Structure of Parallel-Velocity-Shear Driven Mode in Toroidal Plasmas

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Abstract

It is shown that the Fourier-ballooning representation is appropriate for the study of short wavelength drift-like perturbation in toroidal plasmas with a parallel velocity shear (PVS). The radial structure of the mode driven by a PVS is investigated in a torus. The Reynolds stress created by PVS turbulence and proposed as one of the sources for a sheared poloidal plasma rotation is analyzed. It is demonstrated that a finite ion temperature may strongly enhance the Reynolds stress creation ability from PVS driven turbulence. The correlation of this observation with the requirement that ion heating power be higher than a threshold value for the formation of an internal transport barrier is discussed.

I Introduction

Short-wavelength electrostatic drift-like instabilities have long been considered a possible source of anomalous particle and energy transport in tokamak plasmas.[1,2] On the other hand, the turbulence resulting from the nonlinear development of such instabilities may create Reynolds stress and plasma rotation (momentum transport) perpendicular to magnetic field.[3,4] And, the shear of such velocity is proven to be responsible for the improvement of plasma confinement in tokamaks, experimentally and theoretically.[5,6] It is important, therefore, to understand the radial structure of the linear modes, even though anomalous transport and rotation are presumably nonlinear phenomena.

It is observed in recent tokamak experiments that a strongly peaked ion velocity parallel to the magnetic field exists in the region where the plasma confinement improvement is measured.[7,8,9,] Such velocity profile naturally possesses a larger parallel velocity shear (PVS). It is theoretically shown that a PVS drives Kelvin-Helmholtz type instability and enhances ion temperature gradient modes.[10,11] On the other hand, however, due to the intrinsically asymmetric feature of its eigenfunction about the rational surface, the PVS mode has been proposed as one of the driving forces for turbulence which produces Reynolds stress and related fluctuation suppressing poloidal velocity shear.[12] Therefore, the effects of a PVS on plasma transport and confinement improvement have been studied intensively and research interests in it are growing. We believe that scientific insights into the role of PVS in tokamak plasmas will significantly help to understand the physical mechanisms for anomalous transport and enhanced confinement. The momentum-energy transport and particle diffusion from turbulence driven by PVS are studied in Refs. 12 and 13, respectively. The role of a PVS in plasmas with reversed or very weak magnetic shear is studied in Ref. 14. A sheared slab magnetic configuration is used in these works.

The ballooning representations were developed and provided a powerful method for the

investigating of drift type instabilities in a torus.[15] Recently, it is found out that there exist two kinds of mode.[16−18] The first kind occurs only at special points isolated in radial direction and is unlikely to be the source of a universal anomalous transport. The second kind can occur at all radii and, therefore, is more general and better candidate for the source of the anomaly in tokamak transport.

It is pointed out, more recently, by Taylor *et al*. that the conventional ballooning representations[15−19] are not valid in toroidal plasmas with a sheared velocity which is perpendicular to the confinement magnetic field.

In the present work, it is shown with asymptotic expansion that the Fourier-ballooning representation[20] is appropriate for the investigation of drift-type instabilities in a toroidal plasma with a sheared velocity parallel to the magnetic field. PVS driven modes in a torus are concerned for the reasons mentioned above and for simplicity. The mode structure and the modification in eigenvalues, introduced by the toroidal coupling, are studied in detail. The Reynolds stress that is created by PVS driven turbulence and may be one important source for a sheared poloidal rotation of plasma is analyzed.

The remainder of this work is organized as follows. In Sec. II, the physics model is described, and the basic equations are given and linearized. The Fourier-ballooning representation is applied and the ballooning equations of zeroth and first order in the expansion parameter are obtained in Sec. III. These equations are solved analytically, and the eigenvalues and eigenfunctions of the modes are also provided. Sec. IV is devoted to the analysis of the mode structure and Reynolds stress, and the conclusions of this work are summarized in Sec. V.

II   Physics Model and Basic Equations

The geometry is a large aspect ratio torus with circular concentric flux surfaces. The coordinates are (r, θ, ζ), corresponding to the radial, poloidal and toroidal directions. The

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magnetic field is given by $\mathbf{B} = B(\boldsymbol{\zeta} + \frac{\epsilon}{q}\boldsymbol{\theta})$, where $\boldsymbol{\zeta}$ and $\boldsymbol{\theta}$ are unit vectors in ζ and θ directions, respectively, q is the safety factor, $\epsilon = \frac{r}{R} \ll 1$, r and R are the minor and major radius of the flux surface, respectively. Fluid theory is employed to describe the ion motion, and the electrons are adiabatic. An equilibrium PVS, $dv_0/dr = dv_\parallel/dr$, is considered. Electrostatic fluctuations are described by the electrostatic potential ϕ. No equilibrium electric field exists.

The basic equations to describe the evolution of the system are ion continuity equation

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla} \cdot (n_i \mathbf{v}_i) = 0, \tag{1}$$

ion equation of motion

$$m_i n_i \left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \boldsymbol{\nabla} \right) \mathbf{v}_i = e n_i \left(-\boldsymbol{\nabla}\phi + \frac{1}{c}\mathbf{v}_i \times \mathbf{B} \right) - \boldsymbol{\nabla} p_i - \boldsymbol{\nabla} \cdot \Pi_i, \tag{2}$$

and adiabatic ion pressure evolution

$$\frac{\partial}{\partial t} p_i + \mathbf{v}_i \cdot \boldsymbol{\nabla} p_i + \Gamma p_i \boldsymbol{\nabla} \cdot \mathbf{v}_i = 0. \tag{3}$$

Electron response is adiabatic

$$n_e = n_0 \exp\left(\frac{e\phi}{T_e} \right), \tag{4}$$

and quasineutrality condition

$$n_i = n_e \tag{5}$$

is required. Here Γ is the ratio of specific heats and the other symbols have their usual meanings such as m_i and e are mass and charge of an ion, respectively, c is the speed of light, and so on.

