rate and the real frequency of the modes both decrease for  $\hat{s} = 0.15$  when  $|s_2|$  increases.

The mixing length estimation for the transport,  $\gamma \langle l^2 \rangle$ , versus gradient of magnetic shear,  $s_2$ , is given in Fig. 7 for the branches  $D$  and  $D'$ . The parameters are the same as that in Fig. 5 except  $\hat{s} = 0.01$ . The results for branches  $G$  and  $G'$  are similar. It is evident that the transport from  $D'$  branch is much higher than that from  $D$  branch in the vicinity of the surface where the magnetic shear reverses when the magnitude of  $s_2$  is small ( $|s_2| \lesssim 0.02$ ). The former ( $D'$  branch) decreases rapidly when the magnitude of  $s_2$  increases. The reduction comes mainly from shrink of the mode structures as can easily be seen in comparison with

Fig. 5. For  $\hat{s} = 0.15$ , the estimated transport induced by  $D$  branch (not shown here) is more than one order of magnitude higher than shown here for  $s_2 \sim -0.01$ . In addition, when the magnitude of  $s_2$  decreases, there is a same sharp drop at  $s_2 = -0.008$  as that shown in Fig. 3 due to the same reasons discussed there. The estimated transport for  $\hat{s} = 0.15$  gradually decreases for increasing  $|s_2|$  and is comparable with what given here for  $s_2 \lesssim -0.04$ .

## V CONCLUSIONS AND DISCUSSION

Ion temperature gradient (ITG or  $\eta_i$ ) modes are studied with gyrokinetic theory in vicinity of a flux surface where the magnetic shear reverses. The kinetics for wave-particle interaction is analyzed in detail. Four distinct branches with significantly different eigenvalues and eigenfunction structures are found simultaneously unstable for a given set of plasma parameters in the vicinity where magnetic shear is very weak. This observation is very important for similar linear and nonlinear studies with initial value scheme since linear and nonlinear coupling between the branches may lead to very complicated, or even global, structures of the modes in the radial direction.<sup>7-10</sup>

The mixing length estimation for the transport,  $\gamma\langle l^2 \rangle$  is given. It is clearly shown that the transport in the vicinity of the surface where the magnetic shear reverses is almost one order of magnitude lower than the maximum value at  $\hat{s} \sim 0.2$ . The reduction comes from decrease of the mode growth rate or the shrink of the mode structures. It is also demonstrated that the effects of the gradient of magnetic shear have to be taken into account in the vicinity of  $\hat{s} \lesssim 0.2$  for  $s_2 = -0.01$  when transports induced by the instabilities are estimated. Such effects are important for  $\hat{s} \lesssim 0.1$  for the same  $s_2$  value when the instabilities are considered only. The gradient of magnetic shear introduces an abrupt variation of mode width when  $\hat{s}$  increases. For  $\hat{s}$  larger than the critical values, see Fig. 3 for an example, the narrow mode width  $\Delta x \sim \rho_s(qR/\hat{s})^{1/2}$  of standard ITG theory applies. Below the critical  $\hat{s}$  the mode width scales as  $(\rho_s q_0 R / \hat{s}_d)^{1/2}$ .

In addition, it is clearly demonstrated that the gradient of magnetic shear,  $s_2$ , stabilizes the modes or suppresses the turbulent transport except in the vicinity of  $s_2 = 0$  point where it is a driving factor for both the instabilities and transports. As a result, it must be easier to realize transport improvement or an ITB in regions of  $s_2 = 0$  and of  $s_2$  being relatively high in the vicinity of flux surface where the magnetic shear reverses. Therefore, in accordance with the definition of  $s_2$ , Eq. (10), a higher ion (electron) temperature, a flat density profile and a higher curvature are in favor of the formation of an ITB in the vicinity of flux surface where the magnetic shear reverses.

The nonlinear saturation of the local modes may be addressed with an extension of the work of the mode coupling equations in Ref. [12]. In that work the odd parity mode dominated and produced an effective transport barrier until secondary modes broke down the barrier. The problem of  $E_r$ -shear on the mode structure is currently under investigation.

