

The Radial Electric Field in a Tokamak with Reversed Magnetic Shear

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Abstract

Neoclassical theory with the impurity rotational velocity is used to evaluate the radial electric field, E_r , in tokamaks. The result of using the complete matrix method for the deuterium-carbon plasma is compared with a reduced analytic formula for determining E_r [Ernst *et al.*, (1998)]. The analytic formula is shown to overestimate the E_r magnitude and its gradient. Two transport measures of the effect of the E_r shear are compared for the reverse shear and enhanced reversed shear discharges in TFTR [Mazzucato *et al.*, (1996)]. We show that the combined E_r and magnetic shear measure, Υ_s , from linear stability theory gives a higher correlation with the observed transition between the two discharges than the vorticity measure ω_s from E_r shear alone.

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1 Introduction

The radial electric field, E_r , in tokamak is thought to play an important role in achieving various improved plasma confinement modes of tokamak operation, such as the high-confinement mode (H-mode) [1] and the enhanced reversed shear mode (ERS) [2]. Various studies have shown that the radial electric field shear may reduce the turbulent transport level and thus improve the plasma confinement. Therefore, it is important to determine the radial electric field profile in order to study the specific correlation between its structure and the onset of different improved confinement modes.

Experimentally, E_r could be measured using the heavy ion beam probe (HIBP), an expensive method that was used in TEXT [3], but is not practical on high magnetic field ($B \geq 4\text{T}$) machines. The radial electric field could be also determined by measuring the quantities required to infer E_r from the lowest order radial force balance equation for each single ion species,

$$E_r = u_{\phi i} B_\theta - u_{\theta i} B_\phi + \frac{1}{Z_i n_i} \frac{dp_i}{dr}, \quad (1)$$

where ion species has charge Z_i , density n_i , pressure p_i , and poloidal and toroidal fluid velocities $u_{\theta i}$ and $u_{\phi i}$; B_θ and B_ϕ are poloidal and toroidal components of magnetic field in tokamak. Such method has been used in DIII-D to measure the E_r around plasma edge region in an L-H transition experiment [1]. A similar approach has also been employed to determine E_r by measuring the toroidal velocity and calculating the poloidal velocity. This method was developed in the 13M approximation [4] and the 21M approximation [5] by using the conventional multi-ion species neoclassical theory [6]. We used this method in both approximations to compute E_r numerically, and found that the difference in the magnitude of E_r by using the two approximations is less than 5%. In the low impurity density approximation, a relatively simple analytical formula was obtained by Ernst for E_r [4]. We compared the E_r from the analytical formula and the numerical solution. Due

to the relatively high concentration of the impurity (carbon) in the discharges considered here, the analytical result does not agree well with the numerical results. For these reasons, neoclassical theory in 13M approximation is used to compute E_r numerically throughout this paper.

In a conventional tokamak magnetic configuration, the radial profile of the safety factor q usually has its minimum value at, or close to, the magnetic axis and increases monotonically outwards. In reversed magnetic shear (RS) configurations, negative magnetic shear is introduced around the magnetic axis and the minimum of $q(r)$ moves outward. Theory shows that the RS could suppress geodesic curvature driven microinstabilities [7, 8], which then results in improved confinement in central region. In 1996, an even higher performance tokamak discharge from the RS configuration was obtained in TFTR, which occurred after a bifurcation from the RS confinement mode to a state with an internal transport barrier, while there was almost no change in the $q(r)$ profile [2]. This state is therefore called the enhanced reversed shear (ERS) mode. The $q(r)$ profiles for the TFTR RS and ERS discharges are shown in Fig. 1, which are obtained from the U-files of TRANSP runs #88299a08 and #88299a20. One hypothesis is that, in the presence of the RS configuration, the radial electric field evolves to a new state with a sufficiently large shear for the onset of the ERS confinement mode. This hypothesis is based on a widely accepted theory that the sufficiently large $\mathbf{E} \times \mathbf{B}$ flow shear will suppress the turbulent transport by enhancing the decorrelation rate of the fluctuations [9, 10, 11]. This mechanism of shearing of the eddies has been used to explain the role of E_r found at the plasma edge when the L-H transition occurs and the edge transport barrier (ETB) forms [1]. It is interesting to notice that, in the ERS experiment in TFTR, as the calculation in Sec. 2 shows later, an E_r field with a similar profile is now formed around the plasma central region inside the q_{\min} surface where the internal transport barrier (ITB) forms. The correlations between the structure of E_r profile and the enhanced confinement modes make it important to find and evaluate some adequate mea-

sure of E_r shear stabilizing effects. One commonly used measure is the Hahm-Burrell $\mathbf{E} \times \mathbf{B}$ flow shearing rate ω_s [11], which is obtained from the analysis of the two-point correlation function evolution equation. In tokamak plasma, the fluctuation suppression effect depends on the ratio of the flow shearing rate ω_s to the ambient turbulence decorrelation rate. Another measure of the E_r stabilizing effect Υ_s [12] arises from the linear ITG mode analysis, which is the ratio of the $\mathbf{E} \times \mathbf{B}$ flow shear to the magnetic shear. Sufficiently large Υ_s would eventually reduce the ITG mode growth rate, indicating the stabilizing effect of E_r shear in the presence of magnetic shear. E_r shear effect is especially magnified around the q_{\min} surface in ERS discharge, where the disconnection of fluctuation occurs [13] and the internal transport barrier forms. Both measures of E_r shear have been evaluated for RS and ERS discharges in this paper, and their significances are discussed.

The rest of this paper is organized as follows. In Sec. 2, we examine in some detail the calculation of E_r in tokamak. We compare the results from the complete matrix method for E_r with Ernst's approximate analytic formula in Sec. 3. In Sec. 4, the $\mathbf{E} \times \mathbf{B}$ shearing rate ω_s and the parameter Υ_s from stability theory are evaluated and compared for RS and ERS experiments. Finally, in Sec. 5, we give the discussion and conclusions.