The ion velocity $\mathbf{v}_i$ is obtained by the usual drift ordering expansion to the lowest order,

$$\mathbf{v}_i^{(0)} = v_\parallel \widehat{\mathbf{b}} + \frac{c}{B}\widehat{\mathbf{b}} \times \boldsymbol{\nabla}\phi + \frac{c}{eBn}\widehat{\mathbf{b}} \times \boldsymbol{\nabla} p_i, \tag{6}$$

which is obtained by putting the left hand side of Eq. (2) and Π_i equal zero; and to the next

order, the polarization drift

$$\mathbf{v}_i^{(1)} = -\frac{m_i c^2}{eB^2} \left(\frac{\partial}{\partial t} + \mathbf{v}_i^{(0)} \cdot \boldsymbol{\nabla} \right) \boldsymbol{\nabla}_\perp \phi, \tag{7}$$

where $\widehat{\mathbf{b}} = \mathbf{B}/B$ is the unit vector along the magnetic field.

The parallel component of Eq. (2),

$$m_i n_i \left[\frac{\partial v_\parallel}{\partial t} + \widehat{\mathbf{b}} \cdot \boldsymbol{\nabla}(\mathbf{v}_E + v_\parallel \widehat{\mathbf{b}}) \cdot \boldsymbol{\nabla}(v_\parallel \widehat{\mathbf{b}}) \right] = -en_i \boldsymbol{\nabla}_\parallel \phi - \boldsymbol{\nabla}_\parallel p_i, \tag{8}$$

is concerned only in this work, where $\mathbf{v}_E$ represents the second term on the right hand side of Eq. (6).

It is straightforward to get equilibrium relations from Eqs. (1) and (8) by putting the time derivatives equal zero. Thus, from Eq. (1) we obtain the first equilibrium relation

$$\boldsymbol{\nabla} \cdot (n_i v_\parallel \widehat{\mathbf{b}}) + \boldsymbol{\nabla} \cdot \left(\frac{c}{eB} \widehat{\mathbf{b}} \times \boldsymbol{\nabla} p_i \right) = 0, \tag{9}$$

which reduces to

$$\frac{n_i v_\parallel}{B} = \text{const.}$$

along a magnetic field line in cold ion limit.

Equation (8) goes like

$$m_i n_i \widehat{\mathbf{b}} \cdot \left[(v_\parallel \widehat{\mathbf{b}}) \cdot \boldsymbol{\nabla} \left(v_\parallel \widehat{\mathbf{b}} \right) \right] = -\boldsymbol{\nabla}_\parallel p_i. \tag{10}$$

This means that

$$\frac{1}{2} m_i n_i v_\parallel^2 + p_i = \text{const.}$$

or

$$\frac{1}{2} v_\parallel^2 + \frac{T_i}{m_i} \ln n_i = \text{const.}$$

along a magnetic field in the case of

$$n_i = \text{const.}$$

or

$$T_i = \text{const.}$$

respectively.

Following standard linearization procedures and normalizing all the perturbations with the corresponding equilibrium quantities, such as $\widehat{n} = \widetilde{n}/n_0$, $\widehat{p} = \widetilde{p}/p_0$, $\widehat{v}_{\parallel} = \widetilde{v}_{\parallel}/v_0$, we get the linearized equations from Eqs. (1), (9), and (3),

$$\frac{\partial \widehat{n}}{\partial t} + v_0 \widehat{\mathbf{b}} \cdot \boldsymbol{\nabla}(\widehat{n} + \widehat{v}_{\parallel}) + \frac{c}{B}\left[\widehat{b} \times \boldsymbol{\nabla}\phi \cdot \boldsymbol{\nabla}\ln n_0 - 2\widehat{b} \times \left(\boldsymbol{\nabla}\phi + \frac{p_0}{en_0}\boldsymbol{\nabla}\widehat{p}\right)\cdot\boldsymbol{\kappa}\right] - \frac{c}{B\Omega}\frac{d}{dt}\boldsymbol{\nabla}_{\perp}^2\phi = 0,$$
(11)

$$\frac{\partial \widehat{v}_{\parallel}}{\partial t} + v_0\widehat{\mathbf{b}}\cdot\boldsymbol{\nabla}\widehat{v}_{\parallel} - \frac{c}{B}\left[\widehat{b}\times\boldsymbol{\nabla}\phi\cdot(\boldsymbol{\kappa}-\boldsymbol{\nabla}\ln v_0)\right] = -\frac{e}{mv_0}\widehat{\mathbf{b}}\cdot\left(\boldsymbol{\nabla}\phi + \frac{p_0}{en_0}\boldsymbol{\nabla}\widehat{p}\right),$$
(12)

$$\frac{\partial \widehat{p}}{\partial t} + v_0\widehat{\mathbf{b}}\cdot\boldsymbol{\nabla}\widehat{p} + \frac{c}{B}\left[\widehat{b}\times\boldsymbol{\nabla}\phi\cdot\ln p_0 + \Gamma\boldsymbol{\nabla}\cdot\widetilde{\mathbf{v}} = 0\right],$$
(13)

where $\boldsymbol{\kappa} = \widehat{\mathbf{b}}\cdot\boldsymbol{\nabla}\widehat{\mathbf{b}}$ is the curvature vector, Ω is the ion gyrofrequency, and the subscript "i" is dropped for simplicity.

Assuming incompressibility and employing large aspect ratio approximation, we get the following unique equation from Eqs. (11)–(13),

$$\frac{\partial^3\widehat{n}}{\partial t_{\parallel}^3} + v_0\widehat{\mathbf{b}}\cdot\boldsymbol{\nabla}\left\{\frac{c}{B}\frac{\partial}{\partial t_{\parallel}}\widehat{\mathbf{b}}\times\boldsymbol{\nabla}\phi\cdot(\boldsymbol{\kappa}-\boldsymbol{\nabla}\ln v_0) - \frac{e}{mv_0}\frac{\partial}{\partial t_{\parallel}}\widehat{\mathbf{b}}\cdot\boldsymbol{\nabla}\phi+\right.$$

$$\left.\frac{p_0}{mv_0n_0}\widehat{\mathbf{b}}\cdot\boldsymbol{\nabla}\frac{c}{B}\widehat{\mathbf{b}}\times\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\ln p_0\right\} + \frac{c}{B}\frac{\partial^2}{\partial t_{\parallel}^2}\widehat{\mathbf{b}}\times\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\ln n_0-$$

