A Collisionless Self-Organizing Model for the H-mode Boundary Layer

S.M. Mahajan

Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712

Z. Yoshida

Graduate School of Frontier Sciences, The University of Tokyo, Tokyo 113-8656, Japan

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Abstract

It is shown that in a collisionless two-fluid model, a combination of the Hall term and fluid vorticity can lead to the formation of a self-organized singular layer which displays the essential observational features of the thin shear-layer associated with the H-mode tokamak discharges: the layer width is of the order of a poloidal gyro-radius, the poloidal velocity of the order of poloidal Mach number unity, and an electrostatic potential (yielding a negative electric field) of the order of the edge plasma temperature.

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I. INTRODUCTION

Despite the rich and diverse experimental phenomenology [1], as well as an equally rich theoretical literature [2], associated with the shear-flow layer which develops when a tokamak plasma makes a transition to the high confinement mode (H-mode), there is general agreement that the defining characteristics of this layer may be summarized as: (1) There is a relatively large plasma flow in the layer (2) The flow is highly sheared in that the characteristic length scale (the layer width) on which the velocity field builds up is rather short; the numerical measure of the length scale is a poloidal gyro-radius (3) There is a precipitous fall in plasma pressure as we go across this thin layer, and (4) a negative electric field corresponding to an electrostatic potential of the order of the edge plasma temperature permeates the layer. In this paper we present a collisionless two-fluid model in which a singular layer of the above-mentioned variety can readily arise as the plasma self-organizes due to the interaction of the flows and the currents inside the layer. The theory is based on our recent work [3] extended to deal with tokamak-like plasma, i.e., the plasmas embedded in a strong external magnetic field. The necessary elements for the creation of a singular layer are the Hall (in the electron equation of motion) and the fluid vorticity terms (in the ion equation); their combination leads to a high spatial derivative term which introduces a short characteristic length scale (λ_i, the ion skin depth) to the otherwise scaleless magnetohydrodynamics (MHD).

II. TWO-FLUID MAGNETOHYDRODYNAMICS

For simplicity, we consider a quasineutral plasma with singly charged ions. Neglecting their small inertia, the electrons obey

$$\boldsymbol{E} + \boldsymbol{V}_e \times \boldsymbol{B} + \frac{1}{en}\nabla p_e = 0, \tag{1}$$

where $\boldsymbol{V}_e$ and p_e are, respectively, the electron flow velocity and pressure, $\boldsymbol{E}$ ($\boldsymbol{B}$) is the electric (magnetic) field, $-e$ is the electron charge, and n is the number density. The ion equation of motion is

$$\frac{\partial}{\partial t}\boldsymbol{V} + (\boldsymbol{V}\cdot\nabla)\boldsymbol{V} = \frac{e}{M}\left(\boldsymbol{E} + \boldsymbol{V}\times\boldsymbol{B}\right) - \frac{1}{Mn}\nabla p_i, \tag{2}$$

where M is the ion mass ($M \gg$ electron mass), and p_i is the ion pressure. We can eliminate $\boldsymbol{E}$ and $\boldsymbol{V}_e$ using $\boldsymbol{V}_e = \boldsymbol{V} - \boldsymbol{j}/(en)$, $\boldsymbol{j} = \mu_0^{-1}\nabla\times\boldsymbol{B}$ ($\boldsymbol{j}$ is the electric current), and $\boldsymbol{E} = -\partial\boldsymbol{A}/\partial t - \nabla\phi$, where $\boldsymbol{A}$ (ϕ) is the vector (scalar) potential. Choosing an arbitrary length scale L_0 and a magnetic field B_0, we normalize variables as

$$\boldsymbol{x} = L_0\widehat{\boldsymbol{x}}, \quad \boldsymbol{B} = B_0\widehat{\boldsymbol{B}}, \quad t = (L_0/V_A)\widehat{t},$$

$$p = (B_0^2/\mu_0)\widehat{p}, \quad \phi = (L_0 B_0 V_A)\widehat{\phi}, \quad \boldsymbol{V} = V_A\widehat{\boldsymbol{V}};$$