2 Neoclassical Calculation of E_r

The simplest picture of E_r in a toroidally rotating tokamak can be obtained from a Faraday generator. The Faraday generator is a conductor rotating in a perpendicular magnetic field. Due to the rotational electromotive force (EMF), the potential drop $d\mathcal{E}$ across dr is

$$d\mathcal{E} = -E_r dr = -\Omega r B_p dr = -V_\phi B_\theta dr, \quad (2)$$

where E_r is the radial electric field is formed cross the conductor due to the toroidal motion. The magnitude of E_r is simply the product of Ωr and B_p , while its direction depends on the direction of the rotation. The tokamak plasma rotating along toroidal direction may be

viewed as a Faraday generator, and E_r generated in the laboratory reference frame is just the product of the toroidal rotation velocity and the poloidal magnetic field, assuming the distribution of density and pressure of plasma are uniform, and the poloidal rotation has been damped. Usually, the toroidal rotation of the plasma is driven by the neutral beam injection. In modern large tokamaks, the toroidal velocity reaches around 200 km/s, the poloidal magnetic field is about 0.2 T, giving E_r approximately 40kV/m [14]. In DIII-D, the toroidal rotational velocity is high, over 300 km/s, and thus E_r is dominated by the rotational EMF. In TFTR, the rotation speed is lower and the density and temperature gradients are of comparable importance.

When taking into account the nonuniformity of the density and temperature of the plasma, as well as the nonzero poloidal rotation, E_r is modified as given in Eq. (1). Our goal is to calculate Eq. (1) from experimentally measurable quantities. In TRANSP data files there are the radial profiles of density and temperature of all ion species, as well as the toroidal rotation velocity of the impurity species, which is measured by using the Doppler shifted lines of the its charged ions. The poloidal rotation velocity $u_{\theta i}$ in Eq. (1) is not available in TRANSP data files, but is calculated in terms of density and temperature gradients below using the standard neoclassical transport theory.

In 13M approximation, the vector component of distribution function for species i is expanded in terms of fluid velocity \mathbf{u}_i and heat flux \mathbf{q}_i :

$$f^{(1)} = f_M \left(1 + \frac{2\mathbf{v}}{v_{ti}^2} \cdot \left[\mathbf{u}_i + \frac{2\mathbf{q}_i}{5p_i} \left(\frac{v^2}{v_{ti}^2} - \frac{5}{2} \right) \right] \right). \quad (3)$$

Below, for convenience, we follow [4] using \mathbf{q}_i to denote the quantity $\mathbf{q}_i/(\frac{5}{2}p_i)$, which has the same dimension of velocity \mathbf{u}_i . Using this distribution function, we can get a set of closed moment equations. Among the moment equations, the two which involve poloidal flow \mathbf{u}_θ and \mathbf{q}_θ are the parallel momentum balance equations

$$\langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Pi}_i \rangle = \langle \mathbf{B} \cdot \mathbf{F}_{1i} \rangle, \quad (4)$$

$$\langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_i \rangle = \langle \mathbf{B} \cdot \mathbf{F}_{2i} \rangle. \quad (5)$$

In these two equations, the surface averaged ($\langle \dots \rangle$) stresses balance the friction along the magnetic field direction. Here, \mathbf{F}_{1i} is the friction force from the collisions in the moment $m_i \mathbf{v}$, while \mathbf{F}_{2i} is the equivalent heat “friction” force from the collisions in the moment $m_i v^2 \mathbf{v}$. In Eq. (4), we have neglected the parallel electric force term $n_i e_i \langle B E_{\parallel} \rangle = (n_i e_i / n_e e) [\langle \mathbf{B} \cdot \mathbf{F}_{1e} \rangle - \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Pi}_e \rangle]$ which is smaller by a factor of $\mathcal{O}[(m_e/m_i)^{1/2}]$. We have also neglected the inertia term $\langle \mathbf{B} \cdot n_i m_i d\mathbf{u}_i/dt \rangle$ and the fluctuation term $e_i \langle \tilde{n} \tilde{E}_{\parallel} \rangle$ which are smaller by a factor of $\mathcal{O}(\rho_i/L)$ (L is the equilibrium scale length) using the conventional assumption that $\mathbf{u}_i \sim (\rho_i/L) v_{Ti}$, $k_{\perp} \rho_i \sim 1$ and $\tilde{n}/n \sim e\tilde{\phi}/T \sim k_{\parallel}/k_{\perp} \sim \rho_i/L$. We estimate these contributions in the Appendix.

Using the distribution function $f^{(1)}$, the parallel component of stress and friction can be expressed in terms of poloidal flow and parallel flow respectively

$$\begin{pmatrix} \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Pi}_i \rangle \\ \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_i \rangle \end{pmatrix} = \frac{n_i m_i}{\tau_{ii}} \begin{pmatrix} \hat{\mu}_{i1} & \hat{\mu}_{i2} \\ \hat{\mu}_{i2} & \hat{\mu}_{i3} \end{pmatrix} \begin{pmatrix} \hat{u}_{\theta i} \\ \hat{q}_{\theta i} \end{pmatrix} \langle B^2 \rangle, \quad (6)$$

$$\begin{pmatrix} \langle \mathbf{B} \cdot \mathbf{F}_{1i} \rangle \\ \langle \mathbf{B} \cdot \mathbf{F}_{2i} \rangle \end{pmatrix} = \frac{n_i m_i}{\tau_{ii}} \Sigma_j \begin{pmatrix} \hat{l}_{11}^{ij} & -\hat{l}_{12}^{ij} \\ -\hat{l}_{21}^{ij} & \hat{l}_{22}^{ij} \end{pmatrix} \begin{pmatrix} \langle u_{\parallel j} B \rangle \\ \langle q_{\parallel j} B \rangle \end{pmatrix}, \quad (7)$$

where $\hat{\mu}_{aj}$ and \hat{l}_{ij}^{ab} are the normalized neoclassical transport coefficients, while $\hat{u}_{\theta i}$, $\hat{q}_{\theta i}$, $u_{\parallel j}$ and $q_{\parallel j}$ are defined in Eqs. (10) and (11). On the other hand, the perpendicular components of the flows \mathbf{u}_i and \mathbf{q}_i can be written in terms of radial gradient of pressure, temperature, and E_r from the radial momentum balance equations

$$\mathbf{u}_{\perp} = \frac{\mathbf{B}}{B^2} \times \left(\frac{\nabla p}{Z n e} + \nabla \Phi \right), \quad (8)$$

$$\mathbf{q}_{\perp} = \frac{\mathbf{B}}{B^2} \times \frac{\nabla T}{Z e}. \quad (9)$$

