Parallel Current in Anisotropic High–Beta Extremely Low Aspect Ratio Tokamak Plasmas

K.C. Shaing

Institute for Fusion Studies

The University of Texas at Austin

Austin, Texas 78712

Abstract

Parallel current in an anisotropic high–beta extremely low aspect ratio tokamak plasma is calculated from the linearized drift kinetic equation. It is found that it depends on the radial gradients of the parallel plasma pressure and the magnetic field strength. It can be expressed as the sum of the bootstrap current and the Pfirsch–Schlüter current.

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I. INTRODUCTION

Stability plasma β limit in low aspect ratio tokamak plasmas can be very high ($\gtrsim 50\%$).[1−4] Here, β is the ratio of the plasma pressure to the magnetic field pressure. Because of the strong auxiliary heating power, equilibrium plasma pressure in such plasmas is likely to be anisotropic, i.e., parallel (to the magnetic field $\boldsymbol{B}$) plasma pressure $p_{\|}$ is not the same as the perpendicular plasma pressure $p_{\perp}$. It will be interesting to investigate pressure gradient driven parallel plasma current, i.e., bootstrap and Pfirsch–Schlüter in such plasmas.

For simplicity, we only calculate the parallel current in the aspect ratio $A \rightarrow 1$ limit. Thus, the results are valid in the edge region of an extremely low aspect ratio tokamak. In that limit, parallel current is not sensitive to the details of the heating mechanisms. The detailed calculations for A away from unity will be addressed separately. A general expression for the Pfirsch–Schlüter current can be obtained from the momentum equation and the conversation laws of particle and charge. Parallel current can be expressed as the sum of the Pfirsch-Schlüter current and the bootstrap current.

This paper is organized as follows: In Sec. II, elementary force balance in anisotropic pressure equilibrium is discussed. In Sec. III the leading order collisionless solution of the linearized drift kinetic equation is obtained. From this solution, parallel plasma flow and current are calculated. It is shown in Sec. IV that exactly the same expression of the parallel plasma flow can be obtained from the momentum equation and the conservation law of particles. Based on this result, a general expression of the Pfirsch–Schlüter current is derived, and an expression of the bootstrap current is calculated. Concluding remarks are given in Sec. V. The proof that a conventional expression for the radial drift velocity is also valid in anisotropic pressure equilibria is shown in Appendix A.

II. EQUILIBRIUM FORCE BALANCE

Anisotropic pressure equilibrium is maintained by the force balance equation[5,6]

$$\frac{1}{c}\boldsymbol{J}\times\boldsymbol{B}=\boldsymbol{\nabla}\cdot\boldsymbol{p^t}, \tag{1}$$

where c is the speed of light, $\boldsymbol{J}$ is the plasma current density, $\boldsymbol{p}^t=p^t_\parallel\widehat{n}\widehat{n}+p^t_\perp(\boldsymbol{I}-\widehat{n}\widehat{n})$, the superscript t denotes total pressure, $\widehat{n}=\boldsymbol{B}/|\boldsymbol{B}|$, and $\boldsymbol{I}$ is the unit tensor. It is obvious that

$$\boldsymbol{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{p}^t=0. \tag{2}$$

which implies

$$\widehat{n}\cdot\boldsymbol{\nabla}p^t_\parallel=(p^t_\parallel-p^t_\perp)\frac{\widehat{n}\cdot\boldsymbol{\nabla}B}{B}. \tag{3}$$

Equation (1) can be expressed explicitly in terms of $p^t_\parallel$ and $p^t_\perp$:

$$\frac{1}{c}\tau^t\boldsymbol{J}\times\boldsymbol{B}=\boldsymbol{\nabla}_\perp p^t_\perp+(p^t_\parallel-p^t_\perp)\frac{\boldsymbol{\nabla}_\perp B}{B}, \tag{4}$$

where $\tau^t=1+4\pi(p^t_\perp-p^t_\parallel)/B^2$. We can define a quantity $\boldsymbol{K}$ such that[5]

$$\frac{4\pi}{c}\boldsymbol{K}=\boldsymbol{\nabla}\times(\tau^t\boldsymbol{B}). \tag{5}$$

