Nonlinear Splitting of Fast Particle Driven Alfvén Eigenmodes: Observation and Theory

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Abstract

The measured spectra of fast particle driven Alfvén eigenmodes in the JET tokamak plasma are interpreted on the basis of a first principle nonlinear model of near–threshold kinetic instabilities. The observed splitting of the Alfvén modes is shown to be due to the combined effect of resonant wave–particle interaction and collision–like relaxation of the resonant particles. The instability growth rate and the effective collision frequency of the resonant particles are determined.

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Most plasmas in nature and in the laboratory are characterized by a suprathermal component, which can be created by various particle injection and acceleration mechanisms. Examples include cosmic rays in space plasmas and fusion products, radio frequency heated ions and beam injected ions in tokamak plasmas. These fast particles can interact resonantly with waves in the plasma and drive kinetic instabilities. The characteristic time for the evolution of the fast particle population is often much longer than the linear growth time of the unstable wave. Therefore, in addition to the linear stability, it is of fundamental importance to study the nonlinear evolution of the wave–particle system.

Alfvén Eigenmodes (AEs) [1] and the associated kinetic instabilities are extensively studied theoretically and experimentally in magnetic fusion devices as they have the potential to eject energetic alpha particles from the core of a fusion reactor, possibly limiting the reactor operational regime [2]. The physics of AEs is addressed experimentally via active and passive diagnostic techniques [3]–[6], and theoretically via MHD and kinetic models [7]. The recent progress in the linear AE physics has been so significant that information on the background plasma and the fast particle population can be extracted from AE observations [8]. AEs are also suitable to explore fundamental nonlinear wave–particle physics because the fast particle population primarily affects the instability growth rate but not the mode structure or frequency.

In this Letter we apply a general nonlinear model for near–threshold kinetic instabilities [9] to interpret the unexplained observation of the splitting of Alfvén Eigenmodes driven by fast particles in the JET tokamak plasma [6]. The consistency between the nonlinear model and the experimental observations allows us to determine the total growth rate of the modes and the effective collision frequency in the phase space region around the wave–particle resonance.

A number of tokamak experiments have shown that fast particles generated by ion cyclotron resonance heating (ICRH) can drive AE unstable if the energetic ion pressure gradient is large enough to overcome the background plasma damping [6, 10, 12]. Figures 1 and 2 illustrate two examples of toroidal AEs (TAEs) driven by the ICRH–produced fast ions in the JET plasma. The magnetic fluctuation signals are detected on Mirnov pick–up coils localized at the outer side of the torus and sampled at 1 MHz. The spectrograms are recorded in the high performance phase of a JET hot ion H–mode plasma [12].

The two discharges shown in Figs. 1 and 2 were heated by a combination of sub–Alfvénic neutral beam injection (NBI) and of H–minority ICRH. Four antennas fed by generators with slightly different frequencies were used in both discharges to spread the ICRH area over about 30 cm ($R_{\rm ICRH} = 3.05\,$m, 3.14 m, 3.25 m, 3.38 m, with the magnetic axis being $R_0 = 3\,$m at 52.5 s in the case of Fig. 1). In this regime the instability is typically observed when the ICRH power exceeds 4 MW (Fig. 2). As indicated on the spectrograms, the different bands in the 200–500 kHz range correspond to toroidal mode numbers $n = 5$ to 12 shifted by the Doppler effect due to the plasma rotation. The frequency of each band is consistent with the calculated TAE frequency at the center of the corresponding TAE gap, $f_{\rm TAE} = v_A/4\pi q R_0$, where R_0 is the tokamak major radius, q the local safety factor and v_A the Alfvén speed. The overall decrease of $f_{\rm TAE}$ with time in this type of discharge is caused by the density increase due to plasma fueling by NBI.

Numerical calculations show that the radial profiles of the different n–modes peak around $r/a \sim 0.2$ to 0.3 [7]. This is consistent with the computed location of the maximum fast–particle pressure gradient [8] and the radius at which the measured plasma rotation frequency corresponds to the Doppler–induced separation of the different n–bands. The splitting of each n–band into several spectral lines, highlighted in Fig. 1(b,c) is characteristic of a large number of similar JET discharges.

The measurements are taken several seconds after the creation of the plasma discharge, when the safety factor profile is monotonic and quasi–stationary. The characteristic time of the ICRH power waveform and the slowing down time of the fast ions resonating with the AE are both much longer than the AE instability inverse growth rate, typically in the millisecond range. This has two important consequences. Firstly, the observed signal represents a succession of nonlinearly saturated states. Secondly, the spatial structure of the modes can be considered approximately constant during the time window chosen for the specific comparison with the theory, namely from 52.5 s to 52.9 s in the discharge shown in Fig. 2. The mode amplitude in the core and that measured at the edge are then related for each n by a constant factor.