Combined with the incompressibility conditions $\nabla \cdot (n\mathbf{u}) = 0$, $\nabla \cdot \mathbf{q} = 0$, the relation between parallel flow and poloidal flow is obtained

$$u_{\parallel} = V_1 + \hat{u}_{\theta}(\psi)B, \quad u_{\theta} = \hat{u}_{\theta}B_{\theta}, \quad (10)$$

$$q_{\parallel} = V_2 + \hat{q}_{\theta}(\psi)B, \quad q_{\theta} = \hat{q}_{\theta}B_{\theta}, \quad (11)$$

$$V_1 = -\frac{T}{ZB_{\theta}} \left(\frac{1}{p} \frac{dp}{dr} + \frac{Z}{T} \frac{d\Phi}{dr} \right), \quad (12)$$

$$V_2 = -\frac{1}{ZB_{\theta}} \frac{dT}{dr}. \quad (13)$$

Here, V_1 and V_2 represent the driving forces, Φ is the equilibrium electrostatic potential, $\hat{u}_{\theta}(\psi)$ and $\hat{q}_{\theta}(\psi)$ are functions of magnetic flux ψ only. Together with the above parallel momentum equations, a set of equations for poloidal flow is finally deduced

$$\sum_j \left[\begin{pmatrix} \hat{\mu}_{i1} & \hat{\mu}_{i2} \\ \hat{\mu}_{i2} & \hat{\mu}_{i3} \end{pmatrix} \delta_{ij} - \begin{pmatrix} \hat{l}_{11}^{ij} & -\hat{l}_{12}^{ij} \\ -\hat{l}_{21}^{ij} & \hat{l}_{22}^{ij} \end{pmatrix} \right] \begin{pmatrix} \hat{u}_{\theta j} \\ \hat{q}_{\theta j} \end{pmatrix} = \sum_j \begin{pmatrix} \hat{l}_{11}^{ij} & -\hat{l}_{12}^{ij} \\ -\hat{l}_{21}^{ij} & \hat{l}_{22}^{ij} \end{pmatrix} \begin{pmatrix} \hat{V}_{1j} \\ \hat{V}_{2j} \end{pmatrix}, \quad (14)$$

where $\hat{V}_{ij} = \langle V_{ij}B \rangle / \langle B^2 \rangle$.

In our calculation, we consider 2 ion species, deuterium and carbon. Thus, the above equations for poloidal flows become a group of 4 linear algebraic equations with 4 unknowns.

$$(\mathbf{M} - \mathbf{L}) \cdot \mathbf{U} = \mathbf{L} \cdot \mathbf{V}, \quad (15)$$

with

$$\mathbf{M} = \begin{pmatrix} \hat{\mu}_{i1} & \hat{\mu}_{i2} & 0 & 0 \\ \hat{\mu}_{i2} & \hat{\mu}_{i3} & 0 & 0 \\ 0 & 0 & \hat{\mu}_{x1} & \hat{\mu}_{x2} \\ 0 & 0 & \hat{\mu}_{x2} & \hat{\mu}_{x3} \end{pmatrix} \quad \mathbf{U} = \begin{pmatrix} \hat{u}_{\theta i} \\ \hat{q}_{\theta i} \\ \hat{u}_{\theta x} \\ \hat{q}_{\theta x} \end{pmatrix}, \quad (16)$$

$$\mathbf{L} = \begin{pmatrix} \hat{l}_{11}^{ii} & -\hat{l}_{12}^{ii} & \hat{l}_{11}^{ix} & -\hat{l}_{12}^{ix} \\ -\hat{l}_{21}^{ii} & \hat{l}_{22}^{ii} & -\hat{l}_{21}^{ix} & \hat{l}_{22}^{ix} \\ \hat{l}_{11}^{xi} & -\hat{l}_{12}^{xi} & \hat{l}_{11}^{xx} & -\hat{l}_{12}^{xx} \\ -\hat{l}_{21}^{xi} & \hat{l}_{22}^{xi} & -\hat{l}_{21}^{xx} & \hat{l}_{22}^{xx} \end{pmatrix} \quad \mathbf{V} = \begin{pmatrix} \hat{V}_{1i} \\ \hat{V}_{2i} \\ \hat{V}_{1x} \\ \hat{V}_{2x} \end{pmatrix}. \quad (17)$$

In the friction-flow relations (7) [or in Eqs. (15)–(17)], the coupling of the ion friction forces to the electron parallel flow, is neglected since it is of $\mathcal{O}[(m_e/m_i)^{1/2}]$. As a consequence, the small electron-pressure modification does not enter the ion poloidal flow obtained as a solution of Eq. (15).

Specifically, the viscosities are given by [6]

$$\hat{\mu}_{aj} = 1.469 \left(\frac{r}{R_0} \right)^{1/2} \frac{8}{3\sqrt{\pi}} \int_0^{\infty} dx x^4 e^{-x^2} \left(x^2 - \frac{5}{2} \right)^{j-1} \tau_{aa} \nu_{\text{tot}}^a(x), \quad (a = i, x; j = 1, 2, 3), \quad (18)$$

and the normalized friction coefficient matrix is given by [5, 6]

$$\mathbf{L} = \begin{pmatrix} -\sqrt{\frac{6}{7}}\alpha & \frac{9}{8}\alpha & \sqrt{\frac{6}{7}}\alpha & -\frac{3}{14}\sqrt{\frac{6}{7}}\alpha \\ \frac{9}{8}\alpha & -(\frac{77}{32}\alpha + \sqrt{2}) & -\frac{9}{8}\alpha & \frac{81}{98}\sqrt{\frac{6}{7}}\alpha \\ \frac{1}{\sqrt{7}\alpha} & -\frac{9}{8\sqrt{6}\alpha} & -\frac{1}{\sqrt{7}\alpha} & \frac{3}{14\sqrt{7}\alpha} \\ -\frac{3}{14\sqrt{7}\alpha} & \frac{81}{98\sqrt{7}\alpha} & \frac{3}{14\sqrt{7}\alpha} & -(\frac{1189}{196\sqrt{7}\alpha} + \sqrt{2}) \end{pmatrix} \quad (\text{for } T_i \sim T_x), \quad (19)$$

where $\alpha = (n_x Z_x^2)/(n_i Z_i^2)$, which measures the relative strength of the impurity ion species. In Eq. (18), the factor $1.469(r/R)^{1/2}$ is the small r/R expansion of the trapped particle fraction f_t/f_c , and f_t and f_c are defined in [6]. This approximation is valid here since $a/R \sim 0.94/2.6 \sim 0.36 < 1$ for the TFTR. The unnormalized form of the friction coefficient matrix \mathbf{L}^p is given in the appendix, where its symmetry properties are discussed.

