Particle Dynamics and its Consequences in Wakefield Acceleration in a High Energy Collider

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Abstract. The performance of a wakefield accelerator in a high energy collider application is analyzed by use of a nonlinear dynamics map built on a simple theoretical model of the wakefield generated by the laser pulse (or whatever other method) and a code based on this map [1].The crucial figures of merit for such a system other than the final energy include the emittance (that determines the luminosity). The more complex the system is, the more "opportunities" the system has to degrade the emittance (or entropy of the beam). Thus our map guides us to identify where the crucial elements lie that affect the emittance. If the focusing force of the wakefield is strong when there is a jitter in the position (or laser aiming) of each stage coupled with the spread in the individual particle betatron frequencies, particles experience a phase space mixing. This effect sensitively controls the emittance degradation. We investigate these effects both in a uniform plasma and in a plasma channel. We also study the effect of beam loading. Further, we briefly consider collision point physics issues for a collider expected or characteristic of such a construction based on a scenario for the multi-staged wakefield accelerators.

INTRODUCTION

The use of plasma waves excited by laser beams for electron acceleration was proposed by Tajima and Dawson [2]. There are many possible applications of plasma based accelerators and one of them, perhaps the most challenging, is for construction of a linear collider. It is believed that conventional linear colliders can go up to 1 TeV center of mass energy of colliding particles [3]. However, beyond that probably some new technology needs to enter. A feature of plasma based accelerators is their ability to sustain extremely large acceleration gradients ($\sim 100\,\mathrm{GV/m}$). Correspondingly we can hope to achieve high energy gains on short distance. Such a machine, no doubt, consists of a large number of elements to reach the desired energy and forms a complex system. In order to identify the most

important research questions in such a complex problem, it is necessary to design a strawman's system, no matter how simplistic it may be, and to try to analyze and deduce the fundamental properties of such a system. Such a strawman's design was in fact carried out for the first time for a wakefield acceleration-based collider in the last AAC meeting [4]. In Sec.2 of this paper we analyze the performance of Laser Wakefield Accelerator (LWFA) in the case of an initially homogeneous plasma using the map developed in [1]. Considering the target center of mass energy $E_{cm} = 5$ TeV and luminosity $\mathcal{L}_g = 10^{35}$ cm^{-2}s^{-1} in [4], it is shown that for a fixed power P_b of the colliding beams there are only two independent parameters describing the beam at IP: the number of particles per bunch N and the longitudinal beam size σ_z (we assume a round beam: the aspect ratio $R = 1$). All other beam parameters at IP can be expressed as functions of the above. So we know what the requirements on the normalized emittance are.

In Sec.2 and 3, respectively, we analyze two different scenarios with an initially uniform plasma density and with a preformed 'hollow' channel. In Sec.4 we consider the effect of beam loading. In Sec.5 we consider possible impact of such a collider system and its performance on IP physics issues, discovery potential and possible limits on future machines. Conclusions are drawn in Sec.6.

MULTISTAGE ACCELERATION. UNIFORM PLASMA

When we try to accelerate particles to TeV energies, we need to investigate problems associated with the multistaging. In order to carry this out, we obtained a map [1] which describes the one to one correspondence between the entrance coordinates and the exit coordinates of the beam particles during the propagation of the beam trough many accelerating stages and used it to build a systems code. The linearized equations [1] of motion for the longitudinal degrees of freedom are:

$$\delta\Psi_{n+1} = \delta\Psi_n \tag{1}$$

$$\delta\gamma_{n+1} = 2\gamma_p^2\Phi_0(\cos(\Psi_s + \Delta) - \cos(\Psi_s))\delta\Psi_n + \delta\gamma_n, \tag{2}$$

where n enumerates the stage, Ψ_s is the 'synchronous' phase, Δ is the phase slippage per accelerating stage (actually it can also depend on n).

