Bifurcation and Relaxation of Radial Electric Field in Enhanced Reversed Shear Tokamak Plasmas


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Abstract

It is shown that toroidal magnetic field ripple induced thermal and fast ion loss can drive the radial electric field to bifurcate over the local maximum of the parallel (or poloidal) viscosity. The subsequent plasma profile evolution reduces the hot particle density and relaxes the radial electric field. This behavior is consistent with the experimental observations in Enhanced Reversed Shear mode in tokamaks.

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To make an economical nuclear fusion reactor, it is necessary to improve its plasma confinement property. An improved confinement mode (i.e. H-mode) in the edge region of a tokamak and a stellarator was observed.\textsuperscript{1–3} The mechanism for the confinement improvement is due to the bifurcation of the radial electric field $E_r$ and the subsequent turbulence suppression due to the radial gradients of the $E \times B$ and the diamagnetic angular velocity.\textsuperscript{4–6} Here, $E$ is the electric field and $B$ is the magnetic field.

Recently, the improved confinement modes in the core region were also observed.\textsuperscript{7–9} There are two basic types of the core confinement improvement modes. One is the Enhanced–Reversed–Shear (ERS) mode,\textsuperscript{7,8} and the other is the Negative–Central–Shear (NCS) mode.\textsuperscript{9} Both of these phenomena can be explained by the same mechanism that triggers the improved edge confinement mode. Although the magnetic shear is reversed in both modes, i.e. the radial gradient of the safety factor $q$ is positive in the region close to the magnetic axis, the mechanisms that generate $E_r$ are quite different. In the NCS–mode, $E_r$ is generated by the toroidal rotation associated with the uni–directional neutral particle beam momentum injection. In the ERS–mode, however, the uni–directional neutral particle beam momentum is negligible. It is observed experimentally that in the ERS–mode poloidal plasma rotation suddenly increases (or bifurcates) to a large value and subsequently relaxes in the transport time scale.\textsuperscript{10,11} The magnitude of the poloidal rotation excursion in terms of the poloidal Mach number is of the order of unity. While the $E_r$ profile in the NCS–mode can be easily explained by the toroidal momentum input associated with the neutral particle beam injection, the bifurcation and relaxation of $E_r$ in the ERS-mode can not be easily explained. Here, we advance a theory to explain such an important phenomenon without resorting to unquantifiable parameters.

Note that it is quite unusual that in the relaxation process there is no back bifurcation involved in the ERS-mode. Thus, the parameter–dependent nonlinearity in our theory may serve as a paradigm for other similar nonlinear phenomena in nature as well.

It has been shown that toroidal magnetic field ripple induced particle fluxes can drive the radial electric field $E_r$ to bifurcate over the local maximum of the poloidal (or parallel)
viscosity. This mechanism provides an explanation for the bifurcation of $E_r$ observed in ERS mode in tokamaks. The radial electric field, however, relaxes in a few hundred ms after the bifurcation. To explain this relaxation phenomenon, either the rippled induced ion loss has to decrease or the electron loss has to increase or a combination of both. While there are magnetohydrodynamic (MHD) activities observed after the bifurcation, it is difficult to quantify the particle loss associated with this MHD activity unambiguously. It would be better if a theory can be developed based on the reduction of the ion ripple loss after bifurcation to explain the relaxation phenomenon. The reason is ion ripple loss can be more accurately determined than electron loss resulting from MHD activities.

Here, we demonstrate the relaxation of $E_r$ by including both thermal and fast ion ripple losses. Fast ion ripple loss assists bifurcation. After bifurcation, plasma confinement improves and thermal ion density increases. The increase of the thermal ion density makes it harder for neutral beam particles to penetrate. Fast ion density and its ripple loss are therefore reduced after the bifurcation. The radial electric field is then relaxed. This mechanism seems to be applicable for the ERS mode observed in Tokamak Fusion Test Reactor (TFTR).

There is another mechanism for the reduction of ion ripple loss after the bifurcation. It is possible in some devices that ion ripple loss is dominated either by the ripple trapping or by the banana drift. Ripple trapping loss mechanism is associated with collisionless particles that are trapped in the magnetic well created by the toroidal magnetic field ripple. These particles drift off the magnetic surface and cause local particle loss. The banana drift loss mechanism results from the particles that are trapped in the toroidal magnetic well. Because toroidal magnetic field ripple breaks toroidal symmetry, the bounce trajectory of these particles is not closed. This also leads to local particle loss. Both of these loss mechanisms are present in collisionless rippled tokamaks. As ion temperature increases, ion ripple loss could increase first which drives bifurcation of $E_r$ and then reduces to relax $E_r$ if the effective collision frequency is, at the beginning, larger than the $E \times B$ drift frequency of the ripple trapped ions or banana drifting ions and then later becomes smaller than
the drift frequency. For TFTR plasma parameters, this mechanism may not be applicable. However, it could be operative in other devices.