$V_A = B_0/\sqrt{\mu_0 M n}$ being the Alfvén speed. Equations (1) and (2) transform to

$$\frac{\partial}{\partial\widehat{t}}\widehat{\boldsymbol{A}} = \left(\widehat{\boldsymbol{V}} - \varepsilon\widehat{\nabla}\times\widehat{\boldsymbol{B}}\right)\times\widehat{\boldsymbol{B}} - \widehat{\nabla}\left(\widehat{\phi} - \varepsilon\widehat{p}_e\right), \tag{3}$$

$$\begin{aligned}\frac{\partial}{\partial\widehat{t}}(\varepsilon\widehat{\boldsymbol{V}} + \widehat{\boldsymbol{A}}) = \widehat{\boldsymbol{V}}\times\left(\widehat{\boldsymbol{B}} + \varepsilon\widehat{\nabla}\times\widehat{\boldsymbol{V}}\right)& \\ - \widehat{\nabla}\left(\varepsilon\widehat{V}^2/2 + \varepsilon\widehat{p}_i + \widehat{\phi}\right)&,\end{aligned} \tag{4}$$

where the scaling coefficient $\varepsilon = \lambda_i/L_0$ is a measure of the ion skin depth

$$\lambda_i = \frac{c}{\omega_{pi}} = \frac{V_A}{\omega_{ci}} = \sqrt{\frac{M}{\mu_0 n e^2}}.$$

Here, for simplicity, we have assumed the density n to be constant. This is, of course, not true for the H-mode layers where the density falls sharply. It turns out that for a given pressure drop, the effects of the variation of n (even when the variation is on the scale of the layer), though profound in determining the details of the fields in the layer, do not make a qualitative difference in the total change suffered by other observables as we go across the layer. Thus the essential features of the theory, which depend upon the jump conditions across the layer (separating the core plasma from the edge region), will be accessible within the constant n assumption. The details will be given in future work. As pointed out earlier, the Hall term $\varepsilon(\widehat{\nabla}\times\widehat{\boldsymbol{B}})\times\widehat{\boldsymbol{B}}$ of (3), and the vorticity term $\varepsilon(\widehat{\nabla}\times\widehat{\boldsymbol{V}})\times\widehat{\boldsymbol{V}}$ of (4) force the short ion skin depth scale on the system and may be regarded as the source of a singular perturbation to the conventional MHD equations. The combination of these terms plays an essential role in determining the structure of the thin shear-flow layer which may appear at the edge of a high-temperature plasma. We now choose the length scale $L_0 = \lambda_i$ (and hence, $\varepsilon = 1$).

III. SEPARATION OF SELF-FIELDS AND EXTERNAL FIELDS

To apply the theory to a tokamak plasma (embedded in a strong external magnetic field), it is appropriate to decompose the magnetic field $\boldsymbol{B}$ into the self-field component $\boldsymbol{B}_s$, and the externally rooted component $\boldsymbol{B}_h$ with $|\boldsymbol{B}_h| = B_0$ ($\gg |\boldsymbol{B}_s|$). Only $\boldsymbol{B}_s$ is produced by the plasma current $\boldsymbol{j}$ in the region of our interest (a thin boundary layer of the core plasma), while $\boldsymbol{B}_h$ is current-free (curl-free, and thus "harmonic") in that region. From now on, the dynamical part of the field, $\boldsymbol{B}_s$, will be normalized by its representative value B_*. The velocities are, then, normalized by the corresponding Alfvén velocity V_{A*}. The pressure gradient across the boundary layer is maintained by the diamagnetic pressure of the magnetic field, and hence, we have an estimate for the variation of the pressure and the magnetic field across the layer; $\delta p = \delta(|\boldsymbol{B}|^2)/(2\mu_0) \approx (B_0 B_*)/\mu_0$ (in physical units) which reads

$$\frac{B_*}{B_0} = \frac{\beta}{2} \ (\ll 1), \tag{5}$$

where β is the conventional beta ratio evaluated for the pressure maintained at the inner edge of the boundary layer. Formally, we define

$$\widehat{\widehat{\boldsymbol{B}}}_s = \left(\frac{B_0}{B_*}\right)\widehat{\boldsymbol{B}}, \quad \widehat{\widehat{\boldsymbol{V}}} = \left(\frac{B_0}{B_*}\right)\widehat{\boldsymbol{V}}, \quad \widehat{\widehat{\nabla}} = \widehat{\nabla},$$
$$\widehat{\widehat{p}} = \left(\frac{1}{\beta/2}\right)\widehat{p}, \quad \widehat{\widehat{\phi}} = \left(\frac{1}{\beta/2}\right)\widehat{\phi},$$

