Modeling of Plasma Rotation in Radio-Frequency-Wave-Heated Tokamak Plasmas

K. C. Shaing

Institute for Fusion Studies University of Texas Austin, TX 78712

Abstract

It is shown by considering the general properties of solution of the kinetic equation that plasma rotation in Radio-Frequency (RF) wave heated tokamak plasmas can be modeled by the RF-modified plasma viscous force. This approach is qualitatively different from a model that uses a radial current to model the RF effects on plasma rotation.

52.55.Fi

It has been known that bulk plasma rotation can have a significant impact on the plasma confinement in improved confinement modes. For example, in Negative Central Shear (NCS) mode, plasma confinement is probably improved by the toroidal rotation associated with neutral particle beam injection [1]. In both High (H) mode, whether it is naturally-occurred or electrode-induced, and Enhanced Reverse Shear (ERS) mode, poloidal $\boldsymbol{E} \times \boldsymbol{B}$ rotation bifurcation plays an important role in triggering plasmas into improved confinement regime [2-4]. Here, E is the electric field and $\boldsymbol{B}$ is the magnetic field. Recent experiments indicate that plasma confinement in Alcator C-MOD is also closely correlated with toroidal plasma rotation in Radio-Frequency wave (RF) heated tokamak plasmas [5].

A model for plasma rotation in RF-heated tokamak plasmas is presented in Ref. [6]. It models the RF effects on plasma rotation as a radial current $\boldsymbol{J}_{rf}$. This radial current produces a $\boldsymbol{J}_{rf} \times \boldsymbol{B}$ force which in turn drives plasma rotation. Here, we present a simple, but different, methodology to model plasma rotation in RF-heated tokamak plasmas based on the solution of the kinetic equation. We only consider the general properties of the solution to illustrate the model. The detailed calculations will be addressed separately.

For simplicity, we assume that RF-heating process does not introduce a momentum input in any direction and that toroidal symmetry is approximately valid. Of course, these assumptions are not really needed in general to model the problem. We adopt them to focus on the basic issue.

The equilibrium particle distribution f_0 satisfies

$$v_{\|}\boldsymbol{n} \cdot \nabla f_0 = C(f_0), \tag{1}$$

where $v_{\|}$ is the parallel (to $\boldsymbol{B}$) particle speed, $\boldsymbol{n} = \boldsymbol{B}/|\boldsymbol{B}|$ is the unit vector, and C is the collision operator including both Coulomb collision operator and RF-heating operator. Assuming we are interested in the collisionless limit, i.e., trapped particle bounce frequency is faster than the effective Coulomb collision frequency and RF-heating rate, and expanding $f_0 = f_0^{(0)} + f_0^{(1)} + \cdots$, we obtain

$$v_{\|}\boldsymbol{n} \cdot \nabla f_0^{(0)} = 0, \tag{2}$$

$$v_{\parallel}\boldsymbol{n}\cdot\nabla f_0^{(1)} = C\left(f_0^{(0)}\right). \tag{3}$$

From Eq. (2), we conclude $f_0^{(0)} = f_0^{(0)}(E,\mu,V)$ instead of a Maxwellian distribution where $E = v^2/2 + e\Phi/M$ is the total energy, v is the particle speed, e is the charge, Φ is the electrostatic potential, M is the mass, μ is the magnetic moment, and V is the volume enclosed in a flux surface. The exact form of $f_0^{(0)}$ can be determined from the bounce averaged form of Eq. (3). For the purpose of our discussion, knowing $f_0^{(0)} = f_0^{(0)}(E,\mu,V)$ is adequate. We will suppress the superscript in $f_0^{(0)}$ from here on.

It is straightforward to show that f_0 automatically satisfies [7]

$$\langle \boldsymbol{B}\cdot\nabla\cdot\boldsymbol{\pi}\rangle = 0, \tag{4}$$

and

$$\langle \boldsymbol{B}_t\cdot\nabla\cdot\boldsymbol{\pi}\rangle = 0, \tag{5}$$

where the angular brackets denote flux surface average , $\boldsymbol{B}_t =$ is the toroidal magnetic field in Hamada coordinates, $\boldsymbol{B}_t = \Psi'\nabla V\times\nabla\theta$, $\Psi' = \boldsymbol{B}\cdot\nabla\zeta$, ζ is the toroidal angel, θ is the poloidal angle, $\boldsymbol{\pi} = (p_{\parallel} - p_{\perp})(\boldsymbol{nn} - \boldsymbol{I}/3)$ is the total (i.e., ion and electron) CGL (Chew-Goldberger-Low) [8] viscosity tensor, $p_{\parallel}$ is the parallel plasma pressure, $p_{\perp}$ is the perpendicular plasma pressure, and $\boldsymbol{I}$ is the unit tensor. The equilibrium distribution function has no effect on the bulk plasma rotation. Note that the radial force balance may be affected by the RF heating. However, this leads to a modification on the MHD equilibrium.