Using Eqs. (3)–(5), we can show that

$$\frac{1}{c}\boldsymbol{K}\times\boldsymbol{B}=\boldsymbol{\nabla}p^t_\parallel-(p^t_\parallel-p^t_\perp)\frac{\boldsymbol{\nabla}B}{B}. \tag{6}$$

For an equilibrium distribution function $f_0=f_0(\varepsilon,\mu,\psi)$, $p^t_\parallel(\psi,\theta)=p^t_\parallel(B,\psi)$ and $p^t_\perp(\psi,\theta)=p^t_\perp(B,\psi)$, where ε is the particle energy, μ is the magnetic moment, ψ is the poloidal flux function, and θ is the poloidal angle, Eq. (6) reduces to

$$\frac{1}{c}\boldsymbol{K}\times\boldsymbol{B}=\frac{\partial p^t_\parallel}{\partial\psi}\Big|_B\boldsymbol{\nabla}\psi, \tag{7}$$

which is the same as that in Ref. 5. Note that from Eq. (5),

$$\boldsymbol{B}\cdot\boldsymbol{K}=\tau^t\boldsymbol{J}\cdot\boldsymbol{B}. \tag{8}$$

It is obvious from Eq. (5) that

$$\boldsymbol{\nabla} \cdot \boldsymbol{K} = 0. \tag{9}$$

The cross–product of Eq. (6) with $\boldsymbol{\nabla}\psi$ yields

$$\boldsymbol{K} \cdot \boldsymbol{\nabla}\psi = 0. \tag{10}$$

Note that in general $\boldsymbol{J} \cdot \boldsymbol{\nabla}\psi \neq 0$ in anisotropic pressure equilibria

$$\frac{4\pi}{c}\boldsymbol{J} \cdot \boldsymbol{\nabla}\psi = -\frac{I\boldsymbol{B} \cdot \boldsymbol{\nabla}\theta}{\tau^t}\frac{\partial \tau^t}{\partial \theta}, \tag{11}$$

where $I = RB_t, R$ is major radius, and B_t is the toroidal magnetic field strength. However, $\langle \boldsymbol{J} \cdot \boldsymbol{\nabla}\psi \rangle = 0$ where the angular brackets denote flux surface averaging.

The Grad–Shafranov equation for anisotropic pressure equilibrium is derived in Refs. 5–7, and will not be repeated here.

III. SOLUTION OF THE DRIFT KINETIC EQUATION AND PARALLEL CURRENT

The linearized drift kinetic equation in an anisotropic high β tokamak plasma can be written as

$$v_\parallel \widehat{n} \cdot \boldsymbol{\nabla} f_1 + \boldsymbol{v}_d \cdot \boldsymbol{\nabla} f_0 + \frac{v_\parallel B}{\Omega}\widehat{n} \cdot \boldsymbol{\nabla}\left(\frac{v_\parallel \widehat{n} \cdot \boldsymbol{\nabla} \times \widehat{n}}{B}\right)\mu\frac{\partial f_0}{\partial \mu} = C(f_1), \tag{12}$$

where f_1 is the perturbed particle distribution function, $\boldsymbol{v}_d$ is the drift velocity, $v_\parallel$ is the parallel particle speed, f_0 is the equilibrium particle distribution function, $\Omega = eB/Mc$ is the gyrofrequency, $B = |\boldsymbol{B}|$, μ is the magnetic moment, e is the charge, M is the mass and $C(f_1)$ is the collision operator. In general, $f_0 = f_0(\varepsilon, \mu, \psi)$ where $\varepsilon = v^2/2 + e\Phi/M$, and Φ is the electrostatic potential. The collision operator $C(f_1)$ includes both Coulomb collision and heating operator. Note that $\partial f_0/\partial\mu$ term in the drift kinetic equation[8] should be kept here.

The equilibrium distribution function f_0 satisfies

$$v_\parallel \widehat{n} \cdot \boldsymbol{\nabla} f_0 = C(f_0). \tag{13}$$

For simplicity, we only choose the leading order collisionless solution $f_0^{(0)}$ of Eq. (13) to approximate f_0. In that case $\widehat{n} \cdot \boldsymbol{\nabla} f_0^{(0)} = 0$, and $\left\langle C\left(f_0^{(0)}\right)\right\rangle_b = 0$ determines $f_0^{(0)}$. The angular bracket $\langle \quad \rangle_b$ here, denotes bounce averaging.