$$\frac{c}{B}\frac{\partial^2}{\partial t_{\parallel}^2}2\widehat{\mathbf{b}}\times\left[\boldsymbol{\nabla}\phi + \frac{p_0}{en_0}\boldsymbol{\nabla}\left(\frac{c}{B}\widehat{\mathbf{b}}\times\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\ln p_0\right)\right]\cdot\boldsymbol{\kappa} - \frac{c}{B\Omega_c}\frac{d^3}{dt\partial t_{\parallel}^2}\boldsymbol{\nabla}_{\perp}^2\phi = 0,$$
(14)

where

$$\frac{d}{dt} = \frac{\partial}{\partial t_{\parallel}} - \frac{p_0 c}{eBn_0}(1+\eta_i)\widehat{\mathbf{b}}\times\boldsymbol{\nabla}\ln n_0,$$

$$\frac{\partial}{\partial t_{\parallel}} = \frac{\partial}{\partial t} + v_0\widehat{\mathbf{b}}\cdot\boldsymbol{\nabla},$$

and

$$\eta_i = \frac{d\ln T_i}{d\ln n_0}.$$

In a slab, $\widehat{\mathbf{b}} = \text{const.}$, and $\boldsymbol{\kappa} = 0$, thus, Eq. (14) reduces to

$$\left\{1 + \frac{\omega_{*e}v_0'L_nk_{\parallel}}{(\omega-k_{\parallel}v_0)^2} - \frac{\omega_{*e}}{(\omega-k_{\parallel}v_0)} - \left[1 + \frac{\omega_{*e}K}{(\omega-k_{\parallel}v_0)}\right]\left(\frac{c_s^2k_{\parallel}^2}{(\omega-k_{\parallel}v_0)^2} + \rho_s^2\boldsymbol{\nabla}_{\perp}^2\right)\right\}\phi = 0,$$
(15)

where $K = (1 + \eta_i)T_i/T_e$, $\rho_s^2 = T_e/m_i$, and $-i\omega = \partial/\partial t$. This is a equation often applied in literature.

In cold ion limit $(T_i/T_e \ll 1)$, Eq. (14) turns out to be

$$\frac{\partial^2 \phi}{\partial t_\parallel^2} - c_s^2(\widehat{\mathbf{b}} \cdot \boldsymbol{\nabla})^2\phi - \frac{v_0 c_s^2}{\Omega}\widehat{\mathbf{b}} \cdot \boldsymbol{\nabla}\left[\widehat{\mathbf{b}} \times \boldsymbol{\nabla}\phi \cdot (\boldsymbol{\nabla}\ln v_0 - \boldsymbol{\kappa})\right] +$$

$$\frac{\partial}{\partial t_\parallel}\left[\frac{c_s^2}{\Omega}\widehat{\mathbf{b}} \times \boldsymbol{\nabla}\phi \cdot (\boldsymbol{\nabla}\ln n_0 - 2\boldsymbol{\kappa})\right] - \rho_s^2\frac{\partial^2}{\partial t_\parallel^2}\boldsymbol{\nabla}_\perp^2\phi = 0, \tag{16}$$

which is the equation we are going to solve bellow.

Let

$$\phi(r, \theta, \zeta, t) = \phi_n(r, \theta)e^{-i\omega t + in\zeta} = e^{-i\omega t + in\zeta - im\theta}\sum_l \phi_l(r)e^{-il\theta}, \tag{17}$$

and

$$\frac{\partial \phi}{\partial t_\parallel} = -i\omega + \frac{v_0}{qR}\left(inq + \frac{\partial}{\partial\theta}\right),$$

then Eq. (16) becomes

$$\left\{\omega_1^2\rho_s^2\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) - \frac{1}{r^2}(m+l)^2\right] - \left[\omega_1^2 - \left(\frac{c_s}{qR}(nq - m - l)\right)^2\right]\right.$$

$$\left. + \frac{c_s^2 v_0'}{\Omega rqR}(nq - m - l)(m + l) + \frac{c_s^2 n_0'}{\Omega r n_0}(m + l)\omega_1\right\}\phi_l(r) -$$

$$\frac{1}{2}\frac{c_s^2}{\Omega R}\left[\frac{v_0}{qR}(nq - m - l) + 2\omega_1\right]\left[-\left(\frac{d}{dr} + \frac{m+l+1}{r}\right)\phi_{l+1}(r) +\right.$$

$$\left(\frac{d}{dr} - \frac{m+l-1}{r}\right)\phi_{l-1}(r)\right] + \frac{nr}{2R^2 q}\frac{c_s^2}{\Omega R}\left[\frac{v_0}{qR}(nq - m - l) + 2\omega_1\right](\phi_{l+1} + \phi_{l-1}) = 0, \tag{18}$$

where $\omega_1 = \omega - \frac{v_0}{qR}(nq - m - l)$ and the prime ("'") represents derivative with respect to r.

Introducing $x = nq(r) - m$ and manipulating Eq. (18), we get

$$\left\{\omega^2\left[\frac{c_s^2}{R^2\omega_s^2 q^2}\frac{d^2}{dx^2} - k_\theta^2\rho_s^2\right] - \left[\omega^2 - \frac{c_s^2}{R^2 q^2}(x - l)^2\right] + \frac{c_s^2 v_0' k_\theta(x - l)}{\Omega Rq}\right.$$

$$\left. + \frac{c_s^2 n_0' k_\theta\omega}{\Omega n_0}\right\}\phi_l(x) + \frac{c_s^2 k_\theta\omega}{\Omega R}\left[\left(1 + \widehat{s}\frac{d}{dx}\right)\phi_{l+1} + \left(1 - \widehat{s}\frac{d}{dx}\right)\phi_{l-1}\right] +$$

$$\left\{\frac{-2\omega v_0' x}{Rqq' n}(x - l)\left[\frac{c_s^2}{R^2\omega_s^2 q^2}\frac{d^2}{dx^2} - k_\theta^2\rho_s^2 - 1\right] - 2\omega^2 k_\theta^2\rho_s^2\frac{l}{m} +\right.$$

$$\left. \frac{c_s^2 v_0' k_\theta(x - l)}{\Omega Rq}\frac{l}{m}\left(1 + \frac{v_0'' r_0}{v_0'\widehat{s}}\right) - \frac{c_s^2 n_0' k_\theta}{\Omega n_0}\left[\frac{v_0' x(x - l)}{Rqq' n} - \frac{l}{m}\omega + \frac{n_0'' r_0 x}{n_0'\widehat{sqn}}\right]\right\}\phi_l$$