The transfer matrix M for the transverse coordinates is:

$$M = \begin{pmatrix} \cos(\frac{\omega}{\omega_s}\Delta), & \frac{1}{\omega}\sin(\frac{\omega}{\omega_s}\Delta) \\ -\omega\sin(\frac{\omega}{\omega_s}\Delta), & \cos(\frac{\omega}{\omega_s}\Delta) \end{pmatrix} \cdot \begin{pmatrix} 1 & \hat{L} \\ 0 & 1 \end{pmatrix}, \tag{3}$$

where Δ is the phase advance of the particle per stage with respect to the wakefield and $\hat{L}$ is the length of the distance between the stages in units of k_p^{-1} ($1/\omega$ is the betatron length in units k_p^{-1} and it depends on the stage number (because the energy

[1] Notation and normalizations in this paper are the same as in [1]

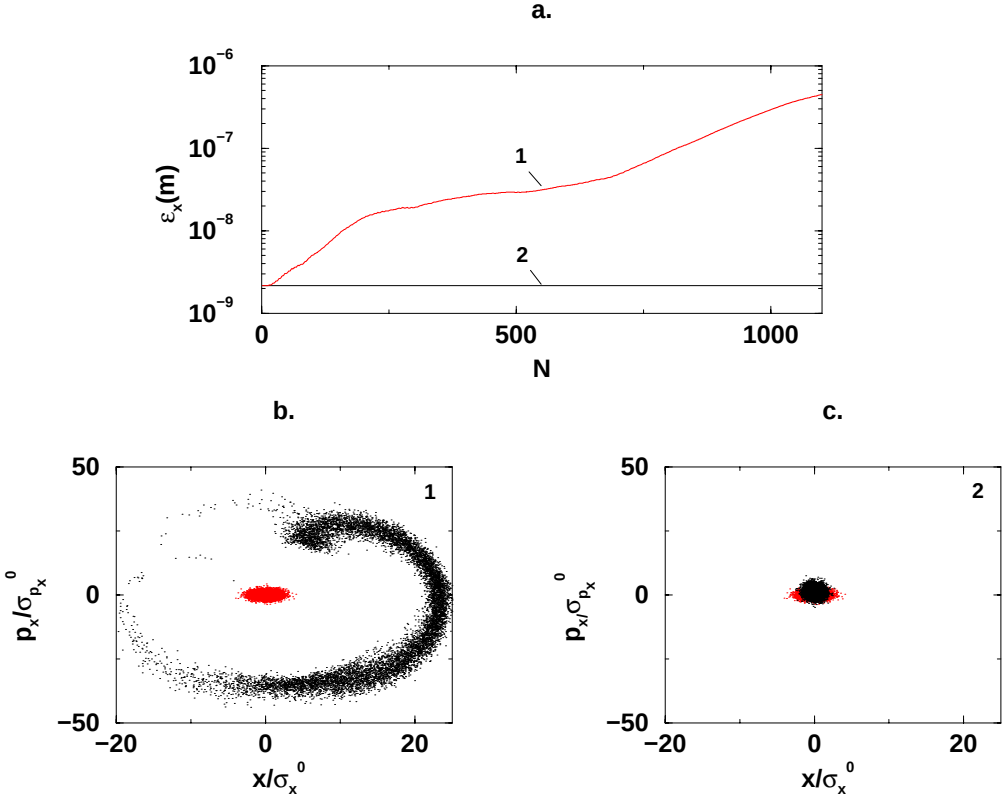

FIGURE 1. The normalized x-emittance ϵ_x vs. stage number N and the transverse phase space before and after the acceleration; 1. γ_p=100, $\hat{L}$=10000, ϵ_x^0=2.2 nm, laser wavelength $\lambda = 1\mu$m, laser spotsize r_s=0.5mm, $a_0 = 0.5$, dislocation size $= 0.1\mu$m, $\delta\gamma/\gamma$=0.01, $\delta\Psi = 0.01$. The total number of stages is N=1100 which gives the required final energy of 2.5 TeV. 2. Channeled case, for parameters see the corresponding section

is increasing) and on the individual particles positions in $(\gamma.\Psi)$ phase space). The matrix (3) can be written also as

$$M = \begin{pmatrix} \cos(\frac{\omega}{\omega_s}\Delta), & \frac{1}{\omega}\sin(\frac{\omega}{\omega_s}\Delta) + \hat{L}\cos(\frac{\omega}{\omega_s}\Delta) \\ -\omega\sin(\frac{\omega}{\omega_s}\Delta), & -\hat{L}\omega\sin(\frac{\omega}{\omega_s}\Delta) + \cos(\frac{\omega}{\omega_s}\Delta) \end{pmatrix} . \tag{4}$$