with the idea that all the normalized dynamical variables are of order unity. In what follows, we shall drop $\widehat{\ }$ to simplify the notation. We consider a one-dimensional system where the fields vary only in the "radial" direction, perpendicular to the magnetic surfaces, i.e., $\boldsymbol{B}_h \cdot \nabla \equiv 0$. We also assume that $\boldsymbol{V}$ is incompressible ($\nabla \cdot \boldsymbol{V} = 0$). Then, we find $\nabla \times (\boldsymbol{V} \times \boldsymbol{B}_h) = 0$, which allows us to write

$$\boldsymbol{V} \times \boldsymbol{B}_h = \left(\frac{\beta}{2}\right)^{-1} \nabla P_i, \tag{6}$$

where P_i is a potential field. Similarly

$$\boldsymbol{V}_e \times \boldsymbol{B}_h = (\boldsymbol{V} - \nabla \times \boldsymbol{B}) \times \boldsymbol{B}_h = -\left(\frac{\beta}{2}\right)^{-1} \nabla P_e \tag{7}$$

with

$$P_e = -P_i - \left(\frac{\beta}{2}\right) \boldsymbol{B}_h \cdot \boldsymbol{B}_s. \tag{8}$$

Equations (6) and (7) represent zero-order diamagnetism. We may, now, rewrite the system (3) and (4) as

$$\begin{aligned} \frac{\partial}{\partial t}\boldsymbol{A}_s &= (\boldsymbol{V} - \nabla \times \boldsymbol{B}_s) \times \boldsymbol{B}_s \\ &\quad + \left(\frac{\beta}{2}\right)^{-1} \nabla (p_e - P_e - \phi), \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{\partial}{\partial t}(\boldsymbol{V} + \boldsymbol{A}_s) &= \boldsymbol{V} \times (\boldsymbol{B}_s + \nabla \times \boldsymbol{V}) \\ &\quad - \left(\frac{\beta}{2}\right)^{-1} \nabla \left[p_i - P_i + \phi + \left(\frac{\beta}{2}\right) \frac{V^2}{2}\right], \end{aligned} \tag{10}$$

where $\nabla \times \boldsymbol{A}_s = \boldsymbol{B}_s$. The most noteworthy feature of equations (6) and (7) is that all the forces which depend upon the externally rooted field have been reduced to effective gradient forces (appearing on the right hand side). Barring the gradient terms, the entire dynamics is expressible in terms of the variables intrinsic to the layer: the self-field, and the flow velocity. This separation is crucial—it is only because of this that we can derive a set of closed equations determining and relating $\boldsymbol{B}_s$ and $\boldsymbol{V}$. We must emphasize that casting external forces as gradient forces is generally not possible in any extended region of the plasma; it is only for the layer dynamics that the assumption $\boldsymbol{B}_h \cdot \nabla \approx 0$ locally holds allowing us this important simplification.

IV. BELTRAMI CONDITIONS AND GENERALIZED BERNOULLI CONDITIONS

We shall consider the most basic, stationary (or slowly evolving) structures of the electromagnetic fields and the associated flows. Taking the curl of (9) and (10), we can cast them in a system of vortex-dynamics equations

$$\frac{\partial}{\partial t}\boldsymbol{\Omega}_j - \nabla \times (\boldsymbol{U}_j \times \boldsymbol{\Omega}_j) = 0 \quad (j = 1, 2) \tag{11}$$

in terms of a pair of generalized vorticities

$$\boldsymbol{\Omega}_1 = \boldsymbol{B}_s, \quad \boldsymbol{\Omega}_2 = \boldsymbol{B}_s + \nabla \times \boldsymbol{V},$$

and the corresponding effective flows

$$\boldsymbol{U}_1 = \boldsymbol{V} - \nabla \times \boldsymbol{B}_s, \quad \boldsymbol{U}_2 = \boldsymbol{V}.$$