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$$+\frac{c_s^2 v_0'}{2\Omega R^2 qq'}\frac{x}{n}(x-l)\Big[\Big(\widehat{s}\frac{d}{dx}+1\Big)\phi_{l+1}-\Big(\widehat{s}\frac{d}{dx}-1\Big)\phi_{l-1}\Big]+$$

$$\frac{c_s^2\omega k_\theta}{\Omega R}\Big(\frac{l+1}{m}\phi_{l+1}+\frac{l-1}{m}\phi_{l-1}\Big)=0, \tag{19}$$

where $\omega_s=(c_s/qR)/(\rho_s q\widehat{s})$. The slow variations of n_0' and v_0' have been invoked here.

Before introducing the Fourier-ballooning representation and further manipulating the equation, it is worthwhile to note that all the terms with a factor x or l besides or instead of $(x-l)$ have order $\mathcal{O}(\frac{1}{n})$ while the rest terms have order $\mathcal{O}(1)$. This is true only when the sheared velocity is parallel to the magnetic field. In other words, there are terms with a factor x or l having order $\mathcal{O}(1)$ when a sheared velocity perpendicular to the magnetic field exists. This essential difference makes the ballooning representations appropriate for the investigation of instabilities in plasmas with a PVS but failed in plasmas with a perpendicular velocity shear. We will come back to this point in the next section.

III   Ballooning Equations

With the Fourier-ballooning representation,[17,18,20] the function $\phi_l(x)$ may be written as

$$\phi_l(x)=\oint dkd\lambda\overline{\phi}(k,\lambda)e^{ik(x-l)-il\lambda}. \tag{20}$$

It is equivalent to the transformations

$$(x-l)\to i\frac{\partial}{\partial k},\qquad \frac{d}{dx}\to ik,$$

$$l\to -i\frac{\partial}{\partial\lambda},\qquad \phi_l(r)\to\overline{\phi}(k,\lambda), \tag{21}$$

$$x\to i\Big(\frac{\partial}{\partial k}-\frac{\partial}{\partial\lambda}\Big),\qquad \phi_{l\pm1}(r)\to e^{\mp i(k+\lambda)}\overline{\phi}(k,\lambda).$$

Substituting Eq. (20) into Eq. (19) gives

$$\Big[\widehat{L}_0+\widehat{L}_1\frac{\partial}{\partial\lambda}-\Omega\Big]\overline{\phi}(k,\lambda)=0, \tag{22}$$

where

$$\widehat{L}_0=\frac{\partial^2}{\partial k^2}+\frac{i\widehat{v}_0'q\widehat{k}_\theta}{\epsilon_n}\frac{\partial}{\partial k}+\Big(\frac{q\widehat{k}_\theta}{\epsilon_n}\Big)^2\widehat{\omega}\Big[\widehat{\omega}(\widehat{k}_\theta\widehat{s})^2k^2+2\epsilon_n P(k,\lambda)\Big], \tag{23}$$

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$$\Omega(\lambda) = -\Big(\frac{q\widehat{k}_\theta}{\epsilon_n}\Big)^2[\widehat{\omega}^2(\widehat{k}_\theta^2 + 1) - \widehat{\omega}], \tag{24}$$

$$\widehat{L}_1 = \frac{1}{n}\Big[\frac{\widehat{k}_\theta \widehat{v}_0'}{\epsilon_n}\Big(1 + \frac{\widehat{v}_0''}{\widehat{v}_0'\widehat{s}}\Big)\frac{\partial}{\partial k} + \frac{i\widehat{k}_\theta^2 \widehat{\omega}q}{\epsilon_n^2}\Big(1 - \frac{\widehat{n}_0''}{\widehat{s}q} - 2\widehat{\omega}\widehat{k}_\theta^2 - 2\epsilon_n \cos(k + \lambda)\Big)\Big], \tag{25}$$

$$P(k, \lambda) = \cos(k + \lambda) + \widehat{s}k\sin(k + \lambda),$$

$$\widehat{v}_0' = \frac{v_0' L_n}{c_s}, \qquad \widehat{k}_\theta = k_\theta \rho_s, \quad \epsilon_n = \frac{L_n}{R},$$

$$\widehat{\omega} = \frac{\omega}{\omega_{*e}}, \quad \widehat{n}_0'' = \frac{n_0'' L_n r_0}{n_0}, \quad \widehat{v}_0'' = \frac{v_0'' L_n r_0}{c_s}.$$

The λ-parameterized eigenvalue $\Omega(\lambda)$ has been generalized to Ω which is independent of λ.

For large n, Eq. (22) may be solved with asymptotic scheme. The lowest order equation is

$$[\widehat{L}_0 - \Omega(\lambda)]\overline{\phi}(k, \lambda) = 0, \tag{26}$$

and the equation to the first order is

$$\Big[\Omega(\lambda) - \Omega + \widehat{L}_1\frac{\partial}{\partial \lambda}\Big]\overline{\phi}(k, \lambda) = 0. \tag{27}$$

Introducing

$$\overline{\phi}(k, \lambda) = \Psi(\lambda)\chi(k, \lambda) \tag{28}$$

with the periodic condition

$$\Psi(\lambda + 2\pi) = \Psi(\lambda), \tag{29}$$

then Eq. (26) is the equation for $\chi(k.\lambda)$ and the Eq. (27) reduces to a equation for $\Psi(\lambda)$,

$$[\Omega(\lambda) - \Omega]\Psi(\lambda) + \overline{L}_1\frac{\partial\Psi}{\partial\lambda} = 0, \tag{30}$$

where

$$\overline{L}_1(\lambda) = \frac{\int dk\chi(k, \lambda)\widehat{L}_1\chi(k, \lambda)}{\int dk\chi(k, \lambda)\chi(k, \lambda)}. \tag{31}$$

A particularly important understanding is that

$$\frac{\partial}{\partial\lambda}(\widehat{L}_1\chi(k, \lambda)) \ll \widehat{L}_1\chi(k, \lambda)\frac{\partial\Psi}{\partial\lambda},$$

in order to scrutinize the plausible localized mode structure with ballooning theory.