So, for N stages

$$\mathcal{M} = M_N M_{N-1} ... M_2 M_1 . \tag{5}$$

We note that because of the common structure of the wakefield in all plasma based accelerators the obtained map, with just slight modifications, can be used to analyze their performance as well.

For a complex system cumulative errors can give rise to a surprising (and often unpleasant) result. We identify that one of the most important such effects stems from the jitter [1] of the aligned wakefield (by whatever mechanism) stage by stage.

The problem here is that up to this point we have not considered possible dislocation of the consequent stages. This, combined with the fact that focusing force is different for different particles can lead to a severe emittance growth. Basically, what happens is that all particles rotate at different angular velocities in the phase space and if there is a stage position shift present, we get a characteristic "banana" shaped distribution (it is "banana" shaped only if the dislocation size is larger than beam size, but in any case the particle distribution gets diluted because of the misalignments). This process critically depends on the magnitude of the betatron frequency spread which means that the typical strength of the focusing force is of great importance. The effect of plasma noise (or other noise, such as laser or the boundary) on the particle dynamics over a stage may be incorporated in a map similar to the stage-by-stage jitter. Such dynamics results in a fuzzy or stochastic [7] map. We consider the case of stage jitter. The dislocation of the aligned position of each stage is given in our code as a stochastic variable which has a Gaussian distribution. The map is modified according to:

$$\begin{pmatrix} \tilde{x}_{n+1} \\ \dot{\tilde{x}}_{n+1} \end{pmatrix} = M_n \begin{pmatrix} \tilde{x}_n - \tilde{\mathcal{D}} \\ \dot{\tilde{x}}_n \end{pmatrix} + \begin{pmatrix} \tilde{\mathcal{D}} \\ 0 \end{pmatrix} , \tag{6}$$

where $\mathcal{D}$ is the misalignment size ($\tilde{\mathcal{D}} = \sqrt{\gamma_n}\mathcal{D}$). The longitudinal degrees of freedom are not affected. Run with random dislocations of magnitude $\sigma_{\mathcal{D}} = 1 \cdot 10^{-7}$ m is presented in Figure 1a and 1b. We see that in this case (corresponds to design I [4]) we have a severe emittance growth (the initial r.m.s. normalized emittance is 2.2 nm). We have to point out that even though we can find cases corresponding to large laser spot sizes which preserve the normalized emittance quite well their practical realization would require a huge laser power probably well above any future experimental limits. The other possible way to cure the situation is to increase the number of stages (and decrease γ_p) which also does not seem plausible (it may be difficult to have more than a thousand stages). To control the laser aiming and beam position better than 0.1 μm during the acceleration process is also not promising. If the initial normalized emittance is larger than the relative growth will be smaller, thus we can hope that, for instance, the design III emittance [4] will work better. This is really the case, Figure 2a and 2b shows basically the best we can do if we go as high as $r_s = 0.5$ mm and using the design III [4] normalized emittance of 330 nm. We see the characteristic dilution of the particle distribution and the corresponding emittance growth which is rather small here. The problem is that the spot-size is large and would require an immense laser power. If we limit ourselves to something a little bit more "reasonable" as $r_s = 0.2$ mm, keeping the other parameters the same, the result is discouraging, the emittance growth is very large in this case (see Figure 2a and 2c). From the computer simulations the conclusion is that in the case of initially homogeneous plasma it is very difficult based on reasonable parameters (laser spot size, dislocation size and number of stages) to avoid a severe emittance growth of the accelerated beam in the presence of small jitters stage-by-stage. The difficulty is primarily due to the fact that the