From the set (11), we can show that, under appropriate boundary conditions, the two total helicities

$$h_j = \frac{1}{2}\int (\mathrm{curl}^{-1}\boldsymbol{\Omega}_j) \cdot \boldsymbol{\Omega}_j \; dx \qquad (j = 1, 2)$$

and the energy

$$E = \frac{1}{2}\int (|\boldsymbol{B}|^2 + |\boldsymbol{V}|^2) \; dx$$

are invariant in time. Notice that these helicities, determined by self-fields alone, are distinct from the version used, for example, by Steinhauer and Ishida [4], where the total magnetic field is used to define them. The coupled Beltrami equations, relating the self-magnetic field and the flow velocity field (in the layer) are now obtained by demanding the variation

$$\delta(E - \mu_1 h_1 - \mu_2 h_2) = 0,$$

where μ_1 and μ_2 are the Lagrange multipliers [5]. For independent variations with respect to $\boldsymbol{A}_s$ and $\boldsymbol{V}$, we obtain the Euler equations ($a = 1/\mu_1$, $b = 1/\mu_2$)

$$
\boldsymbol{B}_s = a(\boldsymbol{V} - \nabla \times \boldsymbol{B}_s), \tag{12}
$$

$$
\boldsymbol{B}_s + \nabla \times \boldsymbol{V} = b\boldsymbol{V}, \tag{13}
$$

which are nothing but the Beltrami conditions [3] expressing the alignment of the respective vorticities and the corresponding flows. As a direct consequence of (12)-(13) and the equilibrium conditions (9)-(10), we obtain a set of generalized "Bernoulli conditions" (c_i, c_e: constants)

$$
p_i - P_i + \phi + \left(\frac{\beta}{2}\right)\frac{V^2}{2} = c_i, \tag{14}
$$

$$
\begin{aligned}
&p_e - P_e - \phi \\
&\quad = p_e + P_i + \left(\frac{\beta}{2}\right)\boldsymbol{B}_h \cdot \boldsymbol{B}_s - \phi = c_e.
\end{aligned} \tag{15}
$$

relating the plasma pressure and the electrostatic field with the self and the externally applied fields. We note that the constancy of the energy density (the sum of the potential and the kinetic energy) implied in (14)-(15) refers to the directions perpendicular, as well as parallel, to the streamlines of $\boldsymbol{V}$. This is an essential difference from the conventional Bernoulli condition. The generalized Bernoulli conditions constitute one of the central results as well as a justification of the present argument of "self-organization." The implied "homogeneity" (or equilibrium) of energy densities in the transverse direction of the ambient streamlines is brought about, possibly, by the existence of fluctuations which average out intensive variables. An interesting consequence is the appearance of a non-trivial "structure" (with a characteristic length scale) in some physical quantities creating a thin boundary layer separating two regions, and enhancing the state of thermal non-equilibrium.

V. STRUCTURE OF SELF-FIELDS IN LAYER

A. Layer Model

The general features of the structure represented by (12)-(13), and (14)-(15) can be illustrated by an analysis in slab geometry (the coordinate x is radial, y is poloidal, and z

is toroidal). We consider a boundary layer 0Δ is scraped-off by a physical boundary. The layer thickness Δ is to be determined by the theory. The fields $\boldsymbol{B}_s$ and $\boldsymbol{V}$ in the boundary layer are determined by solving (12)-(13) with appropriate boundary conditions on $\boldsymbol{B}_s$ and $\boldsymbol{V}$, as well as assuming values for a, b and the width Δ. We note that these equations can be solved without reference to the Bernoulli conditions (14)-(15). Then, the Bernoulli conditions relate the field $\boldsymbol{B}_s$ and $\boldsymbol{V}$ to the pressures p_e, p_i and the electrostatic potential ϕ. When we prescribe the "jumps" of these quantities across the layer, $\boldsymbol{B}_s$ and $\boldsymbol{V}$ must be set to yield the given jumps, and these conditions will demand a consistent set a, b, and Δ, resulting in a totally self-consistent model of the boundary layer. Note that the scale of the singular perturbation ($\epsilon = \lambda_i/L_0$) defines the scale of the boundary layer. In the limit $\epsilon \to 0$, the boundary layer shrinks into a surface, and some physical quantities will have discontinuities through the surface.

B. Beltrami Conditions

We begin the analysis by explicitly solving Beltrami equations. Combining (12) and (13) yields a second order partial differential equation

$$\nabla \times (\nabla \times \boldsymbol{V}) + c_1 \nabla \times \boldsymbol{V} + c_2 \boldsymbol{V} = 0, \tag{16}$$

where $c_1 = (1/a) - b$ and $c_2 = 1 - b/a$. Denoting the curl derivative $(\nabla\times)$ by "curl," (16) becomes

$$(\mathrm{curl} - \Lambda_+)(\mathrm{curl} - \Lambda_-)\boldsymbol{V} = 0, \tag{17}$$

where

$$\Lambda_\pm = \frac{1}{2}\left[-c_1 \mp (c_1^2 - 4c_2)^{1/2}\right]. \tag{18}$$