The procedure outlined here is the same as it is when the PVS is absent.[17] One of the important features of the results is that Eq. (26) is an ordinary differential equation, or an 1D eigenvalue problem, in other words, with λ as a parameter. So that, for large n, the system may be solved asymptotically. It becomes more clear now that the Fourier-ballooning representation is appropriate for the study of short wavelength drift-like perturbations in toroidal plasmas with a PVS. However, as mentioned in the last section, the operator $\widehat{L}_0$ in Eq. (26) would involve derivatives with respect to λ and the Fourier-ballooning representation would not apply if there were a sheared velocity perpendicular to the magnetic field.[19]

Equation (26) is fairly documented, and with the strong coupling approximation and the assumption of $\epsilon_n \ll 1$, it is straightforward to get the eigenfunction,

$$\chi(k, \lambda) = \exp\left[\sigma(k + \Delta k)^2 - \frac{iq}{2\epsilon_n}\widehat{k}_\theta \widehat{v}'_0 k\right], \tag{32}$$

where

$$\sigma = i\widehat{k}_\theta\left(\frac{q}{\epsilon_n}\right)\frac{\widehat{\omega}}{2}\left[(\widehat{k}_\theta\widehat{s})^2 + \frac{\epsilon}{\widehat{\omega}}(2\widehat{s} - 1)\cos\lambda\right]^{\frac{1}{2}},$$

$$\Delta k = \frac{\epsilon_n(\widehat{s} - 1)\sin\lambda}{\widehat{\omega}(\widehat{k}_\theta\widehat{s})^2 + \epsilon(2\widehat{s} - 1)\cos\lambda},$$

and eigenvalue

$$\widehat{\omega} = \widehat{\omega}_0 + \widehat{\omega}_1, \tag{33}$$

where

$$\widehat{\omega}_0 = \frac{1}{2(1 + \widehat{k}_\theta^2)}\left\{1 - \frac{i\widehat{s}\epsilon_n}{q} \pm i\left[(1 + \widehat{k}_\theta^2)\widehat{v}'^2_0 - \left(1 - \frac{i\epsilon_n\widehat{s}}{q}\right)^2\right]^{\frac{1}{2}}\right\},$$

$$\widehat{\omega}_1 \simeq \epsilon_n A \cos\lambda,$$

$$A = \frac{-1}{(1 + \widehat{k}_\theta^2)} \pm \frac{1}{\sqrt{(1 + \widehat{k}_\theta^2)\widehat{v}'^2_0 - \left(1 - \frac{i\epsilon_n\widehat{s}}{q}\right)^2}}\left[\frac{-\epsilon_n(2\widehat{s} - 1)}{2q\widehat{s}\widehat{k}_\theta^2} + \frac{i(1 - \frac{i\widehat{s}\epsilon_n)}{q}}{1 + \widehat{k}_\theta^2}\right].$$

The solution of Eq. (30) with the boundary condition (29) is

$$\Psi(\lambda) = \exp\left\{\left[i2\pi N + \frac{\int_{-\pi}^{\pi}\frac{\Omega(\lambda)d\lambda}{\overline{\overline{L}}_1(\lambda)}}{\int_{-\pi}^{\pi}\frac{d\lambda}{\overline{\overline{L}}_1(\lambda)}}\right]\int^{\lambda}\frac{d\lambda'}{\overline{L}_1(\lambda')} - \int^{\lambda}\frac{\Omega(\lambda')d\lambda'}{\overline{L}_1(\lambda')}\right\}, \tag{34}$$

with the corresponding global eigenvalue

$$\Omega = \frac{i2\pi N + \int_{-\pi}^{\pi} \frac{\Omega(\lambda)d\lambda}{\overline{L}_1(\lambda)}}{\int_{-\pi}^{\pi} \frac{d\lambda}{\overline{L}_1(\lambda)}}, \tag{35}$$

where N is an integer.

The Eqs. (34) and (35) are general expressions for $\Psi(\lambda)$ and Ω. In order to get specific expressions for our problem, we must calculate $\overline{L}_1$ first.

Substituting Eq. (25) into Eq. (31) gives

$$\overline{L}_1(\lambda) = \frac{i\widehat{k}_\theta \widehat{\omega} q}{n\epsilon_n}[(1 - \frac{\widehat{n}_0''}{\widehat{s}q} - 2\widehat{\omega}\widehat{k}_\theta^2) - 2\epsilon_n e^{\frac{1}{8\sigma}} \cos(\lambda - \Delta k + \nu)], \tag{36}$$

where

$$\nu = \frac{\widehat{v}_0'}{2\widehat{\omega}} \left[(k_\theta \widehat{s})^2 + \frac{\epsilon_n(2\widehat{s} - 1)}{\widehat{\omega}} \cos \lambda \right]^{-\frac{1}{2}}.$$

$\Omega(\lambda)$ is given by Eq. (24) with $\widehat{\omega}$ being a function of λ.

In this way, the specific expressions for the eigenfunction $\Psi(\lambda)$ and the eigenvalue Ω may be obtained. Thus, we do the $\epsilon_n << 1$ expansion and get

$$\overline{L}_1(\lambda) = \frac{i\widehat{k}_\theta^2 q}{n\epsilon_n}(a_1 + b_1 \cos \lambda + c_1 \sin \lambda), \tag{37}$$

$$\Omega(\lambda) = -\left(\frac{q}{\epsilon_n \widehat{s}}\right)^2 (\widehat{s}\widehat{k}_\theta)^2 (A_1 + B_1 \cos \lambda), \tag{38}$$

where

$$a_1 = \widehat{\omega}_0 - \frac{\widehat{n}_0''}{\widehat{s}q}\widehat{\omega}_0 - 2\widehat{\omega}_0 \widehat{k}_\theta^2,$$

$$b_1 = \epsilon_n(A - 2e^{\sigma_1} \cos \lambda_v),$$

$$c_1 = -2\epsilon_n e^{\sigma_1} \sin \lambda_v,$$

$$\sigma_1 = \frac{-i\epsilon_n \widehat{s}}{4q(\widehat{s}\widehat{k}_\theta)^2 \widehat{\omega}_0},$$