We would like to emphasize that the details in determining \( E_r \) may vary from device to device. However, extended neoclassical theory that takes the details of the experimental operation into account should be able to determine \( E_r \) with reasonable accuracy\(^4\).\(^{18}\)

For simplicity, we assume only that poloidal Mach number due to \( \mathbf{E} \times \mathbf{B} \) rotation is of the order of the ion thermal speed, i.e., \( M_{pE} = c E_r / (B_p v_{ti}) \sim 1 \) where \( c \) is the speed of light, \( B_p \) is the poloidal magnetic field strength, and \( v_{ti} \) is the ion thermal speed. The Mach number of the poloidal flow \( V_\theta \) is assumed to be less than unity, i.e., \( M_{p\theta} = V_\theta B / (B_p v_{ti}) < 1 \) where \( B = |\mathbf{B}| \). With this assumption the plasma compressibility effects, such as shock formation, can be neglected. It has been demonstrated that qualitative bifurcation behavior is not affected by the compressibility effects. Because \( M_{p\theta} < 1 \), the convective \( \mathbf{V} \cdot \nabla \mathbf{V} \) term in the fluid momentum can be neglected. Here, \( \mathbf{V} \) is the plasma flow velocity. We would like to note that the compressibility effects that result from \( V_\theta B / (B_p v_{ti}) \sim 1 \) are intrinsically nonlinear phenomena of the non–dissipative momentum equation and can not be correctly described by the solution of the drift kinetic equation.

The steady toroidal and poloidal momentum balance equations are\(^1\)\(^2\)\(^12\)

\[
\langle \mathbf{B}_t \cdot \nabla \cdot \Sigma_a \Pi_a \rangle = \langle \mathbf{B}_t \cdot \Sigma_a S_{ma} \rangle, \tag{1}
\]

\[
\langle \mathbf{B}_p \cdot \nabla \cdot \Sigma_a \Pi_a \rangle = 0, \tag{2}
\]

where \( \mathbf{B}_t \) is the toroidal magnetic field, \( \Pi_a \) is the viscous tensor of species \( a \), \( S_{ma} \) is the momentum source, and the angular brackets denote flux surface average. For simplicity, we neglect the toroidal momentum source term in Eq. (1).

For the plasma parameters in current devices, the important ripple loss mechanisms are ripple trapping, banana drift, and collisionless ripple plateau\(^1\)\(^9\). Because the leading order perturbed particle distribution function for these loss mechanisms has no variation along the magnetic line, \( \langle \mathbf{B}_p \cdot \nabla \cdot \Pi_a \rangle = - \langle \mathbf{B}_t \cdot \nabla \cdot \Pi_a \rangle = -(e_a B_p B \Gamma_a^p / c) \) where \( \Gamma_a^p \) is the ripple induced flux, and \( e_a \) is charge\(^2\)\(^0\). The ripple trapping flux for species \( a \) is\(^1\)\(^9\)
\[ \Gamma_i^a = -0.018 n_a \frac{1}{\alpha^3} \left( \frac{c T_a}{e_a B R} \right)^2 \frac{1}{\delta^{1/2}} \times \int dx v_a x^6 e^{-x^2} \frac{P_a'}{P_a} + e_a \Phi'/T_a + (x^2 - 5/2) T_a'/T_a \alpha^2 \left( \frac{c T_a}{\nu_a B R c} x^2 + \frac{\epsilon c E_r}{B r} \right)^2 + \left( \nu_a / \delta \right)^2, \]

where \( \alpha = \epsilon/(N q \delta) \), \( \epsilon = r/R \), \( r \) is the minor radius, \( R \) is the major radius, \( N \) is the number of the toroidal magnetic field coils, \( \delta \) is the ripple well depth, \( \nu_a \) is the collision frequency, \( P_a \) is the plasma pressure, \( T_a \) is the temperature, \( n_a \) is the plasma density, and \( d/dr \). Note that to obtain Eq. (3), we have assumed \( \alpha > 1 \). The banana drift flux is\(^{19}\)