Since the operators $(\mathrm{curl} - \Lambda_\pm)$ commute, the general solution to the "double curl Beltrami equation" (17) is given by the linear combination of two Beltrami fields $\boldsymbol{G}_{\Lambda_\pm}$ satisfying $(\mathrm{curl} - \Lambda_\pm)\boldsymbol{G}_{\Lambda_\pm} = 0$, i.e.,

$$\boldsymbol{V} = C_+ \boldsymbol{G}_{\Lambda_+} + C_- \boldsymbol{G}_{\Lambda_-}, \tag{19}$$

where $C_\pm$ are arbitrary constants. The corresponding magnetic field is given by

$$\boldsymbol{B}_s = (b - \Lambda_+)\, C_+ \boldsymbol{G}_{\Lambda_+} + (b - \Lambda_-)\, C_- \boldsymbol{G}_{\Lambda_-}. \tag{20}$$

These interrelated flow and magnetic fields represent the structure of a "thin layer" which may be generated at the boundary of a plasma where a shock-like jump in the pressure emerges. In slab geometry, we easily find that the sheared vector field

$$\boldsymbol{\mathcal{G}}_{\Lambda_\pm} = \ {}^t(0,\ \ \sin(\Lambda_\pm x + \theta_\pm),\ \ \cos(\Lambda_\pm x + \theta_\pm)) \tag{21}$$

solves $\nabla \times \boldsymbol{G}_{\Lambda_\pm} = \Lambda_\pm \boldsymbol{G}_{\Lambda_\pm}$ ($\theta_\pm$ are arbitrary constants). The four parameters $C_\pm$ and $\theta_\pm$ are to be determined by boundary conditions on $\boldsymbol{B}_s$ and $\boldsymbol{V}$. In the core plasma, we assume $\boldsymbol{V} = 0$ (possibly in some inertial system). Since $\boldsymbol{B}_s$ is the magnetic field generated by the current in the layer, the poloidal field must vanish at the inner boundary [6]. To normalize $\boldsymbol{B}_s$, we set $B_{s,z}(0) = -B_*$ where B_* has been related to the plasma pressures by (5). The sign of $B_{s,z}(0)$ is chosen to indicate that the layer is diamagnetic; starting from a negative value, the self-field approaches zero at the outer boundary of the layer. In summary, we have

$$V_y(0) = 0,\ V_z(0) = 0,\ B_{s,y}(0) = 0,\ B_{s,z}(0) = -1. \tag{22}$$

Stokes theorem also demands [6]

$$B_{s,z}(\Delta) = 0, \tag{23}$$

which will be used to determine Δ. Choosing $\theta_\pm = 0$ in (21) satisfies the boundary conditions $V_y(0) = B_{s,y}(0) = 0$. The other boundary conditions demand

$$C_+ + C_- = 0, \quad C_+ = \frac{1}{\Lambda_- - \Lambda_+}. \tag{24}$$

The flow kinetic energy density is given by

$$V^2(x) = 4C_+^2 \sin^2[x(\Lambda_- - \Lambda_+)/2]. \tag{25}$$

Now the fields $\boldsymbol{B}_s$ and $\boldsymbol{V}$ are determined for the given three parameters $\Lambda_\pm$ (or a and b), and Δ.

C. Bernoulli Conditions

Next we study the Bernoulli conditions. Relevant boundary conditions are

$$p_i(\Delta) = p_e(\Delta) = 0, \quad \phi(\Delta) = 0, \quad P_i(\Delta) = 0. \tag{26}$$

The last two conditions are arbitrary, but are chosen for convenience. Adding both sides of (14) and (15), and writing $p = p_e + p_i$ (total pressure), we obtain

$$p + \left(\frac{\beta}{2}\right) \boldsymbol{B}_h \cdot \boldsymbol{B}_s + \left(\frac{\beta}{2}\right) \frac{V^2}{2} = c_i + c_e. \tag{27}$$