$$\lambda_v = \frac{\widehat{v}_0' \widehat{k}_\theta \widehat{s}}{2\widehat{\omega}_0},$$

$$A_1 = (\widehat{k}_\theta^2 + 1)\widehat{\omega}_0^2 - \widehat{\omega}_0,$$

$$B_1 = \epsilon_n A(2\widehat{\omega}_0 - 1).$$

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Substituting Eqs. (37) and (38) into Eqs. (34) and (35), we get

$$\Psi(\lambda) = \exp\left\{ i2\pi N + i \left(\frac{q\widehat{k}_\theta}{\epsilon_n} \right)^2 n\epsilon_n \left[\frac{2B_1 b_1}{b_1^2 + c_1^2} \left(\tan^{-1}\left(\frac{(a_1 - b_1)\tan\frac{\lambda}{2} + c_1}{\sqrt{a_1^2 - b_1^2 - c_1^2}} \right) - \lambda \right) \right. \right.$$
$$\left. \left. - \frac{B_1 c_1}{b_1^2 + c_1^2} \ln(a_1 + b_1\cos\lambda + c_1\sin\lambda) \right] \right\}, \tag{39}$$

and

$$\Omega = \Omega_0 + \Omega_1 = -\left(\frac{q\widehat{k}_\theta}{\epsilon_n} \right)^2 \left\{ A_1 + \left[\frac{B_1 b_1}{b_1^2 + c_1^2} \sqrt{a_1^2 - b_1^2 - c_1^2} - a_1^2 \right] \right\}, \tag{40}$$

for $N = 0$.

The term in the square bracket on the right hand side of Eq. (40), Ω_1, is the correction for the global eigenvalue, that is introduced by and proportional to the toroidal coupling effect ϵ_n. This is in contrast with the first kind of ballooning instability for which such correction is zero.[15] Therefore, we identify the modes studied here as a second kind ballooning instability which occurs at all radii and is more plausible candidate responsible for the anomaly of transport in magnetically confined plasmas.

Usually, further expansions are made,[17,18] then we write

$$\int^\lambda \frac{d\lambda'}{\overline{L}_1(\lambda')} = -\frac{in\epsilon_n}{\widehat{k}_\theta^2 q a_1} \left[\lambda - \frac{\epsilon_n}{a_1}(A\sin\lambda - 2e^{\sigma_1})\sin(\lambda - \lambda_v) \right],$$

and

$$\Omega(\lambda) = \overline{\Omega} + \Delta\Omega\cos\lambda, \tag{41}$$

with

$$\overline{\Omega} = -\left(\frac{q}{\epsilon_n \widehat{k}_\theta} \right)^2 A_1,$$

$$\Delta\Omega = -\left(\frac{q}{\epsilon_n \widehat{k}_\theta} \right)^2 \epsilon_n A \left[2\widehat{\omega}_0(\widehat{k}_\theta^2 - 1) - 1 \right],$$

and

$$\int^\lambda \frac{\Omega(\lambda')d\lambda'}{\overline{L}_1(\lambda')} = \frac{inq}{\widehat{\omega}_0\epsilon_n(1 - 2a_1)} \left\{ A_1\lambda + \epsilon_n \left[A(2\widehat{\omega}_0 - 1) + \right. \right.$$
$$\left. \left. \frac{A_1 A}{a_1} \right] \sin\lambda - \frac{A_1\epsilon_n}{a_1} e^{\sigma_1}\sin(\lambda - \lambda_v) \right\}. \tag{42}$$

Substituting Eq. (41) into Eq. (35) gives

$$\Omega = \overline{\Omega}. \tag{43}$$

It is the average value of $\Omega(\lambda)$ over λ for $N = 0$. Comparison of Eq. (40) with Eq. (43) reveals that the correction part Ω_1 in the former is missed in the latter. In other words, the first order correction to the eigenvalue is missed with the expansion usually employed.

The eigenfunction under the usual expansion turns out to be (for $N = 0$)

$$\Psi(\lambda) = \exp\left\{ -\frac{inq}{\widehat{\omega}_0^2(1 - 2\widehat{\omega}_0\widehat{k}_\theta^2)} \left[g_1 \sin\lambda - g_2 \sin(\lambda - \lambda_v) \right] \right\}, \tag{44}$$

where

$$g_1 = \left[4A_1(1 - \widehat{\omega}_0\widehat{k}_\theta^2) + \widehat{\omega}_0(1 + 2\widehat{\omega}_0\widehat{k}_\theta^2) \right] A,$$

$$g_2 = 4A_1 e^{\sigma_1}.$$

The characteristics of this function are discussed in detail for $\lambda_v = 0$ in Ref.17. The parameter λ_v represents the PVS effects and changes the mode localization in λ-space.

IV   Mode Structure and Reynolds Stress

According to Eq. (17), the spatial part of the perturbed electrostatic potential may be written as

$$\Phi_n(r, \theta, \zeta) = e^{in\zeta - im\theta} \sum_l e^{-il\theta} \phi_l(r). \tag{45}$$

Substituting Eqs. (20), (28), (32) and (39) into Eq. (45) would give a general expression for $\Phi_n(r, \theta, \zeta)$. We leave such an expression for late numerical analysis and use Eq. (44) instead of Eq. (39) for $\Psi(\lambda)$ at present. By rewriting

$$\chi(k, \lambda) = \chi_0 \exp[\sigma(k - k_0)^2], \tag{46}$$

with

$$k_0 = -\Delta k + \frac{iq}{4\sigma\epsilon_n} \widehat{k}_\theta \widehat{v}_0', \tag{47}$$

we get

$$\Phi_n(r, \theta, \zeta) = e^{in\zeta - im\theta} \sum_l \oint d\lambda dk e^{il(\theta + k + \lambda) + ikx} \Psi(\lambda) \chi(k, \lambda)$$

$$\simeq e^{in\zeta - im\theta} \oint d\lambda e^{ix\theta} \Psi_0 \exp\left\{-\frac{inq}{a_1^2}[\alpha_1 x\lambda + g_1 \sin\lambda - g_2 \sin(\lambda - \lambda_v)]\right\} \chi(-\theta - \lambda, \lambda),$$

where

$$\alpha_1 = \frac{a_1^2}{nq}.$$

With the standard saddle point approximation, the integration over λ may be carried out and we have the mode structure,

$$\Phi_n(r, \theta, \zeta) \simeq e^{in\zeta - im\theta} \Psi_0 e^{-ix\theta} \times$$

$$\chi(-\theta - \lambda_0, \lambda_0) \exp\left\{-\frac{inq}{a_1^2}[\alpha_1 x\lambda_0 + g_1 \sin\lambda_0 - g_2 \sin(\lambda_0 - \lambda_v)]\right\},$$

where $\lambda_0(x)$ is determined by the stationary point condition

$$\alpha_1 x + g_1 \sin\lambda_0 - g_2 \sin(\lambda_0 - \lambda_v) = 0. \tag{48}$$