\[ \Gamma_b^a = -\frac{0.15 n_a N q R^2 \delta^2}{\sqrt{2 \pi \epsilon}} \int_{(W_p/T_a)^2}^{\infty} dx x^6 e^{-x^2} \left( \nu_a \rho_{pa}^2 / 4 R^2 \right) \times \frac{P_a'/P_a + e_a \Phi'/T_a + (x^2 - 5/2) T_a'/T_a}{\left( \frac{c E_r}{B_p v_{ta}} + \frac{\tilde{Q} \rho_{pa} x^2}{2 R} \right)^2 + 0.4 \left( \frac{c E_r}{B_p v_{ta}} + \frac{\tilde{Q} \rho_{pa} x^2}{2 R} \right) \left( \frac{N q \nu_a}{\nu_a v_{ta} / R} \right)^2} + 0.2 \left( \frac{N q \nu_a}{\nu_a v_{ta} / R} \right)^2, \]

where \( \tilde{Q} = (1.5 \delta n q / d \delta n r + 0.7) \), \( \rho_{pa} \) is the poloidal gyroradius, \( v_{ta} = \sqrt{2 T_a / M_a} \) is the thermal speed, \( M_a \) is the mass, \( W_{pa} / T_a = (\nu_s a N q^2)^{1/2} \), and \( \nu_s = (\nu_a R q / v_{ta} \epsilon^{3/2}) \). The collisionless ripple plateau flux is\(^{19}\)

\[ \Gamma_{rp}^a = -\frac{n_a \delta^2 x_{R_p}^a}{6 \langle \delta^2 \rangle} \left[ I_1 \left( \frac{P_a'}{P_a} + \frac{e_a \Phi'}{T_a} \right) + I_2 \frac{T_a'}{T_a} \right], \]

where \( x_{R_p}^a = (3 \sqrt{\pi} / 4) (v_{ta} / R q) \rho_{pa}^2 N q \langle \delta^2 \rangle \), \( \langle \delta^2 \rangle = \int_0^{2 \pi} \int_0^{W_p/T_a} d x e^{-x^2} x^5 / 2 \pi \), and \( I_1 = 2 \int_0^{W_p/T_a} d x e^{-x^2} x^5 \). We are interested in plasmas that consist of thermal ions \((a = i)\), fast ions \((a = H)\), and thermal electrons \((a = e)\). For each of these species, there is a corresponding \( \Gamma_i^a \), \( \Gamma_b^a \), and \( \Gamma_{rp}^a \). During the bifurcation and relaxation processes, ion temperatures, ion densities, and their gradients evolve. These quantities become part of the controlling parameters in the the ERS mode transition.

The nonlinear resonant plasma viscosity due to banana and circulating particles is\(^{21}\)

\[ \langle B \cdot \nabla \cdot \Pi \rangle = \frac{n_a M_a B^2}{\mu_{a} U_{\theta a} + 2/5 \mu_{2a} q_{\theta a} / \rho_{a}}, \]

where \( U_{\theta a} = (v_{ta} / B) [V_{\lambda} / v_{ta} - c E_r / (B p v_{ta}) + c P'_a / (n_a e_a v_{ta} B_p)] \) and \( q_{\theta a} / \rho_{a} = (v_{ta} / B) [q_{\lambda} / (p_{a} v_{ta}) + (5/2) c T_a / (e_a v_{ta} B_p)] \). Because \( V_{\parallel e} \approx V_{\parallel i} \ll v_{te} \) and \( c E_r / (B p v_{te}) \ll 1 \),
electron viscosity is not affected in the regime where \(cE_r/(B_pv_{ti}) \sim 1\), and standard electron viscosity coefficients apply. The ion nonlinear resonant viscosity coefficients in the banana regime are \((\kappa_{b1}, \kappa_{b2}) = \int_{|U_{pm}|}^{\infty} dx x^4 e^{-x^2} K_B(1, x^2)\), and in the plateau–Pfirsch–Schlüter regime are \((\kappa_{ps1}, \kappa_{ps2}) = \int_0^{\infty} dx x^4 e^{-x^2} K_{ps}(1, x^2)\), where \(K_{ps} = (\nu \epsilon^2/2\sqrt{\pi}) \int_{-1}^{1} dy (1 - 3y^2) (\nu_i x/\nu_i) \{(y + U_{pm}/x)^2 + (\nu_i e^{3/2} \nu_T(x)/\nu_i x)^2\}\), \(\nu_T = 3 \nu_D + \nu_E\) is the anisotropy relaxation frequency, \(K_B = 1.46 \sqrt{\nu \nu_D} (8/3\sqrt{\pi}) (1 - 3U_{pm}^2/x^2) \mathbf{F}(1 + U_{pm}^2/x^2)^{-3/2}\), \(\mathbf{F} = (1 - U_{pm}^2/x^2) + (\nu_D/\nu_D) U_{pm}^2/x^2\), \(U_{pm} = [V_{||}/v_{ti} - cE_r/(v_{ti}B_p)]\), \(\nu_D, \nu_E\), and \(\nu_D\) are defined in Ref. 22. The nonlinear viscous coefficients are \(\mu_{1a} = \kappa_{b1}^a \kappa_{ps1}^a / (\kappa_{b1}^a + \kappa_{ps1}^a)\) and \(\mu_{2a} = \kappa_{b2}^a \kappa_{ps2}^a / (\kappa_{b2}^a + \kappa_{ps2}^a) - (5/2) \mu_{1a}\).