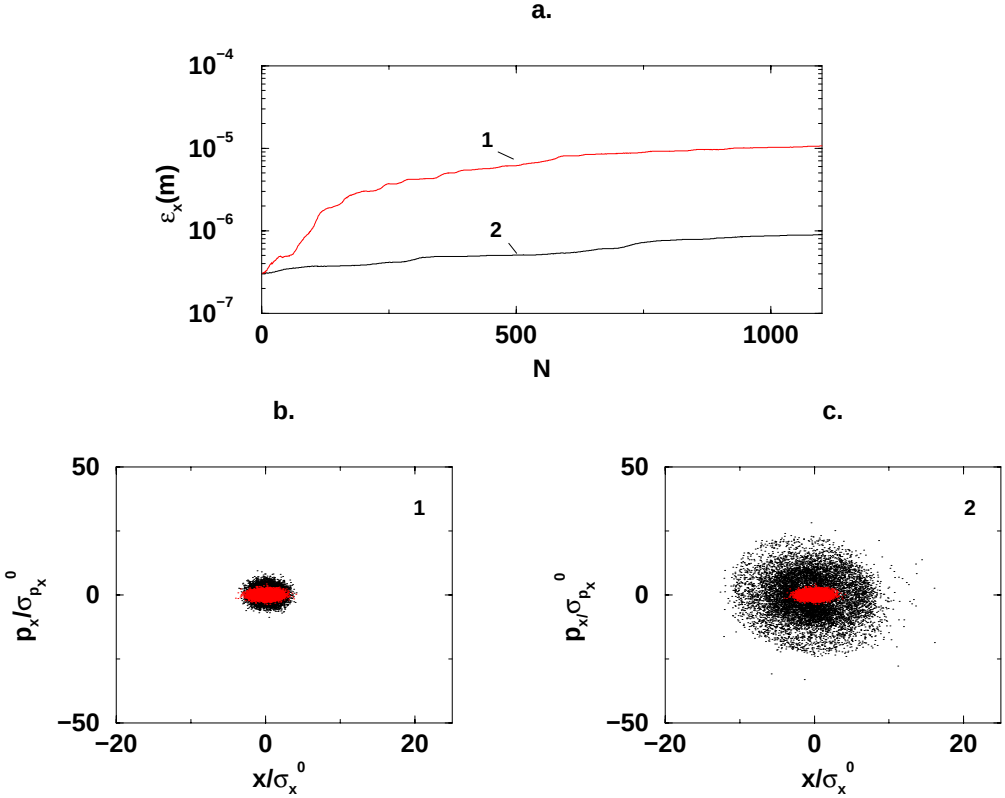

FIGURE 2. The normalized x-emittance vs. stage number and the transverse phase space before and after the acceleration for different laser spot sizes. $\gamma_p = 100$, $\hat{L} = 10000$, $\epsilon_x^0 = 330$nm, $r_s = 0.5$mm (1) and $r_s = 0.2$mm (2), $a_0 = 0.5$, dislocation size=0.1μm, $\delta\gamma/\gamma$=0.01, $\delta\Psi = 0.1$.

wakefield focusing force is too large in this case. We should also remember that the above considerations do not include any nonlinear effects which also contribute to the phase area increase, not to mention that in addition to all of the above we have to come up with a mechanism for guiding of the laser pulse (in the cases of small laser spot size). Otherwise the diffraction limits the acceleration length.