To model plasma rotation, one needs to solve linear drift kinetic equation for the perturbed distribution $f_1 = f_1(E,\mu,V,\theta)$ which satisfies [9]

$$v_{\parallel}\boldsymbol{n}\cdot\nabla f_1 + \boldsymbol{v}_d\cdot\nabla f_0 = C(f_1), \tag{6}$$

where $\boldsymbol{v}_d$ is the particle drift velocity. From f_1 we calculate nonvanishing

$$\langle \boldsymbol{B}\cdot\nabla\cdot\boldsymbol{\pi}\rangle = \langle \boldsymbol{B}_p\cdot\nabla\cdot\boldsymbol{\pi}\rangle, \tag{7}$$

which is required in the poloidal momentum equation

$$\partial\langle NM\boldsymbol{B}_p\cdot\boldsymbol{V}\rangle/\partial t=\langle\boldsymbol{B}\cdot\nabla\cdot\boldsymbol{\pi}\rangle-\psi'\chi'\langle\boldsymbol{J}\cdot\nabla V\rangle/c. \tag{8}$$

Here, N is plasma density, $\boldsymbol{V}$ is plasma flow velocity, c is the speed of light, $\boldsymbol{B}_p=-\chi'\nabla V\times\nabla\zeta$ is the poloidal magnetic field in Hamada coordinates and $\chi'=\boldsymbol{B}\cdot\nabla\theta$. Because we are only interested in the subsonic flow, $\langle\boldsymbol{B}_p\cdot\boldsymbol{V}\cdot\nabla\boldsymbol{V}\rangle$ term is neglected in Eq. (8). The parallel and poloidal viscosity in Eq. (7) include the effects of RF-heating through the equilibrium distribution f_0. They are RF-modified. If we approximate $f_0=f_M+f_{rf}$ with f_M, a Maxwellian distribution and f_{rf} a RF-modified distribution, the viscosity in Eq. (7) will consist of a standard neoclassical viscosity [10] $\langle\boldsymbol{B}\cdot\nabla\cdot\boldsymbol{\pi}\rangle=\langle\boldsymbol{B}_p\cdot\nabla\cdot\boldsymbol{\pi}\rangle_{nc}$ and a contribution from f_{rf}: $\langle\boldsymbol{B}\cdot\nabla\cdot\boldsymbol{\pi}\rangle_{rf}=\langle\boldsymbol{B}_p\cdot\nabla\cdot\boldsymbol{\pi}\rangle_{rf}$. Note that the separation of plasma viscosity into a neoclassical contribution and a RF contribution is purely formal because the Coulomb collision operator and RF collision operator will mix these two effects in a inseparable way. The radial current density term in Eq. (8) is related to $\partial\langle\boldsymbol{E}\cdot\nabla\boldsymbol{V}\rangle/\partial t$ through Ampére's law:

$$\langle\boldsymbol{J}\cdot\nabla V\rangle=-(1/4\pi)\partial\langle\boldsymbol{E}\cdot\nabla\boldsymbol{V}\rangle/\partial t \tag{9}$$

and at the steady state it has no contribution to plasma rotation. At the steady state we have

$$\langle\boldsymbol{B}\cdot\nabla\cdot\boldsymbol{\pi}\rangle=\langle\boldsymbol{B}\cdot\nabla\cdot\boldsymbol{\pi}\rangle_{nc}+\langle\boldsymbol{B}\cdot\nabla\cdot\boldsymbol{\pi}\rangle_{rf}=0, \tag{10}$$

if we approximate $f_0=f_M+f_{rf}$. At the steady state, RF viscosity induced current is balanced by the standard neoclassical viscosity current. Note that the $\langle\boldsymbol{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}\rangle_{\mathrm{rf}}$ induces a radial flux or current caused by the RF heating. However, this radial flux or current has nothing to do with the toroidal rotation because the corresponding force itself is in the parallel direction. It is important to realize it is the components of the force, not the radial current, that are important when modeling plasma rotation.

Note that f_1 does not contribute to $\langle\boldsymbol{B}_t\cdot\nabla\cdot\boldsymbol{\pi}\rangle$ because of the toroidal symmetry. To obtain a non-vanishing toroidal viscosity $\langle\boldsymbol{B}_t\cdot\nabla\cdot\boldsymbol{\pi}\rangle$, one needs to go to higher order in analogous to the standard neoclassical theory [11]. Note that this non-vanishing toroidal

viscosity represents the radial transport of the toroidal momentum. However, as shown here, the magnitude of this viscosity is smaller than the CGL viscosity. The steady state toroidal plasma rotation is then determined by

$$\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\pi} \rangle = \langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\pi} \rangle_{nc} + \langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\pi} \rangle_{rf} + \langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\pi} \rangle_{an} = 0, \tag{11}$$

if we approximate $f_0 = f_M + f_{rf}$. We repeat that the neoclassical and RF viscosity in Eq. (11) are not CGL viscosity, but are higher order perpendicular viscosity. An anomalous toroidal viscosity $\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\pi} \rangle_{an}$ is also included in Eq. (11) to realistically model toroidal flow damping rate. Again we see that radial plasma current density has no effect on the toroidal rotation at the steady state.