The above neoclassical model gives a procedure for calculating the poloidal velocities of the working gas and impurity gas from their density and temperature profiles. The diagram in Fig. 2 summarizes the system in terms of the input, transform, and output information. From the output poloidal flow velocities we can determine E_r if the toroidal velocity of either the working gas or the impurity gas is known. For hydrogenic working gas there is no line emission and there is no direct measurement of the flow velocity. For the carbon component (impurity ion species), there is accurate spectroscopic measurement of its toroidal velocity. From these measurements in TRANSP data files shown as the input in Fig. 3a, we compute the E_r profile.

The driving terms, which are the radial gradients of the pressure and temperature, and the toroidal velocity of the impurity carbon are read and calculated from the TFTR TRANSP data files [2] as shown in Fig. 3a. By numerically calculating the neoclassical transport coefficients and solving the equations at each point along the minor radius of tokamak, we obtain the radial profiles of poloidal flow $u_{\theta i}$ and $u_{\theta x}$ in Fig. 3b. The E_r formula (1) is valid for all ion species, so we use the measured toroidal flow and the calculated poloidal flow of carbon to get E_r . We may also use the data for deuterium to calculate E_r . The toroidal flow of deuterium $u_{\phi i}$ is not available in TRANSP data file for the reason explained earlier,

but it can be inferred from the toroidal flow of carbon by formula

$$u_{\phi i} = u_{\phi x} + (V_{1i} - V_{1x}) + (\hat{u}_{\theta i} - \hat{u}_{\theta x})B_T. \quad (20)$$

Through the above procedure, the input TRANSP data profiles $n_i(r)$, $T_i(r)$, $n_x(r)$, $T_x(r)$, and $u_{\phi x}(r)$, which are shown in Fig. 3a, are transformed into output data profiles of $u_{\theta i}(r)$, $u_{\theta x}(r)$, $u_{\phi i}(r)$, and $E_r(r)$, which are shown in Fig. 3b.

Now we can take a look at the motional EMF parts of E_r and the full E_r itself shown in Fig. 4, which shows, as expected, that the motional EMF parts of E_r and the full E_r differs most around the transport barrier region, where the plasma pressure gradient is the largest.

Once the poloidal velocity $u_{\theta i}$ is solved for all species, the bootstrap current could also be obtained. Hence the above procedure also provides a means to calculate the bootstrap current, which does not depend on the radial electric field E_r due to the neutrality condition. The bootstrap current for the ERS discharge is thus obtained at $t = 2.7$ s, which agrees well with the bootstrap current profile in TRANSP data file.

3 Comparison with the Ernst's E_r Formula

A simple model [4] of E_r has been developed by using a low impurity (carbon) concentration approximation $\alpha \ll 1$. In this approximation, the impurity heat flux \mathbf{q}_x is neglected, and the four linear algebraic equations are decoupled into 2 independent sets of equations,

$$\begin{cases} \hat{u}_{\theta i} = \alpha_1 \hat{q}_{\theta i}, & \alpha_1 = -\frac{\hat{\mu}_{i2}}{\hat{\mu}_{i1}} \\ \hat{\mu}_{i2} \hat{u}_{\theta i} + \hat{\mu}_{i3} \hat{q}_{\theta i} = -\sqrt{2} \frac{\langle q_{\parallel i} B \rangle}{\langle B^2 \rangle} = -\sqrt{2} (\hat{q}_{\theta i} + \hat{V}_{2i}) \end{cases} \quad (21)$$

This leads to

$$\langle (u_{\parallel i} - u_{\parallel x}) B \rangle = \frac{3}{2} \langle q_{\parallel i} B \rangle = \frac{3}{2} \frac{\alpha_2}{1 + \alpha_2} \langle V_{2i} B \rangle, \quad \alpha_2 = (\hat{\mu}_{i3} - \frac{\hat{\mu}_{i2}^2}{\hat{\mu}_{i1}}) \frac{1}{\sqrt{2}}. \quad (22)$$

Using the simple analytic solutions to the above equations, a reduced model of E_r is obtained giving

$$E_r = u_{\phi x} B_{\theta} + \frac{1 - \alpha_1 - \frac{\alpha_2}{2}}{1 + \alpha_2} \frac{1}{Z_i} \frac{dT_i}{dr} + \frac{T_i}{Z_i n_i} \frac{dn_i}{dr}. \quad (23)$$

which depends only on deuterium density, temperature, and carbon toroidal rotation velocity profiles. This expression for E_r is of the zeroth order in the impurity strength parameter α . By keeping some terms proportional to α in solving the above decoupled two sets of equations for $u_{\theta i}$ and $q_{\theta i}$, a more accurate approximate expression for E_r could apply in the case $\alpha \sim 1$,

$$E_r = u_{\phi x} B_{\theta} + \frac{1 - \alpha_1 - \frac{\alpha_2}{2}}{1 + \alpha_2} \frac{1}{Z_i} \frac{dT_i}{dr} + (1 - \alpha_3) \frac{T_i}{Z_i n_i} \frac{dn_i}{dr}, \quad (24)$$

where

$$\alpha_2 = \left(\hat{\mu}_{i3} - \frac{\hat{\mu}_{i2}^2}{\hat{\mu}_{i1}} \right) \frac{1}{\sqrt{2} + \alpha}, \quad \alpha_3 = \frac{\hat{\mu}_{i1} \left(\hat{\mu}_{i3} + \sqrt{2} + \frac{13}{4}\alpha \right) - \hat{\mu}_{i2} \left(\hat{\mu}_{i2} - \frac{3}{2}\alpha \right)}{\left(\hat{\mu}_{i3} + \sqrt{2} + \frac{13}{4}\alpha \right) (\hat{\mu}_{i1} + \alpha) - \left(\hat{\mu}_{i2} - \frac{3}{2}\alpha \right)^2}. \quad (25)$$

Expression (24) is the Ernst's E_r -formula [4] and the case $\alpha_3 = 0$ corresponds to Eq. (23) for almost pure plasma with trace impurity. In Fig. 5a, E_r profiles computed from the above formulas (23) and (24) are compared with E_r obtained in Sec. 2 for ERS discharge ($t = 2.7$ s). It shows that both approximate expressions substantially overestimate the magnitude of E_r . The reason for this is that in the discharge for which we compute E_r here, $\alpha \sim 2 - 8$, which falls outside the region where either approximate expression (23) or (24) applies.