A dimensionless nonlinear equation for \(M_p = -cE_r/(B_pv_{ti})\) can be derived by substituting Eqs. (3)–(6) into Eq. (2) to obtain

\[
\frac{M_i \mu_{1i}}{M_e \mu_{1e}} \left( U_{pm} + \bar{c}_{pi} + \frac{\mu_{2i}}{\mu_{1i}} \bar{c}_{Ti} \right) = -U_{pm} - \mathcal{B} - D_i (\bar{c}_{pi} + M_p + R_i \bar{c}_{Ti}) \\
- D_H \left( \bar{c}_{ph} + M_p \sqrt{\frac{T_i}{T_H}} + R_H \bar{c}_{TH} \right) \\
+ D_e \left( \bar{c}_{pe} - M_p \sqrt{\frac{T_i M_e}{T_e M_i}} + R_e \bar{c}_{Te} \right),
\]

(7)

where \(\mathcal{B} = [c/(eB_p v_{ti})](p_e'/n_e + \mu_{2e} T_e'/\mu_{1e})\), \(\bar{c}_{pa} = cp_a'/(n_a |e_a| B_p v_{ta})\), and \(\bar{c}_{Ta} = cT_a'/(n_a |e_a| B_p v_{ta})\). The nonlinearity that is responsible for the bifurcation is in \(\mu_{1i}\), which has a local maximum at a certain value of \(U_{pm}\). Because parallel flow is usually subsonic, i.e., \(|V_{||}/v_{ti}| \ll 1\), we neglect the difference between \(U_{pm}\), and \(M_p\). This approximation decouples parallel flow \(V_{||}\) and radial electric field. Equation (7) becomes approximately a nonlinear equation for \(E_r\) or \(M_p\). In general, of course, we need to solve coupled equations for \(V_{||}\) and \(M_p\).

Equation (7) is solved numerically by plotting the left– and right–hand side of it versus \(U_{pm}\). The solutions are the intersections of these two curves. We use the ERS–mode like parameters: \(B = 3\ \text{T}, \ R = 2.6\ \text{m}, \ a = 0.8\ \text{m}, \ \epsilon = 0.1, \ N = 20, \ q = 2.5, \ \delta = 1.3 \times 10^{-3}\). The bifurcation process is demonstrated in Fig. 1. The gradients parameters are fixed to be
\( \tau_{Te} = \tau_{pe} = 0 \). \( B = [0.85 + (\mu_{2e}/\mu_{1e})0.85], \tau_{pi} = 0.85, \tau_{Ti} = 0.85, \tau_{pH} = 2.0, \) and \( \tau_{TH} = 0. \)

The L–mode solution is described by the intersection of the solid curve [the left side of Eq. (7)] and the solid line with crosses [the right side of Eq. (7)] for 4 keV ions as shown in Fig. 1. When the hot particle density increases to \( 2.4 \times 10^{12} \text{cm}^{-3} \), which is 10% of the thermal density, there are multiple solutions as can be seen from the intersections between solid curve and solid line with squares in Fig. 1. The solution in the middle is unstable. The solution with the largest value of \( U_{pm} \) is the ERS–mode solution. When ion temperature increases to 6 keV, only the ERS–mode solution exists as indicated by the intersection between dotted curve and dotted line. Once in the ERS state, particle confinement improves because turbulence fluctuations are suppressed by the gradients of \( E \times B \) and diamagnetic angular velocity, and thermal particle density increases. The increased thermal particle density tends to reduce neutral particle penetration and, therefore, hot particle density. When the hot particle density decreases from \( 2.4 \times 10^{12} \text{cm}^{-3} \) to \( 1.2 \times 10^{12} \text{cm}^{-3} \) (solid line), and finally to zero (dotted line), the radial electric field relaxes, as shown in Fig. 2. Note also in the relaxation process, there is no back–bifurcation. This is because the slope of the nonlinear viscosity in the large \( U_{pm} \) region changes in the higher ion temperature regime. Interestingly, no bifurcation process is observed in TFTR in the relaxation phase either.