The data can be interpreted via a nonlinear theory of near–threshold regimes for kinetic instabilities [9]. In this model the perturbed particle distribution function near the resonance reflects a competition between the field of the mode that tends to flatten it and relaxation processes that constantly tend to restore it. When this process is diffusive and is characterized by a step size smaller than the region of interaction with the wave, it can be represented by an effective collision frequency, ν_{eff} [9]. The resulting flux of free energy into the resonance can balance the background dissipation, allowing the mode to persist in the saturated state.

In the hot tokamak plasma experiment reported herein, the fast ion relaxation process due to velocity space diffusion induced by the ICRH wave field is expected to dominate over that due to Coulomb collisions. The fast ions generated by ICRH move along trapped orbits in the tokamak magnetic field and interact with the TAE mode when the resonance condition $\omega - n\,\omega_\phi - \ell\omega_\theta = 0$ is satisfied. Here ω is the mode frequency, n the toroidal mode number and ℓ an integer. The particle toroidal precessional and poloidal bounce frequencies, ω_ϕ and ω_θ, are functions of the particle energy (E), toroidal angular momentum (P_ϕ) and magnetic moment (μ). For simplicity, we consider here one dominant resonance corresponding to one value of ℓ, i.e. a surface in the E, μ, P_φ space. The ICRH creates an energetic hydrogen tail which occupies a narrow region on this surface, determined by $E \sim \mu B_0$, with B_0 being the equilibrium magnetic field. The AE spatial localization further reduces the interaction region, so that a fixed value can be assumed for ν_{eff}.

Close to the instability threshold, the mode linear growth rate, γ, is small compared with both the energetic particle drive, γ_L, and the damping rate due to the background plasma, γ_d, i.e. $\gamma = \gamma_L - \gamma_d \ll \gamma_L, \gamma_d$. As the mode reaches saturation before the resonant particles complete a bounce period in the wave field, the nonlinear correction to γ_L can be calculated by expanding the energetic particle response in powers of the mode amplitude A and retaining only the lowest order term. The magnitude of A is proportional to the square of the nonlinear bounce frequency for a typical resonant particle trapped in the field of the mode [9]. The nonlinear bounce frequency is much smaller than ν_{eff} [9], which implies that the particles move out of resonance before their motion becomes strongly nonlinear.

The corresponding nonlinear correction to γ_L is quadratic in A, while the nonlinear term in the resulting equation for the mode amplitude is cubic in A [9]:

$$\exp(-i\phi)\frac{dA}{dt} = \frac{\gamma}{\cos\phi} A - \frac{\gamma_L}{2}\int_0^{t/2} \tau^2 d\tau \int_0^{t-2\tau} d\tau_1 \exp\left[-\nu_{\text{eff}}^3 \tau^2 \left(\frac{2\tau}{3} + \tau_1\right)\right]$$
$$\times A(t-\tau)A(t-\tau-\tau_1)A^*(t-2\tau-\tau_1). \tag{1}$$

The parameter ϕ in this equation characterizes the contribution of hot particles to the mode frequency. The structure of the nonlinear term indicates that the saturated amplitude scales as $\gamma^{1/2}$. For any $\gamma > 0$, Eq. (1) admits a saturated solution with constant magnitude $|A|$:

$$|A|^2 = 2\,\frac{\gamma}{\gamma_L}\,\frac{\nu_{\text{eff}}^4}{\cos\phi}\left[\int_0^{\infty} \exp\left(\frac{-2\tau^3}{3}\right) d\tau\right]^{-1}. \tag{2}$$

Analytical studies and numerical integration of Eq. (1) show that for sufficiently small γ the mode converges to this saturated state but that this state becomes unstable and bifurcates when γ exceeds a critical value γ_{cr}. In the limit of $\phi \ll 1$, it is found that $\gamma_{\text{cr}} = 0.486\nu_{\text{eff}}$. For $\gamma > \gamma_{\text{cr}}$, Eq. (1) admits a solution in the form of a limit cycle

$$A = A_0 \exp(i\delta\omega t)\Big[1 + \alpha_1\ \exp(i\Delta\omega t) + \beta_1 \exp(-i\Delta\omega t)$$

$$+ \alpha_2 \exp(2i\Delta\omega t) + \beta_2 \exp(-2i\Delta\omega t) + \dots\Big], \tag{3}$$

where A_0 is the amplitude of the main spectral component, $\delta\omega$ the nonlinear frequency shift of the mode and $\Delta\omega$ is the sideband frequency split; α_i and β_i are the amplitudes of the sidebands. Their value depends on $\gamma/\nu_{\rm eff}$ and ϕ; the value of ϕ in particular determines the sideband symmetry, i.e. the ratio $(|\alpha_i| - |\beta_i|)/(|\alpha_i| + |\beta_i|)$.