This behavior of a spuriously-enhanced E_r gradient can be further seen by multiplying α with a parameter ϵ in the friction coefficient matrix (19), i.e., $\alpha \rightarrow \epsilon\alpha$. Varying ϵ from 1 to $\epsilon \rightarrow 0$ when computing E_r through solving equation (15), we see from Fig. 5b that as $\epsilon \rightarrow 0$, the profile of the numerically computed E_r approaches to E_r profile obtained from expression (23). The curve of E_r from the Ernst's formula (24) lies in between the curve of $\epsilon = 1$ and the curves of $\epsilon \rightarrow 0$. Compared with the numerically computed E_r profile ($\epsilon = 1$),

formula (24) overestimates the maximum value of E_r by approximately 56%. The relative difference of E_r gradients ranges from 33% to a factor of 8 near the q_{\min} surface.

During the last operations of the TFTR tokamak new high resolution diagnostics of the poloidal velocity becomes available [15]. A variety of E_r profiles were obtained with certain shots showing a short duration rapid increase of the poloidal velocity u_θ in the ion diamagnetic direction ($u_\theta < 0$). This produces brief periods of positive $E_r(r, t)$ fields thought to be associated with turbulence generation of poloidal shear flow. For the purpose of benchmarking the present E_r calculation and showing one of the qualitatively different E_r regimes, we show in Fig. 6 the results for applying our code to TFTR shot #103794 at $t = 2.00$ s. The Houlberg *et al.*'s [5] NCLASS code calculation for E_r is available at $t = 1.96$ s and is shown for comparison. The average relative error defined by $\langle (E_r^{\text{ZHS}} - E_r^{\text{NCLASS}})^2 \rangle^{1/2} \simeq 2.25$ kV/m using E_r values at 10 independent radial points to calculate the mean. Other time values show similar agreement. The NCLASS code contains refinements such as multiple ionization states and potato orbit correction factors for the near axis points not included in our code, so that exact agreement would not be expected.

4 E_r shearing in RS and ERS Discharges

Using the method in Sec. (2), we calculate E_r in reversed magnetic shear and enhanced magnetic shear experiments [2], as shown in Fig. 7a and Fig. 7b respectively. In both experiments, E_r radial profile has a ‘well’ structure inside the central region where the safety factor q is minimum. A similar ‘well’ structure appeared in DIII-D L-H transition, but there the ‘well’ is located at the plasma edge [1]. The correlation between the location of the E_r ‘well’ and the minimum point of $q(r)$, if any, is uncertain. As time evolves, E_r ‘well’ develops from a rather shallow ‘well’ to much deeper one in both the RS and the ERS discharges. The difference between the two discharges is the magnitude and gradient of the E_r that develops. The E_r in the ERS discharge is significantly larger and steeper than that in the RS discharge

at all the time stages.

One theoretical measure of the effect of E_r shearing rate is the Hahm-Burrell $\mathbf{E} \times \mathbf{B}$ flow shearing rate ω_s

$$\omega_s = \frac{\Delta\psi_0}{\Delta\phi_0} \frac{\partial^2 \Phi_0(\psi)}{\partial \psi^2} \simeq \left| \frac{RB_\theta}{B_\phi} \frac{\partial}{\partial r} \left(\frac{E_r}{RB_\theta} \right) \right|, \quad (26)$$

where $\Delta\psi_0$ and $\Delta\phi_0$ are the ambient radial and toroidal correlation lengths measured in units of poloidal flux and radians respectively [11]. Their study shows that the fluctuation suppression occurs when the flow shearing rate ω_s exceeds the decorrelation rate of the ambient turbulence. At the same times, e.g., $t = 2.7$ s, the ω_s in ERS discharge is about two to three times as large as that in the RS discharge around the central region as seen in Fig. 8a. This suggests that the sufficiently large E_r shear may account for the onset of ERS mode from the RS mode, as suggested in [2]. Note that the question of why E_r is larger at this time may have to do with subtle changes in the turbulence and the core particle fueling and confinement at earlier times. Central particle fueling and confinement is thought to be a common element in the second type of enhanced performance discharges [17].

Another relevant measure of E_r shearing rate is the linear stability theory parameter

$$\Upsilon_s = \frac{\text{flow shear}}{\text{magnetic shear}} = \frac{L_{VE}^{-1}}{L_s^{-1}} \simeq \sqrt{\frac{m_i}{T_e}} \left| \frac{R \partial_\psi (E_r / RB_\theta)}{\partial_\psi \ln q} \right|, \quad (27)$$

which measures the stabilizing effects of $\mathbf{E} \times \mathbf{B}$ flow shear in sheared magnetic fields [12]. Here, L_s and L_{VE} are the magnetic shear and the flow shear scale length respectively. Ion temperature gradient instability analysis has indicated that sufficiently large Υ_s decreases the linear growth rate even to negative values. In Fig. 8b, the profiles of $\Upsilon_s(r)$ for the RS and ERS discharges show large stabilizing values peaked around the shear reversal surface. Eq. (27) formally diverges around the minimum q surface in both experiments, suggesting existence of a strong stabilizing effect from $\mathbf{E} \times \mathbf{B}$ flow shear in small magnetic shear region, and the disconnection of fluctuations across the q_{\min} surface. The role of the disconnected toroidal eigenmodes across q_{\min} is seen by the rotation of the Bloch angle $\Delta\theta = \tau_c d\Omega/dq$