In the collisionless limit, i.e., trapped particle bounce frequency is greater than the effective collision frequency, the leading order equation of Eq. (12) is

$$v_\parallel \widehat{n} \cdot \boldsymbol{\nabla} f_1^0 + \boldsymbol{v}_d \cdot \boldsymbol{\nabla} f_0 + \frac{v_\parallel B}{\Omega} \widehat{n} \cdot \boldsymbol{\nabla} \left(\frac{v_\parallel \widehat{n} \cdot \boldsymbol{\nabla} \times \widehat{n}}{B} \right) \mu \frac{\partial f_0}{\partial \mu} = 0. \tag{14}$$

Noting that

$$\boldsymbol{v}_d \cdot \boldsymbol{\nabla} f_0 = \frac{v_\parallel}{B} \boldsymbol{B} \cdot \boldsymbol{\nabla} \left(\frac{I v_\parallel}{\Omega} \right) \frac{\partial f_0}{\partial \psi}. \tag{15}$$

We integrate Eq. (14) to obtain

$$f_1^0 = -\frac{I v_\parallel}{\Omega} \frac{\partial f_0}{\partial \psi} - \frac{B}{\Omega} \frac{v_\parallel \widehat{n} \cdot \boldsymbol{\nabla} \times \widehat{n}}{B} \frac{\partial f_0}{\partial \mu} + g(\varepsilon, \mu, \psi), \tag{16}$$

where g is an integration constant. Note that in general $I = I(\psi, \theta)$, and $\partial(I\tau^t)/\partial\theta = 0$.[7] To determine g, we need to solve the next order equation of Eq. (12)

$$v_\parallel \widehat{n} \cdot \boldsymbol{\nabla} f_1^1 = C(f_1^0). \tag{17}$$

It is obvious that the detailed expression of g depends on the collision operator $C(f_1)$. However, in the $A \rightarrow 1$ limit, $g = 0$ as long as $C(f_1^0)$ is an even function of $v_\parallel$. Here, we adopt such an assumption. Note that g satisfies

$$\left\langle C\left(f_1^{(0)}\right)\right\rangle_b = 0, \tag{18}$$

The function g is, then, an odd function of $\sigma = v_\parallel/|v_\parallel|$. In the $A \rightarrow 1$ limit, all the particles are trapped. Trapped particles satisfy the reflection boundary condition, i.e., $g_{\sigma=+} = g_{\sigma=-}$ at the reflection point. To satisfy the reflection boundary condition g must vanish. We would also like to note that if $C(f)$ is an even function of σ, our results will remain valid even if $f_0^{(1)}$ is included in Eq. (1).

A general expression for parallel mass flow $V_\parallel$ regardless if g vanishes or not can be obtained by taking the $(Bv_\parallel)$ moment of Eq. (16):

$$BNV_\parallel = -\frac{Ic}{e}\left[\frac{\partial p_\parallel}{\partial\psi} - (p_\parallel - p_\perp)\frac{1}{B}\frac{\partial B}{\partial\psi} + Ne\frac{\partial\Phi}{\partial\psi}\right] + \frac{J_\parallel B}{e}\frac{4\pi}{B^2}(p_\parallel - p_\perp) + B^2 G_K(\psi). \tag{19}$$

where N is plasma density $J_\parallel$ is parallel current density, and $G_K(\psi)$ is related to the moment of g and is a flux function. Both $p_\parallel$ and $p_\perp$ are functions of ψ and θ. Note that Eqs. (16)–(19) are valid for all species. In the limit of $A \to 1, G_k(\psi) \to 0$, and the parallel current $J_\parallel$ is

$$\tau^t J_\parallel B = -c\left[I\frac{\partial p^t_\parallel}{\partial\psi} - I\left(p^t_\parallel - p^t_\perp\right)\frac{1}{B}\frac{\partial B}{\partial\psi}\right]. \tag{20}$$

As can be seen from Eq. (20) that parallel current depends on the radial gradients of the parallel plasma pressure $p^t_\parallel$ and magnetic field B.