We remember that $\chi(k, \lambda)$ is originally the zeroth order eigenfunction in $k-$space, i.e. in Fourier space. With an inverse Fourier transform, finally, the mode structure in the real space turns out to be

$$\Phi_n(r, \theta, \zeta) \simeq e^{in\zeta - im\theta}\{\sum_l \exp[il(\theta + \lambda_0)]e^{\{\frac{(x-l)^2}{4\sigma} + ik_0(x-l)\}} \times$$

$$\exp\{-\frac{1}{\alpha_1}[g_1 \sin\lambda_0 - g_2 \sin(\lambda_0 - \lambda_v)]\}\}. \tag{49}$$

We note in Eq. (49) that the terms in the first brace are the summation over different harmonics including the side-band harmonics, and the last exponential, varying on a longer scale length, gives an envelope of the mode structure. The amplitude of each harmonic is determined by the first exponential in the first brace.

It has been pointed out[12] that the Reynolds stress created by turbulence strongly depends on the asymmetry of the mode structure. Here, the asymmetry comes most probably from

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the k_0 term. In order to make an estimate, let us get an explicit expression for $i2\sigma k_0$ from Eqs. (33) and (47),

$$i2\sigma k_0 \simeq \frac{q}{\widehat{s}}(\widehat{s} - 1)\sin\lambda_0\Big(1 + \frac{\epsilon_n(2\widehat{s} - 1)}{\widehat{k}_\theta\widehat{s}\widehat{\omega}_0}\cos\lambda_0\Big)^{\frac{1}{2}} + \frac{q\widehat{k}_\theta\widehat{v}_0'}{2\epsilon_n}. \tag{50}$$

The dependence of λ_0 on x is rather weak and the main imaginary part in Eq. (50) comes from $\widehat{\omega}_0$ and is of order $\epsilon_n(\ll 1)$. The role of this part in symmetry breaking of the 2D mode structure is emphasized in Ref. 17. The second term in Eq. (50) is of order 1. It is real and introduces a pure shift without deforming the mode structure in x-space in plasmas with cold ions $(T_i/T_e \ll 1)$.

On the other hand, the last term in Eq. (50) has to be changed to

$$\Delta = \frac{q\widehat{k}_\theta\widehat{v}_0'\widehat{\omega}}{2\epsilon_n(\widehat{\omega} + K)},$$

with a significant imaginary part, when finite ion temperature effects are taken into account.[12] In this case, the symmetry breaking deformation of the mode structure, that is introduced by the imaginary part of Δ, is significantly enhanced by a PVS, and a considerable Reynolds stress may be created by the turbulence The imaginary part of Δ and therefore the deformation of the mode structure is proportional to the growth rate of the mode (and therefore to η_i) and the ratio T_i/T_e. All these three quantities are strongly related to the ion heating power and the higher the latter, the higher the former. This result is rather important from the point of view that there is a threshold of ion heating power for the formation of an internal transport barrier (ITB) in experiments.[21]

Numerical calculations are performed to study the features of the mode structure and Reynolds stress in detail. From Eqs. (20), (28), (32), and (39), the 2D structure of the mode may be written as

$$\phi_n(x, \theta) = e^{im\theta}\sum_{l=-L}^{L} e^{-il\theta}\oint d\lambda e^{-il\lambda}\Psi(\lambda)\overline{X}(x - l, \lambda), \tag{51}$$

where

$$\overline{X}(x - l, \lambda) = \frac{1}{\sqrt{-2\sigma}} \exp\left[\frac{(x - l)^2}{4\sigma} - i\Delta_k(x - l)\right], \tag{52}$$

and

$$\Delta_k = \frac{\epsilon_n(\widehat{s} - 1)\sin\lambda}{\widehat{\omega}(\widehat{k}_\theta\widehat{s})^2 + \epsilon_n(2\widehat{s} - 1)\cos\lambda} - \frac{iq\widehat{k}_\theta\widehat{v}_0'}{4\sigma\epsilon_n}. \tag{53}$$

The mode structure in x-space at $\theta = 0$ is shown in Fig. 1. The other parameters are $\epsilon_n = 0.01$, $\widehat{s} = 1.4$, $\widehat{k}_\theta = 0.01$, $q = 1.5$, $\widehat{v}_0' = 1.1$, $\widehat{n}_0'' = 0.05$, $N = 0$, $n = 10$. The real part (Fig. 1(a)) of the function is much higher than its imaginary part (Fig. 1(b)). The structure is composed of one dominant and a few side-band harmonics. The maximum of the dominant harmonic is shifted from the rational surface due to the toroidal coupling and PVS effects. The asymmetry of the mode structure is purely due to the toroidal coupling and is very limited in this case.

In order to show the role of finite ion temperature in the asymmetry creation of the mode structure, that is discussed analytically above, the mode structure is shown again in Fig.2 when the deformation factor Δ_k in Eq. (52) is replaced by

$$\Delta_k = \frac{\epsilon_n(\widehat{s} - 1)\sin\lambda}{\widehat{\omega}(\widehat{k}_\theta\widehat{s})^2 + \epsilon_n(2\widehat{s} - 1)\cos\lambda} - \frac{iq\widehat{k}_\theta\widehat{v}_0'}{4\sigma\epsilon_n}\frac{\widehat{\omega}}{(\widehat{\omega} + K)}. \tag{54}$$

$\eta_i = 2$ and $T_i = T_e$ are used and the other parameters are the same as that in Fig. 1. We note that this is an approximate consideration only since the ballooning equations and their solutions must be changed when the finite T_i effect is taken into account (see Appendix). Even so, it is enough to demonstrate the importance of a finite T_i. In comparison with Fig. 1(a) it is clear that the asymmetry in the mode structure is high now. In addition, there is a deformation in the dominant harmonic and the relative amplitudes of the side-band ones are higher than they are in Fig. 1(a) .