HOLLOW PLASMA CHANNEL

A possible way to avoid these difficulties is the 'hollow channel' design [8] in which we use a preformed vacuum channel in an underdense plasma (the overdensed case was studied in [9]). In this case we get several important features: the focusing force is exactly (because the phase velocity of the wakemode is very close to the speed of light) linear and weak in the channel (the weak focusing is a very important improvement over that of a uniform plasma case); there exists a stable solution for the laser mode; the acceleration gradient is uniform in transverse coordinates within the channel. An obvious drawback is a loss in the magnitude of the acceleration gradient which, of course, is unavoidable in this case. The equations

for the wakefield in the channel are [8]:

$$E_z(\xi, r) = -k_{ch}^2 \int_{\xi}^{\infty} \cos(k_{ch}(\xi - \xi'))\ \Phi_l(a, \xi')\ d\xi'\,, \tag{7}$$

$$E_r(\xi, r) - B_\theta(\xi, r) = \frac{k_{ch} r}{4\gamma_p^2} k_{ch}^2 \int_{\xi}^{\infty} \sin(k_{ch}(\xi - \xi'))\ \Phi_l(a, \xi')\ d\xi', \tag{8}$$

where a is the channel radius, Φ_l is the ponderomotive potential and k_{ch} is given by: $k_{ch} = k_p/\sqrt{1 + k_p a \frac{K_0(k_p a)}{2K_1(k_p a)}}$, where K_0 and K_1 are the modified Bessel functions of zeroth and first order, respectively. For instance, if we choose $k_p a = 1$ then the electric field in z-direction will be reduced by a factor of 0.6 [8] compared to the initially uniform plasma. So, formally there are no major changes to our previous map scheme. There is a reduction in Φ_0 and the magnitude of the focusing changes: $\omega = \frac{k_{ch}}{k_p}\left(\frac{\Phi_0}{2\gamma\,\gamma_p^2}\right)^{1/2}$ (in units of k_p). Importantly, we have a decrease in the magnitude of the focusing force. Making these minor changes to the map, we are able to investigate the accelerator performance in this case.

Run shown on Figure 1a and 1c indicates a significant improvement over the previous design. Here we are able to preserve even design I emittance of 2.2 nm. The stage considered is: $\gamma_p = 150$, the channel radius $a = 30\mu$m, the laser spot size $r_s = 50\mu$m, the plasma density (outside the channel) $n = 5 \cdot 10^{16}\text{cm}^{-3}$ and the laser wavelength $\lambda \sim 1\mu$m. The size of the stage dislocations was increased in this case to 0.5μm. From the graphs we see that the emittance growth of the accelerated beam is now much smaller and the design is more promising. The total number of stages was 900 which gives a total energy of 2.5 TeV. The drift space is 20000 (in units of $1/k_p$). Unfortunately, there is an additional effect: because in reality we have a finite density gradient it leads to a resonant absorption where the local plasma frequency matches the wakefield frequency. This effect has been studied in [10], where an expression for the quality factor of the hollow channel is derived. Possible low values of this factor can limit the acceleration of multiple bunches in a single shot created wakefield. We note that the real problem we have to deal with is the strong focusing of the wakefield. Channel is one way to reduce it, but there may be other possibilities as well, for instance, "flat top" laser pulse profile, etc. Still more work should be done on the subject and on the closely related problem of beam loading to come up with the best possible design of LWFA for collider applications. The purpose of the present paper is to introduce the systems approach to LWFA so that more future work on a complex collider machine may be theoretically carried out in detail.

BEAM LOADING EFFECT

All of our previous considerations are strictly applicable only in the case of weak beams which correspondingly produce weak wakefields. In a real case we may want

to extract a significant portion of the plasma wave energy. This can be accomplished when we load a large number of particles into the wave (from the requirements for high luminosity it already follows that we need relatively intense beams). We now need to take into account the wakefield produced by the accelerated beam itself. The total accelerating field (in a simple 1-d model), including the beam induced wakefield [6], is:

$$E_{z\,tot} = E_w \cos(k_p\xi) - 4\pi \int_{\xi}^{\xi_0} d\xi' \; \rho(\xi') \; \cos k_p(\xi - \xi') \,, \tag{9}$$

where the first term is due to the accelerating plasma wave and the second is the wakefield produced by the accelerated particle bunch (ξ_0 is the position of the bunch head). If we assume a constant bunch density profile, (9) is easily integrated to give

$$E_{z\,tot} = E_w \cos(k_p\xi) - \frac{4\pi N e}{k_p l d_{eff}^2} \sin k_p(\xi - \xi_0), \tag{10}$$

where d_{eff} is the effective transverse beam size (of order $1/k_p$), l is the bunch length and N is the number of electrons in the bunch.