IV. SOLUTION OF THE FLUID EQUATION, PFIRSCH–SCHLÜTER CURRENT, AND BOOTSTRAP CURRENT

The leading order steady state momentum equation, for a single species, is[9]

$$Ne\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{V}\times\boldsymbol{B}\right) = \boldsymbol{\nabla}\cdot\boldsymbol{p}, \tag{21}$$

where $\boldsymbol{V}$ is the plasma flow velocity, $\boldsymbol{E}$ is the electrostatic electric field. The parallel component of Eq. (21) is

$$Ne\,\boldsymbol{B}\cdot\boldsymbol{E} = \boldsymbol{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{p}. \tag{22}$$

For an equilibrium distribution function of the form $f_0 = f_0(\varepsilon,\mu,\psi)$, Eq. (22) is automatically satisfied.[5,9] The perpendicular flow velocity, calculated from Eq. (21), is

$$N\boldsymbol{V}_\perp = -\frac{1}{e}\left(\tau\boldsymbol{J} - \frac{c}{4\pi}\boldsymbol{B}\times\boldsymbol{\nabla}\tau\right) + \frac{\boldsymbol{J}}{e} + \frac{1}{e}(\tau-1)\boldsymbol{J}_\parallel + \frac{c}{eB^2}\boldsymbol{B}\times\boldsymbol{\nabla\psi} \times\left[(p_\perp - p_\parallel)\frac{1}{B}\frac{\partial B}{\partial\psi} - \frac{\partial p_\parallel}{\partial\psi} - Ne\frac{\partial\Phi}{\partial\psi}\right]. \tag{23}$$

The quantity τ is defined as $\tau = 1 + 4\pi(p_\perp - p_\parallel)/B^2$, where $p_\perp$ and $p_\parallel$ are anisotropic pressure for each individual species. Although the local radial flux $N\boldsymbol{V}_\perp \cdot \boldsymbol{\nabla}\psi \neq 0$ in general, $\langle N\boldsymbol{V}_\perp \cdot \boldsymbol{\nabla}\psi\rangle = 0$. Substituting Eq. (23) into $\boldsymbol{\nabla} \cdot (N\boldsymbol{V}) = 0$, and solving for $NV_\parallel$, we obtain

$$\frac{NV_\parallel}{B} = -\frac{\tau - 1}{e}\frac{J_\parallel}{B} + \frac{Ic}{eB^2}\left[(p_\parallel - p_\perp)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel}{\partial \psi} - Ne\frac{\partial \Phi}{\partial \psi}\right] + G_f(\psi), \tag{24}$$

where $G_f(\psi)$ is an integration constant. Note that Eq. (24) and Eq. (19) are identical. To obtain such an agreement one has to keep $\partial f_0/\partial\mu$ in the linearized drift kinetic equation. Since Eq. (23) is valid for all species, an expression for parallel plasma current can be constructed to obtain

$$B\tau^t J_\parallel = Ic\left[(p_\parallel^t - p_\perp^t)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}\right] + B^2 G(\psi). \tag{25}$$

It is obvious that Eq. (25) can also be obtained directly from $\boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$. The integration constant $G(\psi)$ can be determined to be, from $\left\langle BJ_\parallel\right\rangle = 0$,

$$G(\psi) = \frac{\left\langle BJ_\parallel\right\rangle}{\langle B^2/\tau^t\rangle} - \frac{\left\langle \frac{Ic}{\tau^t}[(p_\parallel^t - p_\perp^t)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}]\right\rangle}{\langle B^2/\tau^t\rangle}. \tag{26}$$

Substituting Eq. (26) into (25), we find

$$BJ_\parallel = \frac{\left\langle BJ_\parallel\right\rangle}{\langle B^2/\tau^t\rangle}\frac{B^2}{\tau^t} + \left\{\frac{Ic}{\tau^t}\left[(p_\parallel^t - p_\perp^t)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}\right] - \frac{B^2/\tau^t}{\langle B^2/\tau^t\rangle}\left\langle \frac{Ic}{\tau^t}\left[(p_\parallel^t - p_\perp^t)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}\right]\right\rangle\right\}. \tag{27}$$