over the correlation time $\tau_c = q_{\min}R/c_s$ for the fluctuations at the q_{\min} surface, where $\Omega = u_\phi/R$ [18]. In the presence of sheared rotation, the ballooning angle poloidally convects with the angular speed $d\Omega/dq = \omega_s/s$ where $s = rq'/q$ is the magnetic shear strength [19]. Thus, the turbulence energy source contained in the toroidal curvature is averaged out when the angle shift $\Delta\theta = \tau_c d\Omega/dq$ over the correlation time τ_c for the fluctuations is significant. The divergence of Υ_s also indicates the necessity of a higher order expansion of magnetic shear $s(r) = rq'/q$ around the q_{\min} surface in stability analysis in order to obtain a more adequate expression of Υ_s in the q_{\min} region. We will not go into the details of the theory for the modified expression of Υ_s which takes into account the radial variation of magnetic shear s' here. Instead we estimate the maximum value of Υ_s below which the expression of the present measure Υ_s is valid. The criterion is that the ITG mode width, which increases with L_s , should not exceed the distance between the rational surface where the mode resides and the q_{\min} surface in order for the formula (27) for Υ_s to be valid. Taking the mode width as $\rho_s L_s/L_n$, where $L_s = qR/s$, and expanding s around the q_{\min} surface as $s(r) = s'_{\min}(r - r_{\min})$, the critical mode width Δr is solved from the relation

$$\Delta r = \frac{\rho_s L_s}{2L_n} \simeq \frac{\rho_s q_{\min} R}{s'_{\min} \Delta r L_n}, \quad (28)$$

as

$$\Delta r = \sqrt{\frac{\rho_s q R}{2s' L_n}} \Big|_{r_{\min}}, \quad (29)$$

where all the quantities in the expression are evaluated at the q_{\min} surface r_{\min} . The Δr is then used to estimate the maximum value of Υ_s in the q_{\min} region (Fig. 8b), which is approximated by

$$\Upsilon_s^{\max} = \frac{V'_E}{c_s} \sqrt{\frac{2L_n q R}{s' \rho_s}} \Big|_{r_{\min}}. \quad (30)$$

Here, $V'_E = R\partial_r(E_r/RB_\theta)$. It can be seen in Fig. 8b that the Υ_s profile thus obtained has a higher peak in the ERS discharge than in the RS discharge. In most of the central region inside the q_{\min} surface, Υ_s is also larger in the ERS experiment than that in the RS

experiment at the same time stage after the bifurcation as shown in Fig. 8b. We argue that these differences in Υ_s contribute to the onset of ERS mode from RS mode due to the reduction in the turbulent transport as shown, for example, by the 3D simulation in [12].

5 Discussion

From the TRANSP data files for the two matched high power TFTR discharges [2], E_r profiles are calculated based on the standard neoclassical theory. Comparison with the approximate analytical formula derived by Ernst shows that, due to the relatively high carbon concentration in the discharge, the analytical formula overestimates the magnitude and the gradient of E_r profile, and the full neoclassical viscosity and friction matrices should be used. The comparison of the numerical result of E_r with Ernst's analytical result shows the sensitivity of E_r to the impurity fraction α . We show that for discharges with $\alpha \geq 1$ the full neoclassical coefficient matrix value of E_r is reduced significantly and its gradient is reduced by a significant factor, due to the smoothing effect of the off-diagonal frictional forces. The results are clearly seen by the scaling $\alpha \rightarrow \epsilon\alpha$ with $0 \leq \epsilon \leq 1$ shown in Fig. 5b. Reduction of the maximum magnitude of E_r is important to bring the poloidal Mach number $M_p = E_r/(B_p v_{ti})$ into the linear range of the neoclassical viscosity, which would otherwise fall into the nonlinear regime [20].

Using the calculated E_r profiles, we examined two measures of sheared E_r effects in the reversed magnetic shear configurations. While the turbulence measure ω_s appears to distinguish the ERS discharge from the RS discharge, the stability measure Υ_s reveals more clearly the location where E_r shear effect is most effective and hence where the internal transport barrier occurs. Although both E_r shear and magnetic shear enter the parameter Υ_s , the investigation presented here suggests that the location of the internal transport barrier is dominated by the location of the q_{\min} surface. Also, Υ_s shows the essential difference between the RS and ERS discharges, which is mainly at and inside the q_{\min} surface. This

difference may account for the bifurcation of ERS mode from RS mode. The stabilizing effect of sheared E_r in reversed magnetic shear configuration seems to be more relevantly measured by the stability parameter Υ_s , which demonstrates at least part of the correlation between E_r shear effects and reversed magnetic shear effects, if not all. However, a more complete form of Υ_s needs to be found within the neighborhood of q_{\min} surface in order to resolve the divergence problem with the current expression of Υ_s derived from the second order perturbation theory.

In conclusion, the determination of the radial electric field within the neoclassical model that orders the turbulence effects as negligible requires the use of the full viscosity and friction coefficients of the working gas and impurity ions. Due to the relatively low levels of the fluctuations in the core of the tokamaks the linear stability theory parameter that depends on the ratio of the shear in the radial electric field to the shear in the magnetic rotation transform may control the transport barrier formation.

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Appendix A: Physical Properties of the Friction–Flow Matrix

In neoclassical theory the viscosity matrix \mathbf{M} and friction matrix \mathbf{L} are defined in Eqs. (15)–(19) in a compact, dimensionless form that obscures the symmetries of arising from Newton’s 3rd law. Here we transform to a physical form that recovers the symmetries and estimate the strength of the parallel deuterium–carbon friction force for this discharge in terms of an equivalent parallel electric field strength.