Since all the normalized quantities are order unity, $(\beta/2)V^2/2$ is negligible with respect to other terms (see (25)). In a tokamak-like plasma, the external field is primarily toroidal (normalized $\boldsymbol{B}_h \approx (B_0/B_*)\boldsymbol{e}_z$) and we may approximate $\boldsymbol{B}_h \cdot \boldsymbol{B}_s \approx (B_0/B_*)B_{s,z}$. Since both $B_{s,z}$ and p are zero at $x = \Delta$ (due to (22) and (26)), we find $c_i + c_e = 0$. Remembering that $(\beta/2)(B_0/B_*) = 1$ (cf. (5)), we finally derive

$$p + B_{s,z} = 0 \quad (0x > 0). \tag{28}$$

The obtained electric field, as well as its gradient, is negative; By (14) and (15), and assuming $p_e = p_i$, the radial electric field is given by

$$E_x = -\frac{d}{dx}\phi = -V_y - \frac{d}{dx}\frac{B_{s,z}}{2}. \tag{29}$$

Monotonically decreasing pressure demands diamagnetism (27) in the layer, i.e., $B_{s,z}$ must increase as we go across the layer (from $B_{s,z}(0) = -1$ to $B_{s,z}(\Delta) = 0$). This implies that the last term on the right-hand side of (29) is always negative. Further, from the solutions $V_y = 2C_+ \sin(\Lambda_+ x)$ and $B_{s,z} = -2\Lambda_+ C_+ \cos(\Lambda_+ x)$, we deduce that V_y must be positive when $B_{s,z}$ is negative. We, thus, find that $E_x < 0$, as well as $dE_x/dx < 0$. Similar results are obtained for the varying density case.

VI. SUMMARY AND DISCUSSION

In summary, we have derived a self-consistent model of a self-organized shear-flow layer which can be generated at the edge of a tokamak plasma. The field distribution inside the thin layer is governed by the "collisionless" singular perturbation stemming from a combination of the Hall and the fluid-vorticity terms in the two-fluid MHD. It is shown that the thickness of the layer, and the magnitude of the self-fields are uniquely determined by the plasma pressure at the inner boundary of the layer. The predicted values of the layer width, poloidal flow velocity are in good agreement with experimental observations. In addition, we find that the radial electric field, as well as its gradient, are negative, and the poloidal flow is well approximated by the $\boldsymbol{E} \times \boldsymbol{B}$ value. What we have presented here is a theory of the H-mode layer when it has been formed; we have neither advanced any mechanism which will cause the desired transition nor have we derived any transition thresholds. A close examination of the normalized equations (9) and (10), however, does suggest the possibility of estimating a beta threshold. Putting the time-dependent terms to zero, we notice that the gradient terms (with $|\nabla| = O(1)$) are $2/\beta \gg 1$ times greater than the remaining self-field terms. The Beltrami conditions, derived, by constrained minimization eventually require the remaining terms to independently vanish. Therefore, we arrive at a very interesting state: In these equations two distinctly ordered terms must separately vanish. In fact, it is this separation which allows to create the layer with desired characteristics. In a formal mathematical treatment, the validity of such a procedure demands that the smaller terms may be of an intermediate order between the successive orderings of the larger term. Since there are two length scales, the length scale of the layer $\lambda_i = c/\omega_{pi}$ ($|\nabla| = O(1)$) and the length scale of the bulk plasma ℓ, the next order of the gradient terms is approximately $\lambda_i/\ell \ll 1$. If $\beta/2$ were in the range $1 \gg \beta/2 \gg \lambda_i/\ell$, then the self-field terms will genuinely correspond to an intermediate order, and the separation procedure will be mathematically justifiable. This condition can be interpreted as β threshold for an H-mode like state.

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[5] We point out that the minimization procedure leads to equations which are independent of the external field. It is because of the fact that the external field is rooted outside the layer ($\nabla \times \boldsymbol{B}_h = 0$).

[6] We consider a thin layer surrounding a torus. The currents in the layer produce both the poloidal ($B_{s,y}$) and toroidal ($B_{s,z}$) self-fields. By the Stokes theorem, $B_{s,y}$ must vanish at the inner boundary. The solenoidal current in the layer confines the toroidal field inside the outer boundary, and hence, $B_{s,z}$ vanishes there.

FIGURES

FIG. 1. Profiles of (a) the shear flow and (b) the self-magnetic field ($V_z \equiv 0$). Here, we assume $a = 0.5$ and $b = 1/a$. The radial coordinate x is in the unit of the ion skin depth λ_i. The solution continues to oscillate as x increases. We cut off x at $\Delta = \pi/4$. Figure (c) shows the profiles of the potential ϕ, radial electric field $-d\phi/dx$, and the pressure p.