Figure 3 (left) shows the power spectrum of the saturated solution of Eq. (1) for increasing values of $\gamma/\nu_{\rm eff}$. The first bifurcation splits the original single spectral line ($\gamma/\nu_{\rm eff} = 0.47$) into a line with a number of equidistant sidebands ($\gamma/\nu_{\rm eff} = 0.52$). Then larger values of $\gamma/\nu_{\rm eff}$ produce a period doubling transition which introduces intermediate frequencies at half of the first sideband spacing ($\gamma/\nu_{\rm eff} = 0.59$). Close to the first bifurcation threshold where the splitting starts, the dominant sidebands are given by α_1 and β_1. From Eq. (1), the following relationship between $\gamma_{\rm cr}$, $\nu_{\rm eff}$ and $\Delta\omega$ is found at this bifurcation point

$$\Delta\omega = 1.18\gamma_{\rm cr} = 0.575\nu_{\rm eff}. \tag{4}$$

For the $n = 8$ band of Fig. 2, the experimentally measured $\Delta\omega \cong 1.8 \times 10^3 2\pi s^{-1}$ gives $\gamma_{\rm cr} \cong 1.5 \times 10^3 2\pi s^{-1}$ and $\nu_{\rm eff} \cong 3.1 \times 10^3 2\pi s^{-1}$. The order of magnitude of $\gamma_{\rm cr}/\omega \sim 1\%$ is consistent with the first measurements of the ICRH drive performed for externally driven, low n ($n = 1, 2$) TAE [6]. In order to account for the measured value of $\nu_{\rm eff}$, the 90° pitch–angle scattering rate of the fast ions due to Coulomb collisions would need to be an order of magnitude larger than its calculated value. The enhanced phase space transport can instead be attributed to the particle diffusion in the ICRH wave fields. From the value of $\nu_{\rm eff}$ one can in principle infer the fast particle diffusion coefficient and get information on the ICRH wave field, although an accurate calculation of the ICRH wave–driven $\nu_{\rm eff}$ goes beyond the scope of this paper.

In the experiment, the time between the beginning of the TAE signal (t_0) and the point of the first bifurcation (t_b) is short compared to the total duration of the instability and the fast particle slowing down time. Over this interval, the evolution of the growth rate γ can therefore be approximated by a linear function of time and $\nu_{\rm eff}$ can be treated as a constant

$$\gamma = \frac{t-t_0}{t_b-t_0}\,\gamma_{\rm cr} = 0.847\,\frac{t-t_0}{t_b-t_0}\,\Delta\omega = 0.49\,\frac{t-t_0}{t_b-t_0}\,\nu_{\rm eff}. \tag{5}$$

With this assumption, Eq. (1) was solved numerically, and the evolution of the mode amplitude was computed on a time interval that includes the first bifurcation. In order to provide a seed for instability and to simulate the noise level observed experimentally, a small random source term was added to the right–hand side of Eq. (1). Figure 4 shows the comparison between the experimental data and the simulation results with $\phi = 3\pi/64$ in terms of the amplitude of one of the modes presented in Fig. 2. The evolution of the central line and both the upshifted and downshifted sidebands is well reproduced by the theory over the time interval in which Eq. (5) is valid. Note that the instability and the bifurcation thresholds are insensitive to the chosen small value of ϕ.

In the same discharge, about 150 ms after the first nonlinear splitting, a period doubling transition is observed in the magnetic fluctuation signal (Fig. 3, bottom right). This observation is consistent with the transition observed in the theoretical spectra when $\gamma/\nu_{\rm eff}$ is increased from 0.52 to 0.59 (Fig. 3, bottom left). The second bifurcation can be analyzed similarly to the first one to determine the γ and $\nu_{\rm eff}$. The value of $\nu_{\rm eff}$ does not change between the two bifurcations while γ increases slower than linearly with time. At later times γ is expected to gradually decrease to negative values as the damping from the background plasma and the beam ions increases [13]. This is clearly observed in Fig. 1, where the split modes revert to single lines and eventually disappear as the system becomes stable.