From the Coulomb cross-section for collisions between the deuterium ($Z_i = 1$, $m_i = 2m_H$) working gas and the carbon ($Z_x = 6$, $m_x = 12m_H$) impurity, it is clear that for $T_x = T_i$ the friction force from the relative parallel flow velocities is

$$F_{\parallel 1x} \simeq -l_{11}^{xi} (u_{\parallel x} - u_{\parallel i}) = -\frac{16\sqrt{\pi}}{3} \frac{n_x n_i Z_x^2 Z_i^2 e^4 \lambda}{m_i v_{Ti}^3} \left(1 + \frac{m_i}{m_x}\right)^{-\frac{1}{2}} (u_{\parallel x} - u_{\parallel i}), \quad (\text{A1})$$

which by Newton’s third law is equal and opposite to the force on deuterium due to collisions with carbon

$$F_{\parallel 1i} \simeq -l_{11}^{ix} (u_{\parallel i} - u_{\parallel x}) = -\frac{16\sqrt{\pi}}{3} \frac{n_x n_i Z_x^2 Z_i^2 e^4 \lambda}{m_i v_{Ti}^3} \left(1 + \frac{m_i}{m_x}\right)^{-\frac{1}{2}} (u_{\parallel i} - u_{\parallel x}). \quad (\text{A2})$$

Galilean invariance dictates that the friction force \mathbf{F}_1 is only a function of the relative flow velocities $\mathbf{u}_i - \mathbf{u}_x$ so as to be independent of reference frame, and Newton’s third law gives that $F_{1a} + F_{1b} = 0$ for friction between species a and b . These symmetries are guaranteed by the collision operator. In terms of the physical friction matrix \mathbf{L}^p these two symmetries are $L_{i1}^p + L_{i3}^p = 0$ and $L_{1j}^p + L_{3j}^p = 0$, where $i, j = 1, 2, 3, 4$.

In Eqs. (15)–(19) the forces are normalized with respect to the self-collision frequency of each species. Taking into account that $(m_x n_x / \tau_{xx}) = (m_i n_i / \tau_{ii}) \sqrt{m_x / m_i} \alpha^2$ where $\alpha = (Z_x^2 n_x) / (Z_i^2 n_i)$ the physical components of the \mathbf{L} -matrix in Eq. (19) becomes for the deuterium–

carbon collisions (where $(m_x/m_i)^{1/2} = \sqrt{6}$)

$$\mathbf{L}^p = \frac{n_i m_i}{\tau_{ii}} \begin{pmatrix} -\sqrt{\frac{6}{7}}\alpha & \frac{9}{8}\alpha & \sqrt{\frac{6}{7}}\alpha & -\frac{3}{14}\sqrt{\frac{6}{7}}\alpha \\ \frac{9}{8}\alpha & -\left(\frac{77}{32}\alpha + \sqrt{2}\right) & -\frac{9}{8}\alpha & \frac{81}{98}\sqrt{\frac{6}{7}}\alpha \\ \sqrt{\frac{6}{7}}\alpha & -\frac{9}{8}\alpha & -\sqrt{\frac{6}{7}}\alpha & \frac{3}{14}\sqrt{\frac{6}{7}}\alpha \\ -\frac{3}{14}\sqrt{\frac{6}{7}}\alpha & \frac{81}{98}\sqrt{\frac{6}{7}}\alpha & \frac{3}{14}\sqrt{\frac{6}{7}}\alpha & -\frac{1189}{196}\sqrt{\frac{6}{7}}\alpha + 2\sqrt{3}\alpha^2 \end{pmatrix}. \quad (\text{A3})$$

The symmetries from Galilean invariance and Newton's third law are now self-evident in Eq. (A3). In the diagonal term $L_{22}^p = -\left(\frac{77}{32}\alpha + \sqrt{2}\right)$ the $\sqrt{2}$ contributions arises from the indirect influence of the carbon drag force that distorts the deuterium distribution away from the maxwellian background (field particles) giving a D-D collisional contribution. Likewise, in the diagonal term L_{44}^p the $2\sqrt{3}\alpha^2$ contribution arises from the carbon-carbon collisions due to the disturbance of the background distribution (field particles) by the friction on the deuterium. These self-collisional contributions are the analog of the role played by the electron-electron collisions in the Spitzer conductivity problem. For small α the D-D collisions dominate L_{22}^p , and for large α the C-C collisions dominate L_{44}^p . We can also derive from the self-adjointness and momentum conservation properties of the collision operator all the symmetry relations for the friction coefficients $l_{jk}^{ab} = l_{kj}^{ba}$ and $\sum_a l_{1k}^{ab} = 0$. These relations are easily confirmed in Eq. (A3) as $L_{jk}^p = L_{kj}^p$ and $L_{1k}^p = L_{k1}^p = -L_{3k}^p = -L_{k3}^p$ ($j, k = 1, 2, 3, 4$).

Now for the experiments analyzed here we calculate that near the q_{\min} surface $1/\tau_{ii} = 20/s$ and $\alpha = 4$. Thus, for $|u_{\parallel x} - u_{\parallel i}| \simeq 100$ km/s corresponding to Fig. 3b, we estimate that the parallel frictional acceleration in Eq. (A1) on each particle is equivalent to an electric field

of magnitude

$$E_{\parallel}^{\text{eff}} = \frac{m_i}{e} \frac{\alpha}{\tau_{ii}} |u_{\parallel i} - u_{\parallel x}| \simeq 80 \text{mV/m}. \quad (\text{A4})$$

The contribution to $E_{\parallel}^{\text{eff}}$ from the parallel thermal fluxes lowers the estimation in Eq. (A4) by approximately 10 mV/m. In neglecting the parallel electric field in Eqs. (4) and (5) we assume that $E_{\parallel}^{\text{eff}} \gg E_{\parallel}^{(A)} + \langle \tilde{n} \tilde{E}_{\parallel} \rangle / n_0$ where $E_{\parallel}^{(A)}$ is from the toroidal inductive electric field and $\langle \tilde{n} \tilde{E}_{\parallel} \rangle$ is from the turbulence. The TRANSP data field gives the loop voltage $V_{\ell} = 0.8 \text{ V}$ giving $E_{\parallel}^{(A)} \simeq 8 \text{ mV/m}$. For the maximum turbulence level we take $\tilde{n}/n = 10^{-3}$ and $\tilde{E}_{\parallel} \leq (T_e/qR)(\tilde{n}/n)$ to bound $\langle \tilde{n} \tilde{E}_{\parallel} \rangle / n_0 \leq 8 \text{ mV/m}$.