The term in the curly brackets in Eq. (27) vanishes after flux surface averaging. The variation of the bootstrap current in a flux surface has the form $\tau^t J_b = \langle J_b B\rangle B/\langle B^2/\tau^t\rangle$. We can define a Pfirsch–Schlüter current as

$$BJ_{ps} = \frac{Ic}{\tau^t}\left[(p_\parallel^t - p_\perp^t)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}\right] - \frac{B^2/\tau^t}{\langle B^2/\tau^t\rangle}\left\langle \frac{Ic}{\tau^t}\left[(p_\parallel^t - p_\perp^t)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}\right]\right\rangle, \tag{28}$$

and a bootstrap current

$$\langle BJ_b\rangle = c\left\langle \frac{I}{\tau^t}\left[\frac{\partial p_\parallel}{\partial \psi} + \left(p_\perp^t - p_\parallel^t\right)\frac{1}{B}\frac{\partial B}{\partial \psi}\right]\right\rangle. \tag{29}$$

The sum of the bootstrap current and the Pfirsch–Schlüter current is the same as the parallel current in Eq. (20).

The evolution equation for $(I\tau^t)$ is

$$\frac{\partial(I\tau^t)}{\partial\psi} = -\frac{4\pi}{c}\frac{\boldsymbol{K}\cdot\boldsymbol{\nabla}\theta}{\boldsymbol{B}\cdot\boldsymbol{\nabla}\theta}. \tag{30}$$

Because $\partial(I\tau^t)/\partial\theta = 0$, $(\boldsymbol{K}\cdot\boldsymbol{\nabla}/\boldsymbol{B}\cdot\boldsymbol{\nabla}\theta)$ is a function of ψ only. From Eq. (4), one finds

$$\frac{\boldsymbol{K}_\perp\cdot\boldsymbol{\nabla}\theta}{\boldsymbol{B}\cdot\boldsymbol{\nabla}\theta} = \frac{Ic}{B^2}\left[\frac{\partial p_\parallel^t}{\partial\psi} - \left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial\psi}\right]. \tag{31}$$

With Eq. (31), we obtain

$$-\frac{c}{4\pi}\frac{\partial}{\partial\psi}(I\tau^t) = \frac{K_\parallel}{B} + \frac{Ic}{B^2}\left[\frac{\partial p_\parallel^t}{\partial\psi} - \left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial\psi}\right]. \tag{32}$$

Noting that $K_\parallel = \tau^t J_\parallel$, we can express $K_\parallel/B$ as $B\tau^t J_\parallel/B^2$. Note that in the $A \rightarrow 1$ limit $-(c/4\pi)\partial(I\tau^t)/\partial\psi = 0$ from Eq. (20).

V. CONCLUDING REMARKS

Because the equilibrium parallel face balance equation

$$\frac{\partial p_\parallel^t}{\partial\theta} = (p_\parallel^t - p_\perp^t)\frac{1}{B}\frac{\partial B}{\partial\theta}. \tag{33}$$

When $p_\parallel^t(\psi,\theta) = p_\parallel^t(B,\psi)$ and $p_\perp^t(\psi,\theta) = p_\perp^t(B,\psi)$,[5] and Eq. (33) is reduced to

$$\frac{p_\parallel^t - p_\perp^t}{B} = \frac{\partial p_\parallel^t}{\partial B}. \tag{34}$$

With Eq. (34), the quantity $[(p_\parallel^t - p_\perp^t)\partial B/B\partial\psi - \partial p_\parallel^t/\partial\psi]$ in Secs. III and IV can be simplified to

$$(p_\parallel^t - p_\perp^t)\frac{1}{B}\frac{\partial B}{\partial\psi} - \frac{\partial p_\parallel^t(\psi,\theta)}{\partial\psi} = -\frac{\partial p_\parallel^t(B,\psi)}{\partial\psi}. \tag{35}$$

Therefore, both bootstrap current and Pfirsch–Schlüter current can be expressed in terms of the radial gradient of parallel plasma pressure evaluated at constant B, i.e.,

$$\left\langle J_b B\right\rangle = -c\left\langle \frac{I\partial p_\parallel^t(B,\psi)}{\tau^t\partial\psi}\right\rangle, \tag{36}$$

$$J_{ps}B = -\frac{Ic}{\tau^t}\frac{\partial p_\parallel^t(B,\psi)}{\partial\psi} + \frac{B^2/\tau^t}{\left\langle B^2/\tau^t\right\rangle}\left\langle \frac{Ic}{\tau^t}\frac{\partial p_\parallel^t(B,\psi)}{\partial\psi}\right\rangle. \tag{37}$$