The time scale involved in the bifurcation can be crucely estimated from the characteristic damping frequency of the equation and the rate of the change of the control parameters, i.e. \( T_i \), in our example. The characteristic poloidal flow damping frequency is of the order of \( \nu_{ii}/\epsilon^x \). Here, \( x \) is a number between 0 to 1. The rate of the change of the control parameters depends on the heating rate. If the heating rate is extremely slow so that ion temperature approaches the critical value adiabatically, the bifurcation time is at first of the order the linear damping rate, i.e. \( (\nu_{ii}/\epsilon^x)^{-1} \) and then as \( T_i \) approaches the critical value, the time scale slows down. This is the well–known critical slowing down phenomenon. However, if \( T_i \) changes at a rate faster than the critical slowing down rate, the critical slowing down rate will not be observed. The bifurcation rate is then controlled by \( (\nu_{ii}/\epsilon^x)^{-1} \),
nonlinearity of the system, and the heating rate. Note that \((\nu_\perp / \epsilon^2)^{-1}\) in the parameters of interests is about 20ms for \(x = 0\), well within the experimentally observed time scale in TFTR which is about 20 to 40 ms. The bifurcation time scale is therefore in good agreement with experimental observations.

The relaxation time scale is of the order of the transport time scale to allow plasma density to build up. This is again consistent with the observation.

The theory presented here requires a value of the toroidal magnetic field ripple strength of the order of \(10^{-3}\). In the NCS–mode plasmas, ripple strength is said to be much smaller than this value. It is difficult in NCS plasmas to observe the ERS–mode like bifurcation.

We would like to emphasize that the scenario employed to demonstrate the bifurcation and relaxation process shown in Figs. 1 and 2 is not unique. One could, for example, let gradient parameters \(\tau_{pa}\) and \(\tau_{pT}\) evolve during the bifurcation and the relaxation processes as in the experiments. Here, we are only interested in demonstrating the fundamental physics mechanism. The detailed time dependent transport simulation will be presented later.

In conclusion, we have demonstrated a bifurcation and relaxation mechanism for the radial electric field with ERS–like plasma parameters by including the ripple loss of both thermal and hot particles. Hot particle ripple loss assists bifurcation. After bifurcation, the hot particle density reduces due to improved confinement, and \(E_r\) relaxes. During the relaxation process, no back bifurcation is observed in the regime of interest. This mechanism could be responsible for the evolution of \(E_r\) observed in TFTR ERS mode.

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References


23. The exact expressions for $D_i$, $D_H$, and $D_e$ can be found by comparing Eq. (7) with Eq. (2) after Eqs. (3)–(6) are inserted, for example, $D_i = -[(\Gamma^i_t + \Gamma^i_b + \Gamma^i_{rp})eB_p/(cn_iM_e\mu_i^e v_{ti})]/(\bar{c}_{pi} + M_p + R_i\bar{c}_{Ti})$. The expressions for $D_H$ and $D_e$ are similar to $D_i$ except the species–dependent parameters are changed accordingly.


FIGURE CAPTIONS

FIG. 1. An example of the bifurcation processes. The L–mode solution is the intersection of the solid curve and solid line with crosses for 4 KeV ions. As hot particle density increases to $2.4 \times 10^{12}$ cm$^{-3}$, there are multiple solutions, as indicated by the intersections of the solid curve and solid line with squares. The solution in the middle is unstable. The new stable solution is the ERS–mode solution. When ion temperature increases to 6 KeV, only the ERS–mode solution exists, as indicated by the intersection between the dotted curve and the dotted line.

FIG. 2. An example of the relaxation processes. As the hot particle density decreases after the bifurcation from $2.5 \times 10^{12}$ cm$^{-3}$ to $1.2 \times 10^{12}$ cm$^{-3}$ (solid line), and finally to zero (dotted line), the value of $E_r$ decreases to a L–mode like value.