We employ (10) in our map formulation and obtain

$$\delta\Psi_{n+1} = \delta\Psi_n \,, \tag{11}$$

$$\delta\gamma_{n+1} = 2\gamma_p^2\Phi_0(\cos(\Psi_s + \Delta) - \cos(\Psi_s))\delta\Psi_n + 2\Delta\gamma_p^2\Phi_0 \frac{N}{N_0}\frac{1}{k_p l}\delta\Psi_n \; + \delta\gamma_n \,. \tag{12}$$

The second term in (12) corresponds to the differences in energy gains due to the beam self-wakefield, N/N_0 is the percent of beam loading and $N_0 = \frac{k_p \Phi_0 d_{eff}^2 m_e c^2}{4\pi e^2}$. In our case the second term is much greater then the first term in (12) and it appears that we have a large energy spread due to this. One way to avoid this is to use longer beam length (and then to compensate the loading by proper choice of the initial phase) and another is to decrease the number of particles per bunch (for instance to use microbunching). Yet another possibility is bunch shaping [6] , which however may not be realistically manageable for such short bunches. In any case future studies on this subject are necessary. The above does not take into account the radial self-wakefield [5]. For dense beams it should be included as well and it gives a correction to the radial focusing force. The correction vanishes for the particles in the head of the bunch and is maximum for the particles in the bunch tail. We also need to compensate for this simultaneously with the longitudinal compensation. Fortunately, our operating region in the wake has the required features – acceleration gradient decreases and focusing force increases with ξ. Thus, in principle, such compensation is possible.

COLLISION POINT PHYSICS

One of the main purposes of the next generation high energy accelerators – the Large Hadron Collider (LHC) and the Next Linear Collider (NLC) is to check the predictions of the weak scale supersymmetry (SUSY) which if correct should lead to the discovery of light Higgs particles and light superpartners. Preservation of beam quality during the acceleration is extremely important, however it is not the only problem we have to solve. There are fundamental difficulties associated with the interaction point (IP) physics. Colliding of beams gives rise to the well known phenomenon of beamstrahlung [11]. It is often undesirable because it effectively decreases colliding particles energies and can also lead to a possible photon contamination of the detectors. The way to avoid the former is to either work in the classical regime with a very small beamstrahlung parameter $\Upsilon \ll 1$ or to move to the "quantum suppression" regime [4] characterized by $\Upsilon \gg 1$. We carried out QED simulations based on the CAIN code [12], [4]. This shows that the photon contamination is relatively easy to avoid due to the fact that the photon angular distribution has a very sharp peak (see Figure 3, in this example all the beamstrahlung photons are in a 20 mrad angle around the beam axis) in the forward (backward) direction (with respect to the beam). Secondary particles created via coherent and incoherent pair production are also sharply peaked in the forward (backward) direction. So we just need to place our detectors at large enough angles.

However, there are additional major obstacles to do discovery physics on such a machine. We should be able to extract the new physics signatures from the experimental data. At these energies, however, there are strong contributions from Standard Model processes which create a significant background to deal with. One of the most important ones, due to the high $\gamma - \gamma$ luminosity (see Figure 4), is that of $\gamma\,\gamma \to W^+\,W^-$. We need to impose additional cuts to eliminate background as much as possible and to improve the signal to noise ratio. The analysis presented in [13] shows that in the region of center of mass energies of a few hundreds GeV it should be possible to observe signatures of new physics. We hope to be able to do Higgs physics in this range (if the light Higgs exists) and also to find the lightest supersymmetric partners (if SUSY is correct). A major method to eliminate the background is b-tagging (for Higgs sector). Consider the following example (if $m_h < 2m_Z$): $e^+e^- \to Z \to Z\,h$. The background consists of ZZ and WW and the way to deal with it may be the double b-tagging and jet-jet mass reconstruction followed by visible energy cut. In the search of s-leptons a possible plan is to use polarized electron beams. For instance, $e^-e^- \to \tilde{e}^-\tilde{e}^-$ is practically free of the W-background if we use right handed polarized electron beams [14]. The produced s-electrons can decay in several different modes, all of them ending in neutralino (invisible) production and lepton pairs which leads to a clean missing energy signature of the reaction. High luminosity to be achieved in the high Υ regime is more than welcome in this case. The great opportunity provided by e^+e^- linear machine is that it can be relatively easy changed into e^-e^- or $\gamma\gamma$ linear collider. In this sense