The total parallel plasma current in the $A \to 1$ limit is then

$$\tau^t J_\parallel B = -Ic\frac{\partial p_\parallel^t(B,\psi)}{\partial\psi}. \tag{38}$$

Note that the definitions for bootstrap current and Pfirsch–Schlüter current are not unique as can be seen from the definition given here and that in Ref. 7. One can also define a Pfirsch–Schlüter current as $\left\langle B\tau^t J_{ps}\right\rangle = 0$ instead of $\left\langle BJ_{ps}\right\rangle = 0$. In this case, the Pfirsch–Schlüter current is

$$\begin{aligned} B\tau^t J_{ps} = {} & Ic\left[\left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial\psi} - \frac{\partial p_\parallel^t}{\partial\psi}\right] \\ & - B^2\left\langle B^2\right\rangle^{-1}\left\langle Ic\left[\left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial\psi} - \frac{\partial p_\parallel^t}{\partial\psi}\right]\right\rangle \end{aligned} \tag{39}$$

and

$$\left\langle B\tau^t J_b\right\rangle = -\left\langle Ic\left[\frac{\partial p_\parallel^t}{\partial\psi} + \left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial\psi}\right]\right\rangle. \tag{40}$$

The total parallel current, which is the sum of Eqs. (39) and (40), is

$$BJ_\parallel = \frac{Ic}{\tau^t}\left[-\frac{\partial p_\parallel^t}{\partial\psi} + \left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial\psi}\right].$$

Total parallel current, as it should be, does not depend on the definitions we choose. One way to remove this ambiguity is to examine the electron parallel friction force expression $F_{\parallel e}$. If the electron friction force has the same dependence on $J_\parallel$ as in the isotropic case, namely, $\left\langle BF_{\parallel e}\right\rangle \propto \left\langle BJ_\parallel\right\rangle$, the definition that requires $\left\langle BJ_{ps}\right\rangle = 0$ seems to be a proper choice. If, however, $\left\langle BF_{\parallel e}\right\rangle \propto \left\langle B\tau^t J_\parallel\right\rangle$, $\left\langle B\tau^t J_{ps}\right\rangle = 0$ is the proper one. If $\left\langle BF_{\parallel e}\right\rangle$ has a $J_\parallel$ dependence different from the two possibilities discussed, the definition for Pfirsch–Schlüter should change accordingly. If we assume, in general, $\left\langle BF_{\parallel e}\right\rangle \propto \left\langle B\mathcal{P}J_\parallel\right\rangle$, we can choose $\left\langle B\mathcal{P}J_{ps}\right\rangle = 0$. Here $\mathcal{P}$ is a function of (ψ,θ), and has the property that $\mathcal{P} \to 1$ as $\tau \to 1$. With this definition,

$$
\begin{aligned}
BJ_{ps} = {} & \frac{Ic}{\tau^t}\left[\left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}\right] \\
& - \frac{B^2}{\tau^t}\left\langle \mathcal{P}\,\frac{B^2}{\tau^t}\right\rangle^{-1}\left\langle \frac{Ic}{\tau^t}\mathcal{P}\left[\left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}\right]\right\rangle, \qquad (41)
\end{aligned}
$$

$$
\langle \mathcal{P} B J_b\rangle = \left\langle \frac{Ic}{\tau^t}\mathcal{P}\left[\left(p_\parallel^t - p_\perp^t\right)\frac{1}{B}\frac{\partial B}{\partial \psi} - \frac{\partial p_\parallel^t}{\partial \psi}\right]\right\rangle, \qquad (42)
$$

and the total parallel current is

$$
BJ_\parallel = \frac{B^2}{\tau^t}\frac{\langle \mathcal{P} B J_b\rangle}{\left\langle \mathcal{P}\,\frac{B^2}{\tau^t}\right\rangle} + BJ_{ps},
$$

which is identical to Eq. (20).