References

- [1] K. H. Burrell, E. J. Doyle, P. Gohil, R. J. Groebner, J. Kim, R. J. LaHaye, L. L. Lao, R. A. Moyer, T. H. Osborne, W. A. Peebles, C. L. Rettig, T. H. Rhodes, and D. M. Thomas, Phys. Plasmas **1**, 1536 (1994).
- [2] E. Mazzucato, S. H. Batha, M. Beer, M. Bell, R. E. Bell, R. V. Budny, C. Bush, T. S. Hahm, G. W. Hammett, F. M. Levinton, R. Nazikian, H. Park, G. Rewoldt, G. L. Schmidt, E. J. Synakowski, W. M. Tang, G. Taylor, and M. C. Zarnstorff, Phys. Rev. Lett. **77**, 3145 (1996).
- [3] A. J. Wootton and P. M. Schoch, *International School of Plasma Physics 'Piero Caldirola', Diagnostics for Contemporary Fusion Experiments, Proceedings of the Workshop*, ed P. E. Stott, D. K. Akulina, G. Gorini, E. Sindoni (Editrice Compositori, Bologna, Italy), p. xx+1092, 521-40, (1991).
- [4] D. R. Ernst, M. G. Bell, R. E. Bell, C. E. Bush, Z. Chang, E. Fredrickson, L. R. Grisham, K. W. Hill, D. L. Jassby, D. K. Mansfield, D. C. McCune, H. K. Park, A. T. Ramsey, S. D. Scott, J. D. Strachan, E. J. Synakowski, G. Taylor, M. Thompson, and R. M. Wieland, Phys. Plasmas **5**, 665 (1998).
- [5] W. A. Houlberg, K. C. Shaing, S. P. Hirshman, and M. C. Zarnstorff, Phys. Plasmas **4**, 3230 (1997).
- [6] S. P. Hirshman and D. J. Sigmar, Nucl. Fusion **21**, 1079 (1981).
- [7] M. S. Chance and J. M. Green, Nucl. Fusion **21**, 453 (1981).
- [8] C. Kessel, J. Manickam, G. Rewoldt, and W. M. Tang, Phys. Rev. Lett. **77**, 1212 (1993).
- [9] H. Biglari, P. H. Diamond, and P. W. Terry, Phys. Fluids B **2**, 1 (1990).

- [10] K. C. Shaing, E. C. Crume, Jr., and W. A. Houlberg, *Phys. Fluids B* **2**, 1492 (1990).
- [11] T. S. Hahm and K. H. Burrell, *Phys. Plasmas* **2**, 1648 (1995).
- [12] S. Hamaguchi and W. Horton, *Phys. Fluids B* **4**, 319, (1992).
- [13] Y. Kishimoto, T. Tajima, W. Horton, M. J. LeBrun, and J. -Y. Kim, *Phys. Plasmas* **3**, 1289 (1996).
- [14] Y. Koide, M. Kikuchi, M. Mori, S. Tsuji, S. Ishida, N. Asakura, Y. Kamada, T. Nishitani, Y. Kawano, T. Hatae, T. Fujita, T. Fukuda, A. Sakasai, T. Kondoh, R. Yoshino, and Y. Neyatani, *Phys. Rev. Lett.* **72**, 3662, (1994).
- [15] R. E. Bell, F. M. Levinton, S. H. Batha, E. J. Synakowski, and M. C. Zarnstorff, *Phys. Rev. Lett.* **81**, 1429 (1998).
- [16] G. Rewoldt, M. A. Beer, M. S. Chance, T. S. Hahm, Z. Lin, and W. M. Tang, *Phys. Plasmas* **5**, 1815 (1998).
- [17] D. T. Garnier, E. S. Marmor, C. L. Fiore, J. A. Goetz, S. N. Golovato, M. J. Greenwald, A. E. Hubbard, J. H. Irby, P. J. O'shea, J. J. Ramos, J. E. Rice, J. M. Schachter, P. C. Stek, Y. Takase, R. L. Watterson, S. M. Wolfe, and A. Martynov, *Fusion Energy 1996*, (IAEA, Vienna), 907-912 (1997).
- [18] H. Sugama and W. Horton, *Two-Dimensional Turbulence in Plasmas and Fluids*, ed. R. L. Dewar and R. W. Griffiths, (AIP Conf. Proceedings **414**, New York), 275-285 (1997).
- [19] W. A. Cooper, *Plasma Phys. Control. Fusion*, **30**, 1805 (1988).
- [20] K. C. Shaing, *Phys. Fluids B* **2**, 2847 (1990).

FIGURE CAPTIONS

FIG. 1. The $q(r)$ profiles for the matched TFTR discharges in Mazzucato *et al.* (1996) [2].

FIG. 2. Diagram showing the input data vector, the neoclassical parallel transport matrix elements and the output vector for determining the poloidal flows and the radial electric field.

FIG. 3. The input data and output vectors of the neoclassical transform for the ERS discharge at $t = 2.7$ s. (a) profiles for the deuterium and carbon densities and temperatures and the toroidal angular velocity of carbon taken from TRANSP 88299a20 at $t = 2.7$. (b) the output profiles of the flow velocities and the radial electric field.

FIG. 4. Comparison of the motional electric field component and the pressure gradient component for (a) the deuterium working gas and (b) the carbon impurity gas.

FIG. 5. Comparison of the radial electric field computed from the full 4×4 matrix method with that obtained from the reduced model in Eqs. (23) and (24). (a) the comparison of the two methods of calculating E_r for the full α value; (b) comparison with the scaled $\alpha \rightarrow \epsilon\alpha$ which shows how the full matrices reduce to the analytic models.

FIG. 6. Benchmarking comparison of E_r from NCLASS and E_r from Eqs. (15)–(20) for the TFTR discharge #103794 in TRANSP file at $t = 2.00$ s. The NCLASS E_r at $t = 1.96$ s was provided by Rewoldt.

FIG. 7. The time evolution of $E_r(r, t)$ for (a) the RS discharge and for (b) the ERS discharge at times before ($t = 2.6$ s) and after ($t = 2.7$ s, 2.9 s) the bifurcation.

FIG. 8. (a) Comparison of the Hahm–Burrell shear rate in the RS and ERS discharge at $t = 2.7$ s. (b) Comparison of the linear stability measure Υ_s in Eq. (27) for the RS

and ERS discharges. The sharp peak at the shear reversed layer indicates a strong stabilizing effect on the drift wave turbulence.