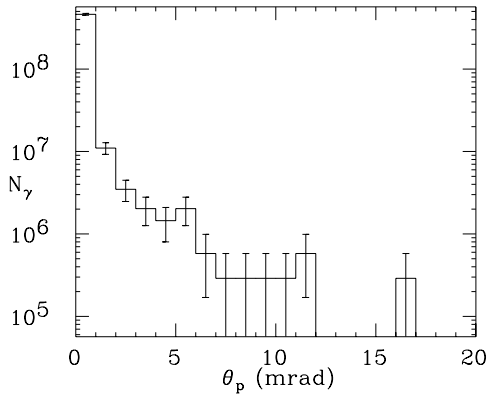

FIGURE 3. The number of photons N_γ vs. θ_p (angle with respect to the beam axis) at IP of a 5 TeV e^+e^- linear collider. (parameters used correspond to design III [4]; this and the next graphs are obtained using the CAIN)

linear colliders will be of great importance even after the start of LHC. In the TeV range the new physics we can expect is largely unknown. The predictions are strongly model dependent and probably modifications will occur after LHC and NLC become operational. If the light Higgs and superpartners are not discovered at 500 GeV center of mass energy we can hope that moving to TeV range will enable us to do this.

CONCLUSION

We studied the cumulative effects of the successive acceleration, transport, and focusing in the laser wakefield (or its sister methods) over many stages. Errors arising from the jitter of each stage or equivalently (in our map approach) the noise in the system can accumulate in such a way to degrade some of the parameters of the beam. The most crucial of these may be the emittance (or the entropy) of the beam. When we have stochastic variables on the wakefield (we chose the stage jitter of the axis of the wakefield in particular), the emittance can significantly increase over the many stages due to the strong focusing of the wakefield. This is probably the most serious effect on the long range behavior of the beams in this kind of accelerator for high energy applications. In order to ameliorate this situation, we consider the weak focusing of a hollow plasma channel and find that in fact the emittance may be well preserved. However, there are many details still need to be investigated. These include the effect of resonance of the wakefield in a channel, and alternative methods to decrease the focusing force. In order to make an efficient accelerator it is desired to have heavy beam loading. In this case the effect of beam loading must be considered. It can have degrading effect on the emittance, as the head and the tail of the beam may experience different forces. Search for the best parameter set in the multidimensional parameter space of a large scale accelerator

should be performed taking into account, to our best notion, future experimental limits and restrictions which might come from them. The work is supported by US DoE.

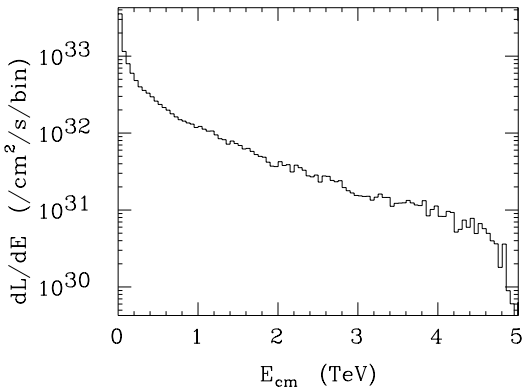

FIGURE 4. The differential $\gamma - \gamma$ luminosity dL/dE vs. the center of mass energy E_{cm} at IP (again design III [4])

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