The Reynolds stress from the turbulence given with Eq. (51) is

$$R \sim \langle \frac{1}{r}\frac{\partial\Phi_n}{\partial\theta}\frac{\partial\Phi_n}{\partial r}\rangle$$

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$$= \langle [\Big(\sum_l (m+l) \oint d\lambda e^{-il\theta - il\lambda} \Psi(\lambda) \overline{X}(x-l, \lambda) \Big)] \times \tag{55}$$

$$[\Big(\sum_l 1(m+l_1) \oint d\lambda e^{-il\theta - il_1\lambda} (x - l_1 - i2\sigma \Delta_k \Psi(\lambda) \overline{X}(x-l_1, \lambda) \Big)] \rangle.$$

The numerical results are given in Fig. 3 as functions of magnetic shear for the same parameters as that in Figs. 1 and 2. The up line is for Fig. 2 and the down line is for Fig. 1. It is clear that the turbulence created Reynolds stress is much higher in plasmas with finite $T_i \simeq T_e$ than it is when $T_i \ll T_e$. This result is in good agreement with above analytic analysis.

V   Conclusions

In this work, it is shown with asymptotic expansion that the Fourier-ballooning representation is appropriate for the study of short wavelength drift-like perturbations in toroidal plasmas with a PVS. The mode driven by a PVS, that belongs to the second kind of drift-type instabilities in a torus and therefore is more general, is investigated. The radial structure of the modes is studied in detail. The Reynolds stress created by PVS turbulence and proposed as one of the sources for a sheared poloidal plasma rotation is analyzed. It is demonstrated that the Reynolds stress creation ability from PVS driven turbulence is proportional to the growth rate of the mode (and therefore to η_i) and the ratio T_i/T_e. All these three quantities are strongly related to the ion heating power and the higher the latter, the higher the former. The correlation of this observation with the requirement that ion heating power be higher than certain threshold values for formation of internal transport barriers is emphasized.

Acknowledgments

Appendix: Finite Ion temperature Effects

The equations for the plasmas with finite ion temperature $(T_i/T_e \neq 0)$ are given in this Appendix. Starting from Eq. (14) and following exactly the procedures outlined after Eq. (16), we get the counterpart of Eq. (22) in plasmas with finite T_i as

$$\left[\widehat{L}_0 + \widehat{L}_1 \frac{\partial}{\partial \lambda} - \Omega \right] \overline{\phi}(k, \lambda) = 0,$$

where

$$\widehat{L}_0 = \frac{\partial^2}{\partial k^2} + \frac{i \widehat{v}_0' q \widehat{k}_\theta \widehat{\omega}}{\epsilon_n (\widehat{\omega} + K)} \frac{\partial}{pa^k} + \left(\frac{q \widehat{k}_\theta}{\epsilon_n} \right)^2 \widehat{\omega} \left[\widehat{\omega}(\widehat{k}_\theta \widehat{s})^2 + 2\epsilon_n P(k, \lambda) \right],$$

$$\Omega(\lambda) = - \left(\frac{q \widehat{k}\theta}{\epsilon_n} \right)^2 (\widehat{k}_\theta \widehat{s})^2 \left[\frac{\widehat{\omega}^2(\widehat{\omega} - 1)}{(\widehat{\omega} + K)} + \widehat{\omega}^2 \widehat{k}_\theta^2 \right],$$

$$\widehat{L}_1 = \frac{1}{n} \left\{ \frac{-iK\widehat{k}_\theta}{\widehat{\omega} q} \frac{\partial^2}{\partial k^2} + \frac{\widehat{k}_\theta \widehat{v}_0'}{\epsilon_n} \left(1 + \frac{\widehat{v}_0''}{\widehat{v}_0' \widehat{s}} \right) \frac{\partial}{\partial k} + \frac{i\widehat{k}_\theta^3 \widehat{\omega} q}{\epsilon_n^2} \left(1 - \frac{\widehat{n}_0''}{\widehat{s} q} - 2\widehat{k}_\theta^2(\widehat{\omega} + K) - \right. \right.$$

$$2\epsilon_n \cos(k + \lambda) \bigg) + \frac{iK\widehat{k}_\theta}{q} \left(\frac{q}{\epsilon_n \widehat{s}} \right)^2 \left[\widehat{k}_\theta^4 \widehat{s}^2 \widehat{\omega}(\widehat{s}^2 k^2 + 1) - \right.$$

$$\left. 2(\widehat{k}_\theta \widehat{s})^2 \epsilon_n \left(P(k, \lambda) + \cos(k + \lambda) \right) - 2\widehat{\omega} \widehat{k}_\theta^2 \right] \bigg\}.$$

It is apparent that, to the lowest order, a finite T_i introduces two changes: one in $\widehat{L}_0$ and the other in $\Omega(\lambda)$. The change in the former is essential for the mode structure modification introduced by $\widehat{v}_0'$. This is discussed in detail in Ref. 12 and will not be repeated here.

The inclusion of a finite T_i makes $\widehat{L}_1$ more complicated than it is for $T_i = 0$. However, the effects introduced by such complication are expected to be quantitative but not qualitative.

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Figure Captions

1. The real (a) and imaginary (b) part of the mode structure in r direction. The parameters are $\hat{s} = 1.4$, $\epsilon_n = 0.01$, $q = 1.5$, $\hat{k}_\theta = 0.01$, $\hat{v}'_\| = 1.1$, $n = 10$, $\theta = 0$. $\hat{n}''_0 = 0.05$, $N = 0$, $K = 0$

2. The same as Fig. 1 except that $K = 3$.

3. Reynolds stress R versus magnetic shear $\hat{s}$ for $K = 0$ (down) and $K = 3$ (up). The other parameters are the same as that in Fig. 1.