Note that because we assume that there is no momentum input from the RF waves, there are no explicit momentum source terms in Eqs. (10) and (11). We could also adopt the finite-orbit-size ordering similar to the one employed in Ref. 12 in Eqs. (1)-(3) and (6), the conclusion would not have changed though.

We can now discuss the difference between our methodology and a model presented in Ref. [6], where plasma rotation in RF-heated tokamak plasmas is modeled by a $\boldsymbol{J}_{rf} \times \boldsymbol{B}$ force where $\boldsymbol{J}_{rf} \times \boldsymbol{B}$ is a radial current density associated with RF-heating. The difference can be easily understood by examining the components of the forces involved in these two models. Indeed, one finds

$$\langle \boldsymbol{B}_p \cdot \boldsymbol{J}_{rf} \times \boldsymbol{B} \rangle = -\langle \boldsymbol{B}_t \cdot \boldsymbol{J}_{rf} \times \boldsymbol{B} \rangle$$

and

$$\langle \boldsymbol{B} \cdot \boldsymbol{J}_{rf} \times \boldsymbol{B} \rangle = 0$$

based on the radial current modeling; while

$$\langle \boldsymbol{B} \cdot \nabla \cdot \boldsymbol{\pi} \rangle = \langle \boldsymbol{B}_p \cdot \nabla \cdot \boldsymbol{\pi} \rangle \neq -\langle \boldsymbol{B}_t \cdot \nabla \cdot \boldsymbol{\pi} \rangle \tag{12}$$

based on the methodology presented here. Note that because the force components are different between these two models, the resultant plasma rotations are also different; a

consequence of Newton's law. Our methodology is consistent with the standard neoclassical theory of the flow damping [13] and models of H-mode and ERS-mode [14,15]. The relation between our model and RF-induced transport theory [16-18] remains to be investigated.

REFERENCES

[1] E. J. Strait, L. L. Lao, M. E. Mauel. R. W. Rice, T. S. Taylor, K. H. Burrell, M. S. Chu, E. A. Lazarus, T. H. Osborne, S. J. Thompson, and A. D. Turnball, Phys. Rev. Lett. **75**, 4421 (1995).

[2] ASDEX Team, Nucl. Fusion **29**, 1959 (1989).

[3] R. J. Taylor, M. L. Brown, B. D. Fried, H. Grote, J. R. Liberati, G. J. Morales, P. Pribyl, D. Darrow, and M. Ono, Phys. Rev. Lett. **63**, 2365 (1989).

[4] F. M. Levinton, M. C. Zarnstorff, S. H. Batha, M. Bell, R. E. Bell, R. V. Budny, C. E. Bush, Z. Chang, E. D. Fredrickson, A. Janos, J. Manickam, S. T. Ramsey, G. L. Schmidt, E. J. Synakowski, and G. Taylor, Phys. Rev. Lett. **75**, 4417 (1995).

[5] J. E. Rice, P. T. Bonoli, J. A. Goetz, M. J. Greenwald, I. H. Hutchinson, E. S. Marmar, M. Porkolab, S. M. Wolfe, S. J. Wukitch, and C. S. Chang, Nucl. Fusion **39**, 1175 (1999).

[6] C. S. Chang, C. K. Phillips, R. White, S. Zweben, P. T. Bonoli, J. E. Rice, M. J. Greenwald, and J. deGrassie, Phys. Plasmas **6**, 1969 (1999).

[7] R. J. Hastie, J. B. Taylor, and F. A. Hass, Ann. Phys. **41**, 302 (1967).

[8] G. F. Chew, M. L. Goldberger, and F. E. Low, Proc. R. Soc. London, Ser. A **236**, 112 (1956).

[9] F. L. Hinton and R. D. Hazeltine, Rev. Mod. Phys. **42**, 239 (1976).

[10] S. P. Hirshman and D. J. Sigmar, Nucl. Fusion **21**, 1079 (1981).

[11] K. T. Tsang and E. A. Frieman, Phys. Fluids **19**, 767 (1976).

[12] K. C. Shaing, R. D. Hazeltine, and M. C. Zarnstorff, Phys. Plasmas **4**, 771 (1997).

[13] S. P. Hirshman, Nucl. Fusion **18**, 917 (1978).

[14] K. C. Shaing and E. C. Crume, Jr., Phys. Rev. Lett. **63**, 2369 (1989).

[15] K. C. Shaing, A. Y. Aydemir, R. Hiwatari, W. A. Houlberg, Y. Ogawa, and M. C. Zarnstorff, Phys. Rev. Lett. **83**, 3840 (1999).

[16] D. A. D'ippolito and J. R. Myra, Phys. Fluids **28**, 1895 (1985).

[17] L. Chen, J. Vaclavik, and G. W. Hammett, Nucl Fusion **28**, 389 (1988).

[18] S. C. Chiu and V. S. Chan, Nucl. Fusion **29**, 1907 (1989).