In conclusion, expressions of the parallel current in the $A \rightarrow 1$ limit for anisotropic, high–β tokamak plasmas are derived. It is found that it depends on the radial gradients of parallel plasma pressure and magnetic field strength. This current can be decomposed in terms of the bootstrap current and the Pfirsch–Schlüter current.

Appendix A: Radial Drift Velocity in Anisotropic High-Beta Tokamak Plasmas

Here we show that the well–known expression for the radial drift velocity[7,8]

$$\boldsymbol{v}_d \cdot \boldsymbol{\nabla}\psi = v_\parallel \widehat{n} \cdot \boldsymbol{\nabla}\left(\frac{I v_\parallel}{\Omega}\right), \tag{A1}$$

is still valid in anisotropic, high–β tokamak plasmas.

Recall that the radial drift velocity can be written as[8]

$$\boldsymbol{v}_d \cdot \boldsymbol{\nabla}\psi = \frac{c\widehat{n} \times \boldsymbol{\nabla}\Phi}{B} \cdot \boldsymbol{\nabla}\psi + \frac{\mu}{\Omega}\widehat{n} \times \boldsymbol{\nabla}B \cdot \boldsymbol{\nabla}\psi + \frac{v_\parallel^2}{\Omega}\widehat{n} \times (\widehat{n} \cdot \boldsymbol{\nabla}\widehat{n}) \cdot \boldsymbol{\nabla}\psi. \tag{A2}$$

For an anisotropic, high–β plasma, the curvature $\widehat{n} \cdot \boldsymbol{\nabla}\widehat{n}$ is

$$\widehat{n} \cdot \boldsymbol{\nabla}\widehat{n} = -\frac{\boldsymbol{B} \times \boldsymbol{K}}{\tau^t B^2} + \widehat{n} \times \left[\tau^t \boldsymbol{B} \times \boldsymbol{\nabla}\left(\frac{1}{\tau^t B}\right)\right], \tag{A3}$$

therefore,

$$\widehat{n} \times (\widehat{n} \cdot \boldsymbol{\nabla}\widehat{n}) \cdot \boldsymbol{\nabla}\psi = -\frac{1}{\tau^t B^2}\widehat{n} \times (\boldsymbol{B} \times \boldsymbol{K}) \cdot \boldsymbol{\nabla}\psi + \frac{1}{\tau^t B^2}\boldsymbol{B} \times \boldsymbol{\nabla}(\tau^t B) \cdot \boldsymbol{\nabla}\boldsymbol{\psi}. \tag{A4}$$

Note that the first term on the right–hand side of Eq. (A4) vanishes. The radial drift velocity is

$$\boldsymbol{v}_d \cdot \boldsymbol{\nabla}\psi = -\frac{I\widehat{n} \cdot \boldsymbol{\nabla}\theta}{B}\left[c\frac{\partial \Phi}{\partial \theta} + \frac{\mu B}{\Omega}\frac{\partial B}{\partial \theta} + \frac{v_\parallel^2}{\tau^t \Omega}\frac{\partial}{\partial \theta}(\tau^t B)\right]. \tag{A5}$$

It is straightforward to show that Eq. (A5) is identical to Eq. (A1) by noting that $\partial(I\tau^t)/\partial\theta = 0$.

REFERENCES

[1] Y.-K. M. Peng and D.J. Strickler, Nucl. Fusion **26**, 769 (1986).

[2] A. Sykes, Nucl. Fusion **32**, 694 (1992).

[3] R.L. Miller, Y.R. Lin-Liu, A.D. Turnbull, V.S. Chan, L.D. Pearlstein, O. Sauter, and L. Villard, Phys. Plasmas **4**, 1062 (1997).

[4] J.E. Menard, S.C. Jardin, S.M. Kaye, C.E. Kessel, and J. Manickam, Nucl. Fusion **37**, 595 (1997).

[5] H. Grad, Phys. Fluids **10**, 137 (1967).

[6] C. Mercier and M. Cotsaftis, Nucl. Fusion **1** 12 (1961).

[7] S.P. Hirshman and D.J. Sigmar, Nucl. Fusion **21**, 1079 (1981).

[8] F.L. Hinton and R.D. Hazeltine, Rev. Mod. Phys. **48**, 239 (1976).

[9] R.J. Hastie, J.B. Taylor, and F.A. Hass, Ann. Phys. **41**, 302 (1967).