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## References

1. Y. Koide, M. Kikuchi, M. Mori, S. Tsuji, S. Ishida, N. Asakura, Y. Kamada, T. Nishitani, Y. Kawano, T. Hatae, T. Fujita, T. Fukuda, A. Sakasai, T. Kondoh, and Y. Neyatani, Phys. Rev. Lett. **72**, 3662 (1994).
2. E.J. Strait, L.L. Lao, M.E. Mauel, B.W. Rice, T.S. Taylor, K.H. Burrell, M.S. Chu, E.A. Lazarus, T.H. Osborne, S.J. Thompson, and A.D. Turnbull, Phys. Rev. Lett. **75**, 4421 (1995).
3. L.L. Lao, K.H. Burrell, T.S. Casper, V.S. Chan, M.S. Chun, J.C. DeBoo, E.J. Doyle, R.D. Durst, C.B. Forest, C.M. Greenfield, R.J. Groebner, F.L. Hinton, Y. Kawano, E.A. Lazarus, Y. R. Lin-Liu, M. E. Mayel, W.H. Meyer, R.L. Miller, G.A. Navratil, T.H. Osborne, Q. Peng, C.L. Rettig, G. Rewoldt, T.L. Rhodes, B.W. Rice, D.P. Schissel, B.W. Stallard, E.J. Strait, W.M. Tang; T.S. Taylor, A.D. Turnbull, R.E. Waltz, and the DIII-D Team, Phys. Plasmas **3**, 1951 (1996).
4. F.M. Levinton, M.C. Zarnstorff, S.H. Batha, M. Bell, R.V. Budny, C. Bush, Z. Change E. Fredrickson, A. Janos, J. Manickam, A. Ramsey, S.A. Sabbagh, G.L. Schmidt, E.J. Synakowski, and G. Taylor, Phys. Rev. Lett. **75**, 4417 (1995).
5. T. Fujita, S. Ide, H. Shirai, *et al.*, Phys. Rev. Lett. **78**, 2377 (1997).
6. J .Q. Dong, Y.Z. Zhang, S.M. Mahajan, and P.N. Guzdar, Phys. Plasmas **4**, 3334 (1997).
7. Y. Kishimoto, J-Y. Kim, W. Horton, T. Tajima, M.J. LeBrun, S.A. Dettrick, J.Q. Li, and S. Sharai, Nucl. Fusion **40**, 667 (2000).
8. Y. Kishimoto and J-Y. Kim, Plasma Phys. Control. Fusion (1999).

9. J. Q. Li, Y. Kishimoto, and T. Tuda, *Plasma Phys. Control. Fusion* **42**, 443 (2000).
10. Y. Idomuro, S. Tokuda, and M. Wakatani, *Phys. Plasmas* **6**, 4658 (1999).
11. J.Q. Dong, P. Guzdar, and Y.C. Lee, *Phys. Fluids* **30**, 2694 (1987).
12. G. Hu and W. Horton, *Phys. Plasmas* **4**, 3262 (1997).

## FIGURE CAPTIONS

FIG. 1. Normalized growth rate (a) and real frequency (b) of the modes versus magnetic shear for  $s_2 = -0.01$ . The four distinct branches,  $D$  (the line without symbols),  $D'$  (the line with squares),  $G$  (the line with diamonds) and  $G'$  (the line with crosses) are clearly shown in the region of  $\hat{s} \lesssim 0.15$ .

FIG. 2. Eigenfunctions of branches  $D$  (a),  $D'$  (b),  $G$  (c) and  $G'$  (d) for  $\hat{s} = 0.01$  and  $s_2 = -0.01$ . The heavy and light lines are the real and imaginary parts, respectively.

FIG. 3. Mixing length estimated transport,  $\gamma \langle l^2 \rangle$ , versus magnetic shear  $\hat{s}$  for branches  $D$  (the line without symbols),  $G$  (the line with diamonds), and  $D'$  (the line with squares), The parameters are the same as that in Fig. 1. The units of  $\gamma \langle l^2 \rangle$  are  $(\rho_s/L_n)(k_y \rho_s)(cT_e/eB)$ .

FIG. 4. Eigenfunctions of branch  $D$  for  $\hat{s} = 0.01$  (a), 0.05 (b), 0.1 (c), 0.15 (d), 0.2 (e) and 0.25 (f). The other parameters are the same as that in Fig. 1.

FIG. 5. Mode growth rate (a) and real frequency (b) of the branches  $D$  (the line with squares) and  $D'$  (the line with crosses) versus the gradient of magnetic shear,  $s_2$ , for  $\hat{s} = 0.01$ . The line without symbols is for  $\hat{s} = 0.15$ .

FIG. 6. The same as that in Fig. 5 but for the branch  $G$  (the line with squares) and  $G'$  (the line with crosses).

FIG. 7. Mixing length estimation for the transport,  $\gamma \langle l^2 \rangle$ , versus gradient of magnetic shear,  $s_2$ , for the branches  $D$  and  $D'$ . The parameters are the same as that in Fig. 5.