This behavior characterizes a large number of JET discharges in which AE are driven by

ICRH fast ions. The strongly nonlinear regime predicted by the theory for large $\gamma/\nu_{\rm eff}$, with an explosive growth of the mode amplitude that could perturb the particle orbits, affecting the ICRH heating process and the plasma performance, is not observed experimentally even for high ICRH power ($P_{\rm ICRH} > 10\,{\rm MW}$). This may be attributed to the fact that the ICRH power produces both the fast particle pressure gradients that drive the instability and the particle phase space diffusion that decorrelates the particles from the resonance. The interdependence of these two effects can prevent the onset of the explosive nonlinear regime and will be the subject of further investigations.

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References

[1] C.Z. Cheng, Liu Chen, and M.S. Chance, Ann. Phys. (N.Y.) **161**, 21 (1985).

[2] F. Romanelli *et al.*, ITER Physics Basis Document, Ch. 5, "Physics of Energetic Ions," submitted to Nucl. Fusion (1998).

[3] K.L. Wong, R.J. Fonck, S.F. Paul, D.R. Roberts, E.D. Fredrickson, R. Nazikian, H.K. Park, M. Bell, N.L. Bretz, R. Budny, S. Cohen, G.W. Hammett, F.C. Jobes, D.M. Meade, S.S. Medley, D. Mueller, Y. Nagayama, D.K. Owens, and E.J. Synakowski, Phys. Rev. Lett. **66**, 1874 (1991).

[4] W.W. Heidbrink, E.J. Strait, E. Doyle, and R. Snider, Nucl. Fusion **31**, 1635 (1991).

[5] A. Fasoli *et al.*, Phys. Rev. Lett. **75**, 645 (1995).

[6] A. Fasoli *et al.*, Plasma Phys. Control. Fusion **39**, B287 (1997).

[7] W. Kerner *et al.*, Nucl. Fusion (to be published, 1998).

[8] L.G. Eriksson *et al.*, Preprint JET-P(98)07 (1998), Phys. Rev. Lett. **81**, in press.

[9] H.L. Berk, B.N. Breizman, M.S. Pekker, Plasma Phys. Rep. **23**, 778 (1997) [in Russian origin: Fizika Plazmy **23**, 842 (1997)].

[10] K.L. Wong, J.R. Wilson, Z.-Y. Chang, G.Y. Fu, E. Fredrickson, G.W. Hammett, C. Bush, C.K. Phillips, J. Snipes, and G. Taylor, Plasma Phys. Contr. Fusion **36**, 879 (1994).

[11] H. Kimura *et al.*, Phys. Lett. A **199**, 86 (1995).

[12] M. Keilhacker and the JET Team, Plasma Phys. Contr. Fusion **37**, A3 (1995).

[13] S.E. Sharapov *et al.*, submitted to Nucl. Fusion.

FIGURE CAPTIONS

FIG. 1. Spectrogram of magnetic activity measured at the plasma edge in a hot–ion H–mode JET discharge (JET shot #40332). $P_{\rm ICRH} \sim 6.5\,{\rm MW}$, $P_{\rm NBI} \leq 16\,{\rm MW}$, $B_T = 3.45\,{\rm T}$, $I_p = 3.7\,{\rm MA}, T_e(0) \sim 7{-}12\,{\rm keV}$ and $\langle n_e \rangle = 2{-}5{\times}10^{19}{\rm m}^{-3}$. The triangular waveform appearing in the spectrum corresponds to the directly coupled perturbation from an external antenna exciter [5]. The gray scale corresponds to the measured value of $\delta B_{\rm edge}$ on a logarithmic scale. The plasma is started at 40 s.

FIG. 2. Spectrogram of magnetic activity measured at the plasma edge in a hot–ion H–mode JET discharge (JET shot #40328). Plasma and machine parameters are similar to Fig. 1.

FIG. 3. Nonlinear splitting of the TAE spectral line and the period doubling bifurcation in snapshots of the mode power spectrum; Left: calculated power spectrum of the saturated solution of Eq. (1) (with $\phi = 3\pi/64$) as parameter $\gamma/\nu_{\rm eff}$ increases; Right: time evolution of experimental spectrum of magnetic activity for the $n = 7$ mode during the increase of $P_{\rm ICRH}$ (JET shot #40328).

FIG. 4. Time evolution of the spectral components of the $n = 8$ mode (JET shot #40328) showing bifurcation and nonlinear splitting at $t = 52.67\,{\rm sec}$. Top: experimental data. Bottom: simulation results with ($\gamma = 1.25 \times 10^4\ (t[s] - 52.56) 2\pi\, s^{-1}$, $\nu_{\rm eff} = 2.8 \times 10^3 2\pi\, s^{-1}, \phi = 3\pi/64$).